



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 The matrix \mathbf{M} represents the sequence of two transformations in the x - y plane given by a stretch parallel to the x -axis, scale factor 14, followed by a rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.

(a) Show that $2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$. [4]

- (b) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by \mathbf{M} . [5]





The unit square S in the x - y plane is transformed by \mathbf{M} onto the rectangle P .

- (c) Find the matrix which transforms P onto S .

[2]





- 2** Prove by mathematical induction that $2025^n + 47^n - 2$ is divisible by 46 for all positive integers n . [6]



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- 3 The quartic equation $x^4 + 7x^2 + 3x + 22 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

- (a) Find the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$.

[2]

- (b) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$.

[2]





(c) Use standard results from the list of formulae (MF19) to find the value of

$$\sum_{r=1}^{10} \left((\alpha^2 + r)^2 + (\beta^2 + r)^2 + (\gamma^2 + r)^2 + (\delta^2 + r)^2 \right). \quad [5]$$





- 4 Let $w_r = r(r+1)(r+2)\dots(r+9)$.

(a) Show that

$$w_{r+1} - w_r = 10(r+1)(r+2)\dots(r+9).$$

[2]

- (b)** Given that $u_r = (r+1)(r+2)\dots(r+9)$, find $\sum_{r=1}^n u_r$ in terms of n .

[3]





(c) Given that $v_r = x^{w_{r+1}} - x^{w_r}$, find the set of values of x for which the infinite series

$$v_1 + v_2 + v_3 + \dots$$

is convergent and give the sum to infinity when this exists.

[3]





- 5** The plane Π has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

- (a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$.

[4]

The point P has position vector $4\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$.

- (b) Find the position vector of the foot of the perpendicular from P to Π .

[4]





- (c) The line l is parallel to the vector $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

Find the acute angle between l and Π .

[3]





- 6 The curve C has equation $y = \frac{x^2 + a}{x + a}$, where a is a positive constant.

(a) Find the equations of the asymptotes of C .

[3]

(b) Find, in terms of a , the x -coordinates of the stationary points on C .

[3]





(c) Sketch C , stating the coordinates of any intersections with the axes.

[3]

.....

(d) Sketch the curve with equation $y = \left| \frac{x^2 + a}{x + a} \right|$.

[1]





- (e) Find the set of values of a for which $\left| \frac{x^2+a}{x+a} \right| = a$ has two real solutions. [4]



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- 7 The curve C has polar equation $r^2 = e^{\sin\theta} \cos\theta$, for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$.

- (a) Find the polar coordinates of the point on C that is furthest from the pole, giving your answers correct to 3 decimal places. [5]

- (b) Find the polar coordinates of the point on C that is furthest from the half-line $\theta = \frac{1}{2}\pi$, giving your answers correct to 3 decimal places. [5]





(c) Sketch C.

(d) Find the area of the region bounded by C , giving your answer in exact form.

[3]





Additional page

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