Marketmaking Middlemen*

Pieter Gautier † Bo Hu‡ Makoto Watanabe§

† ‡ § VU Amsterdam, Tinbergen Institute

June 26, 2019

Abstract

This paper develops a model in which market structure is determined endogenously by the choice of intermediation mode. There are two representative modes of intermediation that are widely used in real-life markets: one is a middleman mode where an intermediary holds inventories which he stocks from the wholesale market for the purpose of reselling to buyers; the other is a market-making mode where an intermediary offers a platform for buyers and sellers to trade with each other. We show that a *marketmaking middleman*, who adopts a mixture of these two intermediation modes, can emerge in a directed search equilibrium and discuss the implications of this on the market structure. Our main insight survives under competing intermediaries.

Keywords: Middlemen, Marketmakers, Platform, Directed Search

JEL Classification Number:D4, G2, L1, L8, R1

*We thank seminar and conference participants at U Essex, U Bern, U Zurich, Goergetown U, Albany, the Symposium on Jean Tirole 2014, the Search and Matching workshop in Bristol, SaM network annual conference 2015/2016 in Aix-en-Provence/Amsterdam, Toulouse School of Economics, Tokyo, Rome, the 2015 IIOC meeting in Boston, the EARIE 2015 in Munich, the 16th CEPR/JIE conference on Applied Industrial Organization, Workshop of the Economics of Platform in Tinbergen Institute, and the 2016 Summer Workshop on Money, Banking, Payments and Finance in Chicago FED for useful comments.

†VU Amsterdam, Tinbergen Institute, Address: Department of Economics, VU Amsterdam, De Boelelaan 1105, NL-1081 HV Amsterdam, The Netherlands, p.a.gautier@vu.nl

‡VU Amsterdam, Tinbergen Institute, Address: Department of Economics, VU Amsterdam, De Boelelaan 1105, NL-1081 HV Amsterdam, The Netherlands, huboshufe@gmail.com

§VU Amsterdam, Tinbergen Institute, Address: Department of Economics, VU Amsterdam, De Boelelaan 1105, NL-1081 HV Amsterdam, The Netherlands, makoto.wtnb@gmail.com

1 Introduction

This paper develops a framework in which market structure is determined by the intermediation service offered to customers. There are two representative modes of intermediation that are widely used in real-life markets. In one mode, an intermediary acts as a *middleman* (or a *merchant*), who is specialized in buying and selling for his own account and typically operates with inventory holdings (e.g. supermarkets, traditional brick and mortar retailers, and dealers in financial and steel markets). In the other mode, an intermediary acts as a *marketmaker*, who offers a marketplace for fees, where the participating buyers and sellers can search and trade with each other and at least one side of the market pays a fee for using the platform (e.g. auction sites, brokers in goods or financial markets, and many real estate agencies).

The market-making mode became more appropriate since new advanced internet technology facilitated the use of online platforms in the late 1990s and early 2000s. In financial markets, an expanded platform sector is adopted in a specialist market, i.e., the New York Stock Exchange (NYSE),[1] and even in a typical dealers' (i.e., middlemen's) market, i.e the NASDAQ. In goods and service markets, the electronic retailer Amazon.com and the online hotel/travel reservation agency Expedia.com, who started as a pure middleman, but now also act as a marketmaker, by allowing other suppliers to participate on their platform as independent sellers. In housing markets, some entrepreneurs run a dealer company (developing and owning luxury apartments and residential towers) and a brokerage company simultaneously in the same market.

Common to all the above examples is that intermediaries operate both as a middleman and a marketmaker at the same time. This is what we call a *marketmaking middleman*. Hence, the first puzzle is to explain the emergence of marketmaking middlemen, i.e., why the middleman or the platform sector has not become the exclusive avenue of trade, despite the recent technological advancements.

We also observe considerable differences in the microstructure of trade in these markets. The NASDAQ is still a more 'middlemen-based' market relative to the NYSE. While some intermediaries in housing markets are marketmaking middlemen, many intermediaries are brokers. Other online intermediaries, such as eBay and Booking.com, are pure marketmakers, who do not buy

[1] In the finance literature, the following terminologies are used to classify intermediaries: brokers refer to intermediaries who do not trade for their own accounts, but act merely as conduits for customer orders, akin to our marketmakers; dealers refer to intermediaries who do trade for their own accounts, akin to our middlemen/merchants. The marketmakers (or specialists) in financial markets quote prices to buy or sell assets as well as take market positions, so they may correspond broadly to our market-making middlemen.

and sell on their own accounts, like Amazon.com and Expedia.com do. They solely concentrate on their platform business. So the second puzzle is to explain what determines the position of an intermediary's optimal mode in the spectrum spanning from a pure marketmaker mode to a pure middleman mode.

We consider a model in which the intermediated-market structure is determined endogenously as a result of the strategic choice of a monopolistic intermediary. In our model, there are two markets open to agents, one is an intermediated market operated by the intermediary, and the other is a decentralized market where buyers and sellers search individually. The intermediated market combines two business modes: as a middleman, the intermediary is prepared to serve many buyers at a time by holding inventories; as a marketmaker, the intermediary offers a platform and receives fees. The intermediary can choose how to allocate the attending buyers among these two business modes.

We formulate the intermediated market as a directed search market in order to feature the intermediary's technology of spreading price and capacity information efficiently – using the search function offered in the NYSE Arca or Expedia/Amazon website or in the web-based platform for house hunters. For example, one can receive instantly all relevant information such as prices, the terms of trade and stocks of individual sellers. In this setting, each individual seller is subject to an inventory capacity of discrete units (normalized to one unit in the model), whereas the middleman is subject to an inventory capacity of a mass K. Naturally, the middleman is more efficient in matching demands with supplies in a directed search equilibrium. The decentralized market represents an individual seller's outside option that determines the lower bound of his market utility.

With this set up, we consider two situations, *single-market search* versus *multiple-market search*. Under single market search, agents have to choose which market to search in advance, either the decentralized market or the intermediated market. This implies that the intermediary needs to subsidize buyers with their expected value in the decentralized market, but once they participate, the intermediated market operates without fear for outside competitive pressure. Given that the middleman mode is more efficient in realizing transactions, the intermediary uses the middleman-mode exclusively when agents search in a single market.

When agents are allowed to search in multiple markets, attracting buyers becomes less costly compared to the single-market search case — the intermediary does not need to subsidize buyers to induce participation. However, the prices/fees charged in the intermediated market must be acceptable relative to the available option in the decentralized market. Otherwise, buyers and sellers can easily switch to the outside market. Thus, under multiple-market search, the outside option creates competitive pressure to the overall intermediated market. In deciding the optimal intermediation mode, the intermediary takes into account that a higher middleman capacity induces more buyers to buy from the middleman, and fewer buyers to search on the platform. This has two opposing effects on its profits. On the one hand, a higher capacity of the middleman leads to more transactions in the intermediated market, and consequently to larger profits. On the other hand, sellers are less likely to trade on a smaller-scaled platform and buyers are more likely to trade with a larger scaled middleman, so that more sellers are available when a buyer attempts to search in the decentralized market. Accordingly, buyers expect a higher value from the less tight decentralized market. This causes cross-markets feedback that leads to competitive pressure on the price/fees that the intermediary can charge, and a downward pressure on its profits. Hence, the intermediary trade-offs a larger quantity against lower price/fees to operate as a larger-scaled middleman. This trade-off determines the middleman's selling capacity and eventually the intermediation mode.

Single-market search may correspond to the traditional search technology for local supermarkets or brick and mortar retailers. Over the course of a shopping trip, consumers usually have to search, buy and even transport the purchased products during a fixed amount of time. Given the time constraint, they visit a limited number of shops — typically one supermarket — and appreciate the proximity provided by its inventory. In contrast, multi-market search is related to the advanced search technologies that are available in the digital economy. It allows the online-customers to search and compare various options easily. Multiple market search is also relevant in the market for durable goods such as housing or expensive items where customers are exposed to the market for a sufficiently long time to ponder multiple available options.

We show that a marketmaking middleman can emerge in a directed search equilibrium. The marketmaking middleman can outperform either extreme intermediation mode. Relative to a pure market-maker, its inventory holdings can reduce the out-of-stock risk, while relative to a pure middleman its platform can better exploit the surplus of intermediated trade. It is this trade-off that answers the two puzzles above. Somewhat surprisingly, our result suggests that an improvement in search technologies induces the intermediary to generate inefficiencies to increase profits. This occurs because platform trade creates more profits but it is at the expense of more frictional matching.

We offer various extensions to our baseline model. First, we introduce non-linear matching functions in the decentralized market, which increases the profitability of middleman even with multi-market search. Second, we introduce an aggregate resource constraint and frictions in the wholesale market, which increases the profitability of using an active platform even with single-market search. Third, we introduce a convex inventory-holding cost function, which reduces the profitability of a middleman, and sellers' purchase/production costs that accrue prior to entering the platform, which reduce the profitability of marketmaker. However, these extensions do not alter our main insight on the emergence of marketmaking middlemen. Forth, we introduce competing intermediaries. As is consistent with the monopoly analysis, we show that an active platform of an incumbent intermediary that charges positive fees can only be profitable when agents search in multi markets and the other intermediary enters with an active platform.

Finally, we provide empirical evidence for our theory. Just as in the last extension with competing intermediaries, we treat Amazon as the centralized market and eBay as the decentralized market. For our chosen product category, Amazon acts as a marketmaking middleman. Specifically, for 32% of the sample, Amazon acts as a middleman; for the other 68%, it acts as a platform. Our empirical evidence strongly supports the model's prediction that Amazon is more likely to sell the product as a middleman when the chance of buyers to meet a seller on eBay is low, the buyers' bargaining power is low, and/or total demand is high.

Our paper is related to the literature on middlemen developed by Rubinstein and Wolinsky (1987).[2] Using a directed search approach, Watanabe(2010, 2018a, 2018b) provides a model of an intermediated market operated by middlemen with high inventory holdings. The middlemen's high selling-capacity enables them to serve many buyers at a time. Because of the lower likelihood of stock-out, it generates a retail premium of inventories. This mechanism is adopted by the middleman in our model. Hence, if intermediation fees were not available, then our model would

[2] Rubinstein and Wolinsky (1987) show that an intermediated market can be active under frictions, when it is operated by middlemen who have an advantage in the meeting rate over the original suppliers. Given some exogenous meeting process, two main reasons have been considered for the middlemen's advantage in the rate of successful trades: a middleman may be able to guarantee the quality of goods (Biglaiser 1993, Li 1998), or to satisfy buyers' demand for a variety of goods (Shevchenko 2004). While these are clearly sound reasons for the success of middlemen, the buyers' search is modeled as an undirected random matching process, implying that the middlemen's capacity cannot influence buyers' search decisions in these models. See also Duffie et al. (2005), Lagos and Rocheteau (2009), Lagos et al. (2011), Weill (2007), Johri and Leach (2002), Masters (2007), Watanabe (2010), Watanabe (2018a), Wright and Wong (2014), Geromichalos and Jung (2018), Lagos and Zhang (2016), Awaya and Watanabe (2018b), Awaya and Watanabe (2018a), Nosal et al. (2015).

be a simplified version of Watanabe where we added an outside market. It is worth mentioning that in Watanabe(2010, 2018a, 2018b), the middleman's inventory is modeled as an indivisible unit, i.e., a positive integer, so that the middlemen face a non-degenerate distribution of their selling units as other sellers do. In contrast, here we model the inventory as a mass, assuming more flexible inventory technologies, so that the middleman faces a degenerate distribution of sales. This simplification allows us to characterize the middleman's profit-maximizing choice of inventory holdings — in Watanabe(2010, 2018b) the inventory level of middlemen is determined by aggregate demand-supply balancing, and in Watanabe (2018a) it is treated as an exogenous parameter. More recently, Holzner and Watanabe (2016) study a labor market equilibrium using a directed search approach to model a job-brokering service offered by Public Employment Agencies, but the choice of intermediation mode is not the scope of their paper.

Our paper is also related to the two-sided market literature.[3] The critical feature of a platform is the presence of a cross-group externality, i.e., the participants' expected gains from a platform depend positively on the number of participants on the other side of it. Caillaud and Jullien (2003) show that even when agents have pessimistic beliefs on the intermediated market, the intermediary can make profits by using "divide-and-conquer" strategies, i.e., subsidizing one group of participants in order to attract another group and extract the ensuing externality benefit. To be consistent with this literature, we develop an equilibrium with an intermediary based on similar pessimistic beliefs. Broadly speaking, if there were no middleman mode, our model would be a directed search version of Caillaud and Jullien (2003) in combination with a decentralized market. Further, our result that the intermediary sometimes induces agents to search more than they like is related to the idea of search diversion in Hagiu and Jullien (2011). They pursue this idea in a model of an information platform that has superior information about the match between consumers and stores and that could direct consumers first to their least preferred store.

Rust and Hall (2003) develop a search model which features the coexistence of different intermediation markets.[4] They consider two types of intermediaries, one is a "middleman" whose market requires costly search and the other is a monopolistic "market maker" who offers a friction-

[3]See, e.g. Rochet and Tirole (2003), Rochet and Tirole (2006), Caillaud and Jullien (2001), Caillaud and Jullien (2003), Rysman (2009), Armstrong (2006), Hagiu (2006), (Weyl, 2010). Related papers from other aspects can be found in Baye and Morgan (2001), Rust and Hall (2003), Parker and Van Alstyne (2005), Nocke et al. (2007), Galeotti and Moraga-González (2009), Loertscher and Niedermayer (2008), Edelman et al. (2015), Hagiu and Wright (2014), Condorelli et al. (2018), and Rhodes et al. (2017). Earlier contributions of this strand of literature are, e.g., Stahl (1988), Gehrig (1993), Yavaş (1994), Yavaş (1996), Spulber (1996), and Fingleton (1997). For platform studies emphasizing matching heterogeneity, see e.g., Bloch and Ryder (2000), Damiano and Li (2008) and De Fraja and Sákovics (2012).

[4]See Ju et al. (2010) who extend the Rust and Hall model by considering oligopolistic market makers.

less market. They show that agents segment into different markets depending on heterogeneous production costs and consumption values, thus these two types of intermediaries can coexist in equilibrium. Their model is very different from ours in many respects. For instance, selling capability and inventory do not play any role in their formulation of a search rule, but it is the key ingredient in our model. As Rust and Hall (2003) state: "An important function of intermediaries is to hold inventory to provide a buffer stock that offers their customers liquidity at times when there is an imbalance between supply and demand. In the securities business, liquidity means being able to buy or sell a reasonable quantity of shares on short notice. In the steel market, liquidity is also associated with a demand for immediacy so that a customer can be guaranteed of receiving shipment of an order within a few days of placement. *Lacking inventories and stockouts, this model cannot be used to analyze the important role of intermediaries in providing liquidity* (page 401; emphasis added)." This is exactly what we emphasize in our model which incorporates Rust and Hall's observation. We show that intermediaries can pursue different types of intermediation modes even when faced with homogeneous agents.

The rest of the paper is organized as follows. Section 2 presents our model of intermediation, and the benchmark case of single-market search. Section 3 extends the analysis to allow for multiple-market technologies and presents the key finding of our paper. Section 4 discusses modeling issues. Section 5 discusses some real-life applications of our theory. Section 6 presents the empirical evidence. Finally, section 7 concludes. Omitted proofs are in the Appendix. Finally, the online appendices contain our extension to allow for competing intermediaries, unobservable capacity and participation fees, and additional details on the empirical analysis.

2 A basic model with single-market search

This section studies the choice of intermediation mode for single-market technologies that serves as a benchmark of our economy. We start with the environment in which the monopolistic intermediary operates.

2.1 The framework

Agents. We consider a large economy with two populations, a mass B of identical buyers and a mass S of identical sellers. Each buyer has unit demand for a homogeneous good, and each seller is able to sell one unit of that good. The consumption value for buyers is normalized to 1.

Sellers can stock the good from a competitive wholesale market at a price equal to the marginal cost $c \in [0, 1)$.

Retail markets. Buyers and sellers can only meet each other in a retail market. There are two retail markets, a *centralized/intermediated market* (C market), which is operated by a monopolistic intermediary, and a *decentralized market* (D market), which serves as the outside option for agents. We consider two different search technologies: single-market search, where agents can attend only one market, and multi-market search where agents can attend both markets sequentially. This section spells out the details of single-market search while Section 3 discusses multi-market search. Below we describe price formation and the trading mechanisms in each market.

Matching and price formation in the decentralized market. In the decentralized market, matching is random and the surplus is split by bilateral bargaining. Denote the population of buyers and sellers that participate in the D market by $B^D \in [0, B]$ and $S^D \in [0, S]$, respectively, and let the buyer-seller ratio of the D market be $x^D = \frac{B^D}{S^D}$. This ratio is the (expected) queue. We assume that if all buyers and sellers participate in the D market ($B^D = B, S^D = S$), then a buyer meets a seller with probability λ^b and a seller meets a buyer with probability $\lambda^s = x^D \lambda^b$. If only a subset of buyers $B^D \leq B$ and sellers $S^D \leq S$ participate, then the matching probabilities are given by $\lambda^b \times \frac{S^D}{S}$ and $\lambda^s \times \frac{B^D}{B}$, respectively.[5] Matched partners follow an efficient bargaining process, which yields a linear sharing rule of the total surplus, with a share of $\beta \in (0, 1]$ for the buyer and a share of $1 - \beta$ for the seller.

Below, we refer to a buyer's value in market j by V^j and a seller's value by W^j. In the D market, the expected value for a buyer is given by V^D,

$$V^D = \lambda^b \frac{S^D}{S} \beta(1 - c), \tag{1}$$

and a sellers' expected value is given by W^D,

$$W^D = \lambda^s \frac{B^D}{B} (1 - \beta)(1 - c). \tag{2}$$

[5]The idea behind $\lambda^b \times \frac{S^D}{S}$ is that if a buyer visits a seller but the seller is not available, i.e., he chose to offer his product in the C market, then the meeting fails. A similar interpretation applies to $\lambda^s \times \frac{B^D}{B}$. By an accounting identity, the number of matched buyers is equal to the number of matched sellers, $B^D \lambda^b \frac{S^D}{S} = S^D \lambda^s \frac{B^D}{B}$. This matching technology, which is linear in the participants on the other side of the market, is a simplified way to formulate the outside option of agents. In Section 4.1, we show that our main insight is valid with general non-linear matching functions where the meeting rate (and the expected value) depends on the relative measures of buyers and sellers.

Matching and price formation in the centralized market. The centralized market is operated by a monopolistic intermediary whose profit-maximizing mode is the focus of the model. The intermediary can perform two different intermediation activities. As a middleman, it purchases a good with mass $K \geq 0$ from the wholesale market at a cost c, and resells it to buyers at a price of $p^m \in [c, 1]$. As a market-maker, it does not buy and sell but instead provides a platform where buyers and sellers can interact with each other for trade after paying the fees. The transaction fees that are charged to buyers and sellers are denoted by f^b, $f^s \in [0, 1]$, respectively, and satisfy $f^b + f^s \leq 1$.[6] Denote the choice of the intermediary by a vector $\mathcal{P}^i$, $i = m, p, h$ (where m refers to middleman, p to platform and h to hybrid). If the intermediary chooses to be a pure middleman, it announces $\mathcal{P}^m = (p^m, K) \in \mathbb{R}^2$ while if it acts as a pure platform it announces $\mathcal{P}^p = (f^b, f^s) \in \mathbb{R}^2$. If the intermediary chooses to adopt a hybrid mode, then $\mathcal{P}^h = (p^m, K, f^b, f^s) \in \mathbb{R}^4$ is announced. Based on $\mathcal{P}^i$, buyers and sellers not only learn if the middleman/platform sector is accessible, but also learn the price/fee in each sector. Below we leave out the superscripts i whenever there is no confusion.

One of the key features of modern intermediaries is that they have an information advantage and they can communicate all relevant price and capacity information to the platform participants. In the first stage, sellers simultaneously post a price $p^s \in [c, 1]$. Owing to the advanced matching technology from the intermediary, the prices and capacities of all the suppliers are publicly observable within the C market. In the second stage, buyers simultaneously decide which supplier to visit. As is standard in the literature, we assume that each buyer can visit at most one supplier, either one of the sellers or the middleman.

Buyers cannot coordinate which supplier to visit. Hence, the platform does not eliminate all frictions. There is a chance that more buyers show up at a given supplier than the supplier can accommodate, in which case some buyers get rationed. Alternatively, fewer buyers may show up at a supplier than the supplier can accommodate, in which case the supplier is rationed. This coordination friction is captured by only considering symmetric equilibria where buyers play identical mixed strategies, denoted by $\sigma = \{\sigma^m, \sigma^s\} \in \mathbb{R}^2$, satisfying $\sigma^m, \sigma^s \in [0, 1]$ and $\sigma^m + \sigma^s = 1$. σ^m (σ^s) represents the probability that a buyer visits the submarket of the middleman (individual sellers).

Suppose that a mass of $B^C > 0$ buyers and $S^C > 0$ sellers participate in the C market. Then

[6] Allowing for participation fees/subsidies, which accrue irrespective of transactions in the C market, will not affect our main result. In Appendix B, we offer such an extended model.

a measure $x^m = B^C \sigma^m$ of buyers visits the middleman sector, and the rest of the buyers visit the market-maker sector, leading to an expected queue of $x^s = \frac{B^C - x^m}{S^c}$ on the platform. Given that we have a continuum of sellers, in a symmetric equilibrium, the probability that an individual buyer visits a particular seller is zero, and the number of buyers visiting a seller, denoted by N, is a random variable, and it follows a Poisson distribution, $\text{Prob}[N = n] = \frac{e^{-x^s} x^{s\,n}}{n!}$, with an expected queue $x^s \geq 0$. [7]

Matching with the middleman Suppose that a measure x^m of the buyers visit the middleman. Since the middleman has capacity K, its expected profit is given by $\min\{K, x^m\}p^m$. The expected value for a buyer who visits the middleman is given by V^m,

$$V^m(x^m, \mathcal{P}^i) = \min\{\frac{K}{x^m}, 1\}(1 - p^m),\ i \in \{m, h\},$$

where $\min\{\frac{K}{x^m}, 1\}$ is the matching probability of a buyer at the middleman. When $K \geq x^m$, the matching probability becomes 1. This is how the advance inventory technologies of the intermediary help to improve the matching efficiency.

Matching with an individual seller on the platform The realized number of matches is a function of the expected queue x^s. A seller with an expected queue $x^s \geq 0$ has a probability $1 - e^{-x^s}$ ($= \text{Prob}[N \geq 1]$) of successfully selling, while each buyer has a probability $\eta^s(x^s) = \frac{1-e^{-x^s}}{x^s}$ of successfully buying. Hence, the expected value of a seller on the platform with a price p^s and an expected queue x^s is given by W^C,

$$W^C(x^s, p^s, \mathcal{P}^i) = x^s \eta^s(x^s)(p^s - f^s - c),\ i \in \{p, h\},$$

and the expected value of a buyer who visits a seller on the platform is given by V^s,

$$V^s(x^s, p^s, \mathcal{P}^i) = \eta^s(x^s)(1 - p^s - f^b),\ i \in \{p, h\}.$$

As for the intermediation mode in the C market, we adopt the following terminology.

Definition 1 (Intermediation Mode) *Suppose $B^C \in (0, B]$ buyers and $S^C \in [0, S]$ sellers participate in the C market. Then we say that the intermediary acts as:*

- *a pure middleman if $x^m = B^C$;*
- *a market-making middleman if $x^m \in (0, B^C)$;*

[7]Suppose there are b buyers and s sellers, where both b and s are positive integers. If each buyer visits each seller with equal probability, a seller gets at least one buyer with probability $1 - (1 - \frac{1}{s})^b$. Taking the limit as b and s go to infinity and $x^s = b/s$ fixed, in a large market, a fraction $1 - e^{-x^s}$ of the sellers will be matched with a buyer. This process generates an urn-ball matching function. See for example Butters et al. (1977).

- *a pure market-maker if* $x^m = 0$.

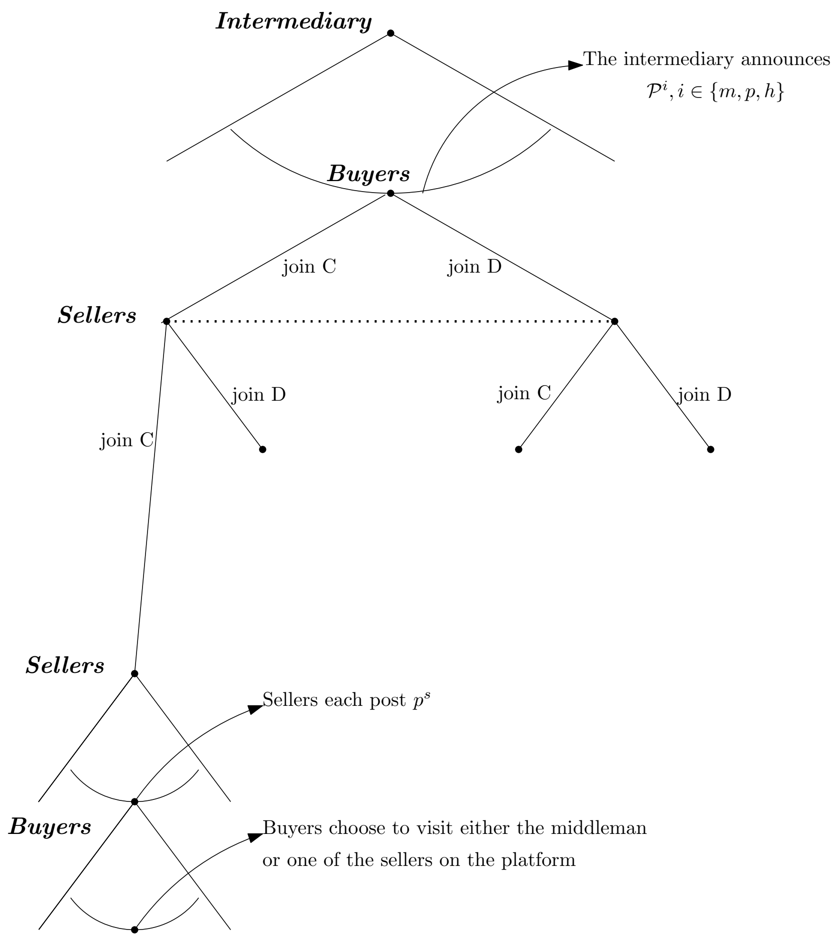


Figure 1: Timing

Timing, strategies and equilibrium concept. The timing of decisions by the buyers, the sellers and the intermediary are as follows.

1. *Announcement stage.* The intermediary announces its intermediation mode and associated plans by $\mathcal{P}^i, i \in \{m, p, h\}$.

2. *Market participation stage.* Observing $\mathcal{P}$, buyers and sellers simultaneously decide which market to participate in, the C or the D market. This gives a distribution of participation in the C market denoted by $\mathcal{N} = \{B^C, S^C\}, B^C \in (0, B], S^C \in [0, S]$. Under single-market search, agents choose either the C or the D market, thus $B^D = B - B^C$ and $S^D = S - S^D$.

3. *Trade stage.* Matching in the D market is random and prices are determined by Nash

bargaining. Trade in the C market follows a directed search game, where sellers first simultaneously post a price. Buyers observe the posted prices of sellers and (or) the announced price of the middleman in $\mathcal{P}$, and they simultaneously decide which supplier to visit, either the middleman or one of the sellers on the platform.

With a continuum of agents on each side of the market, the setting does not correspond exactly to a game. Nevertheless, Figure 1 clearly illustrates the timing and decisions. The intermediary's strategy consists of pricing and inventory decision of the middleman sector as well as the fee setting of the platform: $\mathcal{P}^h = (p^m, K, f^b, f^s)$, with $p^m \in [c, 1]$, $f^b, f^s \in [0, 1]$ and $K \geq 0$ if both modes are adopted, and $\mathcal{P}^m = (p^m, K)$ or $\mathcal{P}^p = \{f^b, f^s)$ if a pure mode is adopted. The seller's strategy consists of a participation decision in the C market given an expected participation distribution $\mathcal{N}$ and the intermediary's announcement $\mathcal{P}$, i.e., $\mathcal{N} \times \mathcal{P} \rightarrow \{0, 1\}$, where we refer to 1 as participating in the C market, and a posted price $p^s \in [c, 1]$ if he decides to join the C market platform. The buyer's strategy also consists of a participation decision in the C market, i.e., $\mathcal{N} \times \mathcal{P} \rightarrow \{0, 1\}$, and in addition, a probability distribution $\sigma = \{\sigma^m, \sigma^s\}$ on which supplier to visit if he joins the C market, satisfying $\sigma^m, \sigma^s \in [0, 1]$, $\sigma^m + \sigma^s = 1$.

