Why Do Bigger Firms Pay More For Performance? Contract Incentives versus Market-induced Incentives

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Abstract

Executives in bigger firms are rewarded with higher incentives in contracts comparing to those who are equally paid but work in smaller firms. To explain, I develop a novel model featuring the competition for managerial skills in a search market. This generates market-induced incentives which enter into the contracting problem between a firm and an executive. I show that market-induced incentives decrease with firm size. As a substitute, more incentive pays are required in bigger firms. The model is estimated on the data of executives in S&P 1500 firms. I use the model to evaluate several regulations that have been proposed or implemented.

Key Words:

executive compensation, firm-size pay premium, dynamic contract, moral hazard, search frictions

1 Introduction

The compensations of executives are closely tied to firm performance in the form of options and stocks. Called the incentive compensation, it accounts for about 70% of the total compensation in $S\&P$ 1500 firms. A salient feature of incentive pays is that they increase with firm size. I document a new evidence that such size premium continues to hold after controlling for executives' total compensation or firm related wealth. That is, compared to their equally wealthy counterparts in smaller firms, executives in bigger firms demand higher incentive rewards. This new fact deserves an explanation and understanding the underlying mechanism would contribute to the current debate about regulations on executive compensations.

My explanation starts with the simple observation that executives are forward looking. They are not only motivated by the incentives offered in the contract, but also motivated by the perspective for higher rewards in the future, called the *market-induced incentives* in this paper. These two sources of incentives substitute each other in the dynamic contract between the firm and the executive. The fact that executives in big and small firms are equally wealthy today does not imply their expected wealth in the future would also be the same. In contrast, firm size represents a "search capital" that executives can explore in the labor market. As long as there are search frictions, managers in bigger firms tend to be wealthier in the future, and by the law of diminishing marginal utility, they are less sensitive to monetary rewards. The market-induced incentives, therefore, are lower for them. As a result, bigger firm executives require higher incentive pays in the contract.

The heterogeneity in market-induced incentives is driven by the search market. In the classical career concern models, the labor market is assumed to be frictionless, hence market-induced incentives only depend on planning horizons.[1]In a more realistic search market setting, the market-induced incentives – embedded in wage offers one expect to receive — certainly depend on the position on the job ladder. In particular, they depend on the firm size. While the intuition would hold in any type of search models, I employ the sequential auction framework (Postel-Vinay and Robin 2002) which allows firms to counter outside offers. It is especially relevant because this type of negotiations usually happens in high skills markets such as the academics market as well as the executives market which is the target of this paper.

An executive is approached by outside firms in a search market. The outside offers can be used to trigger renegotiations with the current firm, where the current and the outside firms bid sequentially to hire the executive. Essentially they enter into a Bertrand competition. The maximum wage firms are willing to bid depends on the productivity of the executive. The productivity is persistent, and it depends on his effort in the past. Therefore, ex ante, the bidding scheme is an incentive scheme —- the executive works harder in order to increase his expected productivity so that firms are willing to bid more.

Market-induced incentives decrease with firm size. Small firms are able to bid up to lower wage levels than big firms due to the scale effect in production. Therefore, a small variation in bidding wages across different productivities generates big incentives in utilities. At higher wage levels where bigger firms can bid, the same variation in bidding wages leads to lower incentives. Because of this, to address the same moral hazard problem, small firms can make use of market-induced incentives while big firms have to

[1]Market-induced incentives have been considered as an alternative way to discipline worker's effort since Fama (1980). The concept is similar as career concerns, though the detailed mechanisms can be different in different models, see Grochulski and Zhang (2017) for a discussion.

provide more contract incentives.

Formally, I consider a dynamic contracting problem in which a risk-neutral firm hires a risk-averse agent/executive whose productivity is observable and persistent over time. The evolution of the executive's productivity depends on his effort and exogenous, idiosyncratic shocks, both of which are unobservable. That is, there is moral hazard. The output is the executive's productivity scaled by the firm size. To explore the interaction of contract and market-induced incentives, I embed this contracting problem into an equilibrium search model.[2]Executives search on-the-job, and use outside offers to renegotiate with the current firm. Hence, the contract is subject to the two-sided limited commitments — executives and firms can terminate the relationship upon receiving better outside values. Despite these complications, the optimal contract exhibits memory and inherits the essential properties of classical infinite repeated moral hazard model (Spear and Srivastava 1987). Moreover, I show that the market-induced incentives decrease with firm size as long as the concavity of the utility function exceeds the scale effect in the production function, which is usually a very weak requirement. For instance, with a CRRA utility the relative risk aversion is required to be bigger than one.

Solving numerically for this optimal contract becomes difficult in the presence of the incentive compatibility constraint, limited-commitment constraints, together with shocks of large support.[3]The firms problem can be written recursively using promised utility (Spear and Srivastava 1987; Rogerson 1985) but one still needs to solve for the promised value in each state of the world in the next period. Following Marcet and Marimon (2017), Mele (2014), Farhi and Werning (2013), Lamadon (2016), I address this issue using the recursive Lagrangian approach. The Lagrangian multiplier represents the weight of the executive in a constructed Pareto problem, and it keeps track of the incentive compatibility constraint, limited commitment constraints and job-to-job transitions.

I bring my model to the ExecuComp dataset of $S\&P$ 1500 U.S. firms from 1992 to 2016. The sample includes top 5 to 8 executives in each firm year. I supplement the ExecuComp dataset with BoardEX dataset to identify the precise spell start and end dates, and whether each spell ends in a job-to-job transition. I provide direct evidence that executives do transit between firms, they have roughly the same job offer arrival rate and they are more likely to transit to bigger firms. I estimate the model's parameters by matching the process of firm profitabilities, executive job turnovers, sensitivity of executive wealth to performance in the data, etc. I find that the model quantitatively captures the pattern that bigger firm pay executives more for performance. I use the model to evaluate several regulations on executive pays that have been proposed or implemented. These exercises are still in progress and will soon be updated.

The rest of the introduction is a review on related literature. In Section 2, I present the data fact. In section 3, I present my model. I characterize the optimal contract and how incentives change with firm size in Section 4. In Section 5, I present the data and estimation. Section 6 decomposes the total incentives based on the estimates. I also studies several counterfactual scenarios to evaluate regulations on executive compensation.

[2]The real world labor market for executive-level jobs, just as labor markets for ordinary jobs, is full of informational and searching frictions. On the one hand, firms who have executive-level vacancies usually hire head hunters to find candidates. Once contact with the candidate is established, numerous ways including but not restricted to talks and interviews with the candidate and his/her previous colleagues are used to figure out whether the candidate is suitable. On the other hand, top managerial positions are limited, and can only be available upon the current executive leaves; at the same time suitable manager candidates are also limited, and can only be hired after negotiation and competing with the current employer. These are the reasons to describe the executive labor market as a search market.

And section 7 concludes.

Related Literature

This paper contributes to the literature that explains the scaling of incentive pays with firm size. One line of research starts from Gabaix and Landier (2008) and Tervio (2008), where competitive assignment models of the managerial labor market, absent an agency problem, are presented to explain why total compensation increases with firm size. By assuming CEO's effort has a multiplicative effect on both CEO utility and firm value, Edmans et al. (2009) embed a moral hazard problem into the competitive assignment model. The model quantitatively generates predictions on wealth performance sensitivities that are consistent with the data. Edmans and Gabaix (2011) extend the model further to risk averse executives. This line of explanation relies on that total compensation increase with firm size, and it can not explain why after controlling for total compensation, the incentive pays still increase with firm size, which is the key question of this paper.

In another line of the literature, Margiotta and Miller (2000) derive and estimate a multi-period principal-agent model with moral hazard. Based on this model, Gayle and Miller (2009) show that large firms face a more severe moral hazard problem, hence higher equity incentives are necessary in order to satisfy the incentive compatibility condition. Gayle et al. (2015)) embed the model of Margiotta and Miller (2000) into a Roy model with human capital accumulations, and they find that the quality of the signal is unambiguously poorer in larger firms, and that explains the most of the pay differential between small and big firms. Compared to these papers, my model highlights the role of a frictional labor market with on-the-job search, which generate cross-sectional variations in market-induced incentives. In this way, my explanation of incentive pays does not rely on the heterogeneity of the moral hazard problems across firms, though this heterogeneity can be easily added. For example, I can assume the effort cost or the hazard ratio of working versus shirking changes with firm size in a proper parametric form. Yet, when these heterogeneities are included, how to identify the two sources of variations would be non-trivial. This is left for future research.

In terms of modeling, this paper builds on and links two strands of literature. One strand is the extensive literature on optimal long-term contracts with private information and(or) commitment frictions, e.g., Townsend (1982), Rogerson (1985), Spear and Srivastava (1987), Phelan and Townsend (1991), Harris and Holmstrom (1982), Thomas and Worrall (1990) and Phelan (1995). Compare to this literature, I model and add market-induced incentives in the incentive compatibility constraint and analyze how this new source of incentives influence the optimal contract.

Another important strand of literature uses structural search models to evaluate wage dispersions. For example, Postel-Vinay and Robin (2002), Cahuc et al. (2006), Lise et al. (2016) among others estimate models with risk-neutral workers and sequential auctions. Compare to this literature, I add a dynamic moral hazard problem which allows me to understand how market frictions influence a long-term contract. The extreme case in my model that firms with size below a threshold only pay a flat wage roughly corresponds to this type of models.[4]

[4]In my model, in some parameter set, firms smaller than a threshold will only pay a flat wage and all incentives come from the market. This case, though not very relevant empirically for the market that I am looking at (almost all executives in my sample are provided with some incentive schemes), it is consistent

This paper is also closely related to Abrahám et al. (2016), which aims to explain wage inequality in general labor market by combining repeated moral hazard and on-the-job search. Other than the differences in topics, there is a critical difference that distinguishes my model from theirs: the productivity (or output) of agents is persistent in my model while is independent in their model, and therefore in my model working hard today rewards the agent in the market tomorrow. This is also where my model is linked to the literature of career concerns (Holmström 1999, Gibbons and Murphy 1992). In the career concern models, the workers productivity level is persistent, yet the market needs to learn it. By exerting effort, the worker can manipulate the market belief and increase the spot wage he receives. In my model, productivity is observed once the executive and the firm meet. Yet due to search frictions, the reward is postponed to the future. Career concern models usually focus on the compensation difference along the time dimension while my focus is cross-sectional. Therefore, I need to model the labor market more realistically as a search market.

2 The Facts

The fact that incentive pays is higher in bigger firms is well documented in the literature and is replicated in table 1 column (1). In the regression, the dependent variable is the log of delta, a standard measurement of incentives in contract, defined as the dollar change in firm related wealth for a percentage change in firm value.[5]The independent variable firm size is measured by market capitalization. It shows for 1% increase in firm size, the incentives increase by 0.6%. Current literature tend to explain this fact by total compensations. Because big firm executives in general have higher total compensations, and given a concave utility function, highly compensated executives require higher incentive pays in dollars to get the same incentives in utiles. Edmans et al.'s (2009) model, for example, is in this spirit.[6]

However, if we plot the heatmap of incentives over two dimensions, the total compensation and the firm size, as in figure 1 (a), and only compare executives with the same total compensation, still those in big firms get higher contract incentives.[7] This finding is confirmed in regression analysis. The positive correlation between incentive pays and firm size exists and is significant after controlling for total compensation in various forms, as shown in table 1 from column (2) to column (4). For 1% increase in the firm scale, the incentive measured by delta increases by 0.3%. This new fact, therefore, deserves explanations.

with other theoretical work such a Grochulski and Zhang (2017). This paves the way to study not only top managers' incentive compensation, but also that of mid-level managers and even rank-and-file employees, as suggested by Edmans et al. (2017).

