Kalman Filters

Hidden Markov Models

Kalman Filtering

Suppose $x_0 = 0$, $x_{t+1} \sim Ux_t + \mathcal{N}(0, Q)$, and $y_t \sim Ox_t + \mathcal{N}(0, R)$, for $0 \leq t < T$, with all $2T$ noise values independent. Based on U, Q, O, R and on a sequence of observations $\mathfrak{y}_\tau = (y_t : 0 \leq t < \tau)$, we estimate $\mathfrak{y}_\tau$ by $K(\vec{y})$, hoping to minimize the expectation of $\|\mathfrak{y}_\tau - K(\vec{y}_\tau)\|^2$.

Well, writing μ_τ, C_τ for the mean and variance of x_τ conditioned on $\mathfrak{y}_\tau$, we have $\mu_0 = 0$, $C_0 = 0$ and

$$
\begin{aligned}
&C_{\tau+1}^{-1}(z - \mu_{\tau+1}, z - \mu_{\tau+1}) = \\
&(UC_\tau U^{-1} + Q)^{-1}(z - U\mu_\tau, z - U\mu_\tau) + \\
&R^{-1}(z - y_\tau, Oz - y_\tau) + \text{const}
\end{aligned}
$$

and

$$C_{tau+1} =$$

Estimating Parameters

Example