



# Cambridge International AS & A Level

CANDIDATE  
NAME
CENTRE  
NUMBER

--	--	--	--	--

CANDIDATE  
NUMBER

--	--	--	--



## FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

---

This document has **20** pages. Any blank pages are indicated.

- 1 (a) Use standard results from the list of formulae (MF19) to find  $\sum_{r=1}^n (r^3 - r)$  in terms of  $n$ , fully factorising your answer. [3]



(b) Express  $\frac{r+3}{r^3-r}$  in the form  $\frac{A}{r-1} + \frac{B}{r} + \frac{C}{r+1}$ , where  $A$ ,  $B$  and  $C$  are constants to be determined, and hence use the method of differences to find  $\sum_{r=2}^n \frac{r+3}{r^3-r}$ . [5]

(c) Deduce the value of  $\sum_{r=2}^{\infty} \frac{r+3}{r^3 - r}$ . [1]



- 2 The cubic equation  $x^3 + bx^2 + cx + d = 0$ , where  $b, c$  and  $d$  are constants, has roots  $\alpha, \beta$  and  $\gamma$ . It is given that

$$\begin{aligned}\alpha + \beta + \gamma &= 2, \\ \alpha^2 + \beta^2 + \gamma^2 &= 3, \\ \alpha^4 + \beta^4 + \gamma^4 &= 5.\end{aligned}$$

- (a) Find the values of  $b$  and  $c$ .

[3]



(b) Find the value of  $d$ .

[5]



- 3 The sequence of positive numbers  $u_1, u_2, u_3, \dots$  is such that  $u_1 < 5$  and, for  $n \geq 1$ ,

$$u_{n+1} = \frac{6u_n + 5}{u_n + 2}.$$

- (a) By considering  $5 - u_{n+1}$ , prove by mathematical induction that  $u_n < 5$  for all positive integers  $n$ .

[5]

DO NOT WRITE IN THIS MARGIN



(b) Show that  $u_{n+1} > u_n$  for  $n \geq 1$ .

[3]



- 4 Let  $k$  and  $m$  be non-zero constants. The matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} k & 0 \\ 0 & m \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

- (a) Give full details of the geometrical transformation in the  $x$ - $y$  plane represented by the matrix  $\mathbf{B}$  in each of the following cases.

- (i)  $m = 1$

[2]

.....

- (ii)  $m = k$

[2]

.....

- (b) Show that the matrix  $\mathbf{ABC}$  is singular.

[6]



\* 0000800000009 \*

DFD

9



© UCL ES 2025



9231/12/Q/N/25

[Turn over



10

- 5 The curve  $C$  has polar equation  $r^2 = \tan 2\theta$ , where  $0 \leq \theta \leq \frac{1}{8}\pi$ .

(a) Sketch  $C$  and state the greatest distance of a point on  $C$  from the pole.

[2]

(b) Find the exact value of the area of the region bounded by  $C$  and the half-line  $\theta = \frac{1}{8}\pi$ .

[4]





- (c) Show that  $C$  has Cartesian equation  $x^4 - 2xy - y^4 = 0$  given that  $0 \leq x \leq \cos\left(\frac{1}{8}\pi\right)$  and  $0 \leq y \leq \sin\left(\frac{1}{8}\pi\right)$ . [4]

[4]

- (d) Using your answer to (b), deduce the exact value of the area bounded by  $C$ , the  $x$ -axis and the line  $x = \cos\left(\frac{1}{8}\pi\right)$ . [2]

[2]

---

---

---

---

---

---

---

---

---

---

---





- 6** The plane  $\Pi$  has equation  $x + 3y + 2z = 1$ .

- (a) Find the perpendicular distance from the origin  $O$  to the plane  $\Pi$ .

[2]

Relative to  $O$ , the points  $A, B, C$  have position vectors

$$-\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

- (b) Find the acute angle between the planes  $OAB$  and  $\Pi$ .

[4]



- (c) Find an equation for the common perpendicular to the lines  $OC$  and  $AB$ .

[8]





- 7 The curve  $C$  has equation  $y = \frac{x^2 + x + 1}{x + 1}$ .

(a) Find the equations of the asymptotes of  $C$ .

[3]

(b) Find the coordinates of any stationary points on  $C$ .

[3]



(c) Sketch  $C$ .

(d) Sketch the curve with equation  $y = \frac{|x|^2 + |x| + 1}{|x| + 1}$ .

[2]



- (e) Find, in exact form, the set of values of  $x$  for which  $\frac{|x|^2 + |x| + 1}{|x| + 1} < 3$ . [3]



## **Additional page**

If you use the following page to complete the answer to any question, the question number must be clearly shown.



\* 0000800000018 \*

DFD



18

**BLANK PAGE**

DO NOT WRITE IN THIS MARGIN



\* 0000800000019 \*

DFD

19

**BLANK PAGE**

DO NOT WRITE IN THIS MARGIN



9231/12/O/N/25



## BLANK PAGE

DO NOT WRITE IN THIS MARGIN

---

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at [www.cambridgeinternational.org](http://www.cambridgeinternational.org) after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