[5]Delta is also known as the "dollar-percentage incentive" or "wealth-performance sensitivity".

[6]Edmans et al.'s (2009) model assumes a multiplicative utility for executives. This implies the more executives are paid, the more incentive pays are required to induce their effort.

[7]The total compensation is the sum of salary and bonus, the value of restricted stocks and options granted, and value of retirement and long-term compensation schemes.These variables will be used throughout the paper. I divide the whole sample into 80×80 cells according to the total compensation and firm size, and compute the mean of $\log(delta)$ within each cell. This is plotted in figure 1.

Table 1: Incentive Pays Increase with Firm Size

	log($delta$)			
	(1)	(2)	(3)	(4)
log(Firm Size)	0.578***	0.295***	0.274***	0.273***
	(250.03)	(112.20)	(104.10)	(103.68)
log(tdc1)		0.7159***		
		(176.18)		
tdc1 Dummies (50)			Yes	
tdc1 Dummies (100)				Yes
Year FEs	Yes	Yes	Yes	Yes
Industry FEs	Yes	Yes	Yes	Yes
Year $\times$ Industry FEs	Yes	Yes	Yes	Yes
Observations	129458	129184	129185	129185

(a) The t statistics are shown in parentheses, and we denote symbols of significance by * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$;
(b) The dependent variable is the log of delta, where delta is the wealth-performance sensitivity defined as the dollar change in firm related wealth for a percentage change in firm value. The key independent variable is the log of firm size, where firm size is measured by firm's market capitalization. The key control variable is the level of total compensation, `tdc1` in ExecuComp dataset. All regressions control for year and industrial dummies and their interaction terms.
(c) Column (1) is a regression of log(delta) on log(firm size), which replicates the cross-sectional regression in the literature. From column (2) to column (4), I add log(tdc1), tdc1 evenly grouped into 50 and 100 categories (then transformed into dummies), respectively.
(d) The data on delta is provided by Coles et al. (2006), Coles et al. (2013), and rest are included in the ExecuComp dataset, both are standard and public available.

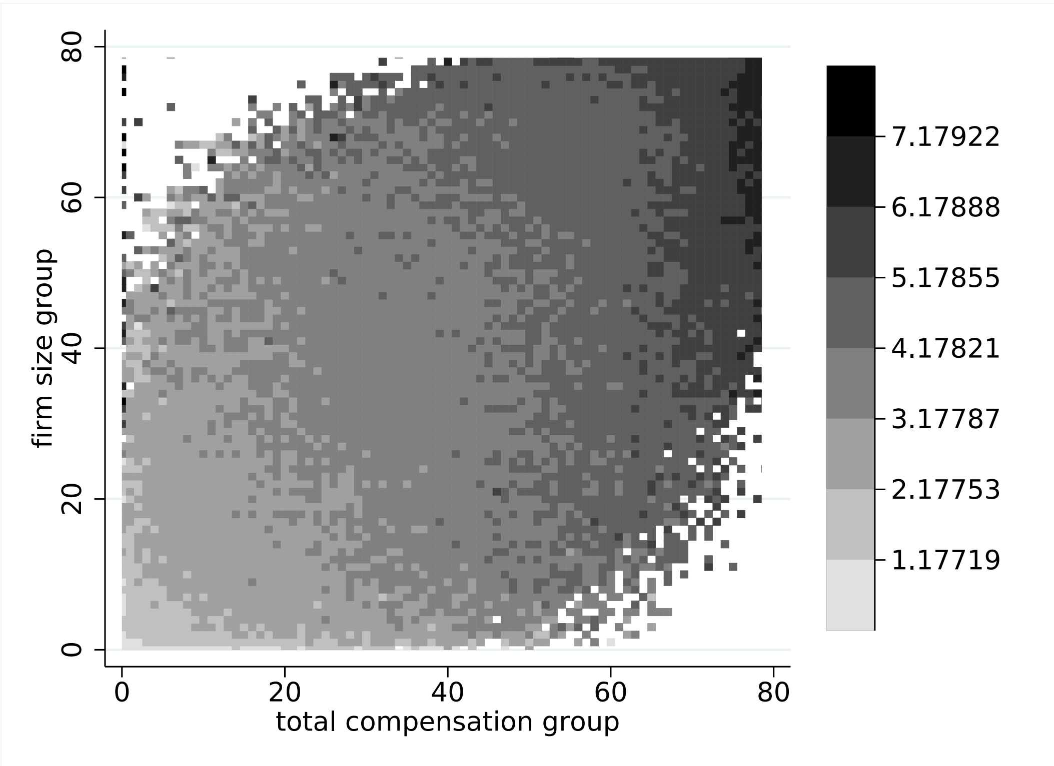

Figure 1: log($delta$) over firm size and total compensation

3 Model

This section lays out the equilibrium model of the executive labor market.

3.1 Preferences and Technologies

Time is discrete, indexed by t and continues forever. In this economy, there is a fixed measure of individuals who aim at executive level jobs. In the following, I will call individuals who are employed *executives*, and those who are not employed *executive candidates*. These candidates are not necessarily being unemployed from other jobs. They are simply not working as an executive, and aim at finding a managerial job.

Individuals want to maximize expected lifetime utility,

$$\mathbb{E}_0 \Sigma_{t=0}^{\infty} (\beta \times (1-\delta))^t (u(w_t) - c(e_t)),$$

where $\beta \in (0,1)$ is the discount factor, $\delta \in (0,1)$ is the death probability, utility of consumption $u : \mathbb{R} \to \mathbb{R}$ is increasing and concave, $c(\cdot)$ is the dis-utility of effort. Effort e_t takes two values, $e_t \in \{0,1\}$, and cost of effort level 0 is normalized to zero. I denote $c(1)$ by c.

Individuals are heterogeneous in productivity which is indexed by $z \in \mathbb{Z} = \{z_1, z_2, ..., z_{n_z}\}$. Individual productivity is observable to himself, to firms that he meets, and can be carried with the individual through job-to-job transitions. Hence, I treat the productivity as general management skills rather than firm-specific skills.[8]The productivity changes over time according to a Markov process $\Gamma_z(z_t, z_{t+1}|e_t)$, which depends on effort e_t. I denote the process by $\Gamma_z(z_t, z_{t+1})$ for $e_t = 1$, and $\Gamma_z^s(|z_t, z_{t+1})$ for $e_t = 0$ (s for shirking).

Once the individual dies, the associated job destroys (if he is matched), and a new executive candidate enters the economy.[9] Executive candidates have constant productivity $z_O \in \mathbb{Z}$, and receive a value W^0. Having a constant productivity during "unemployment phase" is not as restrict as it looks. It merely says when individuals starts their career as an executive, each of them is expected to be as productive as others (their productivities are drawn from the same distribution $\Gamma_z(z_O, z)$). I further define the likelihood ratio

$$g(z, z') \equiv \frac{\Gamma^s(z, z')}{\Gamma(z, z')}.$$

Notice as a likelihood ratio, it satisfies $E[g(z, z')] = \sum_{z'} g(z, z')\Gamma(z, z') = 1$. I assume the Markovian processes are such that

- Taking effort delivers a higher expected productivity:

$$E_\Gamma[z' g(z, z')] < E_\Gamma[z'];$$

- Taking effort is more likely to deliver a higher productivity (Monotone likelihood ratio property, MLRP):

[8]However, firm-specific skills can be included by a productivity discount upon a job-to-job transition. This is left as a future extension.

[9]In an alternative setting, the executive position continues as a vacancy upon breaking up with the executive. In such case, to endogenize the vacancy value, one need to aggregate number of meetings between vacancies and searching workers by a standard aggregate matching function. This goes beyond the scope of this paper.

$$g(z, z') \text{ is non-increasing in } z'.$$

On the other side of the labor market are executive jobs (or firms, represented by its board members). Jobs are characterized by a technological factor, called firm size, denoted by $s \in \mathbb{S} = [\underline{s}, \bar{s}]$. In empirical sections, I use the firms' market capitalization as a proxy for firm size. Firm size is observable to all matched executives and it is permanent and exogenous.[10]

Executives and jobs are imperfectly informed about executive types and the location of the job, which precludes the optimal assignments as in Gabaix and Landier (2008). Yet, when two agents meet, both are informed about each other's types immediately. Search is random and all workers, employed and unemployed, sample from the same job offer distribution $F(s)$. Unemployed workers meet firms with probability λ_0, while employed workers meet other firms with probability λ_1. I treat λ_1, λ_0 and $F(s)$ as exogenous parameters. That is, we are in a partial equilibrium.

A match between a worker of productivity z and a firm of size s produces a flow of output $f(s, z) = y(s)z$, where $y(s)$ is strictly increasing and concave in s. Later whenever it is needed, I use a particular function form $f(s, z) = \alpha sz$, where $\alpha \in (0, 1)$ is the upper bound of the share fraction of the executive. This function form entails that executive's efforts rolled out across the entire firm up to a scale of α and thus have a greater effect in a larger company.[11]

3.2 Timing

An individual is either an employed executive or an executive candidate, and enters each period with the last period productivity, denoted by z. The executive candidates have a constant productivity $z = z_O$, and with probability λ_0 to meet a firm of size s drawn from the offer distribution $F(s)$. The matched individual and firm bargain on a contract. The firm will offer an contract with a value of least W^0. The individual then enters next period as an employed executive. What is essential for our analysis is the value W^0.

An employed executive starts the period with his current productivity z.

1. **Compensation:** The firm will first pay a wage w for this period, in accordance with the contract.

2. **Production:** Then the executives enter the production phase where he chooses an effort level $e \in \{0, 1\}$. His productivity z' is then realized according to $\Gamma(z, z'|e)$, which depends on his effort choice. The firm only observes the output $y(z, s)$, not the executive's effort. That is, there is moral hazard.

[10] From the view of labor search literature, one could interpret firm size here as "the productivity of the job" or "firm type". Instead of using total number of employees, I prefer to use total assets value as a proxy for firm size since the performance of the firm is usually measured by return on assets. If one interpret firm size as the total number of employees, then it can be endogenized by modeling the labor market of ordinary workers, but not by modeling the market of executives as analyzed here.

[11] There has been a discussion on the appropriate production function form for executives. Taking s as firm size, and z as the executive's per unit contribution to shareholder values. An additive production function such as $f(s, z) = s + z$ implies the effect of executives on firm value is independent of firm size. This specification is appropriate for a perk consumption. An multiplicative production function such as $f(s, z) = y(s)z$ where $y(s)$ is increasing in s, is appropriate for executives' actions that can roll out across the entire firm and thus have a greater effect in a larger company. The latter is the function form adopted here.

3. **Leave/Die:** After the production phase, with probability δ he dies, otherwise the executive can choose to leave the job voluntarily.
4. **Sequential Auction :** The executive has the a probability λ_1 to sample a job offer of firm size $s' \sim F(s')$ (s' denotes poaching firm size and s is the current firm size.) The current firm can counter the outside offer and make a take-it-or-leave-it contract offer to the executive. The value of the contract to the executive therefore will be determined by a sequential auction between the current and poaching firms as described below. The executive will enter the next period as employed by either current firm or the poaching firm.