The equilibrium concept is in the spirit of a subgame perfect Nash equilibrium. In equilibrium, we require all players (buyers, sellers, and the intermediary) to make the decision(s) that maximize(s) their expected payoffs at every stage, given their expectations of the future realizations of the variables that impact their payoffs. We also require that expectations are rational. Below, we discuss the equilibrium concept for all stages. Working backward, we start with the directed search equilibrium on the platform.

2.2 Directed search equilibrium

Each individual seller (if any) announces an equilibrium price p^s and faces a corresponding expected queue x^s of buyers. The middleman announces a price p^m and faces a corresponding queue x^m of buyers. Since the measure of buyers visiting individual sellers $S^C x^s$ and the middleman x^m should sum up to the total population of participating buyers B^C, we have a standard accounting identity

$$S^C x^s + x^m = B^C. \tag{3}$$

In equilibrium, all trading decisions are optimal given the intermediary's announcement $\mathcal{P}^i$ and the measure of participating agents $\mathcal{N} = \{B^C, S^C\}$. We follow the directed search literature and characterize the directed search equilibrium using the expected queues x^i, $i = m, s$, and sellers' price p^s.[8]

Definition 2 (Directed search equilibrium) *Given $\mathcal{P}^i$ and $\mathcal{N}$, a directed search equilibrium is a triple (x^s, x^m, p^s) such that:*

- *Buyers only visit sellers that offer them their expected market utility, else they do not participate;*
- *Sellers post a price p^s to maximize profits subject to the constraint that visiting buyers must receive their market utility;*
- *The queues x^s and x^m satisfy the accounting identity (3).*

Buyers' equilibrium strategy. In equilibrium, buyers search optimally and only visit suppliers who offer them their market utility, implying that

$$x^m = \begin{cases} B^C & \text{if } V^m(B^C, \mathcal{P}^h) \geq V^s(0, p^s, \mathcal{P}^h) \text{ or under } \mathcal{P}^m \\ (0, B^C) & \text{if } V^m(x^m, \mathcal{P}^h) = V^s(x^s, p^s, \mathcal{P}^h) \\ 0 & \text{if } V^m(0, \mathcal{P}^h) \leq V^s(\frac{B^C}{S^C}, p^s, \mathcal{P}^h) \text{ or under } \mathcal{P}^p, \end{cases} \tag{4}$$

where $V^i(\cdot)$ is the equilibrium value of buyers in the C market of visiting a seller if $i = s$ and the middleman if $i = m$. Note that the third case in (4) happens only if $S^C > 0$. Combining (3) and (4) gives the counterpart for $x^s \in [0, \frac{B^C}{S^C}]$.

Accordingly, a buyer's market utility is defined by V^C,

$$V^C \equiv \max\{V^s, V^m\}.$$

Here and below, we will not write out the explicit dependence of value functions on $\mathcal{P}^i$ and p^s whenever there is no confusion.

Sellers' equilibrium strategy. To derive the equilibrium price p^s, we follow the standard procedure in the directed search literature. Suppose that a potential deviant seller offers a price $p \neq p^s$ that attracts an expected queue $x \neq x^s$ of buyers. Note that given that we have a continuum of sellers, this deviation has measure zero and does not affect the expected utility of buyers in the C market, V^C.

[8] We can equivalently define the equilibrium using buyers' visiting decision probability σ, since given B^C and S^C, it is a bijection between σ^j and x^j, $j = m, s$.

Since buyers must be indifferent between visiting any seller (including the deviating seller), the market-utility condition holds on and off the equilibrium path and satisfies

$$\eta^s(x)\left(1-p-f^b\right)=V^C, \tag{5}$$

where $\eta^s(x) \equiv \frac{1-e^{-x}}{x}$ is the probability that a buyer is served by this deviating seller. Given market utility V^C, (5) determines the relationship between x and p, which we denote by $x = x\left(p|V^C\right)$. This standard directed search logic yields a downward sloping demand curve faced by the seller: when the seller raises his price p, the expected queue length of buyers x becomes smaller and this corresponds to a lower trading probability for the seller, and vice versa.

Given the search behavior of buyers described above and the market utility V^C, the seller's optimal price must satisfy

$$p^s\left(V^C\right)=\arg\max_p \left(1-e^{-x\left(p|V^C\right)}\right)(p-f^s-c)$$

Substituting out p using (5), the sellers' objective function can be written (with a small abuse of notation) as

$$W^C(x,\mathcal{P})=\left(1-e^{-x}\right)(1-f-c)-xV^C,$$

where $f \equiv f^b+f^s$ and $x = x\left(p|V^C\right)$ satisfies (5). Since choosing a price is isomorphic to choosing a queue, the first order condition is

$$\frac{\partial W^C(x,\mathcal{P})}{\partial x}=e^{-x}(1-f-c)-V^C=0.$$

The second order condition is also satisfied. Arranging the first order condition using (5) and evaluating it at $x^s = x\left(p^s|V^C\right)$, we can back out the equilibrium price $p^s = p^s\left(V^C\right)$ which is equal to

$$p^s=f^s+c+\left(1-\frac{x^s e^{-x^s}}{1-e^{-x^s}}\right)(1-f-c). \tag{6}$$

Accordingly, the buyer's expected value can be written as

$$V^s(x^s,\mathcal{P})=e^{-x^s}(1-f-c), \tag{7}$$

and the seller's expected value can be written as

$$W^C(x^s,\mathcal{P})=(1-e^{-x^s}-x^s e^{-x^s})(1-f-c).$$

Given these expected values, we now turn to the buyers' and sellers' participation decisions.

2.3 Participation equilibrium

Following the intermediary's announcement, each infinitesimal agent has expectations about how all agents will participate in the C market, and in equilibrium those expectations are correct. Our definition of the participation equilibrium is therefore a rational expectation equilibrium which is consistent with the literature, e.g., Caillaud and Jullien (2003) and Hagiu (2006).

Definition 3 (Participation equilibrium) *A participation equilibrium given* $\mathcal{P}^i$ *is a pair* $\mathcal{N} = (B^C, S^C)$ *such that*

$$B^C = B \cdot I\{V^C(\mathcal{P}, \mathcal{N}) \geq V^D(\mathcal{N})\}$$

and

$$S^C = S \cdot I\{W^C(\mathcal{P}, \mathcal{N}) \geq W^D(\mathcal{N})\},$$

where I *is an indicator function which equals* 1 *if the condition in brackets is satisfied, and otherwise equals* 0*. A participation allocation is a mapping* $\mathcal{N}(\cdot)$ *that maps each intermediary announcement* $\mathcal{P}$ *into a participation equilibrium* $\mathcal{N}(\mathcal{P})$*.*

Note that in the above definition, we make it explicit that a buyer's and a seller's value in the C market ultimately depends on the intermediary announcement $\mathcal{P}$ and the distribution of participants $\mathcal{N}$. We also make the tie-breaking assumption that agents choose to participate in the C market if they are indifferent between visiting the C and the D market.

As in the standard two-sided market literature, there exit indirect network externalities in the C market namely that for each $\mathcal{P}$ there may exist multiple participation equilibria. For instance, an equilibrium based on pessimistic beliefs, where no agents join the C market ($\mathcal{N}(\cdot) = \{0, 0\}$), can coexist with another equilibrium based on optimistic beliefs, where all agents join the C market ($\mathcal{N}(\cdot) = \{B, S\}$). As in Caillaud and Jullien (2003), we assume that agents hold pessimistic beliefs on the participation decision of other agents. Under such beliefs, *buyers and sellers coordinate on a participation distribution such that the C market is empty whenever possible.* For a pure platform such as the one considered by Caillaud and Jullien (2003), a divide-and-conquer strategy is the only way to get a positive market share. In our model, however, even without the participation of sellers, there is supply in the C market since the inventory K of the middleman can be announced ex ante. Therefore, in order to break the pessimistic beliefs, the intermediary only needs to convince buyers that if they do join the C market, they have access to the middleman inventory and their expected value is higher than if they would visit the D market. Thus, we can refine the participation equilibrium as follows.

Definition 4 *We say a participation allocation $\mathcal{N}(\cdot)$ is based on pessimistic beliefs if*

$$\mathcal{N}(\mathcal{P}) = \begin{cases} (B, S^C) & \text{if } \mathcal{P} \text{ involves } K = B \text{ and } 1 - p^m \geq \lambda^b \beta(1-c); \\ (0,0) & \text{otherwise;} \end{cases}$$

where $S^C \in \{0, S\}$.

To understand the refinement, note that buyers compare the C market whose platform is empty to a nonempty D market. Buyers have pessimistic expectations when visiting the intermediary, i.e., they believe that all sellers are in the D market. Hence, the only scenario that buyers are willing to join the C market is when the middleman holds enough inventory $K = B$, and charges a price that guarantees more than the expected value in the nonempty D market $1 - p^m \geq \lambda^b \beta(1-c)$. Notice that although S^C is not specified in the definition, it follows from a seller's (optimal) decision that whenever $B^C = B$ and the platform is activated, we have $S^C = S$, and when the platform is not activated, we have $S^C = 0$.[9]

2.4 Optimal intermediation mode

We now move to the optimal choice of intermediation mode. Given the directed search equilibrium and the participation allocation under pessimistic beliefs, the intermediary chooses inventory capacity K, price/fees p^m, f^b and f^s to optimize its business mode.

In what follows, we show that under a single-market search technology, the intermediary optimally shuts down the platform ($S^C = 0$), and serves all buyers as a pure middleman with enough inventory, i.e., $K = B$ and $x^m = B^C = B$. This leads to the following pure middleman profits,

$$\Pi = B(p^m - c),$$

subject to the participation constraint of buyers in the C market,

$$V^m(x^m, \mathcal{P}^m) = 1 - p^m \geq \lambda^b \beta(1-c). \tag{8}$$

The middleman optimally sets $p^m = 1 - \lambda^b \beta(1-c)$. Note that the outside value of buyers is given by $\lambda^b \beta(1-c)$, which is supported by their belief that the D market is non-empty.

[9]Sellers are not able to join the platform if the intermediary only activates the middleman sector.

Now, we show that under single-market search, creating an active platform is not profitable. Suppose that the intermediary deviates and opens a platform with intermediation fees $f = f^b + f^s \leq 1$. Then, the platform generates a non-negative trade surplus $1 - f \geq 0$. Notice to break the pessimistic beliefs, intermediary still needs to hold an inventory of $K = B$ and posts a price $p^m \leq 1 - \lambda^b\beta(1-c)$. According to Definition 3, given that all buyers join the C market, all sellers find it more profitable to join the C market as well: $S^C = S$. The optimal search behavior of buyers implies the following condition for an active platform:

$$V^s(x^s, \mathcal{P}^h) \geq V^m(x^m, \mathcal{P}^h) = 1 - p^m \geq \lambda^b\beta(1-c),$$

where the first inequality guarantees the activeness of the platform in accordance to condition (4), and the second inequality follows directly from (8). Inserting (7) into this expression,yields,

$$f = f^s + f^b < (1 - \lambda^b\beta)(1-c).$$

Then, the intermediary's expected profits consist of the revenue of platform fees, $S(1-e^{-x^s})f$, and the revenue of inventory sales minus inventory cost, $\min\{B, x\}p^m - Bc$. Without going into the details of the optimization problem, observe that

$$\begin{aligned}
\Pi(x^s, x^m, \mathcal{P}^h) &= S(1 - e^{-x^s})f + \min\{B, x^m\}p^m - Bc \\
&< Sx^s f + x^m p^m - Bc \\
&\leq (Sx^s + x^m)\max\{f, p^m\} - Bc \\
&< B(1 - \lambda^b\beta)(1-c) = \Pi,
\end{aligned}$$

for all $x^s \in (0, \frac{B}{S}]$. Hence, opening the platform is not profitable under single-market search.

The intuition behind the occurrence of a pure middleman mode is as follows. Given the frictions on the platform, a larger middleman sector creates more transactions. To achieve the highest possible number of transactions, the intermediary shuts down the platform. In a nutshell, the middleman's capacity is the most efficient way to distribute the good and, if agents search within a single market, the intermediary is guaranteed the highest possible surplus by choosing this mode. The allocation characterized here serves as a benchmark for the rest of our analysis.

Proposition 1 (Pure middleman) *Under single-market search technologies, the intermediary will not open the platform and will act as a pure middleman with $x^m = K = B$, serving all buyers for sure.*

3 Multi-market search

In this section, we extend our analysis to multiple-market search technologies where agents can search in both the C and the D market. To facilitate the presentation of our key idea, we make the assumption that the C market opens prior to the D market.[10] Apart from the fact that this appears to be the most natural setup in our economy, it can be motivated by the first mover advantage of the intermediary: its expected profit is higher if the C market opens before the D market. Hence, this sequence arises endogenously if the intermediary is allowed to select the timing of the market sequence.[11] Formally, the timing in the multi-market search environment is as follows.

1. The intermediary announces $\mathcal{P}^{\rangle}$.
2. After observing $\mathcal{P}^{\rangle}$, buyers and sellers simultaneously decide which market(s) to participate in, the C and/or the D market. This gives a distribution of participation decisions denoted by $\mathcal{N} = \{B^C, S^C, B^D, S^D\}$.[12]
3. The C market opens first where trade follows a directed search game. Then the D market opens where matching is random and prices are determined by Nash bargaining.

Under multi-market search, participating in the C market does not rule out the possibility of trading in the D market. An agent can always reject the trade if it yields negative value. Formally, we have $V^C(\mathcal{P},\mathcal{N}) \geq V^D(\mathcal{N})$ and $W^C(\mathcal{P},\mathcal{N}) \geq W^D(\mathcal{N})$ satisfied for any configuration of $\mathcal{P}^{\rangle}$ and any participation distribution $\mathcal{N}$. According to Definition 3, the only participation equilibrium is the one where all agents first visit the C market and then the D market.

While inducing participation is easy, the more difficult part for the intermediary is to convince agents that trade in the C market is better than in the D market. The complications come from the fact that the terms of trade that the intermediary commits to in the C market affects the market utility of buyers and sellers in the D market. The intermediary takes this into account

[10]If the two markets opened at the same time, we would have to deal with the agents' beliefs about what other agents would choose when they turn out to be matched in both markets. This would give rise to multiple equilibria which complicates the analysis significantly. Our sequential setup avoids this issue. In an infinite horizon model, one can construct a stationary equilibrium relatively easily where the order of the markets does not matter (see Watanabe 2018a).

[11]In a recent study without intermediation, Armstrong and Zhou (2015) show that a seller often makes it harder or more expensive to buy its product later than at the first opportunity.

[12]Extending $\mathcal{N}$ with B^D and S^D is needed, since agents may participate in multiple markets, i.e., $B^C + B^D \geq B$ and $S^C + S^D \geq S$.

and this feed back from the D market makes it optimal to adopt a hybrid where it acts both as a marketmaker and a middlemen. [13]

In this section, we derive an equilibrium under multi-market search where buyers and sellers choose to visit the C and D market sequentially, and the intermediary optimally operates as a marketmaking middleman. We work backward and start with the equilibrium value in the D market.

3.1 Equilibrium values in the D market

Suppose that in equilibrium, all agents join the C market ($B^C = B, S^C = S$) and that the intermediary's prices/fees make it more profitable to trade in the C market than in the D market.[14] Then, the agents who ultimately join the D market are those who failed to trade in the C market.

In order to derive the equilibrium values for B^D and S^D, first assume that an equilibrium exists (we will verify this later). Denote the expected queue at the middleman by x^m, and the expected queue at an individual seller by x^s. Both satisfy the accounting identity (3). Then, the population of matched sellers in the C market is $Sx^s\eta^s(x^s) = S(1-e^{-x^s})$. Hence, sellers who are not matched in the C market automatically join the D market,

$$S^D = S - S(1-e^{-x^s}) = Se^{-x^s}.$$

The population of matched buyers in the C market consists of two groups, the buyers matched with the middleman, $\min\{K, x^m\}$, and the buyers matched with one of the sellers on the platform, $B\eta^s(x^s) = S(1-e^{-x^s})$. Hence, the measure of buyers joining the D market is given by

$$B^D = B - \min\{K, x^m\} - S(1-e^{-x^s}).$$

Inserting B^D and S^D into (1) and (2) gives the equilibrium values for buyers in the D market,

$$V^D(x^m) = \lambda^b e^{-x^s}\beta\,(1-c)\,,$$

while for sellers we obtain:

$$W^D(x^m, K) = \lambda^s\xi\,(x^m, K)\,(1-\beta)\,(1-c)\,,$$

[13] Note further that irrespective of agents' beliefs, an empty D market cannot occur in equilibrium. This is because even when buyers are extremely pessimistic about the D market so that sellers are indifferent between entering and not entering, there will always be sellers who fail to sell in the C market and they will be automatically present in the D market.

[14] Conditions for this to hold are incentive constraints (10), (11) and (12), which are derived below.

where e^{-x^s} is the probability that a seller fails to trade in the C market, and $\xi(x^m, K)$ is the probability that a buyer fails to trade in the C market and it is given by

$$\xi(x^m, K) \equiv 1 - \frac{1}{B}\left(\min\{K, x^m\} + S\left(1 - e^{-\frac{B - x^m}{S}}\right)\right). \tag{9}$$

The buyer visits the middleman sector with probability $\frac{x^m}{B}$ and is served with probability $\min\left\{\frac{K}{x^m}, 1\right\}$. Alternatively he visits the platform with probability $\frac{Sx^s}{B}$ and is served with probability $\eta^s(x^s) = \frac{1-e^{-x^s}}{x^s}$. Hence, the second term of $\xi(x^m, K)$ represents the probability of the buyer to trade in the C market.

3.2 Directed search equilibrium under multi-market search

In this section, we derive the directed search equilibrium for the C market. Relative to single-market search, what is new here is that agents always expect a non-negative value of visiting the D market when deciding whether or not to accept an offer in the C market. Therefore, the prices/fees in the C market must be low enough to induce buyers/sellers to visit *and trade.*

Incentive constraints to trade in the C market. Whenever the platform is active, it must satisfy the following incentive constraints:

$$1 - p^s - f^b \geq V^D, \tag{10}$$

$$p^s - f^s - c \geq W^D. \tag{11}$$

Condition (10) states that the offered price/fee on the platform is acceptable for a buyer only if the offered payoff, $1 - p^s - f^b$, weakly exceeds the expected value that buyers can obtain in the D market, $V^D = \lambda^b e^{-x^s}\beta(1-c)$. The outside payoff is $\beta(1-c)$ if the buyer matches with a seller who has failed to trade in the C market. This happens with probability $\lambda^b e^{-x^s}$. Hence, the larger the platform size x^s, the higher the chance that a seller trades in the C market, and the lower the chance that a buyer can trade successfully in the D market and the lower his expected outside payoff V^D is.

Condition (11) is the incentive constraint for sellers to trade in the C market, which states that the payoff in the C market $p^s - f^s - c$ should be no less than the expected payoff in the D market, $W^D = \lambda^s \xi(x^m, K)(1-\beta)(1-c)$. This payoff depends on a seller's chance of finding a trading partner in the D market $\lambda^s \xi(x^m, K)$. A similar participation constraint must be satisfied in order for buyers to trade in the middleman sector:

$$1 - p^m \geq V^D, \tag{12}$$

where the middleman's price must be acceptable for buyers relative to the expected payoff in the D market. Under conditions (10) to (12), agents are weakly better off trading in the C market. Hence, under our tie-breaking assumption, indifferent buyers trade in the C market whenever they are matched there.

Equilibrium values Given the outside option that buyers can obtain in the D market, the equilibrium value of buyers in the C market equals $V^C = \max\{V^s(x^s), V^m(x^m, K)\}$, where

$$V^s\left(x^s\right) = \eta^s(x^s)\left(1 - p^s - f^b\right) + \left(1 - \eta^s(x^s)\right) V^D(x^m) \tag{13}$$

for an active platform $x^s > 0$ and

$$V^m\left(x^m, K\right) = \min\{\frac{K}{x^m}, 1\}\left(1 - p^m\right) + \left(1 - \min\{\frac{K}{x^m}, 1\}\right) V^D(x^m) \tag{14}$$

for an active middleman sector $x^m > 0$. In this case, if a buyer visits a seller (or a middleman), then he gets served with probability $\eta^s(x^s)$ (or $\eta^m(x^m)$) and his payoff is $1 - p^s - f^b$ (or $1 - p^m$). If not served in the C market, he enters the D market and finds an available seller with probability $\lambda^b e^{-x^s}$, and obtains a payoff of $\beta(1 - c)$.

Similarly, the equilibrium value of active sellers on the platform is given by

$$W^C(x^s, K) = x^s\eta^s(x^s)\left(p^s - f^s - c\right) + \left(1 - x^s\eta^s\left(x^s\right)\right) W^D(x^m, K). \tag{15}$$

A seller trades successfully in the C market platform with probability $x^s\eta^s(x^s)$ and if this occurs, he receives $p^s - f^s - c$. If not successful in the C market, the seller can meet a buyer in the D market with probability $\lambda^s\xi\left(x^m, K\right)$ and obtains a payoff of $\left(1 - \beta\right)\left(1 - c\right)$.

In equilibrium, buyers search optimally and only visit a supplier who offers their market utility. Sellers set an equilibrium price p^s that maximizes profits. Condition (4) in the previous section continues to characterize buyers' optimal search strategy. To derive the equilibrium price p^s, we again follow the standard procedure in the directed search literature.

Consider a seller who deviates to a price $p \neq p^s$ and attracts an expected queue $x \neq x^s$ of buyers, subject to the equilibrium market-utility condition (which holds on and off the equilibrium path):

$$V^C = \eta^s\left(x\right)\left(1 - p - f^b\right) + \left(1 - \eta^s\left(x\right)\right) V^D, \tag{16}$$

where $\eta^s\left(x\right) \equiv \frac{1-e^{-x}}{x}$ is the probability that a buyer is served by this deviating seller. If not served, which occurs with probability $1 - \eta^s\left(x\right)$, he receives V^D. For a given market utility V^C,

(16) determines the relationship between x and p, which we denote by $x = x(p|V)$. This yields a downward sloping demand: when the seller raises his price p, the queue length of buyers x becomes smaller, and vice versa.

Given the search behavior of buyers described above and the market utility V^C, the seller's optimal price must satisfy

$$p^s\left(V^C\right) = \arg\max_p \left\{\left(1 - e^{-x\left(p|V^C\right)}\right)(p - f^s - c) + e^{-x\left(p|V^C\right)} W^D\right\}. \tag{17}$$

The seller trades successfully in the C market platform with probability $1 - e^{-x\left(p|V^C\right)}$ and in that case he receives $p - f^s - c$. Otherwise, the seller can meet a buyer in the D market and he obtains an expected value of W^D.

Substituting out p in (17) using (16), we can rewrite the sellers' objective function as follows:

$$W^C(x) = \left(1 - e^{-x}\right)\left(v\left(x^m, K\right) - f\right) - x\left(V^C - V^D\right) + W^D,$$

where $v\left(x^m, K\right)$ is the intermediated trade surplus, i.e., the total surplus in the C market net of the outside options, and it is defined by

$$v\left(x^m, K\right) \equiv 1 - c - V^D(x^m) - W^D(x^m, K).$$

Since choosing a price p is isomorphic to choosing a queue x, the first order condition is

$$\frac{\partial W^C(x)}{\partial x} = e^{-x}\left(v\left(x^m, K\right) - f\right) - \left(V^C - V^D\right) = 0.$$

The second order condition is also satisfied. Arranging the first order condition using (16) and evaluating it at $x^s = x\left(p^s|V^C\right)$, we obtain the equilibrium price $p^s = p^s\left(V^C\right)$ which can be written as

$$p^s - f^s - c = \left(1 - \frac{x^s e^{-x^s}}{1 - e^{-x^s}}\right)\left(v(x^m, K) - f\right) + W^D. \tag{18}$$

Equation (18) states that the optimal price p^s net of fee f and cost c guarantees the seller a profit that equals the seller's outside value W^D plus a share $1 - \frac{x^s e^{-x^s}}{1 - e^{-x^s}}$ of the intermediated trade surplus that the intermediary is willing to give to buyers and sellers, $v(x^m, K) - f$.

For the platform to be active, the price and fees must satisfy the incentive constraints (10) and (11). Substituting in (18) yields

$$f \leq v(x^m, K), \tag{19}$$

which states that for the platform to be active ($x^s > 0$), the total transaction fee f should not be greater than the intermediated trade surplus, $v\left(x^m, K\right)$. Whenever (10) and (11) are satisfied,

(19) must hold, and whenever (19) is satisfied, (10) and (11) must hold. Hence, (19) is a sufficient condition for an active platform.

Observe that $K > x^m$ cannot be profitable for the intermediary since it only increases the capacity costs. For $K \leq x^m$, the intermediated trade surplus $v(x^m, K)$ can be rewritten as

$$v(x^m, K) = \left[1 - \lambda^b e^{-\frac{B-x^m}{S}} \beta - \lambda^s \left(1 - \frac{K + S(1 - e^{-\frac{B-x^m}{S}})}{B}\right)(1-\beta)\right](1-c),$$

which is decreasing in x^m. This occurs because a larger sized platform (i.e., a lower x^m) crowds out the D market transactions and lowers the outside option of the buyers.

3.3 Intermediation mode

Our next step is to determine the profit for each intermediation mode, denoted by $\tilde{\Pi}(x^m)$.

Pure middleman. If the intermediary does not open the platform then $x^m = B$ and any encountered seller in the D market is always available for trade. Hence, as before, the middleman selects capacity $K = B$, serves all buyers at a price $p^m = 1 - \lambda^b \beta(1-c)$ satisfying (12) and makes profits

$$\tilde{\Pi}(B) = B(1 - \lambda^b \beta)(1-c). \tag{20}$$

Pure market-maker. When the middleman sector is not open, $x^s = \frac{B}{S}$. Given that the equilibrium price p^s at the platform is given by (18), the intermediary charges a fee $f = f^b + f^s$ in order to maximize

$$S\left(1 - e^{-\frac{B}{S}}\right) f,$$

subject to the constraint (19). The constraint is binding and this yields:

$$f = v(0,0) = \left[1 - \lambda^b e^{-x^s} \beta - \lambda^s \xi(0,0)(1-\beta)\right](1-c).$$

where $\xi(0,0) = 1 - \eta^s(x^s)$ according to (9). The profit for the market-maker mode is

$$\tilde{\Pi}(0) = S(1 - e^{-\frac{B}{S}}) v(0,0). \tag{21}$$

Market-making middleman. If the intermediary is a market-making middleman, then $x^m \in (0, B)$ and $x^s \in \left(0, \frac{B}{S}\right)$, satisfying the condition that buyers must be indifferent between visiting the middleman or the platform $V^m(x^m) = V^s(x^s)$. Using the equilibrium values: (13), (14), and (18), this indifference condition generates the following expression for the price $p^m = p^m(x^m)$:

$$p^m = 1 - \lambda^b e^{-x^s} \beta (1 - c) - \frac{x^m e^{-x^s}}{\min\{K, x^m\}} (v(x^m, K) - f). \tag{22}$$

Together with (3), this equation defines the relationship between p^m and x^m. Applying this expression, we can see that condition (12) is eventually reduced to (19). The profit for the marketmaking middleman mode is

$$\tilde{\Pi}(x^m) = \max_{x^m, f, K} \Pi(x^m, f, K) = S(1 - e^{-x^s}) f + \min\{K, x^m\} p^m - Kc$$

subject to (19) and $x^m \in (0, B)$. Note that $K > x^m$ cannot be profitable since it is a mere increase in capacity costs. Profit maximization requires the following.

Lemma 1 *The market-making middleman sets: $K = x^m$ and $f = v(x^m, K)$.*

Proof. See Appendix A.1. ■

The above conditions imply that the intermediary's capacity should satisfy all the forthcoming demands, and the intermediation fee should be set to extract the full intermediation surplus.

Profit-maximizing intermediation mode. We are now in the position to derive the profit-maximizing intermediation mode. To do so, it is important to observe that relative to the pure middleman mode, an active platform with multiple-market search can undermine the D market by lowering the available supply. This influences the middleman's price in the following way. With $v(\cdot) = f$, the incentive constraint (12) is binding, and the middleman's equilibrium price is given by

$$p^m = 1 - \lambda^b e^{-x^s} \beta(1 - c)$$

for any $x^s \geq 0$ (see (22)). This shows that p^m decreases with x^m: the outside option of buyers depends positively on the size of the middleman sector, since a larger scale of the middleman crowds out the platform and increases the chance that a buyer can find an active seller in the D market (who was unsuccessful at the platform). Hence, in order to extend the size of the middleman sector, the intermediary must lower the price p^m. In other words, a larger platform allows for a price increase by reducing agents' outside trade opportunities.

