



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n (8r^3 + 12r^2 + 4r + 3)$ in terms of n , simplifying your answer. [3]

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(b) Show that

$$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$$

and hence use the method of differences to find $\sum_{r=1}^n \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [1]





- (d) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **AB**. [5]

[illegible]



3 Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \cos x) = (-1)^n (x \sin x - (2n-1) \cos x). \quad [7]$$

[illegible]



7

[illegible]



4 The quartic equation $x^4 + x^3 + x^2 + x + 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Show that a quartic equation with roots $2\alpha+1, 2\beta+1, 2\gamma+1, 2\delta+1$ is

$$y^4 - 2y^3 + 4y^2 + 2y + 11 = 0. \quad [4]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The sum $(2\alpha + 1)^n + (2\beta + 1)^n + (2\gamma + 1)^n + (2\delta + 1)^n$ is denoted by S_n .

(b) Find the value of S_2 . [2]

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(c) Given that $S_3 = -22$, find the value of S_4 . [2]

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5 The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$.

[4]

[illegible]

(b) Find the perpendicular distance from the point with position vector $-\mathbf{i}-2\mathbf{k}$ to Π_1 .

[3]

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(c) The plane Π_2 has equation $3x + 2y - z = 14$.

Find a vector equation of the line of intersection of Π_1 and Π_2 .

[4]



6 The curve C has polar equation $r = \sin 3\theta$, for $0 \leq \theta \leq \frac{1}{3}\pi$.

(a) Sketch C and state the equation of the line of symmetry.

[3]

(b) Find the exact value of the area of the region enclosed by C .

[4]





In parts **(c)** and **(d)** you may use the identity $\sin 3\theta \equiv 3 \sin \theta - 4 \sin^3 \theta$.

- (c) Find the maximum distance of a point on C from the initial line. [5]

[illegible]

- (d)** Find a Cartesian equation for C . [3]

[illegible]

- 7 The curve C has equation $y = \frac{x+2}{x^2+3x+1}$.

(a) Find the equations of the asymptotes of C .

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(b) Show that C has no stationary points.

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(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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(d) Sketch the curve with equation $y = \left| \frac{x+2}{x^2+3x+1} \right|$.

[2]





(e) Find in exact form the set of values of x for which $\left| \frac{x+2}{x^2+3x+1} \right| > 2$. [4]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]







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