The timing line is shown in figure 2.

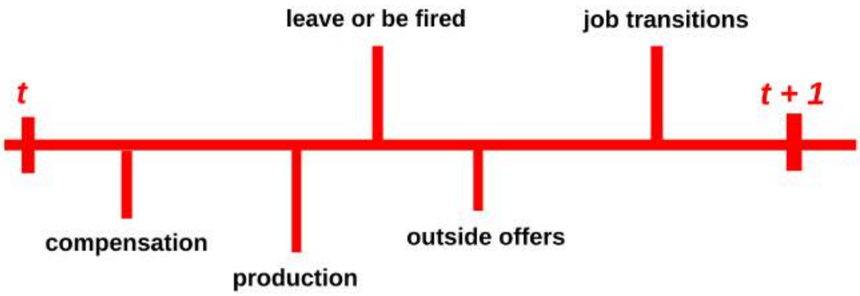

Figure 2: Timing

3.3 Information Structure and Contracts

A contract defines the transfers and actions for the executive and the firm within a match for all future histories. Call $h_t = (z'_t, s_t, s'_t)$ the state of the match at period t, where z'_t is the realized productivity at t, s_t is the current firm size, s'_t is the size of poaching firm and $s'_t = s_O$ if there is no poaching firm. Call $h^t = (h_1, h_2, .., h_t)$ a given history of realizations between h_1 and h_t. The history of productivity and poaching firms is common knowledge to the worker and the firm and fully contractible. A feasible contract in this framework is a plan

$$\{e_t(h^{t-1}), w_t(h^{t-1}), I_t(h^t)\}_{t=0}^{\infty},$$

that satisfied the participation constraints of both sides and incentive compatibility constraint. That is, the firm chooses the wage paid at every history $w_t(h^t)$, and whether to terminate the relationship $I_t(h^t)$, and the executive responds by choosing the effort level $e_t(h^{t-1}) \in \{0, 1\}$.

The executive's action is taking effort or not. Since the effort is not observable, incentives are needed to induce effort. In the following, I assume taking effort $e = 1$ is always optimal. This is true in all the numerical exercises below and is in accordance with the fact that almost all executives in my data are provided with some incentive package.

In allowing for the history of poaching firms to be contractible, countering outside offers are allowed in the contract. Indeed, w_t in the contract depends on the whole history of productivity and outside offers up to t, h^t. In other words, two executives with exactly the same history up to period $t-1$, the same productivity z'_t, but have different outside offers s'_t will get different compensations.

Both sides of the contract have limited commitment. The executive will choose to terminate the relationship if the current firm can not provide more than the outside value, which could be the non-executive job value W^0 or the offer of the poaching firm. The firm will choose to terminate the contract upon the compensation is so high that the firm profit is negative. In the model, given a reasonable support of productivity z such that the value of job is always positive, the only possibility is that when there is a bigger poaching firm. Hence, $I_t(h^t) = 1$ means there is a job-to-job transition and is exogenously determined by the job offer arrival rate and the comparison of (s, s').[12]

In the following, I will use the executive's beginning-of-period expected utility, denoted by V, as a co-state variable to summarize the history of productivities and outside offers. A dynamic contract, defined recursively, is

$$\sigma \equiv \{e(V), w(V), W(z', V) | z' \in \mathbb{Z} \text{ and } V \in \Phi\},$$

where e is the effort level suggested by the contract, w is the compensation, and W is the promised value given for productivity z'.[13]

3.4 Sequential Auction

The optimal contract is between the executive and the firm. In deriving it, the outside offers and its consequence on contracting parties' values should be tracked. So before delving into the contracting problem, I first characterize what happens when there is an outside offer. Here I employ the sequential auction framework (Postel-Vinay and Robin 2002, Cahuc et al. 2006).

Let $\Pi(z, s, V)$ denote the discounted profit of a firm with size s, executive productivity of z, and promised value to the executive V. The maximum the firm would like to give to the executive, the maximum bidding value $\tilde{W}(z, s)$, is defined by

$$\Pi(z, s, \tilde{W}) = 0.$$

The firm would rather destroy the job (or the vacancy value is normalized to 0) if the executive demands a value higher than $\tilde{W}$. It is intuitively clear that given the production function $f(z, s)$ increases in z and s, $\tilde{W}(z, s)$ also increases in z and s. For this reason, I call $\tilde{W}(z, s)$ the *bidding frontier*. $\tilde{W}(z, s)$ increases in s, hence the bigger firm always wins the auction. $\tilde{W}(z, s)$ increases in z, hence there is an incentive effect: if working hard on the current job is not only an input into current production but also an investment in the (inalienable and transferable) human capital, then it is intuitive that the objectives of the firm and the worker become better aligned and the need for short-term compensation incentives decreases.

Specifically, if an executive worked in a firm of size s receives an offer from an outside firm of size s', there are three cases out of the sequential auction.

1. If $s' > s$, then the executive transits to firm s', and his continuation value is the maximum the current firm s can provide, $\tilde{W}(z, s)$. That is, he receives the full

[12] If we allow the domain of z be extended such that for some z the profit is negative even when the firm only offers the non-executive job value W^0, then firing happens. This extension will be added later.

[13] Promised utilities as co-states have been used extensively in models with incentive or participation constraints. Among others, Phelan and Townsend (1991) studied a model of risk-sharing with incentive constraints, Kocherlakota (1996) analyzed the risk-sharing model with the PC described above, Hopenhayn and Nicolini (1997) studied a model of unemployment insurance and Alvarez and Jermann (2000) studied a decentralized version of the above risk-sharing model with debt constraints.

surplus of his former job at firm s. I define the set of such firms as

$$\mathcal{M}_1(z,s,W) \equiv \{s' \in \mathbb{S} | s' > s\}.$$

2. If $s' \leq s$, then the executive stays with his current firm but may use the outside offer to renegotiate up to the frontier of the outside firm $\tilde{W}(z,s')$,

$$\mathcal{M}_2(z,s,W) \equiv \{s' \in \mathbb{S} | \tilde{W}(z,s) > \tilde{W}(z,s'), W < \tilde{W}(z,s')\}.$$

3. In the third case, the outside offer is dominated by the current value and the executive just discards it and continues to work at firm s.

3.5 The Contracting Problem

I can now describe the firm's problem in terms of promised utilities. The firm chooses a wage w, a set of promised values depending on z' which then determines the set of $\mathcal{M}_2$. The expected profit of a match to the firm can be expressed recursively as

$$\begin{aligned}\Pi(z,s,V) = \max_{w,W(z')} \sum_{z'} \Bigg[y(s)z' - w + \beta(1-\delta)\Bigg(\lambda_1 \int_{s' \in \mathcal{M}_2} dF(s')\Pi_1(z',s,\tilde{W}(z',s')) \\ + \Big(1 - \lambda_1 \int_{s' \in \mathcal{M}_1 \cup \mathcal{M}_2} dF(s')\Big)\Pi_1(z',s,W(z'))\Bigg)\Bigg]. \end{aligned} \tag{BE-F}$$

subject to the promise keeping constraint,

$$\begin{aligned} V =& u(w) - c + \beta(1-\delta)\sum_{z'} \Bigg[\lambda_1 \int_{s' \in \mathcal{M}_1} dF(s')\tilde{W}(z',s) + \lambda_1 \int_{s' \in \mathcal{M}_2} dF(s')\tilde{W}(z',s') \\ &+ \Big(1 - \lambda_1 \int_{s' \in \mathcal{M}_1 \cup \mathcal{M}_2} dF(s')\Big) W(z')\Bigg]\Gamma(z,z'), \end{aligned} \tag{PKC}$$

the incentive compatibility constraint,

$$\begin{aligned} &\beta(1-\delta)\sum_{z'} \Bigg[\lambda_1 \int_{s' \in \mathcal{M}_1} dF(s')\tilde{W}(z',s) + \lambda_1 \int_{s' \in \mathcal{M}_2} dF(s')\tilde{W}(z',s') \\ &+ \Big(1 - \lambda_1 \int_{\mathcal{M}_1 \cup \mathcal{M}_2} dF(s')\Big) W(z')\Bigg](1 - g(z,z'))\Gamma(z,z') \geq c. \end{aligned} \tag{IC}$$

and the participation constraints,

$$\tilde{W}(z',s) \geq W(z') \geq W^0(z'). \tag{PC}$$

The firm chooses the current period wage w and the promised utilities $W(z')$ for each state z' tomorrow. These control variables must be chosen to maximize expected returns subject to the promise keeping constraint (PKC), incentive compatibility constraint (IC), and the participation constraints of the firm and the executive (PC).

The promise keeping constraint makes sure that the choices of the firm honors the promise made in previous periods to deliver the value V to the executive. The right hand side of the constraint is the lifetime utility of the executive given the choices made by the firm. (PKC) is also the Bellman equation of an executive with state $\{z, s, V\}$.

The incentive compatibility of the executive differentiates itself from the promise keeping by the term $(1 - g(z, z'))$. It merely says the continue value of taking effort is higher than not taking effort. This create incentives for the manager to pursue the shareholders' interests rather than his own. Since in my data sample, over 99% observations have positive incentive pays, I assume that $e = 1$ is optimal choice of the firm.

Compared to an incentive compatibility in a standard moral hazard problem, (IC) has an incentive part offered by the contract, the *contract incentives*, denoted by Ξ_c,

$$\Xi_c \equiv \beta(1-\delta)\Big(1 - \lambda_1 \int_{\mathcal{M}_1 \cup \mathcal{M}_2} dF(s')\Big) \sum_{z'} W(z')(1 - g(z, z'))\Gamma(z, z'), \tag{1}$$

it also has an incentive part due to the sequential auction in the labor market, the *market-induced incentives*, denoted by Ξ_m

$$\begin{aligned}\Xi_m = \beta(1-\delta)\lambda_1 \times \sum_{z'} \Bigg[& \int_{s' \in \mathcal{M}_1} dF(s')\tilde{W}(z', s) \\ & + \int_{s' \in \mathcal{M}_2} \tilde{W}(z', s') dF(s') \Bigg] (1 - g(z, z'))\Gamma(z, z'). \end{aligned} \tag{2}$$

This opens the channel that market-induced incentives influence the optimal contract of the firm and the executive. Finally, there are limited commitments of the executive and the firm as stated in (PC). Before turning to characterize the optimal contract, I define the equilibrium.

3.6 The Equilibrium

An equilibrium is the executive unemployment value W^0, the value function of employed executives W satisfies (PKC), the profit function of the firms Π and an optimal contract policy $\sigma = \{w, e, W(z')\}$ for $z' \in \mathbb{Z}$ that solves the contracting problem (BE-F) with associated constraints (PKC), (IC) and (PC), the stochastic process of executive productivity Γ follows the optimal effort choice and a distribution of executives across employment states evolving according to flow equations.

The proof of the existence of the equilibrium is an exercise applying Schauder's fixed point theorem and is detailed in Appendix A.

Proposition 1. *The equilibrium exists.*

4 Contract Characterization

Although the dynamic contract problem (BE-F) is complicated by the sequential auction with outside offers, the optimal contract still inherits a neat characterization as in Spear and Srivastava (1987).