Proposition 2 (Market-making middleman/Pure Market-maker) *Given multi-market search technologies, there exists a unique directed search equilibrium with active intermediation. The intermediary will open a platform and act as:*

- *a market-making middleman if* $\lambda^b \beta \leq \frac{1}{2}$ *or if* $\lambda^b \beta > \frac{1}{2}$ *and* $\frac{B}{S} \geq \bar{x} \in (0, \infty)$*;*
- *a pure market-maker if* $\lambda^b \beta > \frac{1}{2}$ *and* $\frac{B}{S} < \bar{x}$*.*

Proof. See Appendix A.2. ■

With multiple-market search technologies, there is cross-market feedback from the D market to the C market, which makes using the platform as part or all of its intermediation activities profitable. Additionally, the intermediary must decide whether or not to operate as a pure market maker. Our results show that the equilibrium mode of the intermediary depends on parameter values. If $\lambda^b \beta \leq \frac{1}{2}$ then the buyers' outside option is low. In this case, the middleman sector generates high enough profits for the market-making middleman mode to be adopted for any value of $\frac{B}{S}$. If instead $\lambda^b \beta > \frac{1}{2}$ then the buyers' outside option is high, and attracting buyers to the middleman sector is costly. In this case, the intermediary will only act as a market-making middleman if $\frac{B}{S}$ is high, since the D market is tight for buyers and they expect a low value from it. The intermediary acts as a a pure market maker if $\frac{B}{S}$ is low, since the buyers expect a high value from the D market. Indeed, the same logic applies to the following comparative statics result.

Corollary 1 (Comparative statics) *Consider a parameter space in which the market-making middleman mode is profit-maximizing. Then, an increase in buyer's bargaining power* β *or buyer's meeting rate* λ^b *in the D market, or a decrease in the buyer-seller population ratio,* $\frac{B}{S}$*, leads to a smaller middleman sector* x^m *and a larger platform* x^s*.*

Proof. See Appendix A.3. ■

4 Extensions

This section considers extensions to the model. As we show below, our main insight, that the benefits of using a platform (as part of) the intermediation business is relatively large when agents can search in multiple markets rather than in a single market only, is robust to these extensions.[15]

[15] For expositional simplicity, we let $c = 0$ and make the tie-breaking assumption that when the middleman is indifferent between $K = x^m$ and $K > x^m$ we set $K = x^m$.

4.1 Matching functions

So far, we assumed a linear matching function in the D market. In this section, we allow for a more general matching function. As is standard in the literature, we assume that the matching function is homogeneous of degree one in B^D and S^D, $M(1, \frac{1}{x^D}) = \frac{M(B^D, S^D)}{B^D}$ and $M(x^D, 1) = \frac{M(B^D, S^D)}{S^D}$, where $x^D = \frac{B^D}{S^D}$ is the buyer-seller ratio in the D market. Then, individual match probabilities depend on the buyer-seller ratio.

$$\lambda^b(x^D) = M(1, \frac{1}{x^D}) \text{ and } \lambda^s(x^D) = M(x^D, 1) = x^D \lambda^b(x^D) \tag{23}$$

where $\lambda^b(x^D)$ is strictly concave and decreasing in x^D.

For single-market search technologies, the result will not be affected by this extension (for instance, under the pessimistic beliefs of Definition 4, the matching probability in the D market is simply replaced by another constant $\lambda^i(x^D)$, $i = b, s$, with $x^D = \frac{B}{S}$). Therefore, we only consider multi-market search technologies. As mentioned before, we let agents exit if they have traded successfully in the C market, because if agents stayed in the D market as in the previous section, then again the analysis would remain essentially unchanged. Then, the population in the D market is given by

$$B^D = B - \min\{x^m, K\} - S(1 - e^{-x^s}) \text{ and } S^D = Se^{-x^s}.$$

With this modification, the buyers' probability to meet an available seller changes from $\lambda^b e^{-x^s}$ to $\lambda^b(x^D)$, and the sellers' probability to meet an available buyer changes from $\lambda^s \xi(x^m, K)$ to $\lambda^s(x^D) = x^D \lambda^b(x^D)$.

In what follows, we derive a condition for a pure middleman mode to be selected under multi-market search technologies. This is the case when, for example, $\lambda^{b\prime}(x^D) = 0$, i.e., when there is no feedback from the D-market to the intermediary's decision in the C market. We proceed with the following steps. First, note that, as before, there is no gain from having excess capacity $K > x^m$. In addition, a pure middleman wants to avoid stockouts ($K < x^m$) if

$$\frac{d\tilde{\Pi}(K)}{dK} = \frac{d}{dK} K \left(1 - \lambda^b(x^D)\beta\right) = 1 - \lambda^b(x^D)\beta + \frac{K}{S}\lambda^{b\prime}(x^D)\beta > 0,$$

for any $x^D = \frac{B-K}{S} \geq 0$, which states that the elasticity of the middleman's price $p^m = 1 - \lambda^b(x^D)\beta$ should satisfy

$$z(K) \equiv -\frac{\partial p^m / \partial K}{p^m / K} = -\frac{K\lambda^{b\prime}(x^D)\beta}{S(1 - \lambda^b(x^D)\beta)} \leq 1.$$

This condition guarantees that a pure middleman should satisfy all the forthcoming demand $K = x^m$.

Second, when all buyers are served by the middleman $x^m = K = B$, the marginal gain of allocating buyers to the platform, measured by the intermediation fee,

$$f = 1 - \lambda^b(x^D)\beta - x^D\lambda^b(x^D)(1-\beta),$$

can not exceed the marginal opportunity cost, measured by the lost revenue in the middleman sector,

$$1 - \lambda^b(0)\beta - K\lambda^{b\prime}(0)\beta\frac{dx^D(K, x^s(K))}{dK}\mid_{x^s(K)=0},$$

where $x^s(K) = \frac{B-K}{S}$ and

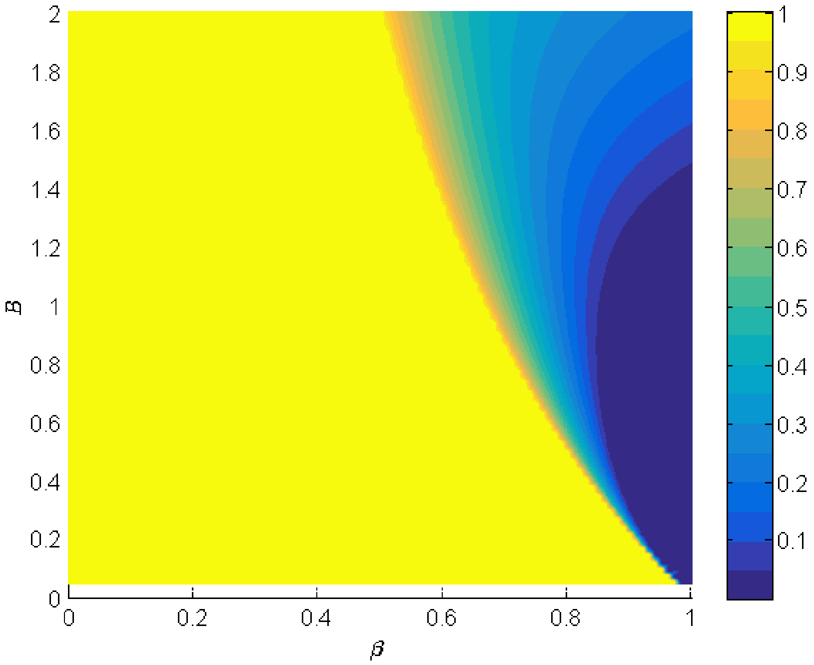
$$\frac{dx^D(K, x^s(K))}{dK}\mid_{x^s(K)=0} = \frac{d}{dK}\frac{B - K - S(1 - e^{-x^s(K)})}{Se^{-x^s(K)}}\mid_{x^s(K)=0} = \frac{-S + (B - K - S)}{S^2e^{-x^s(K)}}\mid_{K=B} = 0.$$

Hence, the intermediary can be a pure middleman even with multiple-market search technologies.

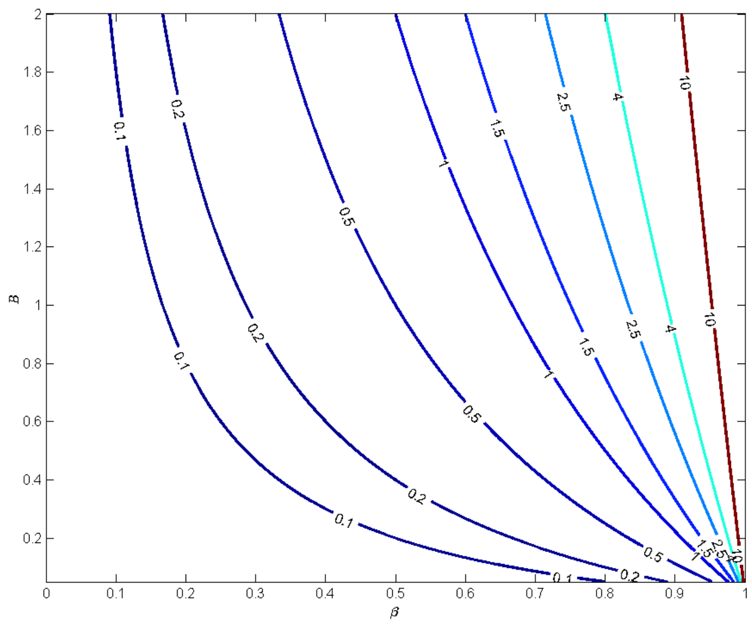
Proposition 3 *With a non-linear matching function in the D market outlined above, a pure middleman mode can be profitable even with multi-market search technologies if the middleman's price is inelastic at full capacity $x^m = K = B$. Otherwise, the intermediary should be a marketmaking middleman or a pure market maker.*

Proof. See Appendix A.4. ■

Figure 2 plots the size of the middleman sector $\frac{x^m}{B}$ and the elasticity of the middleman's price with respect to capacity, evaluated at $x^m = K = B$. It shows that when a pure middleman mode is selected, $\frac{x^m}{B} = 1$, the price is inelastic: $z(B) < 1$, whereas when an active platform is used, the price is elastic: $z(B) > 1$. This confirms that given the appropriate restriction on the meeting rate $\lambda^b(x^D)$, our main conclusion in the baseline model is valid with an alternative assumption that agents exit after successful trade in the C market. It is intuitive that when the middleman's price is elastic, there is strong enough negative feedback from the D market on the price that makes the exclusive use of the middleman sector not profitable.



(a) The optimal size of middleman sector ($\frac{x^m}{B}$) changes with B and β



(b) Price elasticity $z(B)$ changes with B and β

Figure 2: The optimal size of the middleman sector and price elasticity at B under a nonlinear matching function

Note: The upper figure plots the optimal size of the middleman sector, x^m/B, using colors to inform its values, against the mass of buyers B on the vertical axis and the buyer's bargaining power β on the horizontal axis. The lower figure is a contour plot on the price elasticity of p^m with respect to K at $K = B$, $z(B)$, against B on the vertical axis and β on the horizontal axis. All values are calculated based on $S = 1$ and $\lambda^b(x^D) = \frac{1-e^{-x^D}}{x^D}$.

4.2 Endowment economy

In our baseline model, we simplified the middleman's inventory stocking by assuming that the good is supplied by a competitive frictionless wholesale market. In this section, we study the implication of wholesale-market frictions in an endowment economy. Suppose that each seller owns one unit of endowment. In total, a mass of S commodities are available. In the wholesale market, the middleman can access a fraction α of sellers, where $\alpha \in (0,1)$ is exogenous. Then, the middleman's inventory should satisfy the aggregate resource constraint,

$$K \leq \alpha S. \tag{24}$$

In a world with unlimited production capacity, sellers are willing to supply as long as the wholesale price, denoted by p^w, is enough to compensate for the marginal cost; whereas in an endowment economy, sellers are only willing to supply if p^w is high enough to compensate them for the foregone trading opportunities elsewhere. Once contacted by the middleman, sellers choose among selling the endowment to the middleman, or joining the C market platform and/or joining the D market. To simplify the analysis, we abstract from the influence of what sellers can expect from the D market on the determination of the wholesale price, and assume that sellers in the D market receive a zero trade share, $\beta = 1$. Our main conclusion does not depend on this simplification. Then, the middleman's offer to buy from sellers is accepted if and only if

$$p^w \geq W^C(x^s), \tag{25}$$

where $W^C(x^s)$ is the expected value of sellers to operate in the C market platform.

Single-market search. The determination of the intermediation mode depends on the available resources. If $B \leq \alpha S$, then the middleman can stock the full inventory to cover the entire population of buyers. In this case, by closing the platform $S^C = 0$, the middleman makes the highest possible profit, $\Pi = B(1 - \lambda^b)$, with the wholesale price $p^w = 0$, just like in the baseline model. If $B > \alpha S$, then the middleman's inventory will not be enough to cover all buyers, and so the intermediary may wish to use a platform even under a single-market search technology.

The resource constraint (24) makes the analysis under single-market search difficult. This is because by Definition 4, when $B > \alpha S$ the intermediary is unable to attract any transaction as K is restricted to be smaller than B. To continue, we revise the pessimistic beliefs in Definition 4 as follows.

Definition 5 *A revised participation allocation $\mathcal{N}(\cdot)$ is defined as $\mathcal{N}(\cdot) = \{B, S^C\}$ where $S^C \in \{0, S\}$ only when $V^C\Big(\mathcal{P}, (B, S^C)\Big) \geq V^D$, and $\mathcal{N}(\cdot) = \{0, 0\}$ otherwise.*

Now, buyers must decide whether they visit a nonempty C market or a nonempty D market. Beliefs are less pessimistic in the sense that the buyers expect the C market to be nonempty. It's worth pointing out that the results in the benchmark model remain the same under this revised participation allocation.

With the wholesale price p^w determined by the binding constraint (25), the fee f and the price p^m determined by the binding participation constraint of buyers, $V^C = \max\{V^s(x^s), V^m(x^m)\} = V^D = \lambda^b$, the intermediary's problem can be written as the choice of the size of its inventory K and the allocation x^m that maximizes

$$\Pi(x^m, f, K) = (S - K)(1 - e^{-x^s})f + \min\{K, x^m\}\, p^m - Kp^w$$

where $x^s = \frac{B - x^m}{S - K}$, subject to the resource constraint (24). To guarantee non-negative price/fees/profits, we shall assume sufficiently low values of $\lambda^b > 0$ whenever necessary (see the proof of Proposition 4).

As expected, the solution is characterized by the binding resource constraint (24) and an active platform $x^s > 0$ when $B > \alpha S$. Although deactivating the platform would lead to the lowest wholesale price for the middleman $p^w = 0$, this is not profitable. The benefit of fee revenue from the active platform outweighs the cost savings of the middleman. Hence, even under single-market technologies, the aggregate resource constraint can be one reason for the intermediary to open the platform sector in the endowment economy.

Proposition 4 *Consider the endowment economy outlined above with single-market search technology, and the zero trade share of sellers in the D market. The intermediation chooses to be:*

- *a pure middleman if $B \leq \alpha S$;*
- *a market-making middleman with $K = \alpha S \leq x^m$ if $B > \alpha S$.*

Proof. See Appendix A.5. ■

The result $x^m \geq K$ occurs because, in line with the previous setup, an excess inventory means extra costs in the middleman sector and lost revenue on the platform. Figure 3 demonstrates that when $B > \alpha S$, it is possible that the intermediary attracts an excessive number of buyers to the

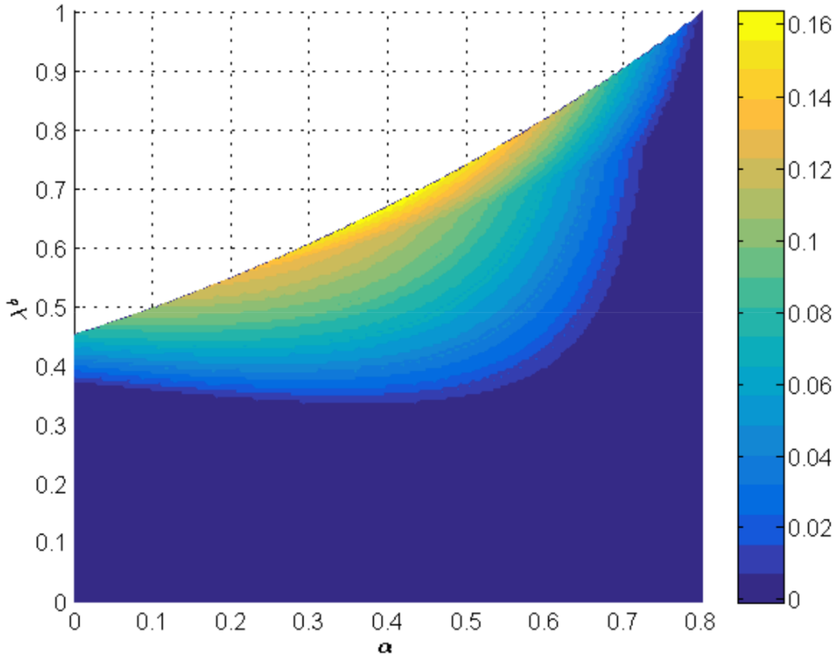


Figure 3: Stockouts of the middleman $(x^m - K)$ under single-market search in endowment economy

Note: The figure plots the level of stockouts, represented by the value of $x^m - K$, against λ^b on the vertical axis and α on the horizontal axis. The figure is drawn with $B = 0.8$ and $S = 1$. We cut out the region where negative profits result for high values of λ^b.

middleman sector $(x^m > K)$ in order to lower the wholesale price in the middleman sector. This results in stockouts. When this occurs, the resource constraint is tight and the outside value of agents is high so that economizing on stocking costs is relatively important.

Multi-market search. With multi-market search technologies, the participation constraint of agents is not the issue but the intermediation fee and the middleman's price should be acceptable relative to the outside value. Hence, the intermediary faces incentive constraints (10) – (12) with an appropriate modification of the match probability in the D market (see the details in the proof of Proposition 5). As before, these conditions are reduced to $f \leq v(x^m, K)$.

To be consistent, we maintain the assumption of a zero trade share of the sellers, $\beta = 1$, in the D market. This assumption now implies that sellers are fully exploited in the C market, thus $p^w = W(x^s) = 0$ for any $x^s \geq 0$.

Under multi-market search, the buyers' outside option depends positively on the number of sellers available in the D market. This has the following consequences. First, just as in the baseline setup, a pure middleman mode can never be profit maximizing. Second, in our endowment economy, the intermediary may wish to stock more inventories than the number of buyers visiting the middleman sector. This is because a larger K will crowd out the supply available in the D market, which will eventually lower the outside value of buyers and increase the profit. Therefore, unlike in all the previous setups, the solution here allows for an excess inventory in the middleman sector.

Proposition 5 *Consider the endowment economy outlined above with a multi-market search technology and a zero trade share of sellers in the D market. The intermediary chooses to be a market-making middleman or a pure market-maker with* $x^m \leq K = \alpha S$.

Proof. See Appendix A.6. ■

Figure 4 shows the occurrence of excess inventory holdings in the middleman sector with high values of λ^b and α. This confirms our intuition that the crowding-out effect of excess inventory is stronger when the buyer's outside value in the D market is higher.

Comparing Proposition 4 and 5, we can summarize the implications of search frictions in wholesale markets represented by α and the agents' search technologies in retail markets on the choice of intermediation mode in our endowment economy as follows.

- For $\alpha S \geq B$, the middleman can stock the full inventory that satisfies all the buyers' demand. As in the benchmark setup, the intermediary chooses to be a pure middleman under single-market search, while it also opens an active platform under multi-market search. Unlike in the previous setup, the middleman holds an excessive amount of inventory.
- For $\alpha S < B$, holding a full inventory is not possible due to an aggregate resource constraint. The intermediary uses a platform irrespective of whether agents search in a single or in multiple markets. Our main insight is still valid. Namely, *the intermediation mode is further away from the pure middleman mode when agents search in multiple markets, rather than in a single market.* The size of the middleman sector, measured by x^m, is smaller under multi-market search than under single-market search technologies.

4.3 Cost functions

Inventory Costs. In the baseline model, we assume zero inventory costs of the middleman. In this section, we consider a convex inventory-cost function $C(K)$ that satisfies $C'(K) \geq 0, C''(K) \geq$

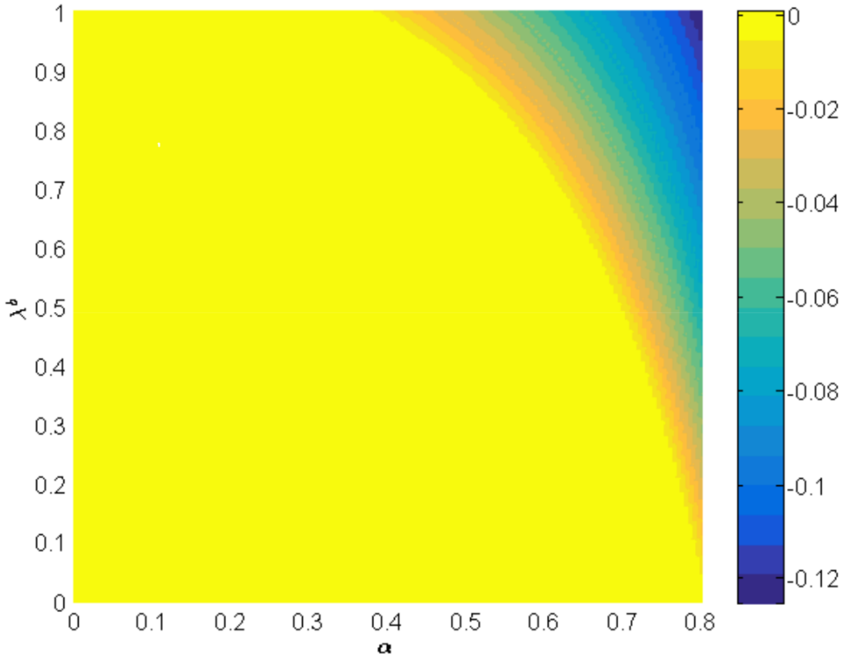


Figure 4: Excessive inventory holdings ($x^m - K$) with multi-market search in endowment economy

Note: The figure plots the level of stockouts, represented by the value of $x^m - K$, against λ^b on the vertical axis and α on the horizontal axis. The figure is drawn with $B = 0.8$ and $S = 1$.

$0, C(0) = 0$, and $C'(K) < 1-\lambda^b$. The last condition guarantees that $C(B) < B(1-\lambda^b)$. We assume $\beta = 1$ for simplicity. We show below that our results continue to hold with positive inventory costs.

As in the baseline model, under single-market search, the pessimistic expectations towards the intermediary requires $K = B$. Thus, the problem of the intermediary can be formulated as

$$\max_{x^m, f} \Pi(x^m) = S(1 - e^{-x^s})f(x^s) + x^m p^m - C(B). \tag{26}$$

subject to $p^m = 1 - \lambda^b$ and $f(x^s) = 1 - \lambda^b e^{x^s}$. The first constraint is given by the buyers' participation constraint in the C market, i.e., $V^m = 1 - p^m \geq \lambda^b$, while the second constraint is given by the buyers' indifference condition, i.e., $V^m = V^s = e^{-x^s}(1 - f)$. The platform fee $f = f(x^s)$ is strictly decreasing (increasing) in x^s (x^m). Intuitively, the tighter the platform, the lower the fee that the intermediary can charge, in order to make buyers indifferent between the platform and the middleman sector. The negative dependence of the platform fee on the platform size favors the middleman mode. Inserting $p^m = 1 - \lambda^b$ and $f(x^s) = 1 - \lambda^b e^{x^s}$ into (26), and taking the derivative with respect to x^m, yields the following first order condition:

$$\frac{\partial \Pi(x^m)}{\partial x^m} = -e^{-x^s} + \lambda^b e^{x^s} + 1 - \lambda^b \equiv \Theta_{Sfoc}(x^m) = 0. \tag{27}$$

Observe in (27) that: $\Theta_{Sfoc}(B) = 0$ and $\Theta'_{Sfoc}(x^m) = -\frac{1}{S}(e^{-x^s} + \lambda^b e^{x^s}) < 0$. Therefore, the pure middleman mode is profit-maximizing.

Under multiple-market search, the objective function is the same as in (26) except that a full inventory $K = B$ is not required to conquer the pessimistic beliefs. Instead, as in the baseline model, the intermediary optimally holds just enough inventory, that is $K = x^m$. The profit maximization problem of the intermediary can be written as

$$\max_{x^m, f} \Pi(x^m) = S(1 - e^{-x^s}) f(x^s) + x^m p^m - C(x^m),$$

subject to $p^m(x^s) = f(x^s) = 1 - \lambda^b e^{-x^s}$ (by (22) and $v(\cdot) = f$ as in Lemma 1). As before, the positive dependence of the middleman's price and the platform fee on the platform size favors the market-maker mode. The first order condition is

$$\begin{aligned} \frac{\partial \Pi(x^m)}{\partial x^m} &= (1 - e^{-x^s})(1 - 2\lambda^b e^{-x^s}) - \frac{\lambda^b x^m e^{-x^s}}{S} - C'(x^m) \\ &\equiv \Theta_{Mfoc}(x^m) = 0. \end{aligned} \tag{28}$$

Observe that $\Theta_{Mfoc}(B) = -\frac{\lambda^b B}{S} - C'(B) < 0$ and

$$\Theta'_{Mfoc}(x^m) = -\frac{e^{-x^s}}{S}\left[1 - \lambda^b e^{-x^s} + 3\lambda^b(1 - e^{-x^s})\right] - \frac{\lambda^b x^m e^{-x^s}}{S^2} - C''(x^m) < 0.$$

Therefore, a market-making middleman is the profit-maximizing mode if

$$\Theta_{Mfoc}(0) = (1 - e^{-\frac{B}{S}})(1 - 2\lambda^b e^{-\frac{B}{S}}) - C'(0) > 0. \tag{29}$$

Otherwise, a pure market-maker mode is selected. Comparing single and multi-market search under positive inventory costs, we find that the conclusion of the benchmark model continues to hold.

Proposition 6 *Consider the convex inventory costs of a middleman as defined above. Then the profit-maximizing intermediation mode under single-market search is a pure middleman, and under multi-market search it involves an active platform.*

The independence between the optimal inventory choice and the positive inventory cost under single-market search is exclusively driven by the pessimistic expectations. Appendix A.7 provides an analysis based on the revised participation allocation (see Definition 5), which shows that a platform is activated even under a single-market search technology, but still, the size of the platform with multi-market search is larger than or equal to that with single-market search. The logic behind this is essentially the same as in the baseline model.

Prior production/purchase before joining the platform. In real-life markets, sellers sometimes need to prepare (produce or purchase) their product for sale prior to market entry. For example, online sellers find it important to display their product's image and keep it ready for delivery before actual transactions occur. A similar issue arises when asset holders are required to commit to their portfolio before trading with their brokers. In these situations, because sellers incur costs irrespective of their success on the platform, attracting sellers to the platform is costly and so the relative profitability of the market-maker mode is reduced. We show, however, that our insight remains valid in such a setting. Interestingly, we also find that a platform can be activated even when the net profit obtained from the platform business is negative.

The only modification that is required now is to introduce a participation constraint for sellers to operate on a platform. Under single-market search, this is irrelevant because the pure middleman mode remains profit maximizing. With multiple-market search, the participation constraint is given by

$$W^C(x^s) - f_p \geq c_E, \tag{30}$$

where $c_E \geq 0$ are the entry costs of sellers to the platform, $f_p \geq 0$ (or $f_p \leq 0$) is a platform participation fee (or subsidy) to be paid by each individual seller, and $W^C(x^s) = \eta(x^s)(p^m - f)$ is the equilibrium value of sellers who participate on the platform. With $\beta = 1$, i.e., zero payoff in the D market for sellers, the intermediary sets $f = p^m = 1 - \lambda^b e^{-x^s}$, satisfying the incentive constraint (6) (note that the participation in the D market does not require prior production/purchase as before), and $f_p = -c_E$. That is, the intermediary should subsidize the entry cost and fully extract the trade surplus in the platform. The profit of a market-making middleman is

$$\tilde{\Pi}(x^m) = S\left[(1 - e^{-x^s})f - c_E\right] + x^m p^m,$$

while the profit of a middleman is

$$\tilde{\Pi}(B) = B(1 - \lambda^b).$$

Comparing these profits, one can find a value of $x^m < B$ (e.g. imagine a neighborhood of $x^m = B - Sx^s \approx B$) and $c_E > 0$ for which the platform profit is negative but $\tilde{\Pi}(x^m) > \tilde{\Pi}(B)$. This leads to the following result.

