MATHEMATICS **9709/01**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2007**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
                          Graph Paper
                          List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
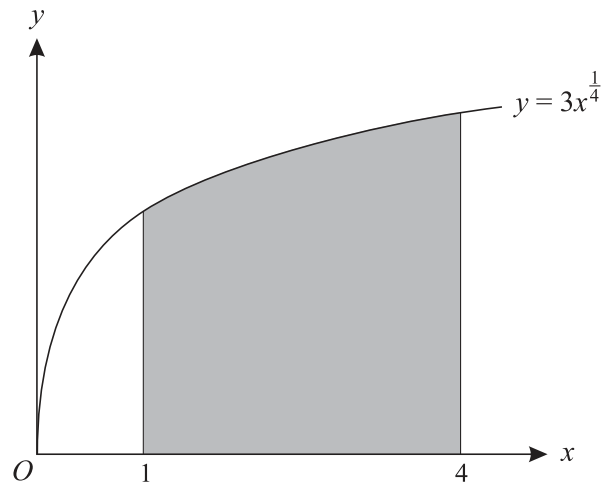The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

[Turn over

1 Find the value of the constant c for which the line $y = 2x + c$ is a tangent to the curve $y^2 = 4x$.   [4]
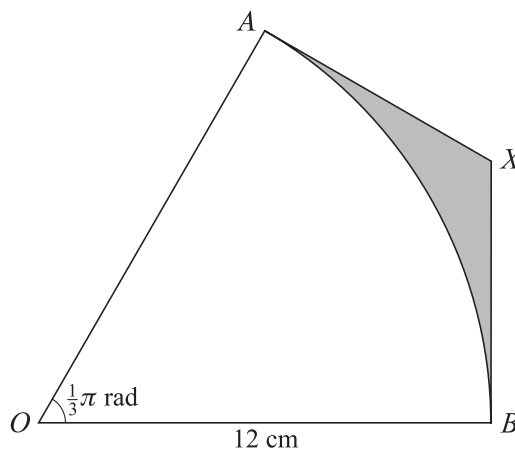
2

The diagram shows the curve $y = 3x^{\frac{1}{4}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$. Find the volume of the solid obtained when this shaded region is rotated completely about the x-axis, giving your answer in terms of π.   [4]

3 Prove the identity $\dfrac{1 - \tan^2 x}{1 + \tan^2 x} \equiv 1 - 2\sin^2 x.$ [4]

4 Find the real roots of the equation $\dfrac{18}{x^4} + \dfrac{1}{x^2} = 4.$ [4]
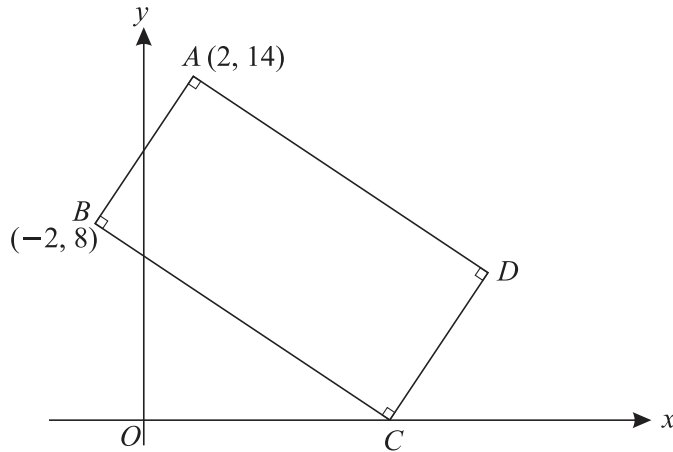
5

In the diagram, OAB is a sector of a circle with centre O and radius 12 cm. The lines AX and BX are tangents to the circle at A and B respectively. Angle $AOB = \frac{1}{3}\pi$ radians.