Proposition 2. *$\Pi(z,s,W)$ is continuous differentiable, decreasing and concave in W, and increasing in z and s. The optimal contract $\{w(z'), W(z')\}$ satisfies*

$$\begin{aligned}
\frac{\partial \Pi(z,s,W)}{\partial W} &= -\frac{1}{u'(w)}, \\
\frac{\partial \Pi(z',s,W(z'))}{\partial W(z')} - \frac{\partial \Pi(z,s,W)}{\partial W} &= -\mu(1-g(z,z')) + \mu_0 - \mu_1, \\
\frac{1}{u'(w(z'))} - \frac{1}{u'(w(z))} &= -\mu(1-g(z,z')) + \mu_0 - \mu_1, \qquad (3)
\end{aligned}$$

where μ is the Lagrangian multiplier for (IC), μ_0 and μ_1 are the multipliers for (PC).

Proof. See Appendix A. □

Proposition 2 equates the principal's and agent's marginal rates of substitution between present and future compensation. They are of the same form as in Spear and Srivastava (1987). However, since W and Π are different from those in Spear and Srivastava (1987), the solution essentially changes.

An immediate result of proposition 2 is how wage $w(z')$ changes depends on how binding the (IC) constraint is, in particular the multiplier μ. For example, when the utility function takes $u(w) = log(w)$, we have

$$w(z') = w(z) + \mu(1-g(z,z')) + \mu_o - \mu_1.$$

It is clear that when the market-induced incentives are higher for bigger firms, the (IC) constraint will be more binding and it will indeed be optimal to offer more incentive pays.

To compare market-induced incentives across firms, let's consider two executives in firms of size $s^1, s^2 \in [\underline{S}, \bar{S}]$ respectively, and $s^1 < s^2$. Suppose they have the same wage and therefore a similar continuation value V.[14] For outside offer with $s' < s^1$, the market-induced incentives will be the same for two executives, both are induced by $\tilde{W}(z', s')$. For $s' > s^1$, the market incentive in firm s^1 will be induced by $\tilde{W}(z', s^1)$ while in firm s^2 will be $\tilde{W}(z', s')$ with $s^2 > s' > s^1$. For $s' > s^2$, the market incentive in firm s^1 will be induced by $\tilde{W}(z', s^1)$ while in firm s^2 will be $\tilde{W}(z', s^2)$. If the incentives associated with bidding frontier of a bigger firm is lower, that is

$$\sum_{z'} \tilde{W}(z', s^1)(1-g(z,z'))\Gamma(z,z') < \sum_{z'} \tilde{W}(z', s^2)(1-g(z,z'))\Gamma(z,z'),$$

then the market-induced incentives would indeed decrease with firm size. Proposition 3 states the conditions for a CRRA utility function.

Proposition 3 (Contract incentive and Firm size)**.** *Consider $s_1, s_2 \in S$, $s_1 < s_2$, effort cost $c \le \bar{c}$ for some $\bar{c} > 0$, and assume the utility function is of the CRRA form*

$$u(w) = \frac{w^{(1-\sigma)}}{1-\sigma}, \sigma > 0,$$

[14] V will note be the same. Because bigger firm allows more potentials to cope with outside offers, when the two executives are of the same wage, the one in bigger firm will have a higher value. For our illustration, the difference in V is ignored. The numerical practice later shows indeed the difference in V is very small and numerically trivial.

then incentives associated with bidding frontier of a bigger firm is lower if

$$\sigma > 1 + \frac{\log \tilde{F}(s_2) - \log \tilde{F}(s_1)}{\log s_2 - \log s_1},$$

where $\tilde{F}(s) = 1 - \beta(1-\delta)\Big(1 - \lambda_1(1 - F(s))\Big)$.

Proof. See Appendix A. □

Intuitively, with the multiplicative production function, an increase in firm size leads to higher bidding wages for all productivities, and meanwhile, the gap between bidding wages for different productivities is also larger. The former effect increases the certainty equivalence resulting in lower market incentives while the latter effect leads to higher incentives. The former effect dominates as long as the concavity of the utility function exceeds the scale effect of the production function. Roughly speaking, proposition 3 states whenever the utility function has enough concavity, the market-induced incentives decrease with firm size. For a CRRA form, the requirement is that the relative risk aversion slighter higher than 1, which is a very weak requirement in this context. The literature almost always estimate/calibrate a higher σ value. For example, a careful calibration study on CEO's by Hall and Murphy (2000) uses σ between 2 and 3. Using an employer-employee matched data from Sweden for the general labor market, Lamadon (2016) estimates a $\sigma = 1.68$.

4.1 Comparative Statics

Next, I examine the model's predictions about market versus contract incentives and the firm size. These predictions hold for a wide range of plausible parameter values. However, because I solve the model numerically, I do not present these predictions as formal propositions. For these practices, I use a model specification the same as that for the estimation in the next section. In particular, the utility function is assumed as in proposition 3. I simulate the model based on the following parameter values: the discount factor $\beta = 0.9$, job arrival rate $\lambda_1 = 0.16$, effort cost $c = 2.5$, and the relative risk averse $\sigma = 2.75$. These parameter values are close to the empirical estimates in the next section. Then I simulate the model and calculate four statistics as follows.

I measure market-induced incentives directly by Ξ_m as defined in (2). This is precisely how market-induced incentives are defined in the model. I calculate the mean of Ξ_m, and how Ξ_m changes with firm size, i.e. the coefficient $\beta_{\Xi-size}$ in the following regression,

$$\Xi_{mit} = \beta_0 + \beta_{\Xi-size}\log(size_{it}) + \beta_1 \log wage_{it} + \epsilon_{0,it}.$$

I measure contract incentives by the wage-performance sensitivity delta, i.e. the dollar change in wage for a percentage change in productivity. This is consistent with the delta in the data as detailed in table 1 and in the next section. However, delta is only a linear measurement of contract incentives while the optimal contract in my model is nonlinear. That is, for every observation in my simulated dataset, there is a nonlinear optimal contract stating a vector of wages in different state of world. To extract the delta information from a nonlinear contract, I regress the vector of log wages on the vector of productivities, and the coefficient of this regression corresponds exactly to the definition of delta. Again, the regression to define delta is done for each observation. I then calculate the mean of delta, and how delta changes with firm size controlling wage level, i.e. coefficient $\beta_{delta-size}$ in the following regression,

$$delta_{it} = \beta_2 + \beta_{delta-size} \log(size_{it}) + \beta_3 \log wage_{it} + \epsilon_{1,it}.$$

Figure 3 plots means of Ξ_m and delta, coefficient $\beta_{delta-size}$ against λ_1. Not surprisingly, as λ_1 increases, the market becomes less frictional, the competition among firms pushes the compensations and induce higher contract incentives measured by delta. At the same time, a higher λ_1 also brings up the market-induced incentives, so Ξ_m increases. What is interesting is how the two sources of incentives change with firm size. With $\lambda_1 > 0$, delta always increases with firm size, and Ξ_m always decreases with firm size. That is, executives in big firms face more contract incentives and less market-induced incentives. Such pattern is amplified as λ_1 increases.

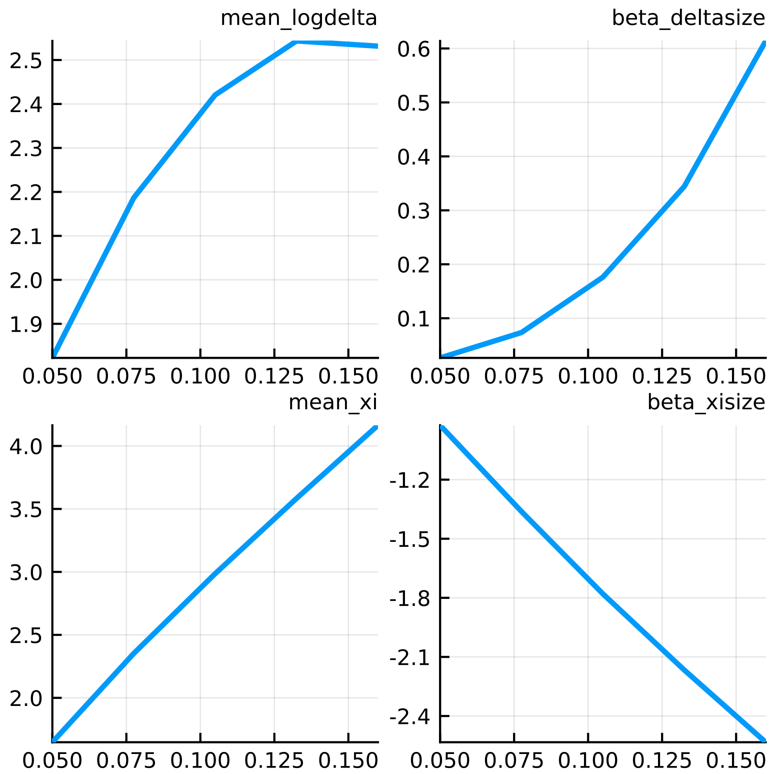

Figure 3: Comparative Statics on λ_1

Figure 4 plots means of Ξ_m and delta, coefficient $\beta_{delta-size}$ against the value of σ. σ can influence both market-induced incentives and contract incentives. On the one hand, a higher σ implies a more concave utility function, thus less variation of wages can generate the same incentive. We observe that delta decreases with σ. On the other hand, the same variation of the bidding frontier is translated into larger market-induced incentives with a higher σ value, making Ξ_m larger. σ also amplifies the differences of market-induced incentives across firm size, therefore, $\beta_{delta-size}$ increases with it.

Finally, the effort cost c only influences the contract incentives. Therefore, the mean of delta increases with firm size while the mean of Ξ_m and β_{Ξ_m-size} are constant. Yet, a higher c implies a relative lower fraction of market-induced incentives compared to c, we thus observe $\beta_{delta-size}$ increases in c. This figure is shown in the appendix.

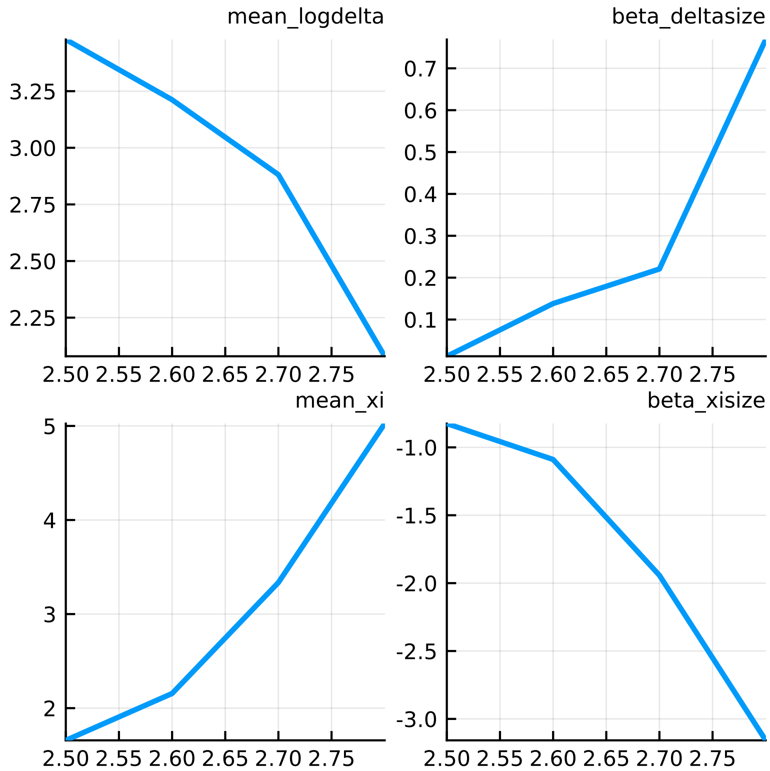

Figure 4: Comparative Statics on σ

5 Estimation

In this section, I bring the model to the data. I first describe how I construct my dataset using information from two databases, and some direct evidence of executives' job-to-job transitions. Then I list model specifications, moments used for estimation and the estimation results. I conclude this section with a comparison of delta in the real data and model-simulated data.