Proposition 7 *Suppose sellers incur production/purchasing costs prior to platform entry. Then, an active platform can be profit maximizing even when the platform entry cost is higher than the platform fee revenue.*

One benefit of having an active platform in the C market for the intermediary is to reduce competition so that it can set a higher price in the middleman sector. This benefit can be the major source of profits for market-making middlemen even when the platform-entry costs are so high that the net profit from the platform business is negative.

4.4 Competing intermediaries

Our framework can be extended to study competing intermediaries. We consider two intermediaries who make a simultaneous choice of platform fees and/or price of their good. In particular, we are interested in whether an active platform with positive fees of an incumbent intermediary, referred to as I, can be profitable when the other intermediary, referred to as E, enters with adopting a pure middleman mode or a pure market-maker mode. To simplify the analysis we abstract away from decentralized market trade, and assume zero marginal costs and zero entry costs.

Single-market search. With single market search, irrespective of the intermediation mode of E (and beliefs of agents on which intermediary to be favorable), I has no strict incentive to activate a platform with positive fees. To see this, let V^E be the buyer's value of buyers of visiting E. If I chooses to be a pure middleman then its profit is Bp^m with price $p^m = 1 - V^E$. If I activates a platform, then, just like in our benchmark setup, the fee should satisfy $f \leq 1 - V^E$ and so its maximum attainable profit with positive fees is strictly less than $B(1-V^E)$. The intuition remains the same as before — with single market search, the middlemen mode is the way to achieve the highest trade surplus.

Multiple-market search with a pure middleman E. To be consistent with the previous analysis, we assume that agents visit I prior to E by default. The idea behind this assumption is that I is a well-established intermediary in the market, whereas E is a newcomer which has no regular customers.

When E is a pure middleman with price $p^E \in [0,1]$, I's price/fee (p^m, f) should satisfy the incentive constraints,

$$p^m \leq p^E \quad \text{and} \quad f \leq p^E,$$

respectively. The major difference from the benchmark is that, as a pure middleman, E would undercut any positive price/fee of I and so an active platform with positive fees can never survive.

Multiple-market search with a pure market-maker E. When E is a pure market-maker with a fee f^E, the incentive constraints become

$$p^m \leq 1 - V^E(f^E) \quad \text{and} \quad f \leq 1 - V^E(f^E).$$

E could either act as a "second source" for intermediation service, or undercut I and be the "sole source". Given the strategic choice of E, a pure middleman I can not exist in equilibrium because it is only profit maximizing when buyers' outside option is zero. However, E has an incentive to undercut I, leading to a positive buyer's value at E. A pure market-making I can neither exist in equilibrium. This follows from the fact that E has an incentive to undercut I as long as the transaction fee is positive; at a fee of zero, E would rather increase the fee to the highest level and extract the full surplus, and I's best response is a pure middleman.

In the online appendix, we show that there exists a pure-strategy equilibrium when undercutting is costly for E. In equilibrium, E operates as the second source, and it adjusts f^E in a way that takes into account the responses of the participating buyers and sellers. Surprisingly, the transaction fee of E affects the intermediation mode of I. To see this, note that a lower f^E improves the outside option for buyers. The buyer now finds it more attractive to visit an individual seller on I's platform rather than I's middleman sector, since even if he is not matched at I, the outside option to trade on the platform of E (with a lower f^E) is a favorable prospect. As such, more buyers switch from I's frictionless middleman to I's frictional platform, leaving more unmatched buyers and less unmatched sellers joining E. Ultimately, this trade-off between more participating buyers (by decreasing f^E) and more participating sellers (by increasing f^E) leads to $f^E < 1$ and a positive value for buyers in equilibrium. Given the positive outside value, an active platform can better exploit I's intermediated surplus as we presented in the benchmark model. Hence, using an active platform with positive fees can be a profitable business mode for an intermediary when the other intermediary also activates a platform.

Proposition 8 *Consider two competing intermediaries, one is an incumbent (I), just like our original intermediary, and the other is an entrant (E) that replaces the D market. Then, I activates a platform with positive fees only when agents search in multiple markets and E also adopts an active platform.*

Proof. See details in the online appendix. ■

5 Examples

Our analysis shows that a marketmaking middleman is more likely to emerge under multi-market search technologies than under single-market technologies. In this section, we offer some real market examples.

Online retailers. The electronic commerce company Amazon.com is traditionally an online retailer, who mainly aims at selling its inventories to customers. In the late 1990s, Amazon was facing fierce competition from local brick and mortar rivals, as well as chain stores such as Walmart, Sears, etc., and especially from eBay. According to the book, *The Everything Store: Jeff Bezos and the Age of Amazon*, Jeff Bezos worried that eBay may become the leading online retailer who attracts the majority of customers. In the summer of 1998, he invited eBay's management team and suggested the possibility of a joint venture or even of buying out their business. This is perhaps Amazon's first attempt to set up an online marketplace. In the end, this trial failed. After several more trials and errors, however, Amazon finally launched their own marketplace in the early 2000s. The entry version of our model in Section 5.3 captures well Amazon's reaction to the entry by eBay.

Amazon's launch of the platform business influenced significantly the book industry. On the one hand, Amazon attracts many of its competitors to join their platform. Indeed, Amazon drove physical book and record stores out of business, and many bookstore owners re-launched their business on the Amazon-website platform. On the other hand, Amazon lowers the chance of buyers to trade outside. As local bookstores disappeared, it became the habit for most book buyers to start their everyday online-shopping using Amazon as the prime site De los Santos et al.

(2012). Overall, these observed phenomena are in line with our theory.[16][17] Not surprisingly, Amazon promoted this shopping pattern to customers in other product categories.

The general picture of the online travel agency industry is similar. Before the rise of Internet, most intermediaries in this industry acted as a pure middleman. In the middleman mode, hotels sell rooms to a middleman in bulk at discounted prices. The middleman then sells them to customers at a markup price. With the online reservation system, a market-making mode became popular, wherein hotels pay a market maker (e.g. Booking.com) commission fees upon successful reservations. The hotels post their services and prices on the platform. Expedia used to be a pure middleman but is nowadays a representative market-making middleman who employs both of these intermediation modes.

Specialist markets. The New York Stock Exchange (NYSE) is a specialist market, which is defined as a hybrid market that includes an auction component (e.g., a floor auction or a limit order book) together with one or more specialists (also called designated market makers). The specialists have some responsibility for the market: as brokers, they pair executable customer orders; and as dealers, they post quotes with reasonable depth Conroy and Winkler (1986).

As for their role as dealers in the exchanges, our model suggests that, at least for less active securities (represented by a lower outside option), the specialists' market can provide predictable immediacy and increase the trading volume and liquidity. This is consistent with the trend to adopt hybrid markets in derivative exchanges and stock markets around the world, especially for thinly-traded securities. For example, several European stock exchanges implemented a program which gives less active stocks an option of accompanying a designated dealer in the auction market. These initiatives were effective not only in enhancing the creation of hybrid specialist markets, but also in increasing trade volumes and reducing liquidity risks (Nimalendran and Petrella 2003,

[16] Nowadays, most buyers and sellers use Amazon as the main website (the first one to visit). On the seller side, according to a survey on Amazon sellers conducted in 2016, more than three-quarters of participants sell through multiple channels, online marketplaces, webstores and bricks-and-mortar stores. The second most popular channel, after Amazon, is eBay, with 73% selling through this marketplace. On the buyer side, according to a recent Reuters/Ipsos poll, 51 percent of consumers plan to do most of their shopping on the Amazon.com.

[17] An alternative (or complementary) to our theory would be a product selection story where Amazon uses the platform for third-party sellers to add new products with the demands too small for Amazon to offer. Once a product is "tested" to be popular enough, Amazon starts to also offer it through the middleman sector. This would be certainly a valid explanation but by far not the exclusive one. First, if this explanation were correct, we should eventually observe that most popular products are listed by Amazon, and most not-so-popular products are listed by independent sellers. In reality, however, many high-demand products are listed by both Amazon and third-party sellers at the same time, and importantly, they are competing with each other. This competition goes against the proposed explanation, but is more in line with our theory. In fact, Amazon could avoid fierce competition with strong competitors operating in the Amazon marketplace, such as GreenCupboards or independent sellers who own 'Buy Boxes', by giving up dealing with such a product in the middleman sector, which should in turn increase their fee revenue.

Anand et al. 2009, Menkveld and Wang 2013, and Venkataraman and Waisburd 2007.)

Another prediction from our analysis is related to the changing competitive environment faced by securities exchanges. As a broader implication, our result that the increased outside pressure goes hand in hand with more decentralized trades, captures the background trend in general: the market for NYSE-listed stocks was highly centralized in 2007 with the NYSE executing 79% of volume in its listings; in 2009, this share dropped to 25% (Securities and Exchange Commission 2010); today, the order-flow in NYSE-listed stocks is divided among many trading venues – 11 exchanges, more than 40 alternative trading systems, and more than 250 broker-dealers in the U.S. (Tuttle 2014). As a more specific implication, we show that the increased pressure from outside markets will scale up the platform component. This is indeed the case. Starting from 2006, the NYSE adopted the new hybrid trading system featuring an expanded platform sector "NYSE Arca", which allows investors to choose whether to trade electronically or by using traditional floor brokers and specialists. The new system is further supplemented by several dark pools, akin to platforms, owned by the NYSE. These strategies are also adopted by NASDAQ which has been thought of as a typical dealers' market. In addition, the use of fees is widely adopted, as is consistent with our theory. For instance, in 2014, the NYSE offered banks a discount of trading costs by more than 80% conditional on their agreement to stay away from the outside dark pools and other off-exchange venues.[18]

Real estate agencies. While intermediaries in housing markets are mostly thought of as brokers, i.e., platforms, the business mode employed by the Trump family is a marketmaking middleman. The Trump Organization holds several hundred thousand square feet of prime Manhattan real estate in New York City (NYC) and some more in other big cities. Besides developing and owning residential real estate, the Trump family operates a brokerage company that deals with luxury apartments, the Trump International Realty. Both of these companies target the same market in NYC. Indeed, the Trump's business mode is a marketmaking middleman – both owning his own residential towers, and offering broker services. According to Forbes, the latter portion of Trump's empire becomes by far his largest business with a valuation of 562 million in 2006. Another example is Thor Equities, a large-scale real estate company, which owns and redevelops retail properties in Soho, Madison Avenue, and Fifth Avenue, and also runs brokerage agencies,

[18] See a report "NYSE Plan Would Revamp Trading" in the Wall Street Journal, 2014. http://www.wsj.com/articles/intercontinental-exchange-proposing-major-stock-market-overhaul-1418844900.

Thor Retail Advisors and Town Residential.

In the endowment economy version of our model, we show that the marketmaking middleman over-invests in inventory with multi-market search, up to the point where the resource constraint is binding. Perhaps, the real estate market in NYC is an appropriate example of this since it is well known to be competitive and tight for house/apartment hunters. In addition, most new developments in big cities are renovations of old houses, and so we can roughly regard the total supply as fixed. Notably, top real estate firms in NYC attempt to expand their business by being engaged in many new joint projects with developers. Mapped into our model, these efforts are aimed at relaxing their resource constraint and increasing their inventory. For example, Nest Seekers, a real estate brokerage and marketing firm in NYC, works tightly with constructors on new developments. They work together from the very early stage of layout design and fund raising (in some cases Nest Seekers offers their own capital) to the later marketing stage. Nest Seekers provides qualified sales and administrative staff to the sales office, prepares pricing schedules, manages all contracts with the brokerage community, and is eventually in charge of the entire marketing process. This co-development business is one step beyond the middleman mode formulated in our theory, but is considered as an alternative way to secure their inventory.[19] This business mode is adopted in many other big real-estate companies in NYC, such as Douglas Elliman, Stribling, and Corcoran.

Finally note that some intermediaries do not only help to promote new developments, but also manage apartment complexes, which constitutes another source of "inventory". For example, Brown Harris Stevens provides residential management service for its customers since cooperative apartments were first introduced to NYC. These cooperative apartments usually contain hundreds of units in one building, and Brown Harris Stevens is then in charge of listing these properties when they are for rent or on sale.

6 Empirical evidence

The model's predictions on the choice of intermediation mode can be empirically tested. As in the last extension of competing intermediaries, we take Amazon as the centralized market and eBay

[19]Strictly speaking, Nest Seekers does not own properties, but becomes the exclusive agent of projects. So far, they have co-developed/marketed more than 30 projects. See https://www.nestseekers.com/NewDevelopments. A report titled "Inside the fight for Manhattans most valuable new development exclusives" by The Real Deal introduces more detailed information on how brokers cooperate with developers, which is available in http://therealdeal.com/2016/03/15/inside-the-fight-for-manhattans-most-valuable-new-development-exclusives/ (visited on July 15, 2016).

as the decentralized market. Our model predicts that Amazon is more likely to sell the product as a middleman when the chance of buyers to meet a seller in the decentralized market, λ^b, is low, the buyers' bargaining power β is low, and the total demand B is high. That is,

$$\Pr(\text{Amazon's middlemen mode is active}) = f(\underset{-}{\lambda^b}, \underset{-}{\beta}, \underset{+}{B}),$$

where $-$ $(+)$ indicates a negative (positive) correlation.

We collected data from *www.amazon.com* and *www.ebay.com*, focussing on one product category, namely pans. We think this product choice is appropriate not only because there are many observations for both eBay and Amazon but also because pans require some minimum consideration and search before a purchase decision is made. In addition, we analyze the theoretically most relevant case where Amazon acts as a marketmaking middleman: For 32% of the sample, Amazon acts as a middleman; for the other 68%, Amazon acts as a market maker. Our data matches each product on sale at Amazon to a list of the offers at eBay. Using information on individual prices and sellers, we construct proxies for key parameters in the model: λ^b, β and B as explained below (see the Online Appendix for more details). Since our data are not experimental, our evidence should be interpreted as suggestive rather than causal.

Table 1 summarizes various cross-sectional regressions of Amazon's intermediation mode, represented by *sellByAmazon*, which is a dummy variable that takes a value of 1 if the product is sold by Amazon. It is an indicator that Amazon is an active middleman for that product. Other variables we use for the linear regression in Column 1 are discussed below. *sellersEbayRelative* is a proxy for λ^b and is defined as the number of sellers on eBay divided by the number of third-party sellers on Amazon. *rank* is a proxy for total demand B and is defined a the sales rank within the product category. Rankings are negatively correlated with sales (e.g., a product with a rank value of 100 is associated with more sales than a product with a rank value 200). *priceDiff* is a proxy for β and is defined as the log of the median price of eBay offers minus the log of the Amazon price. *listedDays* controls for the number of days since the product was first listed on Amazon. Our theoretical model predicts that *sellByAmazon* is negatively correlated to *sellersEbayRelative*, negatively correlated to *log(rank)* and positively correlated to *priceDiff*. As shown in Table 1, all explanatory variables have the expected signs and are statistically significant in all the specifications (including those where we use alternative proxies and probit regressions).[20] To

[20]Recent work by Zhu and Liu (2018) also examines empirically the product choice by Amazon. While their

Table 1: Regressions for Amazon's intermediation mode

	(1) Linear	(2) Linear	(3) Probit	(4) Probit
sellersEbayRelative	-0.00630***		-0.00778***	
	(0.000765)		(0.00119)	
sellersEbayRefined		-0.00159**		-0.00156*
		(0.000595)		(0.000698)
sellersAmazon		-0.000552		-0.000669
		(0.000917)		(0.000996)
log(rank)	-0.103***	-0.107***	-0.105***	-0.110***
	(0.00442)	(0.00463)	(0.00500)	(0.00517)
priceDiff	0.105***	0.111***	0.124***	0.133***
	(0.00820)	(0.00830)	(0.0107)	(0.0111)
log(price)	0.0390***	0.0406***	0.0484***	0.0510***
	(0.00608)	(0.00613)	(0.00680)	(0.00688)
listedDays	0.0602***	0.0660***	0.0717***	0.0784***
	(0.00480)	(0.00486)	(0.00599)	(0.00604)
Observations	6457	6457	6457	6457
Adjusted R^2	0.136	0.130		

Note: Columns (1) and (2) use linear probability model, and columns (3) and (4) use probit model. For probit models, the marginal effects evaluated at the sample mean are reported. *sellersEbayRefined* is the number of sellers on a refined list of sellers on eBay by matching the title of offers with the Amazon product title and restricting the price of offers between 0.5 and 1.5 times the Amazon price. *sellersAmazon* is the number of third-party sellers on Amazon. Robust standard errors are reported in parentheses. Other variables are explained in the main text. * denotes $p < 0.05$, ** denotes $p < 0.01$, *** denotes $p < 0.001$.

quantify the effect of available options on eBay, we find that the chance that Amazon acts as a middleman decreases by 3.7 percent for a one-standard deviation increase in *sellersEbayRelative* (λ^b), and increases by 0.1 percent for a one percent increase in the median eBay price relative to the Amazon price (proxied by *priceDiff*, β). In the Web Appendix, we give more detailed information on the data and we experiment with a number of different specifications, but none of them alters our main results.

7 Conclusion

This paper developed a model in which market structure is determined endogenously by the choice of intermediation mode. We considered two representative business modes of intermediation that are widely used in real-life markets: a market-making mode and a middleman mode. We derived conditions for a mixture of the two modes, a *marketmaking middleman* to emerge.

One implication of our theory is that intermediaries can use a platform to reduce competition with sellers in the decentralized market. However, this is done by inducing consumers to search excessively and so generates inefficiencies. For future research, it would be interesting to examine this from the viewpoint of a regulator.

approach is very different from ours, it is interesting to note some common evidence. For instance, we find that the number of sellers on Amazon is negatively associated with the likelihood of Amazon to act as a middleman. This may reflect a crowding out effect of Amazon on third-party sellers. Similarly, Zhu and Liu (2016) find that Amazon's entry could discourage third-party sellers and eventually force them to leave the platform. Also, our evidence suggests that Amazon is more likely to sell more established products of higher prices. This is also consistent with Zhu and Liu (2018)'s findings that Amazon may be targeting successful products to exploit the surplus from third-party sellers.

Appendices

A Omitted proofs

A.1 Proof of Lemma 1

Using $K \leq x^m$ and (22), the intermediary's problem can be written as

$$\begin{aligned}
\max_{x^m,f,K} \Pi\left(x^m, f, K\right) &= S(1 - e^{-x^s})f + \min\left\{K, x^m\right\} p^m - Kc \\
&= S(1 - e^{-\frac{B-x^m}{S}})f + K(1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta)(1-c) - x^m e^{-\frac{B-x^m}{S}}\left(v(x^m, K) - f\right)
\end{aligned}$$

subject to (19) and

$$0 < K \leq x^m < B.$$

Observe that: $\lim_{x^m \to B} \Pi\left(x^m, f, K\right) = \tilde{\Pi}(B)$ and $\lim_{x^m \to 0} \Pi\left(x^m, f, K\right) = \tilde{\Pi}(0)$, where $\tilde{\Pi}(B) = B(1 - \lambda^b\beta)(1-c)$ is the profit for the pure middleman mode (20) and $\tilde{\Pi}(0) = S(1 - e^{-\frac{B}{S}})f$ is the profit for the pure market-maker mode (21). Hence, we can compactify the constraint set and set up a general problem to pin down a profit-maximizing intermediation mode using the following Lagrangian:

$$\mathcal{L} = \Pi\left(x^m, f, K\right) + \mu_k(x^m - K) + \mu_b(B - x^m) + \mu_v\left(v(x^m, K) - f\right) + \mu_0 K,$$

where the μ's ≥ 0 are the lagrange multiplier of each constraint. In the proof of Proposition 2, we show that the following first order conditions are necessary and sufficient:

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi\left(x^m, f, K\right)}{\partial x^m} + \mu_k - \mu_b + \mu_v \frac{\partial v\left(x^m, K\right)}{\partial x^m} = 0, \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial f} = \frac{\partial \Pi\left(x^m, f, K\right)}{\partial f} - \mu_v = 0, \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi\left(x^m, f, K\right)}{\partial K} - \mu_k + \mu_0 + \mu_v \frac{\partial v\left(x^m, K\right)}{\partial K} = 0. \tag{33}$$

The solution is characterized by these and the complementary slackness conditions of the four constraints.

We now prove the claims in the lemma. First, (32) implies that we must have

$$\mu_v = S(1 - e^{-x^s}) + x^m e^{-x^s} > 0,$$

which implies the binding constraint (19),

$$f = v\left(x^m, K\right) = \left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^s\left\{1 - \frac{K}{B} - \frac{S}{B}(1 - e^{-\frac{B-x^m}{S}})\right\}(1-\beta)\right](1-c).$$

Second, applying μ_v from (32) into (33) gives

$$\mu_k = \left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta + \lambda^b\left(1 - e^{-\frac{B-x^m}{S}}\right)(1-\beta)\right](1-c) + \mu_0 > 0,$$

which implies that $K = x^m$. This completes the proof of Lemma 1. ■

A.2 Proof of Proposition 2

⊙ Active platform. First of all, we show that the platform will always be active (i.e., $x^m < B$) in equilibrium. Substituting μ_k, μ_v into (31),

$$\begin{aligned}
(1-c)^{-1}(\mu_b - \mu_0) &= -e^{-\frac{B-x^m}{S}}\left[1 - \lambda^b e^{-\frac{B-x^m}{S}}\beta - \lambda^b\left\{\frac{B}{S} - (1 - e^{-\frac{B-x^m}{S}})\right\}(1-\beta)\right] \\
&\quad -\lambda^b \frac{x^m}{S} e^{-\frac{B-x^m}{S}} + 1 - \lambda^b\beta + \lambda^b(1 - e^{-\frac{B-x^m}{S}})^2 \\
&\equiv \phi(x^m \mid B, S, \beta, \lambda^b).
\end{aligned} \tag{34}$$

Suppose that the solution is $x^m = B$. Then, (34) yields $\phi(B \mid \cdot) = (1-c)^{-1}\mu_b = -\frac{B}{S}\lambda^b\beta < 0$, which contradicts $\mu_b \geq 0$. Hence, the solution must satisfy $x^m < B$ (which implies $\mu_b = 0$).

$\odot$ Market-making middleman or pure market-maker. Second, we derive the condition for a pure market-maker $x^m = 0$ or a market-making middleman $x^m > 0$. Since $\phi(B \mid \cdot) < 0$, if $\phi(0 \mid \cdot) > 0$, there exists $x^m \in (0, B)$ that satisfies $\phi(x^m \mid \cdot) = 0$, i.e., a market-making middleman. Further,

$$\frac{\partial\phi(x^m \mid \cdot)}{\partial x^m}\mid_{\phi=0} = -\frac{1}{S}\left[1-\lambda^b\beta+\lambda^b(1-e^{-\frac{B-x^m}{S}})^2+2\lambda^b(1-e^{-\frac{B-x^m}{S}})e^{-\frac{B-x^m}{S}}\right]-\frac{\lambda^b}{S}e^{-\frac{B-x^m}{S}}(1-e^{-\frac{B-x^m}{S}})<0.$$

This implies that the allocation of the middleman sector $x^m \in (0, B)$ is unique (if it exists), and that if $\phi(0 \mid \cdot) < 0$ then $\phi(x^m \mid \cdot) < 0$ for all $x^m \in [0, B]$ and the solution must be a pure market maker, $x^m = 0$.

Now, we need to investigate the sign of it:

$$\begin{aligned}\phi(0 \mid B, S, \beta, \lambda^b) &= -e^{-x}\left[1-\lambda^b e^{-x}\beta-\lambda^b\left(x-1+e^{-x}\right)(1-\beta)\right]+1-\lambda^b\beta+\lambda^b(1-e^{-x})^2\\ &\equiv \Theta(x),\end{aligned}$$

where $x \equiv \frac{B}{S}$. Observe that:

$$\Theta(0) = 0 < 1-\lambda^b\beta+\lambda^b = \Theta(\infty),$$

and

$$\frac{\partial\Theta(x)}{\partial x} = e^{-x}\left[1-\lambda^b x+\lambda^b\beta(x-2)+4\lambda^b(1-e^{-x})\right].$$

This derivative has the following properties: $\frac{\partial\Theta(x)}{\partial x}\mid_{x=0} = 1-2\lambda^b\beta$;

$$\frac{\partial\Theta(x)}{\partial x}\mid_{\Theta(x)=0} = 1-\lambda^b\beta(1+e^{-x})+\lambda^b(1-e^{-x})(1+2e^{-x}) \equiv \Upsilon(x).$$

There are two cases.

- When $\lambda^b\beta \leq \frac{1}{2}$, we have $\frac{\partial\Theta(x)}{\partial x}\mid_{x=0} \geq 0$ and $\frac{\partial\Theta(x)}{\partial x}\mid_{\Theta(x)=0} > 0$, implying that no $x \in (0, \infty)$ exists such that $\Theta(x) = 0$. Hence, $\Theta(x) = \phi(0 \mid \cdot) > 0$ for all $x \in (0, \infty)$.
- When $\lambda^b\beta > \frac{1}{2}$, we have $\frac{\partial\Theta(x)}{\partial x}\mid_{x=0} < 0$. Hence, there exists at least one $\bar{x} \in (0, \infty)$ such that $\Theta(x) < 0$ for $x < \bar{x}$ and $\Theta(x) \geq 0$ for $x \geq \bar{x}$. Below we show that such a value has to be unique. For this purpose, observe that:

$$\begin{aligned}&\Upsilon(0) = 1-2\lambda^b\beta < 0 < 1+\lambda^b(1-\beta) = \Upsilon(\infty), \frac{\partial\Upsilon(x)}{\partial x} = \lambda^b e^{-x}(4e^{-x}-1+\beta),\\ &\frac{\partial\Upsilon(x)}{\partial x}\mid_{x=0} = \lambda^b(3+\beta) > 0, \frac{\partial^2\Upsilon(x)}{\partial x^2}\mid_{\frac{\partial\Upsilon(x)}{\partial x}=0} = -4e^{-x}\lambda^b e^{-x} < 0.\end{aligned}$$

  These properties imply that there exists an $x' \in (0, \infty)$ such that $\Upsilon(x) <$ for all $x < x'$ and $\Upsilon(x) \geq 0$ for all $x \geq x'$. This implies that $\bar{x}$ is unique.

To summarize, we have shown that if $\lambda^b\beta \leq \frac{1}{2}$ then the solution is a market-making middleman $x^m \in (0, B)$ for all $x = \frac{B}{S} \in (0, \infty)$. If $\lambda^b\beta > \frac{1}{2}$ then there exists a unique critical value $\bar{x} \in (0, \infty)$ such that the solution is a market-making middleman for $x \geq \bar{x}$ and is a pure market-maker $x^m = 0$ for $x < \bar{x}$.

