



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 Find the roots of the equation $z^3 = 27 - 27i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]





2 Let $I_n = \int_0^1 (1-x)^n \sinh x \, dx$, where n is a non-negative integer.

- (a) Show that, for $n \geq 2$, $I_n = -1 + n(n-1)I_{n-2}$.

[4]

- (b) Find the exact value of I_2 .

[3]





- 3 By considering the binomial expansion of $\left(z - \frac{1}{z}\right)^5$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\operatorname{cosec}^5 \theta = \frac{a}{\sin 5\theta + b \sin 3\theta + c \sin \theta},$$

where a , b and c are integers to be determined.

[6]

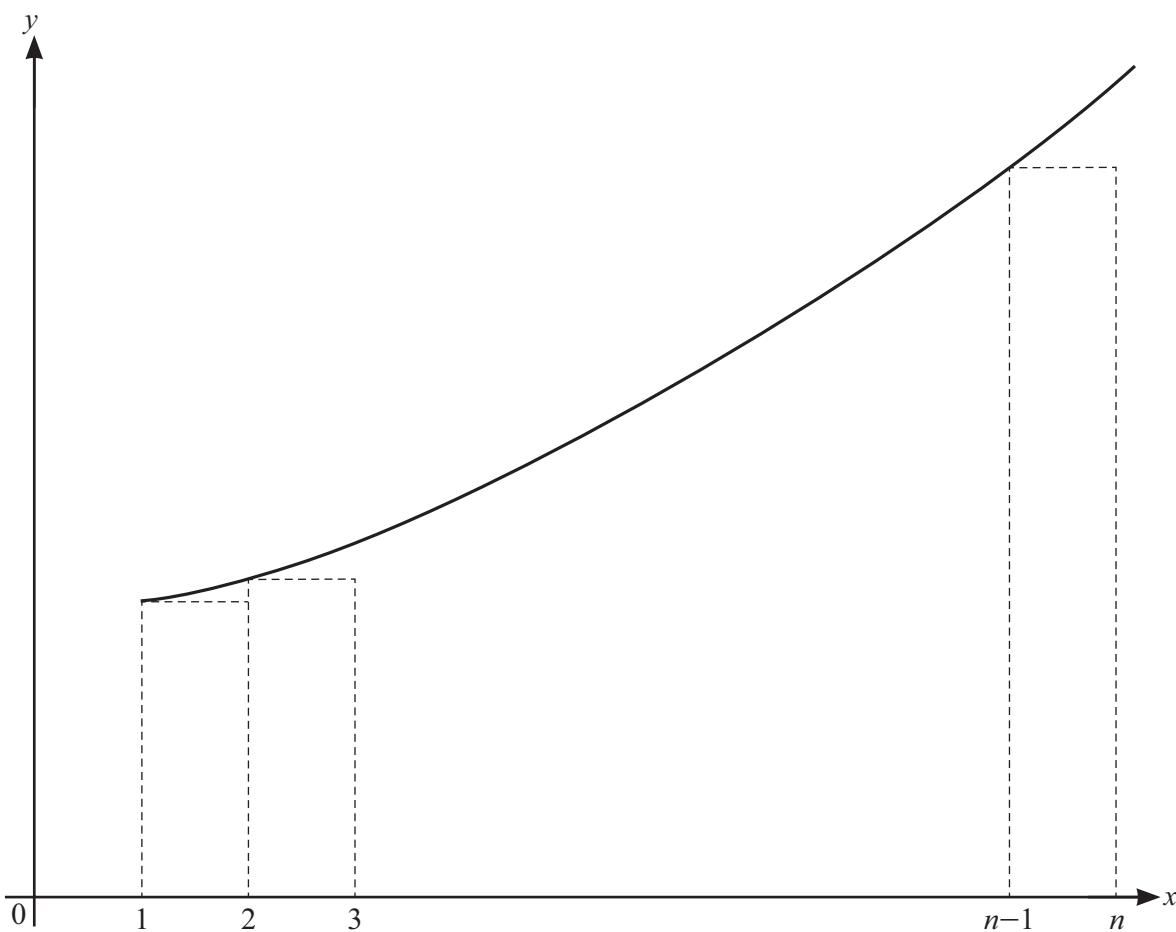
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The diagram shows the curve with equation $y = \frac{1}{\sqrt{x}} e^{\sqrt{x}}$ for $x \geq 1$, together with a set of $n - 1$ rectangles of unit width.

- (a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left(2 + \frac{1}{\sqrt{n}}\right) e^{\sqrt{n}} - 2e. \quad [5]$$





- (b) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}}$. [4]





- 5** Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 6x = e^{-t},$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$.

[10]

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- 6 (a) Starting from the definitions of \tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2 u = \operatorname{sech}^2 u.$$

[3]

- (b) Show that $\frac{d}{dt}(\operatorname{sech}^{-1} t) = -\frac{1}{t\sqrt{1-t^2}}$. [4]





It is given that

$$x = \tanh^{-1} t \quad \text{and} \quad y = t \operatorname{sech}^{-1} t, \quad \text{for } 0 < t < 1.$$

- (c) Show that $\frac{dy}{dx} = -\sqrt{1-t^2} + (1-t^2)\operatorname{sech}^{-1} t$. [4]

- (d) Find $\frac{d^2y}{dx^2}$ in terms of t . [4]





- 7 Find the solution of the differential equation

$$\frac{dy}{dx} - \frac{x+5}{x^2 + 10x + 61} y = 1,$$

given that $y = 0$ when $x = 3$. Give your answer in an exact form.

[10]





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- 8 (a) It is given that λ is an eigenvalue of the non-singular square matrix \mathbf{A} , with corresponding eigenvector \mathbf{e} .

Show that \mathbf{e} is an eigenvector of \mathbf{A}^3 with corresponding eigenvalue λ^3 .

[2]

matrix \mathbf{A} is given by

$$A = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$

- (b) Show that the eigenvalues of \mathbf{A} are -1 , 1 and 5 .

[2]





- (c) Find a matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $\mathbf{A} - 2\mathbf{I} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.

[6]

- (d) Use the characteristic equation of \mathbf{A} to show that $(\mathbf{A} - 2\mathbf{I})^3 = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I}$ where a , b and c are constants to be determined. [3]

[3]





Additional page

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