5.1 Data Sources

Data come from ExecuComp (1992 to 2015); Compustat; BoardEX; Coles et al. (2006) and Coles et al. (2013). ExecuComp and Compustat is the standard data source in this literature. They provide rich information on compensations of top five to eight executives in companies included in the S&P 500, S&P 400 MidCap and S&P SmallCap 600 indexes and comprehensive measures of company financial performance. However, this data source does not contain any information of prior employment history. The start and end dates of the current employment ares also not known.

To keep track of the transitions among employment status, I supplement ExecuComp with information from the database BoardEX. The latter contains details of each executive's employment history, including starting and ending dates, firm names and positions. It also provides extra information on education background, social networks, and more accurate records of age. There is no index to directly link ExecuComp and BoardEX. I construct my sample by firstly matching the executives' first, middle and last names, the date of birth and working experiences (in which year the executive worked in which firms). When the three items are consistent between the observations from the two data sources, the executives are identified as the same person. By this way, I am able to identify 33,988 executives, roughly 2/3 of the whole ExecuComp sample (from year 1992 to year 2015). The table in the appendix compares the basic descriptive statistics of identified and unidentified executives. There is no obvious sample selection due to my data processing. More details about how I construct the dataset are left in the appendix.

Finally, the data provided by Coles et al. (2006) and Coles et al. (2013) include two important variables on contract incentives: firm related wealth and delta. The former is the dollar value of the executive's stock and option portfolio (in \$000*s*), and the latter is the Dollar change in wealth associated with a 1% change in the firms stock price (in \$000*s*).

5.2 Data Description

[A subsection of variables and summary statistics to be added.]

5.3 Some direct evidence on executives' job-to-job transitions

In this section, I will establish several evidence on executives' job-to-job transitions. First, to establish market-induced incentives, we must observe there are job-to-job transitions. In my dataset, I observe stable job-to-job transitions, ranges from 3% to 8% from year to year. In particular, it becomes higher after the financial crisis.

Second, my results rely on a job ladder where executives move from small to big firms. Among the job-to-job transitions that I can observe the size of both the original firm and the target firm, there are significantly larger fraction of executives move from small to bigger firms, as shown in table 2. Among those who does move to smaller firms, I find half of those cases are due to a title change from some non-CEO title to CEO title.[15] The pattern that executives are more likely to transit to bigger firms is also observed across age-groups (table 2) and industries (table 3).

Table 2: Firm Size Changes in Job-to-Job transitions (across age groups)

Age Group	Firm Size Decrease	Firm Size Increase	All
[26, 40)	33	61	94
[40, 50)	259	448	707
[50, 60)	236	394	630
[60, 65)	29	51	80
[65, 70)	8	23	31
[70, 86)	1	4	5
All	681	1229	1910

The third fact is about the search frictions. One of the assumption I have made in the model is that executives in all firms face the same job arrival rate. If the assumption is reasonable, then the observed job-to-job transition rate should decrease with firm size because small firm executives are more likely to accept an offer. To test this, I estimate a Cox model on the duration to job-to-job transitions, controlling for profitability, age,

[15] I find 6253 job-to-job transitions from a CompuStat firm, and only 1910 of them that have the size information of original and target firms. The rest firms are private firms and the size information is not disclosed.

Table 3: Firm Size Changes in Job-to-Job Transitions (across Fama & French 12 industries)

Industry	Firm Size Decrease	Firm Size Increase	All
1	12	32	44
2	7	24	31
3	37	59	96
4	13	13	26
5	8	11	19
6	73	104	177
7	4	12	16
8	15	29	44
9	46	111	157
10	27	43	70
11	38	69	107
12	50	52	102

education dummies, executive title dummies, year and industry dummies. Job-to-job transition is defined as the executive leaves the current firm and works in another firm within 30 days. I also construct alternative job-to-job transitions using 15, 60, 90, and 190 days. The result is shown in table 4 column (1). For one percentage increase in firm size, the job-to-job transition hazard rate decreases by 0.04. I add the interaction terms between firm size and age dummies to test if the career concern would decrease as the executive approaches the retire age. The effect is significant as is shown in column (2). The negative correlation between job-to-job transition hazard rate and firm size monotonically decreases with age. I do the same test for alternative job-to-job transition measurement with different day gaps. The result is very robust.

Table 4: Firm Size and EE (30 days)

	(1)	(2)
$\log(mkcap)$	-0.0416***	-0.0672***
$[30, 40) \times \log(mkcap)$		-0.0792
$[40, 45) \times \log(mkcap)$		-0.00102
$[45, 50) \times \log(mkcap)$		0.00949
$[50, 55) \times \log(mkcap)$		0
$[55, 60) \times \log(mkcap)$		0.123***
$[60, 65) \times \log(mkcap)$		0.180***
$[65, 80] \times \log(mkcap)$		0.118
profit	-0.124****	-0.154***
N	326919	326919

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, **** $p < 0.001$

5.4 Numerical Algorithm, Model Specification and Parameters

I estimate the model's parameters using Simulated Methods of Moments. That is, I use a set of moments that are informative for the model's parameters and minimize the distance between data moments and model-generated moments. My moments are partly coefficients from auxiliary regressions, so the approach could alternatively be presented as Indirect Inference.

The main difficulty resides in solving the contracting problem which requires finding the promised utilities $W(z', s')$ in each state of the world for the next period. This becomes infeasible as soon as reasonable supports are considered for $\mathbb{Z}$ and $\mathbb{S}$. However, the first order condition with respect to W reveals that the promised utility in different states are linked to each other and that it is optimal for the firm to promise identical marginal utilities across sates tomorrow as much as possible. The solution is then characterized by how the promised marginal utility evolve over time subject to multipliers of (IC) and (PC. In Appendix B, I use the recursive Lagrangian approach to solve the contract numerically.

I estimated the model fully parametrically and make several parametric assumptions. Being consistent with the analysis before, I use the constant relative risk aversion utility function $u(w) = \frac{w^{1-\sigma}}{1-\sigma}$. The production function is set as $f(z, s) = \alpha s z$ where α is set to be 0.1%. I model the process of productivity by an $AR(1)$ model,

$$z_t = \rho_0(e) + \rho z_{t-1} + \epsilon_t,$$

where ϵ follows a normal distribution $N(0, \sigma_\epsilon)$, and the mean for effort level $e = 0$ is normalized to zero. The process is transformed to a discrete Markovian Chain using Tauchen (1986) on a grid of 5 points.[16] Furthermore, I set the sampling distribution of firm size $F(s)$ a log-normal distribution with expectation of μ_s and standard deviation of σ_s. Finally, the discount rate β is set to be 0.9 (the model is solved annually), the grid number for Lagrangian multiplier λ is 30, and for firm size s is 20.

Table 5: Parameters

Parameters	Description
δ	death probability
λ_1	offer arrival probability when employed
ρ_z	AR(1) coefficient of productivity shock
μ_z^w	mean of disturbance for $e = 1$
σ_ϵ	standard deviation of disturbance
μ_s	mean of F(s)
σ_s	standard deviation of F(s)
c	cost of efforts
σ	relative risk aversion

5.5 Moments and Identifications

Table 5 lists the complete set of parameters I estimate. I next make a heuristic identification argument that justifies the choice of moments used in the estimation. Appendix gives more details on how I construct these moments. The identification can be divided into three parts.

[16] A grid of only 5 points is for the speed of calculation. However, the simulated moments are very robust to this choice.

The first part of the identification is on the exogenous processes, including the executives' productivities, the exit rate and the offer arrival rate. There are direct links between the model and the data. The exit rate directly informs δ. Likewise, the incidence of job-to-job transitions is monotonically related to λ_1. The parameters of the productivity process, ρ_z, μ_z^w and σ_ϵ, are informed directly by the estimates of an AR(1) process on the profitability of each firm-executive match,

$$\texttt{profit}_{it} = \beta_0 + \rho_z \texttt{profit}_{it-1} + \epsilon_{it,0},$$

where i represents the executive-firm match and t represents the year.

The second part of the identification is on the job offer distribution. In particular, μ_s and σ_s are disciplined by the mean of log firm size and the mean of log wage. Given $\lambda_1 > 0$, the higher μ_s is, the more likely that executives can transit to larger firms, and the higher firms can compete for them. Hence, both $\log(wage)$ and $\log(size)$ are higher. σ_s controls the variation of job offers and has a similar effect as μ_s. The shape of job offer distribution can be further disciplined by the relationship between wage and firm size. I regress log wage on log of firm size, controlling for age dummies (divided into 100 groups),

$$\log(wage_{it}) = \beta_1 + \beta_{wage-size} \log(size_{it}) + \text{age dummies} + \epsilon_{it,1}.$$

The coefficient $\beta_{wage-size}$ is included as a moment to be matched.

The final part of the identification concerns the moral hazard problem, in particular two parameters σ and c. To be consistent with the incentive measurement delta in the data, I construct in the simulated data a delta variable defined by the dollar change in wage for a percentage change in productivity. How large delta is, how delta changes with firm size and wage can be used to inform σ and c. I use four moments. The mean of log delta is the first moment. It measures how large the contract incentives are. The next two moments are coefficients in the following two regressions. The first regression is delta on firm size, controlling for wage dummies,

$$\log(delta_{it}) = \beta_2 + \beta_{delta-size} \log(size_{it}) + \text{wage dummies} + \epsilon_{it,2},$$

the second regression is delta on wage (total compensation), controlling for firm size,

$$\log(delta_{it}) = \beta_3 + \beta_{delta-wage} \log(wage_{it}) + \beta_4 \log(size_{it}) + \epsilon_{it,3}.$$

The moments used are coefficient $\beta_{delta-size}$, which informs how the contract incentives change with firm size, and coefficient $\beta_{delta-wage}$, which informs how the contract incentives change with wage. The last moment I use for this part of identification is the fraction of positive delta, which informs the possibility of corner solutions (i.e. flat wage contract).

How do these four moments identify σ and c? First, both σ and c positively influence the mean of log delta. σ impacts delta via both the market-induced incentives and contract incentives. On the one hand, the larger σ is, the more concave the utility function, the higher delta should be. On the other hand, the larger σ is, the smaller market-induced incentives are, and the higher delta should be. Hence, σ and delta are positive correlated. c and delta are also positive correlated, although c only works through the contract incentives: the higher c is, the larger contract incentives are required to satisfy the incentive compatibility constraint. Yet, σ and c have different influence on how delta changes with firm size. Given other parameter values, a higher σ implies higher market-induced incentives hence a larger $\beta_{delta-size}$. A higher c does not change market-induced incentives while making the contract incentives larger, and hence gives a smaller $\beta_{delta-size}$. This difference distinguishes the two parameters. Finally, in the model there are parameter values such that the market-induced incentives are large enough to fulfill the incentive compatibility constraint and therefore delivers $delta = 0$. The fraction of positive delta helps to avoid too many of such corner solution . In all the regressions mentioned above, I also controlled for industry and year dummies for real data.

