



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n (8r^3 + 12r^2 + 4r + 3)$ in terms of n , simplifying your answer. [3]

[illegible]



(b) Show that

$$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$$

and hence use the method of differences to find $\sum_{r=1}^n \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [1]





2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}.$$

- (a) Give full details of the geometrical transformation in the x - y plane represented by \mathbf{A} . [2]

[illegible]

- (b) Give full details of the geometrical transformation in the x - y plane represented by \mathbf{B} . [2]

[illegible]

The triangle DEF in the x - y plane is transformed by \mathbf{AB} onto triangle PQR .

- (c) Show that the triangles DEF and PQR have the same area. [2]

[illegible]



- (d) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **AB**. [5]

[illegible]



3 Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \cos x) = (-1)^n (x \sin x - (2n-1) \cos x). \quad [7]$$

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The background is white, and there are no margins or other markings present.



This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.





4 The quartic equation $x^4 + x^3 + x^2 + x + 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Show that a quartic equation with roots $2\alpha+1, 2\beta+1, 2\gamma+1, 2\delta+1$ is

$$y^4 - 2y^3 + 4y^2 + 2y + 11 = 0. \quad [4]$$

[illegible]



The sum $(2\alpha + 1)^n + (2\beta + 1)^n + (2\gamma + 1)^n + (2\delta + 1)^n$ is denoted by S_n .

(b) Find the value of S_2 . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Given that $S_3 = -22$, find the value of S_4 . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





5 The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

This image shows a full page of white paper with ten horizontal dashed lines, evenly spaced from top to bottom. These lines are typical of primary-ruled notebook paper used for teaching handwriting or basic writing skills. There are no margins, text, or other markings on the page.

(b) Find the perpendicular distance from the point with position vector $-\mathbf{i}-2\mathbf{k}$ to Π_1 . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(c) The plane Π_2 has equation $3x + 2y - z = 14$.

Find a vector equation of the line of intersection of Π_1 and Π_2 .

[4]



6 The curve C has polar equation $r = \sin 3\theta$, for $0 \leq \theta \leq \frac{1}{3}\pi$.

(a) Sketch C and state the equation of the line of symmetry.

[3]

(b) Find the exact value of the area of the region enclosed by C .

[4]





In parts **(c)** and **(d)** you may use the identity $\sin 3\theta \equiv 3 \sin \theta - 4 \sin^3 \theta$.

- (c) Find the maximum distance of a point on C from the initial line. [5]

[illegible]

- (d)** Find a Cartesian equation for C . [3]

[illegible]



(a) Find the equations of the asymptotes of C .

[2]

[3]

(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation $y = \left| \frac{x+2}{x^2+3x+1} \right|$.

[2]





(e) Find in exact form the set of values of x for which $\left| \frac{x+2}{x^2+3x+1} \right| > 2$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]







Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

