Perturbative Analysis of SGD

0. Datatype of the Loss Landscape

$$
\mathbb{E}\,(\text{SGD Test Loss}) = \bullet \; - \; \eta \binom{T}{1}\left(\bullet\!-\!\!-\!\bullet \right)
$$

$$
+ \; \eta^2 \binom{T}{2}\left(\bullet\!-\!\bullet\!-\!\bullet + \tfrac{1}{2}\,\bullet\;\overset{\frown}{\bullet\;\bullet} \right) + \eta^2 \binom{T}{1}\left(\tfrac{1}{2}\,\bullet\!\overset{\frown}{=}\!\bullet \right)
$$

$$
- \; \eta^3 \binom{T}{3}\left(\bullet\!-\!\bullet\!-\!\bullet\!-\!\bullet + \tfrac{1}{2}\,\bullet\;\overset{\frown}{\bullet\;\bullet}\;\bullet + \tfrac{1}{2}\,\bullet\;\bullet\;\overset{\frown}{\bullet\;\bullet} + \right.
$$
$$
\left. \tfrac{1}{2}\,\bullet\;\overset{\frown}{\bullet\;\bullet}\;\bullet + \tfrac{1}{2}\,\bullet\;\bullet\;\overset{\frown}{\bullet\;\bullet} + \tfrac{1}{6}\,\bullet\;\bullet\;\overset{\frown}{\bullet\;\bullet} \right)
$$

$$
- \; \eta^3 \binom{T}{2}\left(\tfrac{1}{2}\,\bullet\!-\!\overset{\frown}{\bullet\;\bullet} + \tfrac{1}{2}\,\overset{\frown}{\bullet\;\bullet}\!-\!\bullet + \tfrac{1}{6}\,\bullet\;\overset{\frown}{\bullet\;\bullet} + \tfrac{1}{6}\,\bullet\;\bullet\;\overset{\frown}{\bullet} \right)
$$

$$
- \; \eta^3 \binom{T}{1}\left(\tfrac{1}{6}\,\bullet\!\overset{\equiv}{\frown}\!\bullet \right) + o(\eta^3)
$$

$$
= \; \bullet \; - \; \eta \binom{T}{1}\left(\bullet\!-\!\!-\!\bullet \right)
$$

$$
+ \; \eta^2 \binom{T}{2}\left(\tfrac{3}{2}\,\bullet\!-\!\bullet\!-\!\bullet \right) + \eta^2 \binom{T}{1}\left(\tfrac{1}{2}\,\bullet\!\overset{\frown}{=}\!\bullet \right)
$$

$$
- \; \eta^3 \binom{T}{3}\left(\tfrac{5}{2}\,\bullet\!-\!\bullet\!-\!\bullet\!-\!\bullet + \tfrac{1}{2}\,\bullet\;\bullet\;\bullet\;\bullet + \tfrac{1}{6}\,\bullet\;\bullet\;\bullet\;\bullet \right)
$$

$$
- \; \eta^3 \binom{T}{2}\left(\tfrac{1}{2}\,\bullet\!-\!\overset{\frown}{\bullet\;\bullet} + \tfrac{1}{2}\,\overset{\frown}{\bullet\;\bullet}\!-\!\bullet + \tfrac{1}{6}\,\bullet\;\overset{\frown}{\bullet\;\bullet} + \tfrac{1}{6}\,\bullet\;\bullet\;\overset{\frown}{\bullet} \right)
$$

$$
- \; \eta^3 \binom{T}{1}\left(\tfrac{1}{6}\,\bullet\!\overset{\equiv}{\frown}\!\bullet \right) + o(\eta^3)
$$