5.6 Results

Table 6 reports the targeted values of moments in the data and the corresponding values in the estimated model. The last two columns lists the parameter estimates and the standard errors. While I arranged moments and parameters along the identification argument made in the previous subsection, all parameters are estimated jointly.

Table 6: Moments and Estimates

Moments	Target	Model	Estimates	Standard Error
Exit Rate	0.0691	0.0691	$\delta = 0.0691$	0.0012
EE Rate	0.0523	0.055	$\lambda_1 = 0.2759$	0.0017
$\hat{\rho_z}$	0.8111	0.5499	$\rho_z = 0.7$	0.0036
Mean(z)	0.1284	0.1763	$\mu_z^w = 0.06$	0.0006
Var(z)	0.0141	0.0141	$\sigma_z = 0.12$	0.0014
Mean(log(wage))	7.17714	6.5241	$\mu_s = 1.7847$	0.228385
Mean(log(size))	7.44379	8.7934	$\sigma_s = 1.3982$	0.0314657
$\beta_{wage-size}$	0.370295	0.3196		
Mean(log(delta))	4.01842	3.8080		
$\beta_{delta-size}$	0.297673	0.2941	$c = 1.91385$	0.0259
$\beta_{delta-wage}$	0.717209	2.1228	$\sigma = 2.50748$	0.0046
Mean($delta > 0$)	0.994725	0.9844		

Overall, the model provides a decent fit to the data. In particular, it quantitatively captures the negative correlation between delta (the contract incentives) and firm size, which is the key target of the model. For this, a job arrival rate $\lambda_1 = 0.2759$ is required which also matches the job-to-job transition rate in the data. The magnitude of λ_1 indicates that, on average, the executive will receive an outside offer every three years, which seems reasonable. The levels of $\log(wage)$, $\log(size)$ and coefficient $\beta_{wage-size}$ are matched reasonably well, showing the on-the-job search and sequential auction in the model capture the main features of the executive labor market. The magnitude of μ_s and σ_s indicates that most offers are provided by relative small firms though there are high variations in firm size.

While the dynamic contract can provide a reasonable predict on the mean of $\log(delta)$, it misses the target of $\beta_{delta-wage}$. A further inspection of the simulated data and the real world data shows that in the real data, there is a large number of observations which have a small `tdc1` and a high delta, while in the simulated data, a low wage is usually associated with a small delta. Hence, there are some heterogeneity of executive-firm match that are not captured in the model. Finally, the exogenous processes on productivity and outside offers are matched quite well.

5.7 Compare Data and Model-Simulated delta

I conclude this section by providing some visual comparisons of delta from the dataset and the simulated data based on the estimates. The comparison are along two dimensions: wage and firm size. In both the real data and simulated data, I create a variable wage group which divides the sample into 100 groups accroding to the value of wage. Similarly, I create a variable size group based on the firm size. In this way, data is segmented into 100×100 cells. I calculate the average $\log(delta)$ of each cell and exclude those cells with less than 10 observations.

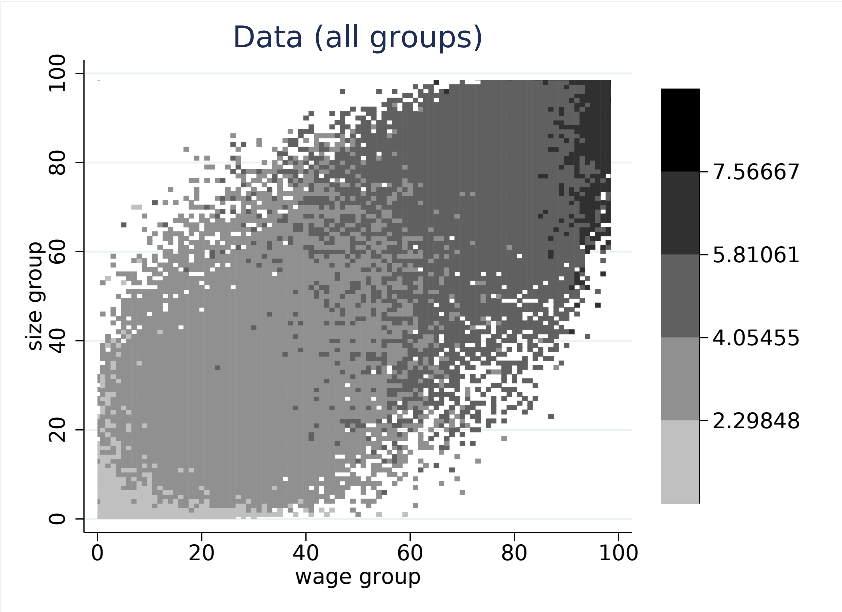

Figure 5: $\log(delta)$ in Data

Figure 5 shows $\log(delta)$ from the real data, figure 6 shows $\log(delta)$ of the simulated data, and figure 7 shows $\log(delta)$ of the simulated data when market-induced incentives are ignored.[17] It is clear that in all figures, delta increases with wage in a similar manner. The upper-left corner represents executives in big firms with low total compensations. This part is missing in the real data because only top 5 to 8 executives within the firm are included in the data. Importantly, comparing figures 6 and 7, we find that the simulated data with market-induced incentives have some variations along the size dimension within each wage group while in the model when market-induced incentives are ignored, delta only changes with wage.

How delta changes with size is not so clear in these overview plots. So I zoom into wage groups: wage group 15 to 20, 50 to 55, 80 to 85. They are shown in figure 8, 9, and 10, respectively. In each figure the plots from up to down are $\log(delta)$ from data, simulated data with market-induced incentives, and simulated data without market-induced incentives. As in the real data, the simulated data with market-induced incentives has

[17]Figure 7 is plotted by solving a model where in the contract problem market-induced incentives are removed from the objective and all the constraints.

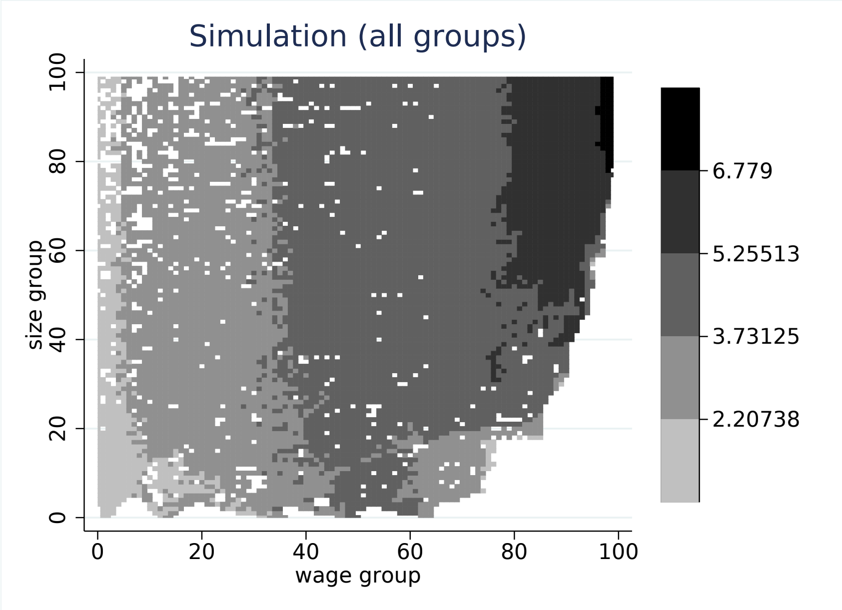

Figure 6: $\log(delta)$ in Simulated Data (with market-induced incentives)

enough variation of delta along firm size within each wage group. In contrast, in the simulated data ignoring market-induced incentives, there is little variation along firm size within each wage group, though the variation along wage group exists. Moreover, it seems the model (with market-induced incentives) can fit the data better for low and medium wage levels. There are still large variations along the firm size for high wages in the data, while the model does not have enough market-induced incentives to generate that large variations.

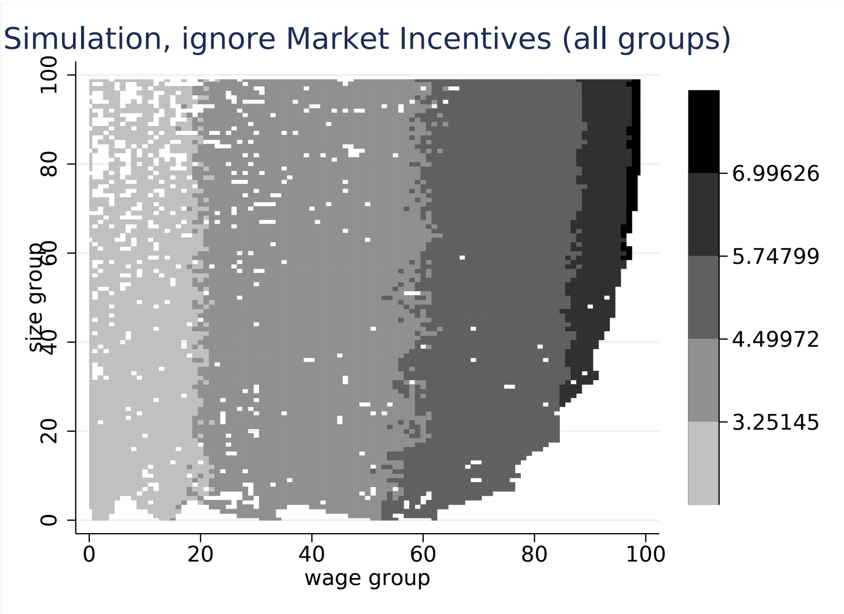

Figure 7: log($delta$) in Simulated Data (without market-induced incentives)