$\odot$ Second order condition. Finally, we verify the second order condition. Define $\mathbf{X} \equiv [x^m, f, K]$ and write the binding constraints as

$$h_1(\mathbf{X}) = v(x^m, K) - f,\ h_2(\mathbf{X}) = x^m - K.$$

The solution characterized above is a maximum if the Hessian of $\mathcal{L}$ with respect to $\mathbf{X}$ at the solution denoted by $(\mathbf{X}^*, \mu^*)$ is negative definite on the constraint set $\{\mathbf{w} : \mathbf{Dh}(\mathbf{X}^*)\mathbf{w} = 0\}$ with $\mathbf{h} \equiv [h_1(\mathbf{X}), h_2(\mathbf{X})]$. This can be verified by using the bordered Hessian matrix, denoted by H.

$$
\begin{aligned}
H &\equiv \begin{bmatrix} 0 & D\mathbf{h}(\mathbf{X}^*) \\ D\mathbf{h}(\mathbf{X}^*)^T & D^2_{\mathbf{X}}\mathcal{L}(\mathbf{X}^*, \mu^*) \end{bmatrix} \\
&= \begin{bmatrix}
0 & 0 & \frac{\partial h_1}{\partial x^m} & \frac{\partial h_1}{\partial f} & \frac{\partial h_1}{\partial K} \\
0 & 0 & \frac{\partial h_2}{\partial x^m} & \frac{\partial h_2}{\partial f} & \frac{\partial h_2}{\partial K} \\
\frac{\partial h_1}{\partial x^m} & \frac{\partial h_2}{\partial x^m} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f\partial x^m} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K\partial x^m} \\
\frac{\partial h_1}{\partial f} & \frac{\partial h_2}{\partial f} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m\partial f} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f^2} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K\partial f} \\
\frac{\partial h_1}{\partial K} & \frac{\partial h_2}{\partial K} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m\partial K} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial f\partial K} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial K^2}
\end{bmatrix} \\
&= \begin{bmatrix}
0 & 0 & -\frac{\lambda^b}{S}e^{-x^s}(1-c) & -1 & \frac{\lambda^s}{B}(1-\beta)(1-c) \\
0 & 0 & 1 & 0 & -1 \\
-\frac{\lambda^b}{S}e^{-x^s}(1-c) & 1 & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{x^m}{S}e^{-x^s} & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m\partial K} \\
-1 & 0 & \frac{x^m}{S}e^{-x^s} & 0 & 0 \\
\frac{\lambda^s}{B}(1-\beta)(1-c) & -1 & \frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m\partial K} & 0 & 0
\end{bmatrix}
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} &= -\frac{1}{S}e^{-x^s}v + \left(-\frac{1}{S}\frac{x^m}{S}\lambda^b e^{-x^s}\beta + 2\left(1+\frac{x^m}{S}\right)e^{-x^s}\frac{\lambda^b}{S}e^{-x^s} - \frac{\lambda^b}{S}\left(1-e^{-x^s}\right)e^{-x^s}\right)(1-c), \\
\frac{\partial^2\mathcal{L}(\mathbf{X}^*,\mu^*)}{\partial x^m\partial K} &= -\left(\frac{\lambda^b}{S}e^{-x^s}\beta + \left(1+\frac{x^m}{S}\right)e^{-x^s}\frac{\lambda^s}{B}(1-\beta)\right)(1-c).
\end{aligned}
$$

The determinant is given by $|H| = -\frac{1}{S}\left[e^{-x^s}v(x^m, K^*) + \frac{x^m}{S}\lambda^b e^{-x^s}\beta(1-c) + 3\lambda^b e^{-x^s}\left(1-e^{-x^s}\right)(1-c)\right] < 0$. Thus, the sufficient condition is satisfied. This completes the proof of Proposition 2. ■

A.3 Proof of Corollary 1

In (34), we have:

$$
\begin{aligned}
\frac{\partial\phi(x^m \mid ., ., \beta, .)}{\partial\beta}\,|_{(\phi(x^m|\cdot)=0)} &= -\lambda^b(1-e^{-2x^s}) - \lambda^b e^{-x^s}\left(\frac{B}{S} - 1 + e^{-x^s}\right) < 0, \\
\frac{\partial\phi(x^m \mid B, ., ., .)}{\partial B}\,|_{(\phi(x^m|\cdot)=0)} &= \frac{1}{S}\left[1 + \lambda^b(1-\beta) - \lambda^b e^{-2x^s} + \lambda^b e^{-x^s}(1-e^{-x^s})\right] > 0 \\
\frac{\partial\phi(x^m \mid ., S, ., .)}{\partial S}\,|_{(\phi(x^m|\cdot)=0)} &= -\frac{x^s}{S}\left[1 + \lambda^b(1-\beta) - \lambda^b e^{-2x^s} + \lambda^b e^{-x^s}\left(\frac{B}{x^s} - e^{-x^s}\right)\right] < 0 \\
\frac{\partial\phi(x^m \mid ., ., ., \lambda^b)}{\partial\lambda^b}\,|_{(\phi(x^m|\cdot)=0)} &= -\frac{1-e^{-x^s}}{\lambda^b} < 0.
\end{aligned}
$$

Hence, since $\frac{\partial\phi(x^m|\cdot)}{\partial x^m}\,|_{(\phi(x^m|\cdot)=0)} < 0$ (see the proof of Proposition 2), it follows that: $\frac{\partial x^m}{\partial\beta} < 0$; $\frac{\partial x^m}{\partial B} < 0$; $\frac{\partial x^m}{\partial S} > 0$; $\frac{\partial x^m}{\partial\lambda^b} < 0$. This completes the proof of Corollary 1. ■

A.4 Proof of Proposition 3

The proof takes steps that are very similar to the ones we made in the proof of Proposition 1. With the non-linear matching function, the intermediary's profit function is modified to

$$
\begin{aligned}
\Pi(x^m, f, K) &= S(1-e^{-x^s})f + \min\{K, x^m\}p^m \\
&= S(1-e^{-\frac{B-x^m}{S}})f + K(1-\lambda^b(x^D)\beta) - x^m e^{-\frac{B-x^m}{S}}(v(x^m, K) - f),
\end{aligned}
$$

where $x^D = \frac{\max\{B-\min\{x^m,K\}-S(1-e^{-x^s}),0\}}{Se^{-x^s}}$, and the surplus function to

$$
v(x^m, K) = 1 - \lambda^b(x^D)\beta - \lambda^s(x^D)(1-\beta).
$$

With these profit and surplus functions, the constraints and the Lagrangian remain unchanged, and the first orders are given by (31) – (33) (the second order conditions are presented below). As before, (32) implies that we must have

$$\mu_v = S(1 - e^{-x^s}) + x^m e^{-x^s} > 0,$$

and the binding constraint (19). Further, substituting μ_v from (32) into (33) gives

$$\mu_k = \mu_0 + 1 - \lambda^b(x^D)\beta + \frac{K}{Se^{-x^s}}\lambda^{b\prime}(x^D)\beta + \frac{1-e^{-x^S}}{e^{-x^s}}\left(\lambda^{b\prime}(x^D)\beta + (\lambda^b(x^D) + x^D\lambda^{b\prime}(x^D))(1-\beta)\right). \quad (35)$$

Substituting μ_k, μ_v into (31) gives,

$$\begin{aligned}\mu_b &= \mu_0 - e^{-x^s}\left(1 - \lambda^b(x^D)\beta - \lambda^b(x^D)x^D(1-\beta)\right) + 1 - \lambda^b(x^D)\beta + \frac{B-K}{S}\frac{K}{Se^{-x^s}}\lambda^{b\prime}(x^D)\beta \\ &\quad + \frac{B-K}{S}\frac{1-e^{-x^S}}{e^{-x^s}}\left(\lambda^{b\prime}(x^D)\beta + (\lambda^b(x^D) + x^D\lambda^{b\prime}(x^D))(1-\beta)\right). \end{aligned} \quad (36)$$

Suppose now that $x^m = B$ and $K > 0$. Then, $\mu_k > 0$ in (35) if and only if

$$1 - \lambda^b(x^D)\beta + \frac{K}{S}\lambda^{b\prime}(x^D)\beta > 0,$$

and $\mu_b \geq 0$ in (36) if and only if

$$\frac{B-K}{S}\left[(1-\beta)\lambda^b(x^D) + \frac{K}{S}\lambda^{b\prime}(x^D)\beta\right] \geq 0,$$

with $x^D = \frac{B-K}{S}$. Both of these conditions are satisfied only when $K = B$ (which implies $x^D = 0$, satisfying the latter condition) and

$$1 - \lambda^b(0)\beta + \frac{B}{S}\lambda^{b\prime}(0)\beta > 0 \quad (37)$$

(satisfying the former condition with $x^D = 0$). Under this condition, the solution is unique, $K = B = x^m$, $x^s = 0$ and $f = v(B, B)$. Hence, we have shown that the solution can be a pure middleman $x^s = 0$ only if (37) holds and otherwise the solution must be $x^s > 0$ (either a marketmaking middleman or a pure marketmaker).

Finally, we verify the second order condition. With the modified profit and surplus functions, as before, the bordered Hessian matrix is given by

$$\begin{aligned} H &\equiv \begin{bmatrix} 0 & D\mathbf{h}(\mathbf{X}^*) \\ D\mathbf{h}(\mathbf{X}^*)^T & D^2_{\mathbf{X}}L(\mathbf{X}^*, \mu^*) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & \frac{\partial h_1}{\partial x^m} & \frac{\partial h_1}{\partial f} & \frac{\partial h_1}{\partial K} \\ 0 & 0 & \frac{\partial h_2}{\partial x^m} & \frac{\partial h_2}{\partial f} & \frac{\partial h_2}{\partial K} \\ \frac{\partial h_1}{\partial x^m} & \frac{\partial h_2}{\partial x^m} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f \partial x^m} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K \partial x^m} \\ \frac{\partial h_1}{\partial f} & \frac{\partial h_2}{\partial f} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial f} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f^2} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K \partial f} \\ \frac{\partial h_1}{\partial K} & \frac{\partial h_2}{\partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial f \partial K} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial K^2} \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & \frac{\partial v(\mathbf{X}^*)}{\partial x^m} & -1 & \frac{\partial v(\mathbf{X}^*)}{\partial K} \\ 0 & 0 & 1 & 0 & -1 \\ \frac{\partial v(\mathbf{X}^*)}{\partial x^m} & 1 & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^{m2}} & \frac{B}{S} & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} \\ -1 & 0 & \frac{B}{S} & 0 & 0 \\ \frac{\partial v(\mathbf{X}^*)}{\partial K} & -1 & \frac{\partial^2 L(\mathbf{X}^*,\mu^*)}{\partial x^m \partial K} & 0 & 0 \end{bmatrix} \end{aligned}$$

with

$$\begin{aligned} \frac{\partial^2 L(\mathbf{X}^*, \mu^*)}{\partial x^{m2}} &= -\frac{1}{S}f - B\frac{\partial^2 \lambda^b}{\partial x^{m2}}(\mathbf{X}^*)\beta - (2 + \frac{B}{S})\frac{\partial v(\mathbf{X}^*)}{\partial x^m}, \\ \frac{\partial^2 L(\mathbf{X}^*, \mu^*)}{\partial x^m \partial K} &= -\frac{\partial \lambda^b}{\partial x^m}\beta - B\frac{\partial^2 \lambda^b}{\partial x^m \partial K}\beta - \frac{B}{S}\frac{\partial v(\mathbf{X}^*)}{\partial K}. \end{aligned}$$

The determinant is $|H| = -\frac{1}{S}(1 - \lambda^b(0)\beta) - \frac{B}{S^2}\lambda^{b\prime}(0)\beta < 0$. This completes the proof of Proposition 3. ■

A.5 Proof of Proposition 4

As stated in the main text, for $\alpha S \geq B$ the intermediary can achieve the highest possible profit by choosing to be a pure middleman. What remains here is to prove the proposition for $\alpha S < B$. Applying the analysis in the previous section, we derive that the seller value equals, $W^C(x^s) = (1-e^{-x^s}-x^s e^{-x^s})(1-f)$ and the indifferent condition for buyers becomes, $V^m(x^m) = V^s(x^s)$ where $V^m(x^m) = \min\{\frac{K}{x^m}, 1\}(1-p^m)$ and $V^s(x^s) = e^{-x^s}(1-f)$. The binding participation constraint for buyers implies that $p^m = 1 - \frac{\lambda^b}{\min\{\frac{K}{x^m},1\}}$ and $f = 1 - \frac{\lambda^b}{e^{-x^s}}$, and the binding condition, (25), implies that $p^w = (1-e^{-x^s}-x^s e^{-x^s})\frac{\lambda^b}{e^{-x^s}}$.

To guarantee $f \geq 0$, it is sufficient to assume that

$$\lambda^b \leq e^{-\frac{B-\alpha S}{S}}.$$

This also guarantees $p^m - p^w = 1 - \lambda^b(1 - \frac{1-e^{-x^s}-x^s e^{-x^s}}{e^{-x^s}}) > 0$ and that profits are non negative.

Using all these expressions of prices and fee, we can write the profit function as

$$\Pi(x^m, K) = (S-K)(1-e^{-x^s})(1-\frac{\lambda^b}{e^{-x^s}}) + \min\{K, x^m\} - x^m\lambda^b - K(1-e^{-x^s}-x^s e^{-x^s})\frac{\lambda^b}{e^{-x^s}},$$

where $x^s = \frac{B-x^m}{S-K}$. Differentiation yields

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} = \frac{S}{S-K}\frac{1-e^{-x^s}}{e^{-x^s}}\lambda^b + \frac{\partial \min\{K, x^m\}}{\partial x^m} - e^{-x^s}, \tag{38}$$

which is positive if $\min\{K, x^m\} = x^m$. Hence, the solution has to satisfy $x^m \geq K$.

Observe that: $\lim_{x^m \to B} \Pi(x^m, K) = \Pi$ and $\lim_{x^m \to 0} \Pi(x^m, K) = \tilde{\Pi}(0)$, where $\Pi = \alpha S - B\lambda^b$ is the profit for the pure middleman mode and $\tilde{\Pi}(0) = S(1-e^{-\frac{B}{S}})(1-\frac{\lambda^b}{e^{-\frac{B}{S}}})$ is the profit for the pure market-maker mode. Hence, as before, we can find a profit-maximizing intermediation mode using the following Lagrangian:

$$\mathcal{L} = \Pi(x^m, K) + \mu_k(x^m - K) + \mu_b(B - x^m) + \mu_0 K + \mu_s(\alpha S - K).$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi(x^m, K)}{\partial x^m} + \mu_k - \mu_b = 0, \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi(x^m, K)}{\partial K} - \mu_k + \mu_0 - \mu_s = 0, \tag{40}$$

where

$$\frac{\partial \Pi(x^m, K)}{\partial K} = e^{-x^s} + x^s e^{-x^s} - \frac{S}{S-K}\frac{1-e^{-x^s}}{e^{-x^s}}\lambda^b x^s.$$

Suppose $x^m = B$. Then, we must have $\mu_k = 0$ (since $B > \alpha S \geq K$) and so (39) implies we also must have $\frac{\partial \Pi(x^m,K)}{\partial x^m} |_{(x^m=B)} = \mu_b \geq 0$. However, $\frac{\partial \Pi(x^m,K)}{\partial x^m} |_{(x^m=B)} = -1 < 0$, a contradiction. Hence, the solution must satisfy $x^m < B$ (and $\mu_b = 0$), i.e., an active platform.

Summing up the two first order conditions with $\mu_b = 0$,

$$\begin{aligned}
\mu_s - \mu_0 &= \frac{\partial \Pi(x^m, K)}{\partial K} + \frac{\partial \Pi(x^m, K)}{\partial x^m} \\
&= x^s e^{-x^s} + (1-x^s)\frac{S}{S-K}\frac{1-e^{-x^s}}{e^{-x^s}}\lambda^b \\
&= -x^s \frac{\partial \Pi(x^m, K)}{\partial x^m} + \frac{S}{S-K}\frac{1-e^{-x^s}}{e^{-x^s}}\lambda^b > 0
\end{aligned}$$

where the last inequality follows from (39) and $\mu_b = 0$ that implies $\frac{\partial \Pi(x^m,K)}{\partial x^m} = -\mu_k \leq 0$. This implies $\mu_s > 0$, i.e., the binding resource constraint (24), which implies $K = \alpha S$. This completes the proof of Proposition 4. ■

A.6 Proof of Proposition 5

In our endowment economy, the middleman's inventory purchase influences market tightness not only in the C market platform, but also in the D market. Given all sellers are in the D market, the probability that a buyer meets a seller available for trade in the D market changes from $\lambda^b e^{-x^s}$ to $\lambda^b \frac{S-K}{S} e^{-x^s}$. With this change and using the analysis of multi-market search shown in the previous section, the value of sellers, $W^C(x^s) = (1 - e^{-x^s} - x^s e^{-x^s})(v(x^m, K) - f)$ and the middleman's price, $p^m = 1 - \lambda^b \frac{S-K}{S} e^{-x^s} - \frac{x^m e^{-x^s}}{\min\{x^m, K\}}(v(x^m, K) - f)$, where $v(x^m, K) = 1 - \lambda^b \frac{S-K}{S} e^{-x^s}$. Substituting these expressions into the profit function, it becomes immediate that the profit is strictly increasing in the fee f. Hence, the incentive constraints are binding, $f = v(x^m, K)$. Using this result, we can write the profit function as

$$\Pi(x^m, K) = (S-K)(1 - e^{-x^s})(1 - \lambda^b \frac{S-K}{S} e^{-x^s}) + \min\{K, x^m\}(1 - \lambda^b \frac{S-K}{S} e^{-x^s}),$$

where $x^s = \frac{B - x^m}{S - K}$. Differentiation yields

$$\begin{aligned} \frac{\partial \Pi(x^m, K)}{\partial x^m} &= -e^{-x^s} \left(1 - \lambda^b \frac{S-K}{S} e^{-x^s} - \lambda^b \frac{S-K}{S}(1 - e^{-x^s})\right) - \frac{\min\{K, x^m\}}{S} \lambda^b e^{-x^s} \\ &\quad + \frac{\partial \min\{K, x^m\}}{\partial x^m} \left(1 - \lambda^b \frac{S-K}{S} e^{-x^s}\right), \end{aligned}$$

which is negative if $\min\{K, x^m\} = K$. Hence, the solution has to satisfy $x^m \leq K$.

Suppose $x^m = B$. Then,

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} |_{x^m = B} = -\frac{B}{S} \lambda^b < 0.$$

Hence, the solution has to be $x^m < B$, i.e., an active platform.

The Lagrangian the becomes,

$$\mathcal{L} = \Pi(x^m, K) + \mu_0 x^m + \mu_k (K - x^m) + \mu_s (\alpha S - K).$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi(x^m, K)}{\partial x^m} + \mu_0 - \mu_k = 0, \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial K} = \frac{\partial \Pi(x^m, K)}{\partial K} + \mu_k - \mu_s = 0, \tag{42}$$

where

$$\begin{aligned} \frac{\partial \Pi(x^m, K)}{\partial K} &= x^s e^{-x^s} \left(1 - \lambda^b \frac{S-K}{S} e^{-x^s} + x^s \lambda^b \frac{S-K}{S}(1 - e^{-x^s})\right) + \frac{x^s x^m}{S} \lambda^b e^{-x^s} \\ &\quad + \left(1 - e^{-x^s}\right)\left(1 - 2\lambda^b \frac{S-K}{S} e^{-x^s}\right) + \frac{x^m}{S} \lambda^b e^{-x^s}. \end{aligned}$$

Combining (41) and (42),

$$\frac{\partial \Pi(x^m, K)}{\partial x^m} + \frac{\partial \Pi(x^m, K)}{\partial K} = x^s e^{-x^s} \left[\left(1 - \lambda^b \frac{S-K}{S} e^{-x^s}\right) + \frac{S-K}{S}(1 - e^{-x^s})\lambda^b + \frac{x^m}{S}\lambda^b\right] = \mu_s - \mu_0,$$

which implies $\mu_s > 0$ and $K = \alpha S$. This completes the proof of Proposition 5. ■

A.7 An analysis on the extension with inventory costs

⊙ Single-market search. The analysis in this section is under the participation allocation in Definition 5. With single-market search, the problem of a middleman can be described as

$$\tilde{\Pi}(K) = \max_{p^m, K} \min\{K, B\} p^m - C(K) \quad \text{s.t. } V^m = 1 - p^m \geq \lambda^b.$$

The solution is $p^m = 1 - \lambda^b$ and $K = B$ (as $C'(K) < 1 - \lambda^b$ for $K < B$), and so the middleman's profit is $\tilde{\Pi}(B) = B(1 - \lambda^b) - C(B)$. When a platform is open, individual sellers solve:

$$\max_x (1 - e^{-x})(p - f^s) \text{ s.t. } V^s = \frac{1 - e^{-x}}{x}(1 - p - f^b)$$

The optimal solution is described as $V^s = e^{-x}(1 - f)$ with $f \equiv f^b + f^s$. In an equilibrium with a market-making middleman, it has to hold that $x = x^s$ and $V^s = V^m = 1 - p^m \geq \lambda^b$, which implies $p^m = 1 - \lambda^b$ and $f = 1 - \lambda^b e^{x^s}$. Notice that optimally $K = x^m$. Hence, the problem of a market-making middleman is given by (43), with $\lim_{x^m \to B} \Pi(x^m) = \tilde{\Pi}(B)$.

$$\max_{x^m, f} \Pi(x^m) = S(1 - e^{-x^s})f(x^s) + x^m p^m - C(x^m). \tag{43}$$

subject to $p^m = 1 - \lambda^b$ and $f(x^s) = 1 - \lambda^b e^{x^s}$. The first order condition becomes

$$\frac{\partial \Pi(x^m)}{\partial x^m} = -e^{-x^s} + \lambda^b e^{x^s} + 1 - \lambda^b - C'(x^m) \equiv \Theta_{Sfoc'}(x^m) = 0. \tag{44}$$

Observe in (44) that: $\Theta_{Sfoc'}(B) = -C'(B) < 0$ and $\Theta'_{Sfoc'}(x^m) = -\frac{1}{S}(e^{-x^s} + \lambda^b e^{x^s}) - C''(x^m) < 0$. Therefore, a market-making middleman is profit-maximizing if

$$\Theta_{Sfoc'}(0) = 1 - e^{-\frac{B}{S}} + \lambda^b(e^{\frac{B}{S}} - 1) - C'(0) > 0. \tag{45}$$

Otherwise, the pure market-maker mode is selected.

⊙ Multi-market search. As in the benchmark, $K = x^m$ in equilibrium. With multi-market search, the only modification is to introduce the marginal inventory cost $C'(x^m)$ in the first order condition:

$$\Theta_{Mfoc}(x^m) \equiv (1 - e^{-x^s})(1 - 2\lambda^b e^{-x^s}) - \frac{\lambda^b x^m e^{-x^s}}{S} - C'(x^m) = 0. \tag{46}$$

where the optimal intermediation mode is determined as detailed in the main text.

⊙ Comparison. Comparing (45) and (46),

$$\Theta_{Mfoc}(0) \quad = \quad \Theta_{Sfoc'}(0) - \lambda^b \left[2e^{-x^s}(1 - e^{-x^s}) + e^{x^s} - 1\right] < \Theta_{Sfoc}(0),$$

implying that whenever a middleman sector is active, i.e., $x^m > 0$, with multi-market search, it has to be active with single-market search as well.

Furthermore, observe that

$$\begin{aligned} \Theta_{Mfoc}(x^m) &= (1 - e^{-x^s})(1 - 2\lambda^b e^{-x^s}) - \frac{\lambda^b x^m e^{-x^s}}{S} - C'(x^m) \\ &< (1 - e^{-x^s})(1 - 2\lambda^b e^{-x^s}) - C'(x^m) \\ &= \Theta_{Sfoc'}(x^m) - \lambda^b \left[2e^{-x^s}(1 - e^{-x^s}) + e^{x^s} - 1\right] \\ &< \Theta_{Sfoc'}(x^m). \end{aligned} \tag{47}$$

This implies that the marginal profit of increasing the size of the middleman sector is smaller with multi-market search than with single-market search. That is, whenever a market-making middleman mode is selected, the platform sector x^s $(= \frac{B - x^m}{S})$ is always larger with multi-market search than single-market search. The logic behind this is essentially the same as in the baseline model, and is summarized as follows.

Proposition 9 *Consider the convex inventory costs of a middleman defined in the main text. Then, a platform is activated even under a single-market search technology. Still, the size of the platform with multi-market search is larger than or equal to that with single-market search.*

B Web Appendix: Competing Intermediaries

B.1 Set-ups

This appendix is an extension on competing intermediaries that explains the emergence of market-making middlemen in a duopoly. We maintain our assumptions on buyers/sellers and replace the decentralized market by a second intermediary. More specifically, we consider two intermediaries that are open to agents, call them an incumbent I and an entrant E, and assume that buyers and sellers can only meet via an intermediary. We analyze both the single and the multiple markets search.

Intermediary I. Intermediary I operates in the same way as the C market intermediary in the benchmark model. It can combine the middleman and the marketmaker modes and it is subject to coordination frictions. The middleman has an inventory advantage compare to an individual seller — it is able to hold a continuum of inventory to lower the out-of-stock risk. For simplicity, we assume the middleman has continuous access to the production technologies. That is, the middleman can produce as it obtains an order from a buyer. Effectively, the out-of-stock probability is zero.[21]

Intermediary E. We consider two scenarios. In one scenario, E is a pure middleman, who has continuous access to the production technologies and strategically sets its inventory price $p^E \in [0, 1]$. In another scenario, E operates as a pure marketmaker who owns the random matching technologies as in the D market of the benchmark model, and it makes profits by extracting a transaction fee denoted by $f^E \in [0, 1]$.[22] The matched buyer and seller engage in an efficient bargaining process. And we focus on the case that the buyer gets the full trading surplus ($\beta = 1$). The intermediation mode of E, either a pure middleman or a pure marketmaker, is exogenously given.

Timing. The timing of the game is as follows. (1) Two intermediaries set fees/prices. Intermediary I decides whether or not to activate the middleman sector and/or the platform and announces a transaction fee $f^I \in [0, 1]$ charged to a seller if the platform is open, and an inventory price $p^m \in [0, 1]$ charged to a buyer if the middleman sector is open. Intermediary E announces a transaction fee $f^E \in [0, 1]$ charged to a seller if it is a pure market-maker or an inventory price $p^E \in [0, 1]$ if it is a pure middleman. Let's denote these prices/fees by $\mathcal{P} = \{f^I, p^m, f^E\}$ or $\{f^I, p^m, p^E\}$. (2) Observing $\mathcal{P}^\rangle$, buyers and sellers simultaneously decide whether to participate in both (or one out of two) intermediaries, yielding a distribution of buyers/sellers across intermediaries. (3) At intermediary I, the participating buyers, sellers and the middleman (if any) are engaged in a directed search game as specified in the main text. At intermediary E, agents

[21] An alternative interpretation of the inventory technologies is that individual sellers have to *produce in advance* (up to the inventory constraint normalized to 1 unit) whereas middlemen can *produce to orders*. Suppose the middleman has to "produce in advance" and holds a mass K of inventory. Given a zero inventory cost, it is a weakly dominant strategy to have K larger or equal to the mass of visiting buyers. This gives a zero out-of-stock probability.

[22] Without loss of generality, we assume the transaction fee is imposed on the seller.

search randomly and follow the efficient sharing rule for the trade surplus if E is a pure platform, or buyers trade directly with E if E is a pure middleman.

The equilibrium concept. Let $\mathcal{N} = \{B^I, B^E, S^I, S^E, x^m\}$ denote a distribution of buyers and sellers across intermediaries and between sectors within intermediary I, where B^I (B^E) is the mass of buyers visiting I (E), S^I (S^E) is the mass of sellers visiting I (E), and x^m is the mass of buyers visiting the middlemen sector of I. Following Definition 3, we say $\mathcal{N}$ is an equilibrium distribution given $\mathcal{P}^{\rangle}$ if

$$B^C = B \cdot I\{V^C(\mathcal{P}, \mathcal{N}) \geq V^D(\mathcal{N})\}$$

and

$$S^C = S \cdot I\{W^C(\mathcal{P}, \mathcal{N}) \geq W^D(\mathcal{N})\},$$

and x^m satisfies the optimal search decision of buyers at I which can be easily modified from (4).[23] We can then define a market allocation as a mapping $\mathcal{N}(\cdot)$ that associates to each price/fee $\mathcal{P}^{\rangle}$ an equilibrium distribution of buyers/sellers $\mathcal{N}(\mathcal{P})$. Hence, $\mathcal{N}(\cdot)$ generates a reduced-form price-setting game between intermediaries where a Nash equilibrium can be defined.

Definition 6 *An equilibrium of the game with a market allocation $\mathcal{N}(\cdot)$ is a price vector $\mathcal{P} = \{f^I, p^m, f^E\}$ and an associated distribution of buyers and sellers $\mathcal{N}(\mathcal{P})$ where neither intermediary I nor E has an incentive to deviate under $\mathcal{N}(\cdot)$.*

B.2 Single-market Search

In this section, we show that, under single-market search, I chooses to be a pure middleman in equilibrium. First of all, the expected value of buyers to search on E, V^E, is taken as given in this analysis, so the specific value of V^E does not matter.[24] Second, we discuss the optimal mode of I. If I is a pure middleman, then it makes a profit of Bp^m with $p^m = 1 - V^E$. If I activates a platform, it must satisfy the participation constraints,

$$\eta^s(x^s)(1 - p^s) \geq V^E,$$
$$p^s - f^I \geq 0.$$

Under these conditions, it holds that

$$f^I \leq 1 - V^E.$$

Hence, the resulting profit of I satisfies $S(1 - e^{-x^s})f^I + x^m p^m < (Sx^s + x^m)\max\{f^I, p^m\} \leq B(1 - V^E)$. That is, for I the pure middleman mode dominates any other modes with an active platform.

[23] x^m is meaningful if there are buyers visiting the I intermediary, $B^I > 0$. If $B^I = 0$, x^m is trivially zero.

[24] V^E is given as follows. If E is a middleman with a price p^E, then $V^E = 1 - p^E$. If E is a market-maker with a fee f^E, then $V^E = e^{-\frac{B}{S}}(1 - f^E)$ under unfavorable beliefs towards I and $V^E = 0$ under favorable beliefs towards I.

B.3 Multi-market search: *E* is a pure middleman.

In multi-market search, we assume that I opens prior to E. As has been shown in the main text, irrespective of beliefs, buyers and sellers will always first visit I. However, it is only when fees/prices of I are comparable to that of E that agents will indeed trade at I (so I is a *first source*). Otherwise, they will forgo trading opportunities at I and trade at E instead. Then, I becomes inactive, and E is the *sole source* of intermediation.

