



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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This document has **20** pages. Any blank pages are indicated.

- 1 (a) Use standard results from the list of formulae (MF19) to find  $\sum_{r=1}^n (8r^3 + 12r^2 + 4r + 3)$  in terms of  $n$ , simplifying your answer. [3]



(b) Show that

$$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$$

and hence use the method of differences to find  $\sum_{r=1}^n \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$ .

[5]

(c) Deduce the value of  $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$ .

[1]



**2** The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}.$$

- (a) Give full details of the geometrical transformation in the  $x$ - $y$  plane represented by A.

[2]

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- (b) Give full details of the geometrical transformation in the  $x$ - $y$  plane represented by  $\mathbf{B}$ .

[2]

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.....  
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The triangle  $DEF$  in the  $x$ - $y$  plane is transformed by  $\mathbf{AB}$  onto triangle  $PQR$ .

- (c) Show that the triangles  $DEF$  and  $PQR$  have the same area.

[2]

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- (d) Find the equations of the invariant lines, through the origin, of the transformation in the  $x$ - $y$  plane represented by  $\mathbf{AB}$ . [5]



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- 3 Prove by mathematical induction that, for every positive integer  $n$ ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \cos x) = (-1)^n(x \sin x - (2n-1)\cos x).$$

[7]

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- 4 The quartic equation  $x^4 + x^3 + x^2 + x + 1 = 0$  has roots  $\alpha, \beta, \gamma, \delta$ .

- (a) Show that a quartic equation with roots  $2\alpha + 1, 2\beta + 1, 2\gamma + 1, 2\delta + 1$  is

$$y^4 - 2y^3 + 4y^2 + 2y + 11 = 0.$$

[4]

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The sum  $(2\alpha+1)^n + (2\beta+1)^n + (2\gamma+1)^n + (2\delta+1)^n$  is denoted by  $S_n$ .

- (b)** Find the value of  $S_2$ . [2]

- (c) Given that  $S_3 = -22$ , find the value of  $S_4$ . [2]



- 5 The plane  $\Pi_1$  has equation  $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$ .

- (a) Find an equation for  $\Pi_1$  in the form  $ax + by + cz = d$ .

[4]

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- (b) Find the perpendicular distance from the point with position vector  $-\mathbf{i} - 2\mathbf{k}$  to  $\Pi_1$ .

[3]





(c) The plane  $\Pi_2$  has equation  $3x + 2y - z = 14$ .

Find a vector equation of the line of intersection of  $\Pi_1$  and  $\Pi_2$ .

[4]





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- 6** The curve  $C$  has polar equation  $r = \sin 3\theta$ , for  $0 \leq \theta \leq \frac{1}{3}\pi$ .

- (a) Sketch C and state the equation of the line of symmetry.

[3]

(b) Find the exact value of the area of the region enclosed by  $C$ .

[4]

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In parts (c) and (d) you may use the identity  $\sin 3\theta \equiv 3 \sin \theta - 4 \sin^3 \theta$ .

- (c) Find the maximum distance of a point on  $C$  from the initial line.

[5]

- (d) Find a Cartesian equation for  $C$ .

[3]

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- 7 The curve  $C$  has equation  $y = \frac{x+2}{x^2+3x+1}$ .

(a) Find the equations of the asymptotes of  $C$ .

(a) Find the equations of the asymptotes of C.

[2]

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- (b) Show that  $C$  has no stationary points.

[3]





(c) Sketch  $C$ , stating the coordinates of the intersections with the axes.

[3]

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(d) Sketch the curve with equation  $y = \left| \frac{x+2}{x^2 + 3x + 1} \right|$ .

[2]





- (e) Find in exact form the set of values of  $x$  for which  $\left| \frac{x+2}{x^2+3x+1} \right| > 2$ . [4]



Additional page

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