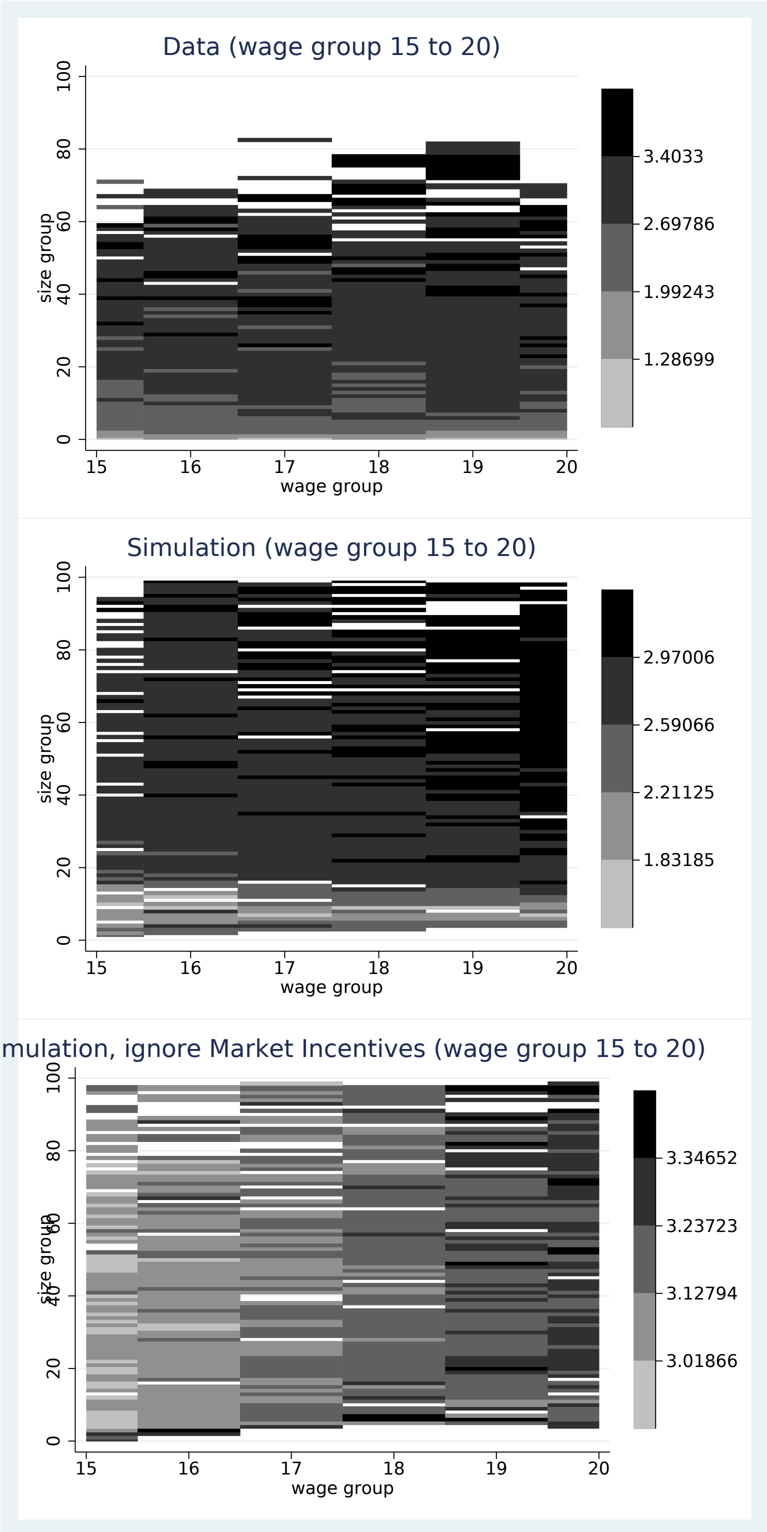

Figure 8: $\log(delta)$ in wage group 15 to 20

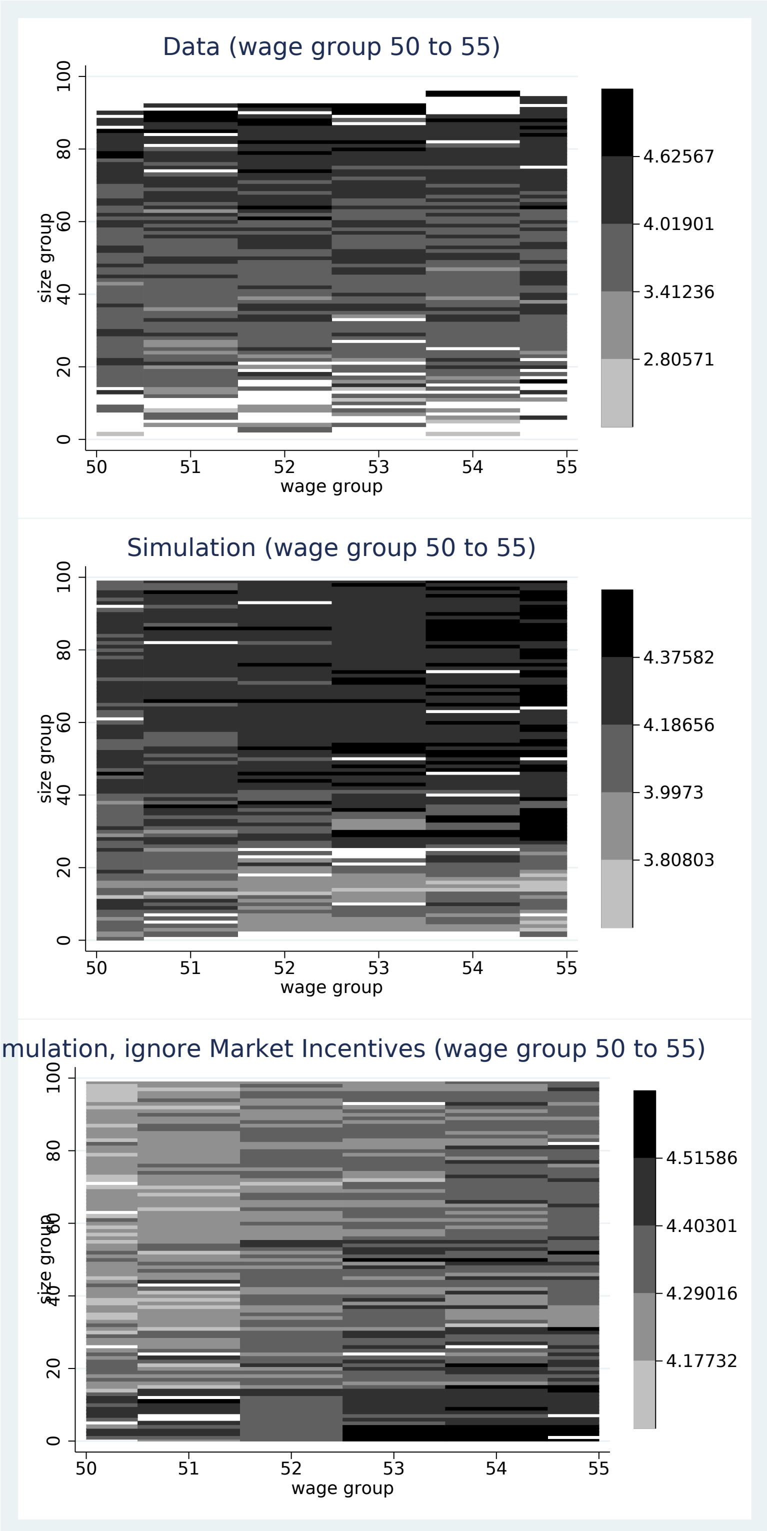

Figure 9: $\log(delta)$ in wage group 50 to 55

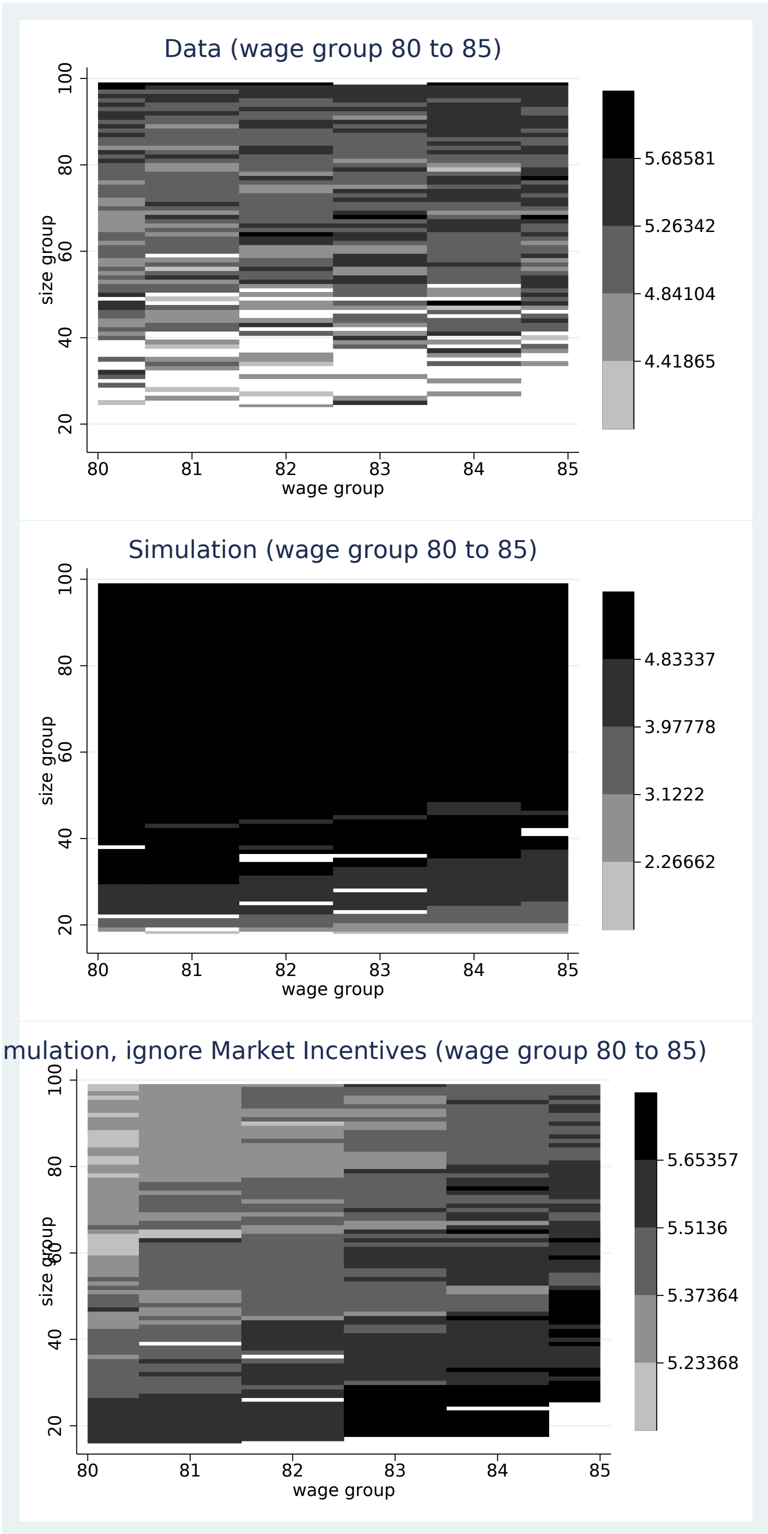

Figure 10: $\log(delta)$ in wage group 80 to 85

6 Quantitative Analysis

6.1 Decomposition

In this section, I do two decompositions. In terms of utilities, I decompose the total incentives into market-induced incentives and contract incentives. In terms of incentive pays delta, I simulate a counterfactual scenario where executives ignore market-induced incentives at all, and then compare the delta with and without market-induced incentives. I show that excluding market-induced incentives will greatly increase the incentive pays in small firms.