In the case that E exogenously acts as a pure middleman, an active platform of I has to satisfy the following incentive constraints:

$$\begin{aligned} 1 - p^s &\geq 1 - p^E \\ p^s - f^I &\geq 0. \end{aligned}$$

These constraints imply: $f^I \leq p^E$. Similarly, an active middleman sector of I has to satisfy $p^m \leq p^E$. If $\max\{p^m, f\} \leq p^E$, then trade can occur in either one of the sectors, and so I can be active. The profit function of I is

$$S(1 - e^{-x^s})f + x^m p^m.$$

Noting $x^s = \frac{B - x^m}{S}$, we see from this expression that the profit maximization requires that $x^m = B$ with $p^m = f = p^E$. Hence an active platform is not profitable for I facing a pure middleman competitor. Since the two intermediaries compete with price, any equilibrium must be subject to the Bertrand undercutting, leading to $p^m = p^E = 0$ and zero profits.

B.4 Multi-market Search: E is a pure market-maker

When E is a pure market-maker, the expected value of buyers to search on E is modified to

$$V^E = \lambda^b e^{-x^s}(1 - f^E),$$

which is the same as in the baseline setup, except that f^E is subtracted from the total surplus. With this modification, as long as

$$\max\{p^m, f^I\} \leq 1 - V^E, \tag{48}$$

trade can happen in at least one of the sectors and so I can be a first source.

B.4.1 The best response of intermediary *I*

We start with a characterization of the best response of intermediary I, which is adapted from Proposition 2.

Proposition 10 *Under multi-market search technologies, given an $f^E \in [0, 1]$, I's best response involves the optimal price/fee $p^m = f^I = 1 - \lambda^b e^{-x^s}(1 - f^E)$, and the optimal intermediation mode of*

- *a pure middleman $x^m = B$ if $f^E = 1$;*
- *a pure market-maker $x^m = 0$ if $\lambda^b e^{-B/S} \geq \frac{1}{2}$ and $f^E \leq 1 - \frac{1}{2\lambda^b e^{-B/S}}$;*

- *a market-making middleman $x^m \in (0, B)$ if $\lambda^b e^{-B/S} < \frac{1}{2}$, or $\lambda^b e^{-B/S} \geq \frac{1}{2}$ and $f^E > 1 - \frac{1}{2\lambda^b e^{-B/S}}$, and x^m is characterized by $f^E = b^I(x^m)$, where $b^I(x^m)$ is given by*

$$b^I(x^m) \equiv 1 - \frac{S(1-e^{-x^s})}{(2S(1-e^{-x^s}) + x^m)\lambda^b e^{-x^s}}. \tag{49}$$

Proof. See B.5.1. ■

It shows that when $f^E = 1$, we are back to the single-market search scenario where agents do not have outside options. So a pure middleman with $p^m = 1$ is optimal. When $f^E < 1$, there is cross-market feedback from E to I, which makes using the platform as part or all of I's intermediation activities profitable. Additionally, I must decide whether it wants to operate as a pure market maker. Our result shows that it depends on buyers' outside option values. If $\lambda^b e^{-B/S} < \frac{1}{2}$, then the buyers' outside option value is low. In this case, the middleman sector generates high enough profits for the market-making middleman mode to be adopted for any value of f^E. If instead $\lambda^b e^{-B/S} \geq \frac{1}{2}$, then the buyers' outside option value is high, and attracting buyers to the middleman sector is costly. In this case, the intermediary will act as a market-making middleman if $f^E > 1 - \frac{1}{2\lambda^b e^{-B/S}}$, where buyers expect a low value from E market, and as a pure market maker if $f^E \leq 1 - \frac{1}{2\lambda^b e^{-B/S}}$, where buyers expect a high value from the E market.

When the mixed mode is activated, $b^I(\cdot)$, the best response of I defined by (49), characterizes the optimal intermediation structure that I is willing to pursue. $b^I(x^m)$ is monotonically increasing in x^m, implying that as f^E decreases, I's optimal mode moves towards a pure platform. Eventually (in the case that $\lambda^b e^{-B/S} \geq \frac{1}{2}$), as f^E approaches $1 - \frac{1}{2\lambda^b e^{-B/S}}$, I's optimal mode becomes the pure market-maker.[25]

Armed with the characterization in Proposition 10, we can rule out any pure strategy equilibrium where I either acts as a pure middleman or as a pure market-maker. A pure middleman I does not arise in equilibrium because, given Proposition 10, I only adopts a pure middleman mode with $p^m = 1$ when $f^E = 1$. But facing $p^m = 1$, E would rather set $f^E = 1 - \varepsilon$, for some $\varepsilon > 0$ to become the only active intermediary and makes a profit of $B\lambda^b f^E > 0$.[26]

Turn to the case when I is a pure market-maker. According to the incentive constraints, when $V^E = \lambda^b e^{-B/S}(1 - f^E) \leq 1 - f^I$, E is the second source; otherwise, I becomes inactive and E is the sole active source. Consider an equilibrium candidate where E sets a fee $f^E \in [0, 1]$ and a pure market-maker I sets a fee $f^I \equiv 1 - \lambda^b e^{-B/S}(1 - f^E)$.[27] In this proposed equilibrium, E wants to undercut I whenever possible, or deviate to $f^{E\prime} = 1$ to get the whole trading surplus from its transactions. On the one hand, if $f^E > 0$, it is profitable for E to undercut I by setting $f^{E\prime} = f^E - \varepsilon$. As such, E becomes the sole source and makes a profit of $B\lambda^b f^{E\prime}$. On the other hand, if $f^E = 0$, then E would rather take the full surplus of each transaction by deviating to

[25] This is so because $\lim_{x^m \to 0} b^I(x^m) = 1 - \frac{1}{2\lambda^b e^{-B/S}}$, $\lim_{x^m \to B} b^I(x^m) = 1$. And if $1 - \frac{1}{2\lambda^b e^{-B/S}} < 0$, then $\lim_{f^E \to 0} x^m > 0$.

[26] The best response analysis of E when I is a pure middleman is as follows. When I is a pure middleman, E can only make transactions by setting a fee f^E low enough so that buyers' incentive constraint to trade at I is violated, i.e., $1 - p^m < \lambda^b(1 - f^E)$. That is, to undercut I, E sets $f^E = 1 - \frac{1-p^m}{\lambda^b} - \varepsilon$ for some $\varepsilon > 0$, as long as this leads to a non-negative f^E. In this way, I becomes inactive and E makes a profit of $B\lambda^b f^E$.

[27] According to Proposition 10, f^I is required to satisfy the best response of I. Any other fee level would lead to a deviation of I.

$f^{E\prime} = 1$ and make a profit of $B\lambda^b e^{-B/S} > 0$. We summarize these observations in the following corollary.

Corollary 2 *There does not exist a pure strategy equilibrium where I operates in a pure (middleman or marketmaker) mode.*

Corollary 2 shows that the same logic of the benchmark model holds in an environment when the outside market is operated by another strategic player. E has an incentive to undercut I, and this gives positive outside values to buyers. To lower buyers' outside values, I activates its platform. Furthermore, Corollary 2 claims even stronger: The pure market-maker mode can not be in equilibrium. This follows the intuition of the classical Bertrand-Edgeworth game. Since the matching probability is less than one at the market-maker sector, it is a profitable deviation for E to set $f^E = 1$ to abstract full trading surplus from agents that are not matched at I.

B.4.2 The best response of intermediary E

We are now at the position of examining the strategies of E. According to Corollary 2, pure intermediary mode of I cannot be in equilibrium. According to Proposition 10, a mixed mode I's optimal strategies always features $p^m = f^I$ irrespective of f^E. Let's refer to the price/fee level by ψ. We discuss the best responses of E under $\psi = p^m = f^I$.

We start with a condition that determines E being the sole or the second source. With the price/fee level of ψ, the incentive constraint (48) becomes

$$f^E \geq 1 - \frac{1}{\lambda^b e^{-x^s}}(1-\psi),$$

for some $x^s \in [0, B/S]$. The right-hand side takes the minimum value at $x^s = B/S$. Therefore, as long as

$$f^E \geq 1 - \frac{1}{\lambda^b e^{-B/S}}(1-\psi), \tag{50}$$

E is the second source for some $x^m \in [0.B]$.

Let's first explore the scenario that E is the sole active intermediary. To do so, E needs to undercut ψ set f^E slightly lower than the right hand side of (50) and makes a profit of

$$\Pi^E_{sole}(\psi) = B\lambda^b\Big(1 - \frac{1}{\lambda^b e^{-B/S}}(1-\psi) - \varepsilon\Big). \tag{51}$$

Observe that when ψ is low, i.e., $\psi < 1 - \lambda^b e^{-B/S}$, E will suffer a loss by undercutting, $\Pi^E_{sole} < 0$.

Turn to the scenario that E is the second source. E chooses an f^E that satisfies (50), and its profit maximization problem is

$$\Pi^E_{2nd}(\psi) = \max_{f^E \in [1-\frac{1}{\lambda^b e^{-B/S}}(1-\psi),1]} \Big(B - x^m - S(1-e^{-x^s})\Big)\lambda^b e^{-x^s} f^E, \tag{52}$$

subject to (3) and (4). The first step to solve the problem is to note that it is in E's interest to have buyers be indifferent between I's two sectors, $V^m = V^s$. This is formally stated in the following lemma.

Lemma 2 *Given $p^m = f^I = \psi$, the optimal solution of problem (52) features*

$$V^m(\psi) = V^s(x^m, \psi, f^E). \tag{53}$$

Proof. Let $\{\hat{f}^E, \hat{x}^m\}$ denote the optimal solution. Suppose at the optimum, $V^m(\psi) > V^s(\hat{x}^m, \psi, \hat{f}^E)$, then $\hat{x}^m = B$. It follows that

$$V^s(\hat{x}^m, \psi, \hat{f}^E) = 1 - f^I = 1 - \psi = 1 - p^m = V^m(\psi),$$

which contradicts with the assumption that $V^m(\psi) > V^s(\hat{x}^m, \psi, \hat{f}^E)$. Suppose at the optimum, $V^m(\psi) < V^s(\hat{x}^m, \psi, \hat{f}^E)$. That is,

$$V^s = e^{-B/S}(1-\psi) + (1 - e^{-B/S})\lambda^b e^{-B/S}(1 - \hat{f}^E) > 1 - \psi.$$

This implies that $\hat{f}^E < 1 - \frac{1-\psi}{\lambda^b e^{-B/S}}$. But then E gains a higher profit by deviating to $\tilde{f}^E = 1 - \frac{1-\psi}{\lambda^b e^{-B/S}}$. At $\tilde{f}^E$, E maintains $\hat{x}^m$ while extracting higher fees from each transaction, thus gains a higher profit according to (52). ■

The intuition is as follows. On the one hand, $V^m > V^s$ leads to a pure middleman incumbent, leaving zero market share to E, which is certainly the worse case for E. On the other hand, $V^m < V^s$ means E needs to set f^E unnecessarily low to attract buyers to I's platform — E could increase f^E without changing the distribution of buyers/sellers yet obtain higher profits.

Inserting the expression of $V^i, i = m, s, E$, into (53), we have

$$\lambda^b e^{-x^s}(1 - f^E) = 1 - \psi. \tag{54}$$

Condition (54) describes the following fact: Increasing f^E leads to less favorable outside values for buyers on I's platform, hence more buyers visit I's middleman sector (x^m increases), and there are more unmatched sellers left for E (e^{-x^s} increases). The implication becomes clear once we substitute for f^E from (54) and insert into (52), which yields

$$\Pi^E_{2nd}(\psi) = \max_{x^m \in [0,B]} \Big(B - x^m - S(1 - e^{-x^s})\Big)\Big(\lambda^b e^{-x^s} - (1-\psi)\Big). \tag{55}$$

By choosing an f^E, E essentially chooses an x^m to balance its demand and supply. A higher x^m implies less buyers join E after trading at I, i.e., $B - x^m - S(1 - e^{-x^s})$ decreases in x^m. At the same time, more sellers join E since the matching probability is now lower for sellers at I's platform, i.e., e^{-x^s} increases in x^m. Therefore, the intermediation structure of I, x^m, determines the supply and demand at intermediary E.

From E's perspective, the optimal x^m depends on the equilibrium price level ψ. When ψ is high, it is profitable to have more buyers at I's platform by lowering x^m and ultimately increase participating buyers on E. E can achieve this by decreasing f^E. When ψ is low, E finds it less profitable to have more participating buyers, and the optimal f^E should be higher.

Finally, note that a pure strategy equilibrium does not exist with $\psi \leq 1 - \lambda^b$. According to Proposition 10, the best response of I features $\psi = 1 - \lambda^b e^{-x^s}(1 - f^E) \geq 1 - \lambda^b$. The equality holds only when $f^E = 0$ and $x^m = B$. But Corollary 2 shows that $x^m = B$ cannot be in equilibrium. Hence, we focus on $\psi > 1 - \lambda^b$ for the best response analysis of E.

Lemma 3 *There does not exist a pure strategy equilibrium with $\psi \leq 1 - \lambda^b$.*

The following proposition characterizes the best response of E given that I sets $f^I = p^m = \psi \in (1-\lambda^b, 1]$.

Proposition 11 *Under multi-market search technologies, for given $\psi = f^I = p^m \in (1-\lambda^b, 1]$, E's optimal strategy is as follows:*

- *For $\psi \in (1-\lambda^b, 1-\lambda^b e^{-B/S}]$, E works as the second source since $\Pi^E_{2nd} > 0 \geq \Pi^E_{sole}$, with an $x^m \in (0, B)$ and satisfies*

$$1-\psi = \lambda^b e^{-x^s}\Big(1 - \frac{B - x^m - S(1-e^{-x^s})}{S(1-e^{-x^s})}\Big); \tag{56}$$

- *For $\psi \in (1-\lambda^b e^{-B/S}, 1]$, E chooses between the second and the sole source depending on which is more profitable, both of which generate positive profits, and if E operates as the second source, then $x^m \in [0, B)$ that satisfies*

$$1-\psi \geq \lambda^b e^{-x^s}\Big(1 - \frac{B - x^m - S(1-e^{-x^s})}{S(1-e^{-x^s})}\Big); \tag{57}$$

  - *In particular, if $\frac{B}{S} \leq -\log\frac{1}{4}$, then for $\psi \in (1-\phi(B,S,\lambda^b), 1]$, where $\phi(B,S,\lambda^b) \equiv \lambda^b e^{-B/S}\Big(1 - \frac{B-S(1-e^{-B/S})}{S(1-e^{-B/S})}\Big)$, E optimally chooses to be the sole source as $\Pi^E_{sole} > \Pi^E_{2nd} > 0$.*

Proof. See B.5.2. ■

The intuition is very clear: When the price/fee of I is low, further undercutting is not profitable for E even though this means more demands. In contrast, when the price of I is high, undercutting the incumbent and acts as the sole source may become profitable for E. In particular, when the market is relatively tight ($\frac{B}{S}$ is low), the residual demands for a second source E are very limited, and the sole source strategy becomes more profitable.

Equation (56) is the key element of the best response of E operating as the second source. It describes the optimal market structure (represented by x^m) that E would like to choose. Consider a range of $[\underline{x}^m, B]$ where $\underline{x}^m \geq 0$ ensures that the right hand side of (56) is non-negative. It then follows that

$$\frac{\partial \psi}{\partial x^m} = -\frac{1}{S}\Big(1 - \psi + \frac{\lambda^b e^{-x^s}(1-e^{-x^s} - x^s e^{-x^s})}{(1-e^{-x^s})^2}\Big) < 0,$$

for $x^m \in [\underline{x}^m, B]$. This corresponds exactly to the intuition above: as ψ increases, E finds it's more profitable to compete with I, and E lowers f^E to make the middleman sector of I less favorable and x^m decreases.

B.4.3 Equilibrium analysis

We start with a lemma on the non-existence of a pure strategy equilibrium where E is the sole intermediation service.

Lemma 4 *There does not exist a pure strategy equilibrium where E is the only active intermediary.*

To see why, suppose E is the sole source and then I makes a zero profit. Then I has a profitable deviation by setting ψ between $1-\lambda^b$ and $1-\lambda^b e^{-B/S}$ so that I can make a positive profit.

Now let's discuss an equilbrium where E is the second source. The equilibrium should jointly solve the optimal responses of two intermediaries (49) and (56), together with the equilibrium conditions (3) and (54). Inserting (54) into (56) gives an alternative expression for $b^E(x^m)$

$$f^E = b^E(x^m) \equiv \frac{B - x^m - S(1-e^{-x^s})}{S(1-e^{-x^s})}, \tag{58}$$

that facilitates our analysis.

The equilibrium $x^m \in (0, B)$ if it exists should solve for $b^I(x^m) = b^E(x^m)$. Proposition 12 gives sufficient conditions for the existence and uniqueness of the equilibrium.

Proposition 12 *Define* $x \equiv \frac{B}{S}$ *and assume either* $2(1-e^{-x}) - x > 0$ *and* $\lambda^b < \frac{1-e^{-x}}{2e^{-x}(2(1-e^{-x})-x)}$, *or* $2(1-e^{-x}) - x \leq 0$. *Then there exists a unique pure strategy equilibrium that features a market-making middleman* I *as the first source and a market-maker* E *as the second source if*

$$1 - \lambda^b e^{-B/S} \geq \psi > 1 - \lambda^b, \tag{59}$$

where $\psi = 1 - \lambda^b e^{-x^s}\left(1 - \frac{x^s-(1-e^{-x^s})}{(1-e^{-x^s})}\right)$, $x^s = \frac{B-x^m}{S}$, *and* $x^m \in (0, B)$ *solves*

$$b^I(x^m) = b^E(x^m). \tag{60}$$

The equilibrium is characterized by $\mathcal{N} = \{B, B - x^m - S(1-e^{-x^s}), S, Se^{-x^s}, x^m\}$, *and* P *that* $f^I = p^m = \psi, f^E = 1 - \frac{1-\psi}{\lambda^b e^{-x^s}}$. *And both intermediaries make positive profits.*

Proof. See B.5.3. ■

Figure 5 demonstrates an equilibrium by two variables, the price/fee level represented by f^E, and the market structure represented by x^m. It plots the two best response functions $b^I(x^m)$ in (49) and $b^E(x^m)$ in (58), and we have marked the function values as x^m approaches 0 and B.[28] The interaction of the two best responses gives the equilibrium f^E and x^m. The equilibrium distribution and price variables can be derived accordingly, as stated in the proposition.

The figure illustrates the comparative statics. Let's consider exogenous changes of the buyer's meeting rate λ^b and the buyer population B. First, as λ^b increases to $\lambda^{b\prime}$, $b^I(x^m)$ moves upward while $b^E(x^m)$ does not move, leading to a smaller x^{m*}. This is illustrated in Figure 6. Second, for an exogenous change of B, it follows that $\frac{\partial b^E(x^m)}{\partial B} > 0$ and $\frac{\partial b^I(x^m)}{\partial B} < 0$ for $x^m \in (0, B)$. That is, as the population of buyers B increases, $b^I(x^m)$ moves down while $b^E(x^m)$ moves upward, leading to a higher x^{m*}. This is illustrated in Figure 7 (the mass of buyers increases from B to B'). Similar comparative statics can be done on the seller population S. We summarize these observations in Corollary 3.

[28] We make use of the following observations:

$$\lim_{x^m \to 0} b^I(x^m) = 1 - \frac{1}{2\lambda^b e^{-B/S}}, \quad \lim_{x^m \to B} b^I(x^m) = 1,$$
$$\lim_{x^m \to 0} b^E(x^m) = \frac{B - S(1-e^{-B/S})}{S(1-e^{-B/S})}, \quad \lim_{x^m \to B} b^E(x^m) = 0.$$

We have plotted one particular scenario that $1 - \frac{1}{2\lambda^b e^{-B/S}} > 0$ and $\frac{B-S(1-e^{-B/S})}{S(1-e^{-B/S})} < 1$. But these restrictions are not required for the existence of an equilibrium.

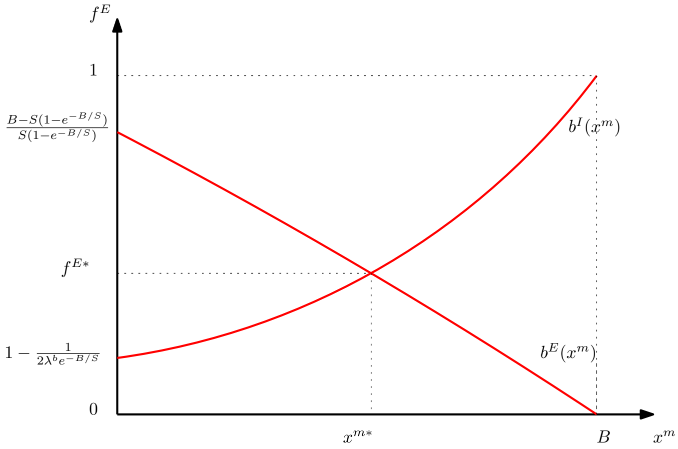


Figure 5: Equilibrium under multi-market search

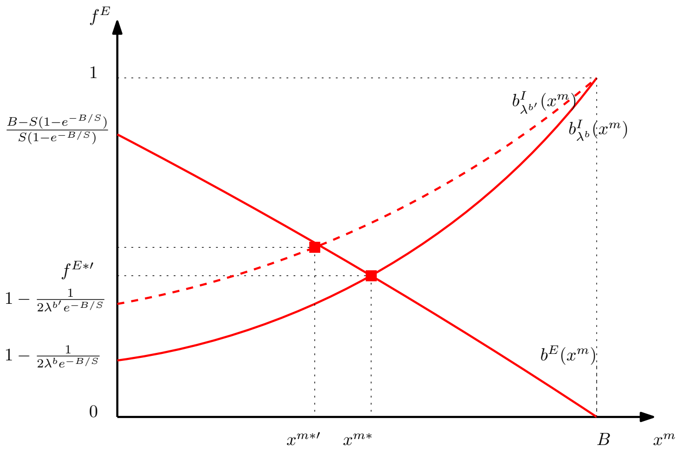


Figure 6: Comparative Statics w.r.t. λ^b

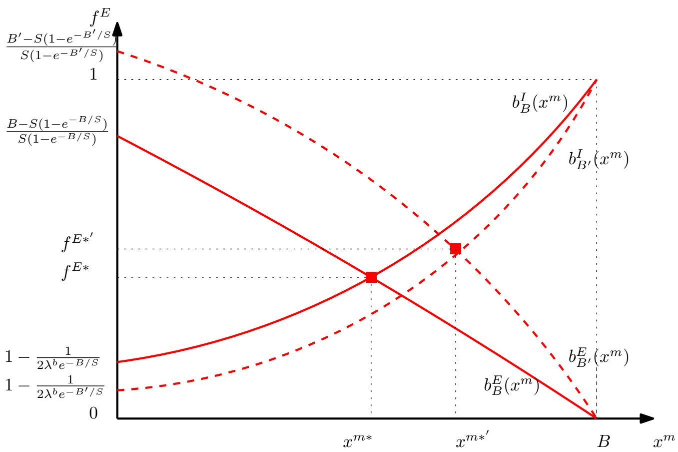


Figure 7: Comparative Statics w.r.t. B

Corollary 3 (Comparative statics) *Consider a parameter space in which a pure strategy equilibrium exists. Then, an increase in the buyer's meeting rate λ^b at intermediary E, or a decrease in the buyer-seller population ratio B/S, leads to a smaller middleman sector x^m and a larger platform x^s of intermediary I in equilibrium.*

Proof. See B.5.4. ■

Numerically, it is easy to verify that sufficient condition (59) is satisfied at least for some reasonable parameter values. For example, taking $B = S = 1$, and setting a grid of λ^b with two decimals from 0.01 to 0.99, then (59) holds for all λ^b grid points smaller than 0.95. For λ^b grid points between 0.95 and 0.98, despite the sufficient condition (59) is violated, it can be verified that being the second source is still more profitable than being a sole source for E, so a pure strategy equilibrium still exist. For the grid point $\lambda^b = 0.99$, E finds it more profitable to undercut I and become the sole source. Then there does not exist a pure strategy equilibrium.

When the pure strategies equilibrium does not exist, by applying Theorem 5 of Dasgupta et al. (1986), we show that because Π^I (Π^E) is bounded and weakly lower semi-continuous in f and p^m (in f^E), and $\Pi^I + \Pi^E$ is upper semi-continuous, there exists a mixed strategy equilibrium.

Proposition 13 *There exists a mixed strategy equilibrium under multi-market search.*

Proof. See B.5.5. ■

Given that in the mixed strategy equilibrium $f^E < 1$ happens with positive probability, according to Proposition 10, I activates its platform with positive probability in equilibrium.

Corollary 4 *In the mixed strategies equilibrium, I's platform is activated with positive probability.*

B.5 Omitted proofs

B.5.1 Proof of Proposition 10

The intermediary's problem can be written as

$$\Pi\left(x^m, f^I\right) = \max_{x^m, f^I} S(1-e^{-\frac{B-x^m}{S}})f^I + x^m(1-V^E(x^m, f^E)) - x^m e^{-x^s}(1-V^E(x^m, f^E)-f^I)$$

subject to $f^I \leq 1-V^E(x^m, f^E)$ and $0 < x^m < B$. We can further compactify the constraint set to $x^m \in [0, B]$ since $\lim_{x^m \to B} \Pi\left(x^m, f^I\right) = \tilde{\Pi}(B)$ and $\lim_{x^m \to 0} \Pi\left(x^m, f, K\right) = \tilde{\Pi}(0)$, where $\tilde{\Pi}(B)$ is the profit for the pure middleman mode and $\tilde{\Pi}(0)$ is the profit for the pure market-maker mode. Then we pin down a profit-maximizing intermediation mode using the following Lagrangian:

$$\mathcal{L} = \Pi\left(x^m, f^I\right) + \mu_0 x^m + \mu_b(B - x^m) + \mu_f\left(1 - V^E(x^m, f^E) - f^I\right),$$

where the μ's ≥ 0 are the Lagrange multiplier of each constraint. The following first order conditions are necessary:

$$\frac{\partial \mathcal{L}}{\partial x^m} = \frac{\partial \Pi\left(x^m, f^I\right)}{\partial x^m} + \mu_0 - \mu_b - \mu_f \frac{\partial V^E\left(x^m, f^I\right)}{\partial x^m} = 0, \tag{61}$$

$$\frac{\partial \mathcal{L}}{\partial f^I} = \frac{\partial \Pi\left(x^m, f^I\right)}{\partial f^I} - \mu_f = 0, \tag{62}$$

The solution is characterized by these and the complementary slackness conditions of the three constraints. (62) implies that we must have $\mu_f = S(1-e^{-x^s}) + x^m e^{-x^s} > 0$, which implies constraint $f^I = 1 - V^E(x^m, f^E)$.

If $f^E = 1$, then $V^E(x^m, f^E = 1) = \lambda^b e^{-x^s}(1-f^E) = 0$. The intermediary's problem can be written as $\Pi\left(x^m\right) = \max_{x^m} S(1-e^{-x^s}) + x^m$, where $\Pi\left(x^m\right)$ is concave in x^m. The first order condition with respect to x^m is $1 - e^{-x^s} = 0$. Therefore, at the optimal $x^m = B$.

Next, if $f^E < 1$, we show that I's platform is active, i.e., $x^m < B$ at the optimum. Substituting μ_f into (61), we have

$$\begin{aligned}\mu_b - \mu_0 &= (1-e^{-x^s})(1-\lambda^b e^{-x^s}(1-f^E)) - (x^m + S(1-e^{-x^s}))\frac{\lambda^b}{S}e^{-x^S}(1-f^E) \\ &\equiv \phi(x^m \mid B, S, \lambda^b, f^E).\end{aligned} \tag{63}$$

At $x^m = B$, (63) yields $\phi(B \mid \cdot) = \mu_b = -\frac{B}{S}\lambda^b(1-f^E) < 0$, which contradicts to $\mu_b \geq 0$. Hence, the solution must satisfy $x^m < B$ (which implies $\mu_b = 0$). At $x^m = 0$, (63) yields $\phi(0 \mid \cdot) = -\mu_0 = (1-e^{-B/S})(1-2\lambda^b e^{-B/S}(1-f^E))$, which requires $f^E \leq 1 - \frac{1}{2\lambda^b e^{-B/S}}$. This leads to the conditions in the proposition. Set $\mu_b = \mu_0 = 0$, we have $\phi(x^m \mid B, S, \lambda^b, f^E) = 0$ according to (63). This gives condition (49) for $x^m \in (0, B)$. Finally, it is straightforward to verify the second order condition using the Hessian of $\mathcal{L}$ with respect to $[f^I, x^m]$.

