

# **Cambridge International AS & A Level**

CANDIDATE  
NAME

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CENTRE  
NUMBER

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## **FURTHER MATHEMATICS**

**9231/22**

Paper 2 Further Pure Mathematics 2

**October/November 2020**

**2 hours**



You must answer on the question paper.

You will need: List of formulae (MF19)

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### **INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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This document has **16** pages. Blank pages are indicated.

- 1 Find the Maclaurin's series for  $\tan(x + \frac{1}{4}\pi)$  up to and including the term in  $x^2$ . [5]

- 2 A curve has equation  $y = \cosh x$ , for  $0 \leq x \leq \frac{1}{2}$ .

Find, in terms of  $\pi$  and  $e$ , the area of the surface generated when the curve is rotated through  $2\pi$  radians about the  $x$ -axis. [6]

- 3 Find all the roots of the equation  $(w+1)^6 = 1$ , giving your answers in the form  $x + iy$  where  $x$  and  $y$  are real and exact. [4]

- 4** Find the solution of the differential equation

$$x \frac{dy}{dx} + 2y = e^x$$

for which  $y = 3$  when  $x = 1$ . Give your answer in the form  $y = f(x)$ .

[8]

- 5** The curve  $C$  has equation

$$y^2 + (xy + 1)^2 = 5.$$

- (a) Show that, at the point  $(1, 1)$  on  $C$ ,  $\frac{dy}{dx} = -\frac{2}{3}$ . [3]

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- (b)** Find the value of  $\frac{d^2y}{dx^2}$  at the point  $(1, 1)$ . [5]

- 6** Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 102 \cos 3t,$$

given that, when  $t = 0$ ,  $x = 1$  and  $\frac{dx}{dt} = 0$ .

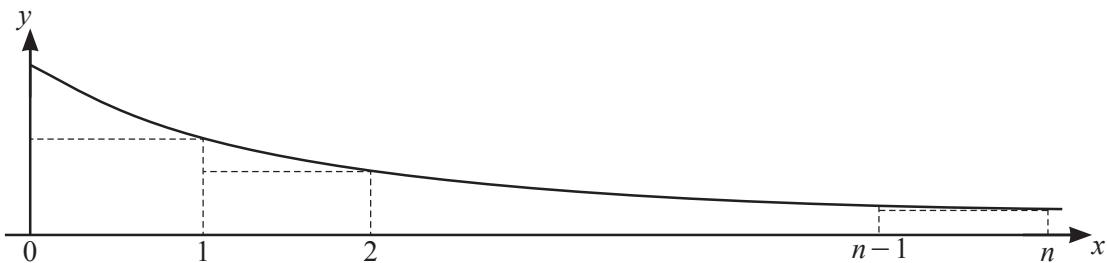
[11]



- 7 (a) Show that  $\sum_{r=1}^n z^{2r} = \frac{z^{2n+1}-z}{z-z^{-1}}$ , for  $z \neq 0, 1, -1$ . [2]

- (b) By letting  $z = \cos \theta + i \sin \theta$ , show that, if  $\sin \theta \neq 0$ ,

$$1 + 2 \sum_{r=1}^n \cos(2r\theta) = \frac{\sin((2n+1)\theta)}{\sin \theta}. \quad [5]$$



The diagram shows the curve  $y = \frac{1}{\sqrt{x^2 + x + 1}}$  for  $x \geq 0$ , together with a set of  $n$  rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r^2 + r + 1}} < \ln\left(\frac{1}{3} + \frac{2}{3}n + \frac{2}{3}\sqrt{n^2 + n + 1}\right). \quad [10]$$



- 9** It is given that  $a$  is a positive constant.

(a) Show that the system of equations

$$\begin{aligned} ax + (2a+5)y + (a+1)z &= 1, \\ -4y &= 2, \\ 3y - z &= 3, \end{aligned}$$

has a unique solution and interpret this situation geometrically.

[3]

The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

- (b) Show that the eigenvalues of  $\mathbf{A}$  are  $a$ ,  $-1$  and  $-4$ . [2]

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- (c) Find a matrix  $\mathbf{P}$  such that

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \mathbf{P}^{-1}. \quad [5]$$

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- (d) Use the characteristic equation of  $A$  to find  $A^{-1}$ . [6]

Additional Page

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