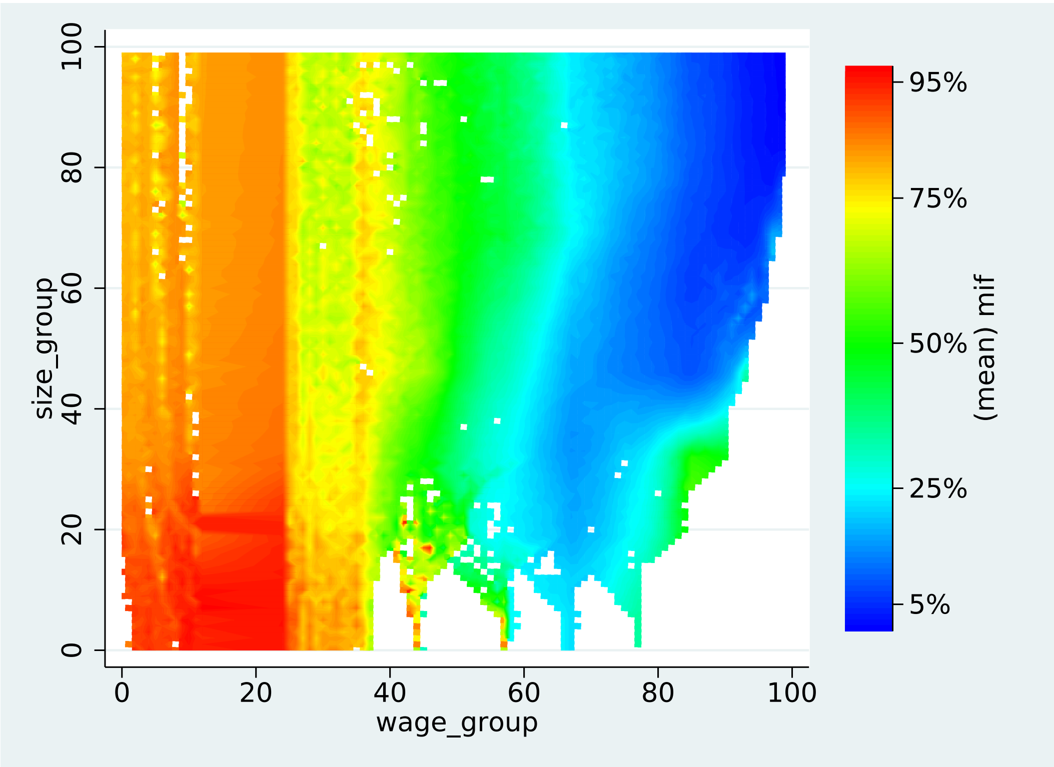

Figure 11: The Fraction of Market Incentives along Firm Size and Wage

Figure 11 plots the fraction of market-induced incentives Ξ_m in total incentives $\Xi_c + \Xi_m$ along the dimensions of firm size and wage. The simulation uses the estimated parameter values. Again, in the simulated data I divide the firm size into 100 groups, divide wage levels into 100 groups, compute the mean of market-induced incentives fraction in each cell and then draw a heat map over the 100×100 cells. The color in the figure represents the value, with red as higher and blue as lower. The blank space indicates a cell with less than 10 observations.

There are two observations. First, market-induced incentives decrease in wage. This is because as wage increases, there are less outside offers that can improve upon the current value. This holds for executives in all firms, though executives in small firms are not likely to get high wages (which gives the blank space in the right-lower corner). Second, within a wage group, the fraction of market-induced incentives decreases in firm size. This is exactly the reason that my model can explain that bigger firms have to provide more contract incentives.

Next, I simulate a counterfactual scenario where market-induced incentives are removed from the contracting problem. The delta simulated from this model is denoted by $delta_c$. This is the delta if all incentives have to be provided by the contract. I then calculate the ratio of $\log(delta)/\log(delta_c)$ and plot it in figure 12. This ratio represents the contribution of the market-induced incentives in $delta$. It shows that almost for each wage group, for the first 30 size groups (out of 100 groups), when market-induced incentives are included, the value of $\log(delta)$ is about 50% to 70% of the $\log(delta_c)$. For size group 30 to 50, the ratio is around 90%. The implication is, if there were no market-induced incentives, small firms would have to pay much higher contract incentives, while the impact on medium and large firms is much smaller.

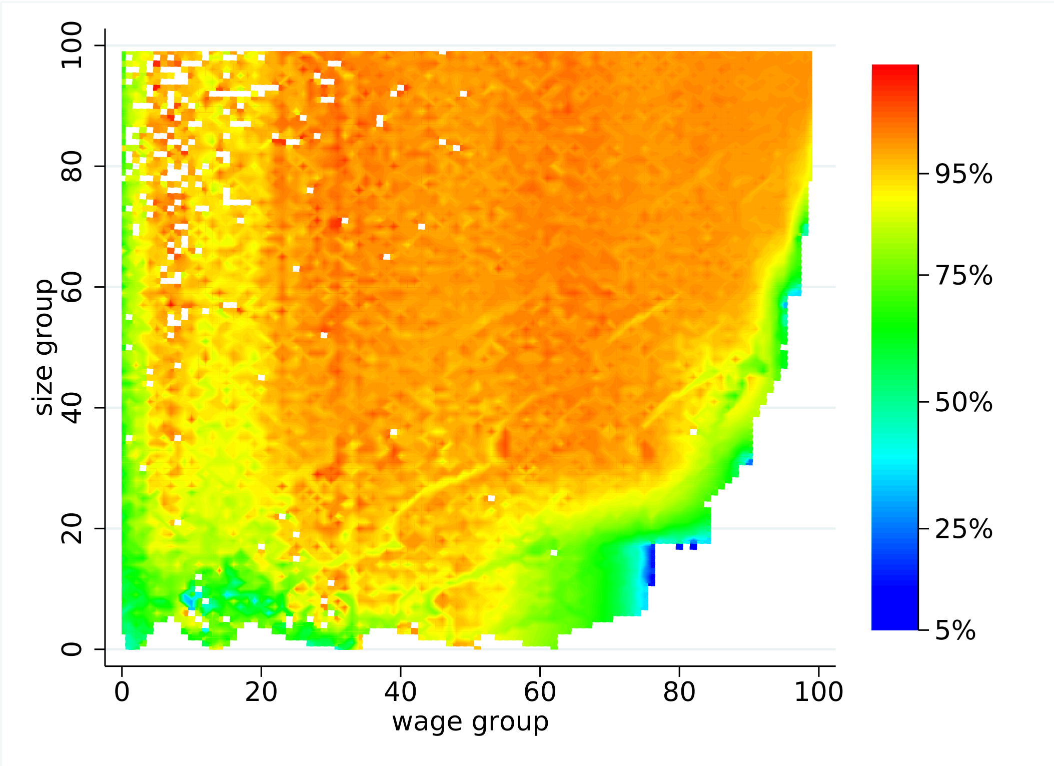

Figure 12: The Ratio of log Delta with and without market-induced incentives along Firm Size and Wage

6.2 Counter-factual analysis

I now use the model to evaluate several specific regulations on executive pays that have been proposed or implemented, including pay ratio restrictions, restrictions on incentive pays.

[to be added]

Since the model involves the incentive problem within each firm, and also connects to the market competition, I explore the contagion effect of good/bad governance. In the model, there are two parameters indicating the quality of governance: effort cost c represents how severe the principle-agent problem is in the firm, and α in the production function represents firm's willingness to pay for the executive, loosely corresponds to the entrenchment.

[to be added]

7 Conclusions

By the law of diminishing utility, wealthy agents are costly to motivate. In a dynamic relationship, the wealth is traced from a life-time utility. Executives in bigger firms are expected to be wealthier, less sensitive to monetary rewards in the future than their equally paid counterparts in smaller firms. Thus, market-induced incentives are lower in bigger firms. This implies higher contract incentive pays are required from big firms. With this intuition, this paper provides a framework for empirically analyzing market-induced and contract incentives in a frictional labor market. I use this framework to investigate why bigger firms pay more for performance. I show that in the counter-factual that when market-induced incentives are ignored, the incentive pays would be much higher in small and medium firms.

Appendix A. Proofs

[to be added]

Appendix B. Computing Algorithm

To further characterize the optimal solution, we resort to the tools developed by Marcet and Marimon (2017, hereafter MM).[18] In dynamic contracting problems with forward looking constraints such as the IC constraint here, the solution does not satisfy the Bellman equation. MM suggest to study a recursive Lagrangian. Under standard general conditions there is a recursive saddle-point functional equation (analogous to a Bellman equation) that characterizes a recursive solution to the planners problem. The recursive formulation is obtained after adding a co-state variable λ_t summarizing previous commitments reflected in past Lagrange multipliers. The time-consistent continuation solution is obtained by using the endogenous λ_t as the vector of weights in the objective function.

Proposition 4 (Marcet and Marimon)**.** *Define Pareto Frontier by*

$$P(z,s,\lambda) = \sup_W \Pi(z,s,W) + \lambda W,$$

where Π and W are defined as in (BE-F) and (PC), and $\lambda > 0$ is a Pareto weight assigned to the executive. Then there exist positive multipliers of $\{\mu, \mu_0(z'), \mu_1(z')\}$ that solve the following problem

$$P(z,s,\lambda) = \inf_{\mu,\mu_0(z'),\mu_1(z')} \sup_w h(z,s,\lambda,w) + \hat{\beta} \sum_{z'} P(z',s,\lambda')\Gamma(z,z'),$$

where multiplier μ corresponds to the incentive compatibility constraint, multipliers $\mu_0(z'), \mu_1(z')$ correspond to participation constraints,

$$h(z,s,\lambda,w) = y(s)z' - w + \lambda u(w) - (\lambda+\mu)c,$$

Pareto weight evolves according to

$$\lambda' = \lambda + \mu(1 - g(z,z')) + \mu_0(z') + \mu_1(z'),$$

and

$$\hat{\beta} = \beta(1-\delta)(1 - \lambda_1 \sum_{\mathcal{M}_1 \cup \mathcal{M}_2} F(s')).$$

The optimal contract $\{w, W(z')\}$ follows that

$$u'(w) = \frac{1}{\lambda}, \tag{4}$$

$$W(z') = W(z',s,\lambda'). \tag{5}$$

[18] This approach has been used in many applications. A few examples are: growth and business cycles with possible default (Marcet and Marimon (1992), Kehoe and Perri (2002), Cooley, et al. (2004)); social insurance (Attanasio and Rios-Rull (2000)); optimal fiscal and monetary policy design with incomplete markets (Aiyagari, Marcet, Sargent and Seppala (2002), Svensson and Williams (2008)); and political-economy models (Acemoglu, Golosov and Tsyvinskii (2011)).

Proof. See Appendix A. □

Proposition 4 can be illustrated intuitively using the Pareto weight of the executive λ and the multiplier μ of the incentive constraint. Suppose the match starts with a $\lambda^{(0)}$, and assume the participation constraints are not binding so that $\mu_0 = \mu_1 = 0$. $\lambda^{(0)}$ has to satisfy $W(z_O, s, \lambda^{(0)}) = W^0$. To deal with the moral hazard, the optimal contract indicates a $\mu^{(0)}$. Then depending on the realization of z', the weight of the executive will be updated to

$$\lambda^{(i)} = \lambda^{(i-1)} + \mu^{i-1}(1 - g(z, z')) \text{ for } i \text{ in } 1, 2...$$

The evolve of λ continues as such till the match breaks. When there is an outside offer such that the executive moves from his current firm to the outside firm, then the new match starts with a $\lambda^{(n)}$ such that $W(z, s', \lambda^{(n)}) = \tilde{W}(z, s)$, where I have denoted the current productivity by z, current firm by s, and the outside firm by s'. It means the new match will assign a new weight to the executive so that he gets the continuation value $\tilde{W}(z, s)$. Then the new Pareto weight will evolve again as illustrated above. In a nutshell, proposition 4 allows us to solve the optimal contract in the space of Pareto weight λ instead of in the space of the promised utility. At any moment, we can transfer from the metrics of λ back to the metrics of utilities using (4) and (5).

The advantage of this method is I do not need to find the promised utilities $W(z')$ in each state of the world for the next period. Instead, λ and μ are enough to trace all $W(z')$. Moreover, λ corresponds to the total compensation level (wage level), while μ corresponds to how much contract incentive is provided in the optimal contract. The two multipliers are enough to understand both theoretically and numerically why keeping the same wage level (the same λ), incentive pays increase with firm size (μ increases with firm size).

[Incomplete Reference]

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