B.5.2 Proof for proposition 11

First of all, from (51) and (55), it is straightforward to see that $\Pi^E_{2nd} \leq 0$ if $\psi \leq 1 - \lambda^b$; and $\Pi^E_{sole} \leq 0$ if $\psi \leq 1 - \lambda^b e^{-B/S}$. These observations give the signs of the profits in all cases in the proposition.

Second, we then discuss the optimal x^m when E acts as the second source using the following first order condition condition:

$$\frac{\partial \Pi^E_{2nd}(x^m, \psi)}{\partial x^m}| = -(1-e^{-x^s})(\lambda^b e^{-x^s} - (1-\psi)) + \frac{B - x^m - S(1-x^{-x^s})}{S}\lambda^b e^{-x^s} = 0. \tag{64}$$

Since $\Pi^E_{2nd}(x^m)$ is continuously differentiable on $[0, B]$, the maximum point is either $x^m = 0$, $x^m = B$ or some $\hat{x}^m$ such that $\frac{\partial \Pi^E_{2nd}(x^m, \psi)}{\partial x^m}|_{x^m = \hat{x}^m} = 0$. With $\psi > 1 - \lambda^b$, we know that E can obtain a positive profit as a second source. However, $\Pi^E_{2nd}(x^m = B) = 0$, thus $x^m = B$ does not give the maximum. When $\frac{\partial \Pi^E_{2nd}}{\partial x^m}|_{x^m=0} > 0$, that is $1 - \lambda^b e^{-B/S}\left(1 - \frac{B - S(1-e^{-B/S})}{S(1-e^{-B/S})}\right) > \psi$, $x^m = 0$ does not satisfy the necessary condition. For an $\hat{x}^m$ that satisfies the first order condition, we rearrange (64) to get (56). These observations give the first point of the proposition.

The following discussion depends on the sign of $\phi(B,S,\lambda^b) \equiv \lambda^b e^{-B/S}\Big(1 - \frac{B-S(1-e^{-B/S})}{S(1-e^{-B/S})}\Big)$. If $\phi(B,S,\lambda^b) < 0$, then for $\psi \in [1-\lambda^b e^{-B/S}, 1]$, E compares Π^E_{sole} and Π^E_{2nd} to decide which is more profitable. If $\phi(B,S,\lambda^b) \geq 0$, then again for $\psi \in (1-\lambda^b e^{-B/S}, 1-\phi(B,S,\lambda^b))$, both the sole source and the second source can be profitable and E chooses the more profitable one. For $\psi \in [1-\phi(B,S,\lambda^b), 1]$, we provide a sufficient condition that as a second source E optimally chooses $x^m = 0$. Yet in this case, the second source profit is strictly dominated by the sole source profit:

$$\Pi^E_{2nd}(\psi) = (B - S(1-e^{-B/S})(\lambda^b e^{-B/S} - (1-\psi)) < B(\lambda^b e^{-B/S} - (1-\psi) - \varepsilon) = \Pi^E_{sole}(\psi).$$

The following elaborates the condition that $x^m = 0$ is optimal in this case. Rearrange (64) to make that the function is in terms of x^s, and refer to it as $\omega(x^s)$:

$$\omega(x^s) \equiv -(1-e^{-x^s})(\lambda^b e^{-x^s} - (1-\psi)) + (x^s - (1-e^{-x^s}))\lambda^b e^{-x^s}. \tag{65}$$

It is straightforward to verify that $\omega(0) = 0$. Under $\psi \geq 1-\phi(B,S,\lambda^b)$, we have $\omega(B/S) \leq 0$. To show $x^m = 0$ is optimal, it is enough to show that $\frac{\partial \Pi^E_{2nd}}{\partial x^m} < 0$ for $x^m \in (0,B)$, or equivalently $\omega(x^s) < 0$ for $x^s \in (0, B/S)$. Suppose there exists an $x^s_1 \in (0, B/S)$ such that $\omega(x^s_1) = 0$. Then either there also exists another $x^s_2 \in (x^s_1, B/S)$ such that $\omega(x^s_2) = 0$, or $\omega'(x^s_1) = 0$. The latter can be easily excluded since the x^s's that allows $\omega(x^s) = \omega'(x^s)$ does not give $\omega(x^s) = \omega'(x^s) = 0$. To derive a condition for the contradiction in the former case, notice

$$\omega'(x^s) = -e^{-x^s}(\lambda^b e^{-x^s} - (1-\psi)) + 3(1-e^{-x^s})\lambda^b e^{-x^s} - x^s\lambda^b e^{-x^s},$$

and $\omega'(x^s)|_{x^s=0} = 1 - \lambda^b - \psi < 0$ under $\psi > 1-\lambda^b$. Further, let's define

$$\Omega(x^s) \equiv \omega'(x^s)|_{\omega(x^s)=0} = -(\psi - (1-\lambda^b e^{-x^s})) + 2(1-e^{-x^s})\lambda^b e^{-x^s}.$$

If both x^s_1 and x^s_2 exist, then we shall have $\Omega(x^s_1) > 0 > \Omega(x^s_2)$. And this is impossible if

$$\Omega'(x^s) = \lambda^b e^{-x^s}(4e^{-x^s} - 1) \geq 0.$$

This condition is guaranteed by $\frac{B}{S} \leq -\log(\frac{1}{4})$. Indeed, for $\frac{B}{S} \in (0, -\log(\frac{1}{4})]$, we have $\phi(B,S,\lambda^b) > 0$. We therefore proved that $x^m = 0$ is optimal for $\psi \in [1-\phi(B,S,\lambda^b), 1]$. ■

B.5.3 Proof for proposition 12

Define

$$g(x^m) \equiv b^I(x^m) - b^E(x^m).$$

First, notice $g(x^m)$ is continuous differentiable with respect to x^m. Taking the limits of x^m to 0 and B, and define $x \equiv \frac{B}{S}$. Under the stated assumption about x and λ^b in the proposition, we have

$$\begin{aligned}
\lim_{x^m \to 0} g(x^m) &= \lim_{x^m \to 0} b^I(x^m) - \lim_{x^m \to 0} b^E(x^m) \\
&= 1 - \frac{1}{2\lambda^b e^{-x}} - \frac{x - (1-e^{-x})}{(1-e^{-x})} \\
&= \frac{2\lambda^b e^{-x}(2(1-e^{-x}) - x) - (1-e^{-x})}{2\lambda^b e^{-x}(1-e^{-x})} < 0.
\end{aligned}$$

and

$$\begin{aligned}
\lim_{x^m \to B} g(x^m) &= \lim_{x^m \to B} b^I(x^m) - \lim_{x^m \to B} b^E(x^m) \\
&= 1 - 0 > 0.
\end{aligned}$$

According to the Intermediate Value Theorem, there exists an $x^m \in (0,B)$ such that $b^I(x^m) = b^E(x^m)$.

Second, the equilibrium x^m is unique. This is because g is monotone increasing in x^m on $(0, B)$. Taking the first order derivate with respect to x^m, we have

$$g'(x^m) = b^{I'}(x^m) - b^{E'}(x^m) > 0,$$

where

$$b^{I'}(x^m) = \frac{(1 + 2(1 - e^{-x^s}))S(1 - e^{-x^s}) + x^m}{(2S(1 - e^{-x^s}) + x^m)^2 \lambda^b e^{-x^s}} > 0,$$
$$b^{E'}(x^m) = -\frac{1 - e^{-x^s} - x^s e^{-x^s}}{S(1 - e^{-x^s})^2} < 0.$$

Thirdly, if $\psi \in (1 - \lambda^b, 1 - \lambda^b e^{-B/S}]$, then according to Proposition 11, E has no incentive to deviate to the sole source since $\Pi^E_{sole} \leq 0$.

Finally, according to Proposition 10 and Proposition 11, both intermediaries make positive profits. ■

B.5.4 Proof for corollary 3

Let's consider a marginal increase in λ^b, B and S in turns.

Consider an increase in λ^b: $\lambda^{b\prime} = \lambda^b + \varepsilon$ with $\varepsilon > 0$. We show the equilibrium market structure under $\lambda^{b\prime}$ follows $x^{m*\prime} < x^{m*}$, where x^{m*} is the equilibrium queue length under λ^b and $x^{m*\prime}$ is the equilibrium queue length under $\lambda^{b\prime}$. We denote the best response function under λ^b by $b^i_{\lambda^b}(x^m)$ and that under $\lambda^{b\prime}(x^m)$ by $b^i_{\lambda^{b\prime}}(x^m)$, $i = I, E$. Since $\frac{\partial b^E(x^m)}{\partial \lambda^b} = 0$, $\frac{\partial b^I(x^m)}{\partial \lambda^b} > 0$, for $x^m \in (0, B)$, we have $b^E_{\lambda^{b\prime}}(x^m) = b^E_{\lambda^b}(x^m)$, and $b^I_{\lambda^{b\prime}}(x^m) > b^I_{\lambda^b}(x^m)$.

Suppose $x^{m*\prime} = x^{m*}$, then

$$b^I_{\lambda^{b\prime}}(x^{m*\prime}) > b^I_{\lambda^b}(x^{m*\prime}) = b^I_{\lambda^b}(x^{m*}) = b^E_{\lambda^b}(x^{m*}) = b^E_{\lambda^{b\prime}}(x^{m*}) = b^E_{\lambda^{b\prime}}(x^{m*\prime}).$$

But $b^I_{\lambda^{b\prime}}(x^{m*\prime}) > b^E_{\lambda^{b\prime}}(x^{m*\prime})$ implies that $x^{m*\prime} = x^{m*}$ is not in equilibrium.

Suppose $x^{m*\prime} > x^{m*}$, then

$$b^I_{\lambda^{b\prime}}(x^{m*\prime}) > b^I_{\lambda^b}(x^{m*\prime}) > b^I_{\lambda^b}(x^{m*}) = b^E_{\lambda^b}(x^{m*}) = b^E_{\lambda^{b\prime}}(x^{m*}) > b^E_{\lambda^{b\prime}}(x^{m*\prime}).$$

Again, $b^I_{\lambda^{b\prime}}(x^{m*\prime}) > b^E_{\lambda^{b\prime}}(x^{m*\prime})$ implies that $x^{m*\prime} > x^{m*}$ can not be in equilibrium.

Consider an increase in B: $B' = B + \varepsilon$ with $\varepsilon > 0$. We show the equilibrium market structure under B' follows $x^{m*\prime} > x^{m*}$. Since

$$\frac{\partial b^E(x^m)}{\partial B} = \frac{1 - e^{-x^s} - x^s e^{-x^s}}{S(1 - e^{-x^s})^2} > 0,$$
$$\frac{\partial b^I(x^m)}{\partial B} = -\frac{2S(1 - e^{-x^s})^2 + x^m}{(2S(1 - e^{-x^s}) + x^m)^2 \lambda^b e^{-x^s}} < 0,$$

for $x^m \in (0, B)$, we have $b^E_{B'}(x^m) > b^E_B(x^m)$, and $b^I_{B'}(x^m) < b^I_B(x^m)$.

Suppose $x^{m*\prime} = x^{m*}$, then

$$b^I_{B'}(x^{m*\prime}) < b^I_B(x^{m*\prime}) = b^I_B(x^{m*}) = b^E_B(x^{m*}) < b^E_{B'}(x^{m*}) = b^E_{B'}(x^{m*\prime}).$$

But $b^I_{B'}(x^{m*\prime}) < b^E_{B'}(x^{m*\prime})$ implies that $x^{m*\prime} = x^{m*}$ can not be in equilibrium.

Suppose $x^{m*\prime} < x^{m*}$, then

$$b^I_{B'}(x^{m*\prime}) < b^I_B(x^{m*\prime}) < b^I_B(x^{m*}) = b^E_B(x^{m*}) < b^E_{B'}(x^{m*}) < b^E_{B'}(x^{m*\prime}).$$

Again, $b^I_{B'}(x^{m*\prime}) < b^E_{B'}(x^{m*\prime})$ implies that $x^{m*\prime} < x^{m*}$ can not be in equilibrium.

Consider an increase in S: $S' = S + \varepsilon$ with $\varepsilon > 0$. We show the equilibrium market structure under S' follows $x^{m*\prime} < x^{m*}$. Since

$$\frac{\partial b^E(x^m)}{\partial S} = -\frac{(1 - e^{-x^s} - x^s e^{-x^s})x^s}{S(1 - e^{-x^s})^2} < 0,$$
$$\frac{\partial b^I(x^m)}{\partial S} = \frac{x^s \left[2(1 - e^{-x^s})^2 + \frac{x^m}{S}(1 - \frac{1 - e^{-x^s}}{x^s})\right]}{S(2(1 - e^{-x^s}) + x^m/S)^2 \lambda^b e^{-x^s}} > 0,$$

for $x^m \in (0, B)$, we have $b^E_{S'}(x^m) < b^E_S(x^m)$, and $b^I_{S'}(x^m) > b^I_S(x^m)$.

Suppose $x^{m*\prime} = x^{m*}$, then

$$b^I_{S'}(x^{m*\prime}) > b^I_S(x^{m*\prime}) = b^I_S(x^{m*}) = b^E_S(x^{m*}) > b^E_{S'}(x^{m*}) = b^E_{S'}(x^{m*\prime}).$$

Sut $b^I_{S'}(x^{m*\prime}) > b^E_{S'}(x^{m*\prime})$ implies that $x^{m*\prime} = x^{m*}$ can not be in equilibrium.

Suppose $x^{m*\prime} > x^{m*}$, then

$$b^I_{S'}(x^{m*\prime}) > b^I_S(x^{m*\prime}) > b^I_S(x^{m*}) = b^E_S(x^{m*}) > b^E_{S'}(x^{m*}) > b^E_{S'}(x^{m*\prime}).$$

Again, $b^I_{S'}(x^{m*\prime}) > b^E_{S'}(x^{m*\prime})$ implies that $x^{m*\prime} > x^{m*}$ can not be in equilibrium. This completes the proof of corollary 3.■

B.5.5 Proof for proposition 13

Consider a game between I, who selects $(f, p^m) \in [0, \bar{f}] \times [0, \bar{p}]$ with a payoff $\Pi^I = \Pi^I(f, p^m \mid f^E)$, and E, who selects $f^E \in [0, 1]$ with a payoff $\Pi^E = \Pi^E(f^E \mid f, p^m)$. Here, we set $\bar{f}, \bar{p} > 1$ and $f > 1$ ($\bar{p} > 1$) leads to an inactive platform (middleman sector). We apply Theorem 5 of Dasgupta and Maskin (1986) to show there exists a mixed strategy equilibrium.

Given Theorem 5 of Dasgupta et al. (1986), it is sufficient to show that Π^I (Π^E) is bounded and weakly lower semi-continuous in f and p^m (in f^E), and $\Pi^I + \Pi^E$ is upper semi-continuous. Clearly, Π^I (Π^E) is bounded in $(f, p^m) \in [0, \bar{f}] \times [0, \bar{p}]$ (in $f^E \in [0, 1]$).

Both of the profit functions are continuous except at

$$\min\{f, p^m\} = 1 - V^E(f^E), \tag{66}$$

where $V^E(f^E)$ is evaluated at $x^m = 0$. So we shall pay attention to this discontinuity point.

First, we show that $\Pi^I(f, p^m \mid f^E)$ is weakly lower semi-continuous in (f, p^m). Give the discontinuous point in (66), we have

$$\Pi^I(f, p^m \mid f^E) = \begin{cases} S(1 - e^{-x^s})f + x^m p^m, & \text{if } \min\{f, p^m\} \le 1 - V^E(f^E) \\ 0 & \text{otherwise,} \end{cases}$$

where in the second situation, the price/fee of I is not competitive to the fee of E, hence agents will trade via E, rather than I, and so I will become inactive. Consider some $f_\varepsilon \in [0, 1]$, and some $f, p^m > 0$ such that $\min\{f, p^m\} = 1 - V^E(f^E)$. For any sequence $\{(f^{(j)}, p^{m(j)})\}$ converging to (f, p^m) such that no two $f^{(j)}$'s, and no two $p^{m(j)}$'s are the same, and $f^{(j)} \le f, p^{m(j)} \le p^m$, we must have $\min\{f^{(j)}, p^{m(j)}\} \le 1 - V^E(f^E)$. Hence,

$$\lim_{j \to \infty} \Pi^I(f^{(j)}, p^{m(j)} \mid f^E) = \Pi^I(f, p^m \mid f^E),$$

satisfying the definition of weakly lower semi-continuity (see Definition 6 in page 13 of Dasgupta and Maskin, 1986, or condition (9) in page 384 of Maskin, 1986).

Second, we shall show that $\Pi^E(f^E \mid f, p^m)$ is lower semi-continuous in f^E. Consider a potential discontinuity point $f_0 \in (0, 1)$ satisfying (66) such that

$$\Pi^E(f^E \mid f, p^m) = \begin{cases} B\lambda^b f^E, & \text{if } f^E < f_0 \\ (B - x^m - S(1 - e^{-x^s}))\lambda^b f^E & \text{if } f^E \ge f_0. \end{cases}$$

Clearly, this function is lower semi-continuous, since for every $\epsilon > 0$ there exists a neighborhood U of f_0 such that $\Pi^E(f^E \mid \cdot) \ge \Pi^E(f_0 \mid \cdot) - \epsilon$ for all $f^E \in U$.

Finally, we prove the upper semi-continuity of $\Pi^I + \Pi^E$. For this purpose, consider all sequences of $\{f^{(j)}, p^{m(j)}, f^{E(j)}\}$ that converges to $\{\hat{f}, \hat{p}^m, \hat{f}^E\}$ that satisfies $\min\{\hat{f}, \hat{p}^m\} = 1 - V^E(\hat{f}^E)$.

Consider first an extreme in which case $\min\{f^{(j)}, p^{m(j)}\} \le 1 - V^E(f^{E(j)})$ for all j. As the equilibrium is that I is visited prior to E, we must have

$$\lim_{j \to \infty} \Pi^I(f^{(j)}, p^{m(j)} \mid f^{E(j)}) + \Pi^E(f^{E(j)} \mid f^{(j)}, p^{m(j)}) = \Pi^I(\hat{f}, \hat{p}^m \mid \hat{f}^E) + \Pi^E(\hat{f}^E \mid \hat{f}, \hat{p}^m).$$

Consider next the other extreme in which $\min\{f^{(j)}, p^{m(j)}\} > 1 - V^E(f^{E(j)})$ for all j. Then, in the equilibrium only E is active and we must have

$$\lim_{j\to\infty} \Pi^I(f^{(j)}, p^{m(j)} \mid f^{E(j)}) + \Pi^E(f^{E(j)} \mid f^{(j)}, p^{m(j)}) = B\lambda^b \hat{f}^E.$$

If $\hat{f} \geq p^{\hat{m}}$, then

$$\Pi^I(\hat{f}, \hat{p}^m \mid \hat{f}^E) + \Pi^E(\hat{f}^E \mid \hat{f}, \hat{p}^m) = B\hat{p}^m = B(1 - \lambda^b(1 - \hat{f}^E)) > B\lambda^b \hat{f}^E.$$

If $\hat{f} < p^{\hat{m}}$, then

$$\begin{aligned} \Pi^I(\hat{f}, \hat{p}^m \mid \hat{f}^E) + \Pi^E(\hat{f}^E \mid \hat{f}, \hat{p}^m) &= B(1 - e^{-\frac{B}{S}})\hat{f} + B\lambda^b e^{-\frac{B}{S}} \hat{f}^E \\ &> B[(1 - e^{-\frac{B}{S}}) + \lambda^b e^{-\frac{B}{S}}]\hat{f}^E > B\lambda^b \hat{f}^E. \end{aligned}$$

Thus,

$$\lim_{j\to\infty} \Pi^I(f^{(j)}, p^{m(j)} \mid f^{E(j)}) + \Pi^E(f^{E(j)} \mid f^{(j)}, p^{m(j)}) < \Pi^I(\hat{f}, \hat{p}^m \mid \hat{f}^E) + \Pi^E(\hat{f}^E \mid \hat{f}, \hat{p}^m).$$

As these two extreme cases give the upper and lower bounds, respectively, all other sequences give some limits in between. Therefore,

$$\lim_{j\to\infty} \Pi^I(f^{(j)}, p^{m(j)} \mid f^{E(j)}) + \Pi^E(f^{E(j)} \mid f^{(j)}, p^{m(j)}) \leq \Pi^I(\hat{f}, \hat{p}^m \mid \hat{f}^E) + \Pi^E(\hat{f}^E \mid \hat{f}, \hat{p}^m),$$

for any of the sequences converging to $\{\hat{f}, \hat{p}^m, \hat{f}^E\}$, and so $\Pi^I + \Pi^E$ is upper semi-continuous. This completes the proof of Proposition 13. ■

C Web Appendix: Participation fees

In this Additional Appendix, we show that our main result does not change in a version of our model where the middleman's supply is not observable in the participation stage, but instead the intermediary can use participation fees/subsidy. Suppose now that in the first stage the intermediary announces a set of fees $F \equiv \{f^b, f^s, g^b, g^s\}$ for the platform, where $f^b,\ f^s \in [0,1]$ is a transaction fee charged to a buyer or a seller, respectively, and $g^b,\ g^s \in [-1,1]$ is a registration fee charged to a buyer or a seller, respectively.

As is consistent with the main analysis, we follow the literature of two-sided markets and assume that agents hold pessimistic beliefs on the participation decision of agents on the other side of the market (Caillaud and Jullien, 2003). Agents believe that the intermediary would never supply anything at all unless the C market attracts some buyers. This is the worst situation for the intermediary. A pessimistic belief of sellers means that sellers believe the number of buyers participating in the C market is zero whenever

$$\lambda^b\beta > -g^b,$$

where $\lambda^b\beta$ is the expected payoff of buyers in the D market and $-g^b$ is the payoff buyers receive in the C market (it is a participation subsidy when $g^b < 0$).

Single-market search: To induce the participation of agents under those beliefs, the best the intermediary can do is to use a divide-and-conquer strategy, denoted by h. To divide buyers and conquer sellers, referred to as $h = D_bC_s$, it is required that

$$D_b \quad : \quad -g^b \geq \lambda^b\beta, \tag{67}$$
$$C_s \quad : \quad W - g^s \geq 0. \tag{68}$$

The divide-condition D_b tells us that the intermediary should subsidize the participating buyers so that they receive at least what they would get in the D market, *even if the C market is empty.* This makes sure buyers will participate in the C market whatever happens to the other side of the market. The conquer-condition C_s guarantees the participation of sellers, by giving them a nonnegative payoff – the participation fee $g^s \geq 0$ should be no greater than the expected value of sellers in the C market, $W = W(x^s)$. Observing that the intermediary offers buyers enough to participate, sellers understand that all buyers are in the C market, the D market is empty, and so the expected payoff from the D market is zero. Here, the expected value of sellers in the C market W is defined under the sellers' belief that the intermediary will select the capacity level optimally given the full participation of buyers.

Similarly, a strategy to divide sellers and conquer buyers, referred to as $h = D_sC_b$, requires that

$$D_s \quad : \quad -g^s \geq \lambda^s(1-\beta), \tag{69}$$
$$C_b \quad : \quad V - g^b \geq 0. \tag{70}$$

where $V = \max\{V^s(x^s), V^m(x^m)\}$ is the expected value of buyers in the C market.

Given the participation decision of agents described above, the intermediary's problem of determining the intermediation fees $F = \{f^b, f^s, g^b, g^s\}$ for $h = \{D_bC_s, D_sC_b\}$ is described as

$$\Pi = \max_{F,h}\{Bg^b + Sg^s + \max_{p^m,K} \Pi(p^m, f, K)\},$$

subject to (67) and (68) if $h = D_bC_s$, or (69) and (70) if $h = D_sC_b$. Here, Bg^b and Sg^s are participation fees from buyers and sellers, respectively, and $\Pi(\cdot)$ is the expected profit in the C market described above. Under either of the divide-and-conquer strategies, the choice of participation fees g^i, $i = b, s$, does not influence anyone's behaviors in the C market. The choice of transaction fees affects the expected value of agents and thus the participation fees and intermediary's profits. However, it does not alter the original solution, a pure middleman, remains optimal.

Proposition 14 *With unobservable capacity and with participation fees, the intermediary sets $f > 1$, $p^m = 1$ and $K = B$. All the buyers buy from the middleman, $x^m = B$, and the platform is inactive, $x^s = 0$. The intermediary makes profits,*

$$\Pi = B - \min\{B\lambda^b\beta, S\lambda^s(1-\beta)\},$$

guaranteeing the participation of agents by $h = D_bC_s$ if $\beta < \frac{1}{2}$ and $h = D_sC_b$ if $\beta > \frac{1}{2}$.

Proof. Consider first $h = D_bC_s$. Then, by (67) and (68), $g^b = -\lambda^b\beta$ and $g^s = W$. For $f > 1$, no buyers go to the platform $x^s = 0$ and all buyers are in the middleman sector $x^m = B$, yielding $g^s = W = 0$. By selecting $K = B$ and $p^m = 1$, the intermediary makes profits,

$$\Pi = -B\lambda^b\beta + \Pi(p^m, 1, B) = (-\lambda^b\beta + 1)B.$$

To show that this is indeed the maximum profit, we have to check two possible cases. Suppose $f = f^b + f^s \le 1$ and $K = 0$. Then, $x^s = \frac{B}{S}$ and $x^m = 0$, and $g^s = W(B/S) \ge 0$, if there is a non-negative surplus in the platform for buyers, $f^b + p^s \le 1$, and for sellers, $f^s \le p^s$. The resulting profit satisfies

$$\begin{aligned} Bg^b + Sg^s + \Pi(p^m, f, 0) &= -B\lambda^b\beta + S(1-e^{-\frac{B}{S}})(p^s - f^s) + S(1-e^{-\frac{B}{S}})f \\ &= -B\lambda^b\beta + S(1-e^{-\frac{B}{S}})(f^b + p^s) \\ &< -B\lambda^b\beta + B = \Pi \end{aligned}$$

for all $f^b + p^s \le 1$. Hence, this is not profitable.

Suppose $f = f^b + f^s \le 1$ and $K \in (0, B]$, and both sectors have a non-negative surplus to buyers, i.e., $p^m \le 1$ and $f^b + p^s \le 1$. This leads to $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up requirement (3) and the indifferent condition (4). Then, $g^s = W(x^s) \ge 0$, and the resulting profit is

$$\begin{aligned} &Bg^b + Sg^s + \Pi(p^m, f, K) \\ &\quad = -B\lambda^b\beta + S(1-e^{-x^s})(p^s - f^s) + S(1-e^{-x^s})f + \min\{K, x^m\}p^m \\ &\quad < -B\lambda^b\beta + Sx^s(f^b + p^s) + x^m p^m \\ &\quad \le -B\lambda^b\beta + (Sx^s + x^m)\max\{f^b + p^s, p^m\} \\ &\quad \le -B\lambda^b\beta + B = \Pi \end{aligned}$$

for all $f^b + p^s \leq 1$ and $p^m \leq 1$. Hence, this is not profitable either. All in all, no deviation is profitable for $h = D_b C_s$.

Consider next $h = D_s C_b$. Then, by (69) and (70), $g^s = -\lambda^s(1-\beta)$ and $g^b = V$. When $f > 1$, no one go to the platform $x^s = 0$ and all buyers are in the middleman sector $x^m = B$ as long as $p^m \leq 1$. This yields $g^b = V = V^m(B) \geq 0$ and $\Pi(p^m, f, B) = Bp^m$ with $K = B$. The profits are

$$\Pi = -S\lambda^s(1-\beta) + B(1-p^m) + \Pi(p^m, f, K) = -S\lambda^s(1-\beta) + B.$$

To show that this is indeed the maximum profit, we have to check two possible cases. Suppose $f = f^b + f^s \leq 1$ and $K = 0$. Then, $x^s = \frac{B}{S}$ and $x^m = 0$, and $g^b = V = V^s(B/S) \geq 0$, if there is a non-negative surplus in the platform for buyers, $f^b + p^s \leq 1$, and for sellers, $f^s \leq p^s$. This leads to

$$\begin{aligned} Sg^s + Bg^b + \Pi(p^m, f, 0) &= -S\lambda^s(1-\beta) + B\frac{1-e^{-\frac{B}{S}}}{\frac{B}{S}}(1-p^s-f^b) + S(1-e^{-\frac{B}{S}})f \\ &= -S\lambda^s(1-\beta) + S(1-e^{-\frac{B}{S}})(1-p^s+f^s) \\ &< -S\lambda^s(1-\beta) + B = \Pi \end{aligned}$$

for all $f^s \leq p^s$. Hence, this is not profitable.

