



Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- 1 The curve C has polar equation $r = e^{\frac{4}{3}\theta}$ for $0 \leq \theta \leq \pi$.

Find the length of C . Give your answer in an exact form.

[4]



- 2 Find the roots of the equation $z^4 = 8 - 8i\sqrt{3}$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]



- 3 The variables x and y are such that $y = 2$ when $x = -1$ and

$$x^2y + (x+y)^3 = 3.$$

- (a) Show that $\frac{dy}{dx} = \frac{1}{4}$ when $x = -1$.

[3]



(b) Find the value of $\frac{d^2y}{dx^2}$ when $x = -1$.

[6]



- 4 (a) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^6$, where $z = \cos\theta + i\sin\theta$, use de Moivre's theorem to show that

$$\cos^6 \theta = a \cos 6\theta + b \cos 4\theta + c \cos 2\theta + d,$$

where a, b, c and d are constants to be determined.

[5]



(b) Find the exact value of $\int_0^{\frac{1}{2}\pi} \cos^6 2x \, dx$.

[3]



- 5 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 5 \cos x.$$

[7]



- (b) For large positive values of x and for any initial conditions, show that the solution to part (a) can be approximated by

$$y \approx R \sin(x + \phi),$$

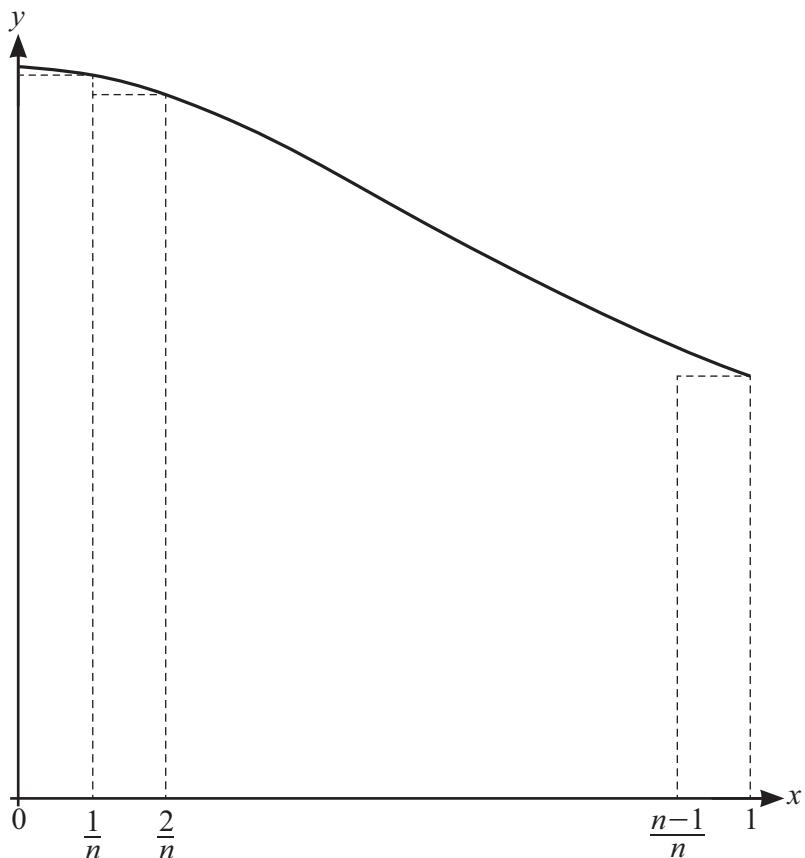
where the constants R and ϕ are to be determined.

[3]



- 6 (a) Use the substitution $x = \frac{1}{2}\sqrt{2} \sinh u$ to find $\int \frac{1}{\sqrt{2x^2+1}} dx$. [3]

.....



The diagram shows the curve with equation $y = \frac{1}{\sqrt{2x^2+1}}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}} < \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3}). \quad [5]$$

.....



- (c) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}}$. [4]

- (d) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}}$. [1]





- 7 (a) Show that $\frac{d}{dx} \left(\frac{1}{2}x\sqrt{4-x^2} + 2 \sin^{-1}\left(\frac{1}{2}x\right) \right) = \sqrt{4-x^2}$. [3]

(b) Find the solution of the differential equation

$$2\frac{dy}{dx} + \frac{y}{2+x} = 2\sqrt{2-x}$$

for which $y = \frac{1}{2}$ when $x = 1$. Give your answer in an exact form.

[8]



* 0000800000013 *

DFD

13



© UCL ES 2025



9231/22/Q/N/25

[Turn over

- 8 (a) Find the values of k for which the system of equations

$$\begin{aligned}x - y + 2kz &= 1, \\ kx + y + 2z &= 2, \\ 2x - y + z &= 3,\end{aligned}$$

does not have a unique solution.

[3]

- (b) Given that $k = -\frac{1}{2}$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [3]

.....
.....
.....
.....
.....

- (c) Given instead that $k = -1$, show that the system of equations in part (a) is also inconsistent. Interpret this situation geometrically. [4]



The matrix A is given by

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix}.$$

- (d) Use the characteristic equation of \mathbf{A} to show that $\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{A}$ where p and q are integers to be determined. [5]

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN





Additional page

If you use the following page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

