



Cambridge International AS & A Level

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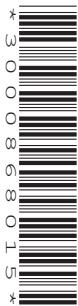


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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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1 Find the set of values of k for which the system of equations

$$\begin{aligned}x + 5y + 6z &= 1, \\kx + 2y + 2z &= 2, \\-3x + 4y + 8z &= 3,\end{aligned}$$

has a unique solution and interpret this situation geometrically.

[4]





2 It is given that

$$x = 1 + \frac{1}{t} \quad \text{and} \quad y = \cos^{-1} t \quad \text{for } 0 < t < 1.$$

- (a) Show that $\frac{dy}{dx} = \frac{t^2}{\sqrt{1-t^2}}$. [2]

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- (b) Show that $\frac{d^2y}{dx^2} = -t^a(1-t^2)^b(2-t^2)$, where a and b are constants to be determined. [4]

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- 3 A curve has equation $y = e^x$ for $\ln \frac{4}{3} \leq x \leq \ln \frac{12}{5}$. The area of the surface generated when the curve is rotated through 2π radians about the x -axis is denoted by A .

- (a) Use the substitution $u = e^x$ to show that

$$A = 2\pi \int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1+u^2} du. \quad [2]$$

- (b) Use the substitution $u = \sinh v$ to show that

$$A = \pi \left(\frac{904}{225} + \ln \frac{5}{3} \right). \quad [6]$$





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4 The matrix A is given by

$$\mathbf{A} = \begin{pmatrix} -11 & 1 & 8 \\ 0 & -2 & 0 \\ -16 & 1 & 13 \end{pmatrix}.$$

- (a) Show that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of \mathbf{A} and state the corresponding eigenvalue. [2]

- (b) Show that the characteristic equation of \mathbf{A} is $\lambda^3 - 19\lambda - 30 = 0$ and hence find the other eigenvalues of \mathbf{A} . [3]





(c) Use the characteristic equation of \mathbf{A} to find \mathbf{A}^{-1} .

[4]





5 Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + x = t^2 + t + 1,$$

given that, when $t = 0$, $x = 12$ and $\frac{dx}{dt} = -6$.

[10]

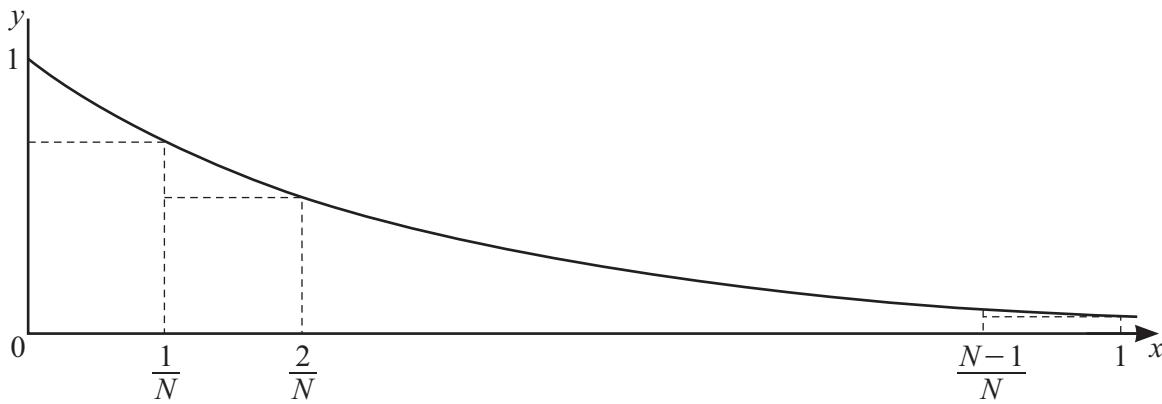
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The diagram shows the curve with equation $y = \left(\frac{1}{2}\right)^x$ for $0 \leq x \leq 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{2}\right)^x dx > L_N$, where

$$L_N = \frac{1}{2N(2^{\frac{1}{N}} - 1)}. \quad [4]$$





(b) Use a similar method to find, in terms of N , an upper bound U_N for $\int_0^1 \left(\frac{1}{2}\right)^x dx$. [4]

(c) Find the least value of N such that $U_N - L_N \leq 10^{-3}$. [2]





(d) Given that $\int_0^1 \left(\frac{1}{2}\right)^x dx = \frac{1}{2 \ln 2}$, use the value of N found in part (c) to find upper and lower bounds for $\ln 2$. [4]

- 7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

is $\frac{1}{4}x + \frac{1}{4}\sqrt{x^2 + 16}$.

[4]





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(b) Hence find the solution of the differential equation

$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

for which $y = 6$ when $x = 3$.

[6]



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- 8 (a) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^7$, where $z = \cos\theta + i\sin\theta$, use de Moivre's theorem to show that

$$\cos^7 \theta = a \cos 7\theta + b \cos 5\theta + c \cos 3\theta + d \cos \theta,$$

where a, b, c and d are constants to be determined.

[5]





$$\text{Let } I_n = \int_0^{\frac{1}{4}\pi} \cos^n \theta d\theta.$$

(b) Show that

$$nI_n = 2^{-\frac{1}{2}n} + (n-1)I_{n-2}. \quad [4]$$

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- (c) Using the results given in parts (a) and (b), find the exact value of I_9 .

[5]





Additional page

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