  (i) Find the exact length of AX, giving your answer in terms of $\sqrt{3}$.   [2]

  (ii) Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$.   [3]

6

The diagram shows a rectangle $ABCD$. The point A is $(2, 14)$, B is $(-2, 8)$ and C lies on the x-axis. Find

   (i) the equation of BC, [4]

   (ii) the coordinates of C and D. [3]

7 The second term of a geometric progression is 3 and the sum to infinity is 12.

   (i) Find the first term of the progression. [4]

An arithmetic progression has the same first and second terms as the geometric progression.

   (ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

8 The function f is defined by $f(x) = a + b \cos 2x$, for $0 \leqslant x \leqslant \pi$. It is given that $f(0) = -1$ and $f\left(\frac{1}{2}\pi\right) = 7$.

   (i) Find the values of a and b. [3]

   (ii) Find the x-coordinates of the points where the curve $y = f(x)$ intersects the x-axis. [3]

   (iii) Sketch the graph of $y = f(x)$. [2]

9 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}.$$

   (i) Given that C is the point such that $\overrightarrow{AC} = 2\overrightarrow{AB}$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]

The position vector of the point D is given by $\overrightarrow{OD} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$, where k is a constant, and it is given that $\overrightarrow{OD} = m\overrightarrow{OA} + n\overrightarrow{OB}$, where m and n are constants.

   (ii) Find the values of m, n and k. [4]

**10** The equation of a curve is $y = 2x + \dfrac{8}{x^2}$.

   (i) Obtain expressions for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$.   [3]

   (ii) Find the coordinates of the stationary point on the curve and determine the nature of the stationary point.   [3]

   (iii) Show that the normal to the curve at the point $(-2, -2)$ intersects the x-axis at the point $(-10, 0)$.   [3]

   (iv) Find the area of the region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 2$.   [3]

11
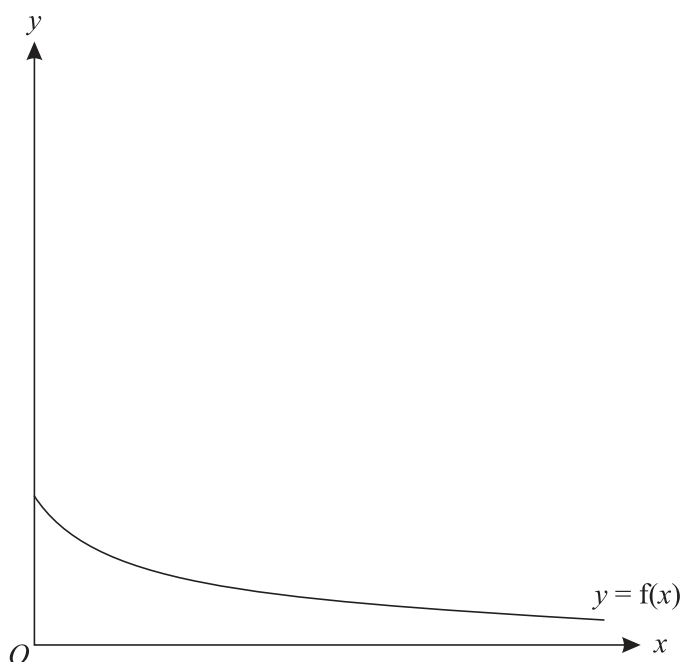
The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f} : x \mapsto \dfrac{6}{2x + 3}$ for $x \geqslant 0$.

   (i) Find an expression, in terms of x, for $\mathrm{f}'(x)$ and explain how your answer shows that f is a decreasing function.   [3]

   (ii) Find an expression, in terms of x, for $\mathrm{f}^{-1}(x)$ and find the domain of f^{-1}.   [4]

   (iii) Copy the diagram and, on your copy, sketch the graph of $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs.   [2]

The function g is defined by $\mathrm{g} : x \mapsto \frac{1}{2}x$ for $x \geqslant 0$.

   (iv) Solve the equation $\mathrm{fg}(x) = \frac{3}{2}$.   [3]