Suppose $f = f^b + f^s \leq 1$ and $K \in (0, B]$, and both sectors have a non-negative surplus to buyers, i.e., $p^m \leq 1$ and $f^b + p^s \leq 1$. This leads to $x^m \in (0, B)$ and $x^s \in (0, \frac{B}{S})$ that satisfy the add-up constraint (3), $Sx^s + x^m = B$, and the indifferent condition (4), $V^s(x^s) = V^m(x^m)$. Then, $g^b = V = V^s(x^s)$, and the resulting profit is

$$\begin{aligned} &Sg^s + Bg^b + \Pi(p^m, f, K) \\ &\quad = -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + \min\{K, x^m\}p^m. \end{aligned}$$

There are two cases. Suppose $K \geq x^m$. Then, the indifferent condition (4) implies that

$$p^m = 1 - \frac{1-e^{-x^s}}{x^s}(1-p^s-f^b).$$

Applying this expression to the profits, we get

$$\begin{aligned} &Sg^s + Bg^b + \Pi(p^m, f, K) \\ &\quad = -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + x^m\left(1 - \frac{1-e^{-x^s}}{x^s}(1-p^s-f^b)\right) \\ &\quad = -S\lambda^s(1-\beta) + (B-x^m)\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + x^m \\ &\quad = -S\lambda^s(1-\beta) + S(1-e^{-x^s})(1-p^s+f^s) + x^m \\ &\quad < -S\lambda^s(1-\beta) + B \end{aligned}$$

for all $f^s \leq p^s$. Suppose $K < x^m$. Then, the indifferent condition implies that

$$p^m = 1 - \frac{x^m}{K}\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b).$$

Applying this expression to the profits, we get

$$\begin{aligned}
&Sg^s + Bg^b + \Pi(p^m, f, K) \\
&\quad = -S\lambda^s(1-\beta) + B\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + K\left(1 - \frac{x^m}{K}\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b)\right) \\
&\quad = -S\lambda^s(1-\beta) + (B-x^m)\frac{1-e^{-x^s}}{x^s}(1-p^s-f^b) + S(1-e^{-x^s})f + K \\
&\quad = -S\lambda^s(1-\beta) + S(1-e^{-x^s})(1-p^s+f^s) + K \\
&\quad < -S\lambda^s(1-\beta) + B
\end{aligned}$$

for all $f^s \leq p^s$. Hence, any deviation is not profitable for $h = D_sC_b$.

Finally, since the intermediary makes the maximum revenue B for either h, which side should be subsidized is determined by the required costs: noting $B\lambda^b = S\lambda^s$, we have $B\lambda^b\beta \gtreqless S\lambda^s(1-\beta) \iff \beta \gtreqless \frac{1}{2}$. This completes the proof of Proposition 14. ■

Multi-market search: With multiple-market search, any non-positive registration fee can ensure that agents are in the C market, since the participation to the C market is not exclusive. Hence, attracting one side of the market becomes less costly. By contrast, conquering the other side becomes more costly, since the conquered side still holds the trading opportunity in the D market. The D_sC_b condition with multiple-market search is

$$\begin{aligned}
D_s &: \; -g^s \geq 0, \\
C_b &: \; \max\{V^s(x^s), V^m(x^m)\} - g^b \geq \lambda^b e^{-x^s}\beta(1-c).
\end{aligned}$$

The divide-condition D_s tells that now a non-positive fee is sufficient to convince one side to participate. The conquer-condition C_b now needs to compensate for the outside option in the D market. Similarly, the D_bC_s condition becomes

$$\begin{aligned}
D_b &: \; -g^b \geq 0, \\
C_s &: \; W(x^s) - g^s \geq \lambda^s\xi(x^s, x^m)(1-\beta)(1-c).
\end{aligned}$$

Participation fees are designed to induce buyers and sellers' participation. Once agents join the C market, the participation fees become sunk costs, and will not influence their trading decision.

The intermediary's problem of choosing $F = \left\{f^b, f^s, g^b, g^s\right\}$ together with $h = \{D_bC_s, D_sC_b\}$ and $p^m, K \in [0, B]$ are described as

$$\Pi = \max_{F,h,K}\left\{Bg^b + Sg^s + \max_{p^m}\Pi(p^m, f, K)\right\}, \tag{71}$$

where $\Pi(p^m, f, K) = S(1-e^{-x^s})f + \min\{K, x^m\}p^m - Kc$. Besides the divide-and-conquer constraints, this maximization problem is also subject to the incentive constraints as described in the main text.

Proposition 15 *In the extended problem described in (71) with unobservable capacity, participation fees and multiple-market search, the determination of the profit-maximizing intermediation mode is identical to the one described in Proposition 2, with $g^i = 0$, $i = s, b$.*

Proof. It suffices to prove that the solution is $g^i = 0$, $i = s, b$ for each intermediation mode, since then the problem (71) will become identical to the one we have already solved in the main text. For a pure middleman mode ($x^m = B$), the intermediary sets $g^b = 0$ to divide buyers, with $p^m = 1 - \lambda^b \beta(1-c)$ satisfying (12). For a pure market-maker mode ($x^s = 0$), either with $D_b C_s$ or $D_s C_b$, the intermediary sets the transaction fee to satisfy the binding incentive constraint (19), $f = v(0,0) = \left[1 - \lambda^b e^{-B/S} - \lambda^s \xi(0,0)\right](1-c)$, and $g^b = g^s = 0$.

For a hybrid mode, the intermediary's problem is subject to the incentive constraint (19), and p^m satisfying (22) so that buyers are indifferent between the two modes. We can rewrite the maximization problem (71) as a two-stage problem over a vector $\mathbf{X} \equiv (x^m, f, K) \in \mathbb{X}$, where $\mathbb{X} \equiv [0, B] \times [0, 1] \times [0, K]$:

$$\begin{aligned} \text{Stage 1:} \quad & \max_{(f,K)} Bg^b(\mathbf{X}) + Sg^s(\mathbf{X}) + \Pi(x^m(f,K), f, K) \\ & s.t.\ 0 \le f \le v(x^m(f,K), K),\ 0 \le K \le B. \\ \text{Stage 2:} \quad & \max_{x^m} \Pi(x^m, f, K) \\ & s.t.\ f \le v(x^m, K),\ 0 \le x^m \le B, \end{aligned} \tag{$\mathcal{A}$}$$

where $g^b(\mathbf{X})$ and $g^s(\mathbf{X})$ are given by the binding divide-and-conquer conditions,

$$g^b(\mathbf{X}) = 0,\ g^s(\mathbf{X}) = \left(1 - e^{-x^s} - x^s e^{-x^s}\right)(v(x^m, K) - f),$$

if $h = D_b C_s$, or

$$g^s(\mathbf{X}) = 0,\ g^b(\mathbf{X}) = e^{-x^s}(v(x^m, K) - f).$$

if $h = D_s C_b$. As our objective is to prove $g^i(\mathbf{X}) = 0$, $i = s, b$, all that remains here is to show that $f = v(x^m, K)$ at the solution. However, it is immediate that the objective function in ($\mathcal{A}$) is strictly increasing in f and any change in f ($< v(x^m, K)$) does not influence the other constraints. Hence, as in the original problem, we must have $f = v(x^m, K)$. This completes the proof of Proposition 15. ■

D Web Appendix: Empirical Appendix

Data and Variables. From *www.amazon.com* and *www.ebay.com*, we retrieve all paginated results listed in the category of Amazon called: *"All Pans"*, which is a subcategory of *"Home & Kitchen: Kitchen & Dining: Cookware"*. This subcategory includes 400 pages of more than 9000 products as of August 2018.[29] For each pan, we obtain the price (*price*), the sales rank in the category *"Home & Kitchen"* (*rank*), the listing days since the first listed date on Amazon by either Amazon or some other sellers on Amazon's market-maker platform (*listedDays*), the number of third-party sellers that sell this product on Amazon (*sellersAmazon*), whether the product is sold by Amazon itself (*sellByAmazon*) and the title of the product.

Sellers could offer various prices for a product on Amazon. We obtain price information from the default page Amazon displays when users search for a product. This gives us the price at which the majority of transactions are processed. Amazon does not publish sales data but does provide a sales ranking for each product. Since ranking information is provided at different levels of categories, in order to make the sales ranking as comparable as possible, we adopt the ranking at the highest possible level *"Home & Kitchen"*. This gives us the variable *rank*.

The title of the product is used to link each product on Amazon to the outside option available at eBay as the theory develops. For each product collected on Amazon, we search its "Amazon product name" on eBay to obtain all related offers. As a proxy for the buyers' matching probability in the decentralized market λ^b, we count the number of all the offers shown in eBay's raw search result. We call this variable *sellersEbayAll*. Admittedly, this is a very noisy measure. EBay tends to provide a long list with offers that are only loosely related to the product. For example, in some cases a pan offered on eBay only matches with some key features of a pan offered on Amazon such as size but it does not match other features such as materials. In this case, we compare the similarity between the eBay product title and the Amazon product title. In some other cases, the titles are similar but the products turn out to be different. For example, searching a pan on eBay only yields an offer of the lid of the same pan on Amazon. To solve this issue, we use the following two-step procedure. We first select offers with product names similar to the Amazon product name.[30] We then refine the list by restricting the offer price between 0.5 and 1.5 times the Amazon price. The rationale for this procedure is that if the offer price is far away from the Amazon offer, the product is likely to vary in quality or could even be a distinct product. Counting the number of sellers in this refined list leads to another proxy for λ^b, *sellersEbayRefined*. This is a more precise measure for the relevant number of sellers on eBay. We will use *sellersEbayRefined* in our main regressions, and use *sellersEbayAll* as a robustness check.

As an alternative proxy for λ^b, we could use the number of sellers on eBay relative to that of Amazon,[31]

$$sellersEbayRelative = \frac{sellersEbayRefined}{sellersAmazon}.$$

[29] The URL of the list of all pans is *https://www.amazon.com/pans/b?node=3737221* (visited on August 24, 2018).

[30] Here, we use the Fuzzy String Matching Library in Python which computes a score between 0 and 100, with 100 indicating the exact matching. The function `fuzz.token_set_ratio()` computes the score and only selects offers with a score higher than 80. We also tried other criteria scores such as 60 and 90. The results are robust.

[31] We use the number of third-party sellers (that is excluding Amazon if Amazon sells) in the denominator.

This measure proxies the relative success probability of meeting a seller on eBay versus Amazon. It is constructed based on a typical buyer's online shopping experience. When a buyer discovers dozens of sellers on Amazon, it is relatively less likely that he can find even better offers outside Amazon, so the perceived outside value of going to eBay is low. In contrast, for a buyer who observes only few sellers on Amazon, the expected payoff of searching on the outside market is high. *sellersEbayRelative* is therefore likely to be positively correlated with λ^b.

As a proxy for buyers' bargaining power in the outside market, β, we compute the price difference between eBay and Amazon. For each product, we find the median price in the refined eBay offer list and compute the log of this price minus the log of Amazon price. This defines the variable *priceDiff*. We expect this variable to be negatively correlated with β (recall that a higher β implies a larger share of the surplus for the buyer so a lower price in the decentralized market).

Descriptive Analysis. We collected information for 9066 products on Amazon and found matched eBay offers for 7944 of them. Variables may have missing values leading to a smaller sample size. For example, ranking information might be provided not in the aggregate category *"Home & Kitchen"* but in some subcategories with incomplete ranking. We did try different (sub)categories to extract ranking information, and it turned out that *"Home & Kitchen"* gave us the largest valid sample with 7942 observations. In the regressions below, we exclude products without any matched eBay offers to avoid missing *priceDiff*, and exclude products without any third-party sellers on Amazon to avoid missing *sellersEbayRelative*. Finally, we only collect offers for brand new products.

Table 2: Summary Statistics

Variables	Obs.	Mean	Std.	Min.	Max.
sellByAmazon	9066	0.32	0.46	0.00	1.00
listedDays	8168	1759.54	1458.12	8.00	6864.00
price	8856	64.03	107.00	0.01	2118.83
rank	7942	440711	314288	28	2581111
sellersAmazon	9066	3.70	5.36	0.00	77.00
sellersEbayAll	9066	14.69	14.13	0.00	60.00
sellersEbayRefined	9066	6.53	8.05	0.00	43.00
sellersEbayRelative	8487	3.30	5.35	0.00	43.00
priceDiff	7944	0.07	0.74	-5.08	6.86
sellersEbayAll_60	9066	20.88	15.87	0.00	62.00
sellersEbayRefined_60	9066	10.54	10.53	0.00	48.00
sellersEbayRelative_60	8487	5.59	7.69	0.00	44.00
priceDiff_60	8349	0.07	0.72	-5.08	6.66

Note: The table reports summary sample statistics for the merged scraped data from www.amazon.com and www.ebay.com. The last four variables *sellersEbayAll_60*, *sellersEbayRefined_60*, *sellersEbayRelative_60*, *priceDiff_60* are defined on a dataset constructed by searching only the first 60 characters of Amazon product title in eBay's search engine. They are used in robustness checks. Finally, we calculate the statistics of each variable with all valid observations in the dataset.

Table 2 presents summary statistics for our main variables of interest. For 32% of the products in our sample, Amazon acts as a middleman; for the other 68% products, Amazon acts as a pure platform. On average, the products have been on sale at Amazon for almost 5 years, although this varies across products from several days to 18 years. There is a large variation in the price and ranking. The maximum price is as high as \$2118.83. The mean price is \$64, the 25th percentile is \$18.9, the 75th percentile is \$72.9. The number of third-party sellers for a product ranges from 0 to 77 with a mean of 3.7 sellers. The number of sellers on eBay is much larger with a mean of 14.69 for *sellersEbayAll* and a mean of 6.53 for *sellersEbayRefined*. On average, the number of sellers on eBay is more than three times as high as the number of sellers on Amazon. Finally, variables with suffix 60 come from another dataset constructed for robustness checks and will be discussed later.

Table 3: Correlations among proxy variables

	logRank	*priceDiff*	*sellerEbay Refined*	*sellerEbay All*	*sellerEbay Relative*
logRank	1.0000				
priceDiff	-0.1359	1.0000			
sellersEbay Refined	-0.1462	-0.0901	1.0000		
sellersEbay All	-0.0595	-0.0797	0.7063	1.0000	
sellersEbay Relative	0.06839	-0.1203	0.6790	0.4866	1.0000

In table 3, the linear correlations among proxies are very weak. Correlations among proxies for different parameters are around 0.1.

Robustness Checks. We shall pursue a number of robustness checks. A first concern is that our result could be driven by the way that we count the number of eBay offers. To address this issue, instead of refining the list of eBay offers, we use the raw list of eBay offers to calculate the number of sellers, *sellersEbayAll*, and replace *sellersEbayRelative* by *sellersEbayAll/sellersAmazon*. Our results are robust to this change as shown in Table 4: although the coefficients of *sellersEbayAll* and *sellersEbayRelative* become smaller, they remain negative. The coefficient of *sellersEbayAll* now becomes non-significant, while *sellersEbayRelative* is still statistically significant. Relative to the result summarized in Table 1, the coefficients of the other variables remain almost the same.

A second concern is a bias caused by using the eBay search engine. We find that the number of offers provided by the eBay search engine is negatively correlated with the length of search text. In general, the longer the search text is, the lower the number of results that the eBay search engine can provide. Hence, the longer the product name is, the less likely it can find good matches in its database. This implies that we may ignore good matches if we provide a very long product name with too much information. For example, the same product may have different product titles by different sellers emphasizing different product features, such as size and color of the pan. In some cases, eBay can not give any offer when searching the whole Amazon product title, but does give the right offers when searching part of the Amazon product title. More importantly, there

Table 4: Regressions for Amazon's intermediation mode using the raw eBay search results

	(1) Linear	(2) Linear	(3) Probit	(4) Probit
sellersEbayRelative	-0.00354***		-0.00478***	
	(0.000428)		(0.000613)	
sellersEbayAll		-0.000226		-0.000439
		(0.000382)		(0.000426)
sellersAmazon		-0.000691		-0.000765
		(0.000915)		(0.000996)
log(rank)	-0.101***	-0.106***	-0.102***	-0.108***
	(0.00444)	(0.00461)	(0.00500)	(0.00513)
priceDiff	0.100***	0.112***	0.122***	0.136***
	(0.00844)	(0.00847)	(0.0111)	(0.0112)
log(price)	0.0346***	0.0402***	0.0440***	0.0503***
	(0.00614)	(0.00622)	(0.00689)	(0.00698)
listedDays	0.0606***	0.0664***	0.0719***	0.0788***
	(0.00479)	(0.00485)	(0.00596)	(0.00604)
Observations	6457	6457	6457	6457
Adjusted R^2	0.135	0.129		

Note: This table reports the robustness check using the raw eBay search results reflected in *sellersEbayAll* and *sellersEbayRelative*. Except for this change, the specification is the same as before.

exists anecdotal evidence showing that the product title on Amazon is longer if it is registered by Amazon itself rather than by third-party sellers. If this is true, we may have spurious correlations. To solve this issue, we construct a second dataset by searching all product names using only the first 60 characters on eBay.[32]

The variables *sellersEbayAll_60*, *sellersEbayRefined_60*, *sellersEbayRelative_60* and *priceDiff_60* are constructed in this new dataset. As shown in the last four rows in the summary statistics Table 2, the average number of sellers for each product becomes larger. For example, in terms of the length of the raw search list, the average increases from 14.69 to 20.88. However, the relative prices between eBay and Amazon do not change much. The results using this new dataset are reported in Table 5 and yield similar relationships as our main ones. There are more observations in the regressions because some Amazon product titles which had no eBay offer before can now be matched. As in our previous regressions, the coefficients of other variables remain almost unchanged.

Table 5: Regressions for Amazon's intermediation mode using first 60 characters to search eBay offers

	(1) Linear	(2) Linear	(3) Probit	(4) Probit
sellersEbayRelative	-0.00450***		-0.00488***	
	(0.000595)		(0.000816)	
sellersEbayRefined		-0.000352		-0.000170
		(0.000499)		(0.000556)
sellersAmazon		-0.00587***		-0.00708***
		(0.000847)		(0.00112)
log(rank)	-0.101***	-0.0950***	-0.102***	-0.0984***
	(0.00423)	(0.00460)	(0.00468)	(0.00515)
priceDiff	0.114***	0.126***	0.134***	0.146***
	(0.00819)	(0.00849)	(0.0105)	(0.0106)
log(price)	0.0431***	0.0503***	0.0529***	0.0586***
	(0.00602)	(0.00614)	(0.00658)	(0.00667)
listedDays	0.0611***	0.0646***	0.0741***	0.0747***
	(0.00449)	(0.00480)	(0.00566)	(0.00576)
Observations	6822	6822	6822	6822
Adjusted R^2	0.138	0.100		

Note: This table reports the robustness check based on eBay search results using only the first 60 characters of the Amazon product title. Except for using new variable reflecting this change, *sellersEbayAll_60*, *sellersEbayRefined_60*, *sellersEbayRelative_60* and *priceDiff_60*, the specification remains the same as before.

[32]In our data, the median length of product title is 65, the minimum is 9, and the maximum is 245. We also tried to cut the first 50 or 80 characters. The results are similar.

References

Acemoglu, D. and Shimer, R. (1999). Holdups and efficiency with search frictions. *International Economic Review*, 40(4):827–849.

Anand, A., Tanggaard, C., and Weaver, D. G. (2009). Paying for market quality. *Journal of Financial and Quantitative Analysis*, 44(6):1427–1457.

Armstrong, M. (2006). Competition in two-sided markets. *The RAND Journal of Economics*, 37(3):668–691.

Armstrong, M. and Zhou, J. (2015). Search deterrence. *The Review of Economic Studies*, 83(1):26–57.

Awaya, Y., K. I. and Watanabe, M. (2018a). Speculative bubbles in an economy with flippers. *mimeo.*

Awaya, Y., Z. C. and Watanabe, M. (2018b). Intermediation and reputation. *mimeo.*

Baye, M. R. and Morgan, J. (2001). Information gatekeepers on the internet and the competitiveness of homogeneous product markets. *American Economic Review*, 91(3):454–474.

Biglaiser, G. (1993). Middlemen as experts. *The RAND journal of Economics*, pages 212–223.

Bloch, F. and Ryder, H. (2000). Two-sided search, marriages, and matchmakers. *International Economic Review*, 41(1):93–116.

Burdett, K., Shi, S., and Wright, R. (2001). Pricing and matching with frictions. *Journal of Political Economy*, 109(5):1060–1085.

Butters, G. R. et al. (1977). Equilibrium distributions of sales and advertising prices. *Review of Economic Studies*, 44(3):465–491.

Caillaud, B. and Jullien, B. (2001). Competing cybermediaries. *European Economic Review*, 45(4-6):797–808.

Caillaud, B. and Jullien, B. (2003). Chicken & egg: Competition among intermediation service providers. *RAND journal of Economics*, pages 309–328.

Condorelli, D., Galeotti, A., and Skreta, V. (2018). Selling through referrals. *Journal of Economics & Management Strategy*, 27(4):669–685.

Conroy, R. M. and Winkler, R. L. (1986). Market structure: The specialist as dealer and broker. *Journal of Banking & Finance*, 10(1):21–36.

Damiano, E. and Li, H. (2008). Competing matchmaking. *Journal of the European Economic Association*, 6(4):789–818.

Dasgupta, P., Maskin, E., et al. (1986). The existence of equilibrium in discontinuous economic games, i: Theory. *Review of Economic Studies*, 53(1):1–26.

De Fraja, G. and Sákovics, J. (2012). Exclusive nightclubs and lonely hearts columns: non-monotone participation in optional intermediation. *Journal of Economic Behavior & Organization*, 84(2):618–632.

De los Santos, B., Hortaçsu, A., and Wildenbeest, M. R. (2012). Testing models of consumer search using data on web browsing and purchasing behavior. *American Economic Review*, 102(6):2955–80.

Duffie, D., Gârleanu, N., and Pedersen, L. H. (2005). Over-the-counter markets. *Econometrica*, 73(6):1815–1847.

Edelman, B., Wright, J., et al. (2015). Price restrictions in multi-sided platforms: Practices and responses. *Harvard Business School NOM Unit Working Paper*, (15-030).

Fingleton, J. (1997). Competition among middlemen when buyers and sellers can trade directly. *The journal of industrial economics*, 45(4):405–427.

Galeotti, A. and Moraga-González, J. L. (2009). Platform intermediation in a market for differentiated products. *European Economic Review*, 53(4):417–428.

Gehrig, T. (1993). Intermediation in search markets. *Journal of Economics & Management Strategy*, 2(1):97–120.

Geromichalos, A. and Jung, K. M. (2018). An over-the-counter approach to the forex market. *International Economic Review*, 59(2):859–905.

Guerrieri, V., Shimer, R., and Wright, R. (2010). Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862.

Hagiu, A. (2006). Pricing and commitment by two-sided platforms. *The RAND Journal of Economics*, 37(3):720–737.

Hagiu, A. and Jullien, B. (2011). Why do intermediaries divert search? *The RAND Journal of Economics*, 42(2):337–362.

Hagiu, A. and Wright, J. (2013). Do you really want to be an ebay? *Harvard Business Review*, 91(3):102–108.

Hagiu, A. and Wright, J. (2014). Marketplace or reseller? *Management Science*, 61(1):184–203.

Hendershott, T. and Menkveld, A. J. (2014). Price pressures. *Journal of Financial Economics*, 114(3):405–423.

Holzner, C. and Watanabe, M. (2016). Understanding the role of the public employment agency. *Tinbergen Institute Discussion Paper 16-041/VII.*

Johri, A. and Leach, J. (2002). Middlemen and the allocation of heterogeneous goods. *International Economic Review*, 43(2):347–361.

Ju, J., Linn, S. C., and Zhu, Z. (2010). Middlemen and oligopolistic market makers. *Journal of Economics & Management Strategy*, 19(1):1–23.

Lagos, R. and Rocheteau, G. (2007). Search in asset markets: Market structure, liquidity, and welfare. *American Economic Review*, 97(2):198–202.

Lagos, R. and Rocheteau, G. (2009). Liquidity in asset markets with search frictions. *Econometrica*, 77(2):403–426.

Lagos, R., Rocheteau, G., and Weill, P.-O. (2011). Crashes and recoveries in illiquid markets. *Journal of Economic Theory*, 146(6):2169–2205.

Lagos, R. and Zhang, S. (2016). Monetary exchange in over-the-counter markets: A theory of speculative bubbles, the fed model, and self-fulfilling liquidity crises. *NBER working paper.*

Li, Y. (1998). Middlemen and private information. *Journal of Monetary Economics*, 42(1):131–159.

Loertscher, S. and Niedermayer, A. (2008). Fee setting intermediaries: on real estate agents, stock brokers, and auction houses. Technical report.

Maskin, E. (1986). The existence of equilibrium with price-setting firms. *The American Economic Review*, 76(2):382–386.

Masters, A. (2007). Middlemen in search equilibrium. *International Economic Review*, 48(1):343–362.

Menkveld, A. J. and Wang, T. (2013). How do designated market makers create value for small-caps? *Journal of Financial Markets*, 16(3):571–603.

Mittal, H. (2008). Are you playing in a toxic dark pool?: A guide to preventing information leakage. *The Journal of Trading*, 3(3):20–33.

Moen, E. R. (1997). Competitive search equilibrium. *Journal of political Economy*, 105(2):385–411.

Nimalendran, M. and Petrella, G. (2003). Do thinly-tradedstocks benefit from specialist intervention? *Journal of banking & finance*, 27(9):1823–1854.

Nocke, V., Peitz, M., and Stahl, K. (2007). Platform ownership. *Journal of the European Economic Association*, 5(6):1130–1160.

Nosal, E., Wong, Y.-y., and Wright, R. (2015). More on middlemen: Equilibrium entry and efficiency in intermediated markets. *Journal of Money, Credit and Banking*, 47(S2):7–37.

Parker, G. G. and Van Alstyne, M. W. (2005). Two-sided network effects: A theory of information product design. *Management science*, 51(10):1494–1504.

Peters, M. (1991). Ex ante price offers in matching games non-steady states. *Econometrica (1986-1998)*, 59(5):1425.

Peters, M. (2000). Limits of exact equilibria for capacity constrained sellers with costly search. *Journal of Economic Theory*, 95(2):139–168.

Pissarides, C. A. (2000). *Equilibrium unemployment theory*. MIT press.

Rhodes, A., Watanabe, M., Zhou, J., et al. (2017). Multiproduct intermediaries and optimal product range. Technical report, University Library of Munich, Germany.

Rochet, J.-C. and Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the european economic association*, 1(4):990–1029.

Rochet, J.-C. and Tirole, J. (2006). Two-sided markets: a progress report. *The RAND journal of economics*, 37(3):645–667.

Rubinstein, A. and Wolinsky, A. (1987). Middlemen. *The Quarterly Journal of Economics*, 102(3):581–593.

Rust, J. and Hall, G. (2003). Middlemen versus market makers: A theory of competitive exchange. *Journal of Political Economy*, 111(2):353–403.

Rysman, M. (2009). The economics of two-sided markets. *Journal of economic perspectives*, 23(3):125–43.

Securities and Exchange Commission (2010). Securities exchange act release no. 34-61358, 75 fr 3594, concept release.

Securities and Exchange Commission (2013). Equity market structure literature review part i: Market fragmentation. *US Securities and Exchange Commission.*

Securities and Exchange Commission (2014). Equity market structure literature review part ii: High frequency trading. *US Securities and Exchange Commission.*

Shevchenko, A. (2004). Middlemen. *International Economic Review*, 45(1):1–24.

Spulber, D. F. (1996). Market making by price-setting firms. *The Review of Economic Studies*, 63(4):559–580.

Spulber, D. F. (1999). *Market microstructure: intermediaries and the theory of the firm.* Cambridge University Press.

Stahl, D. O. (1988). Bertrand competition for inputs and walrasian outcomes. *The American Economic Review*, pages 189–201.

Tuttle, L. A. (2014). OTC trading: Description of non-ats otc trading in national market system stocks. *SSRN Working Paper 2412100.*

Venkataraman, K. and Waisburd, A. C. (2007). The value of the designated market maker. *Journal of Financial and Quantitative Analysis*, 42(3):735–758.

Watanabe, M. (2010). A model of merchants. *Journal of Economic Theory*, 145(5):1865–1889.

Watanabe, M. (2018a). Middlemen: A directed search equilibrium approach. *Tinbergen Institute working paper.*

Watanabe, M. (2018b). Middlemen: The visible market-makers. *The Japanese Economic Review*, 69(2):156–170.

Weill, P.-O. (2007). Leaning against the wind. *The Review of Economic Studies*, 74(4):1329–1354.

Weyl, E. G. (2010). A price theory of multi-sided platforms. *American Economic Review*, 100(4):1642–72.

Wright, R. and Wong, Y.-Y. (2014). Buyers, sellers, and middlemen: Variations on search-theoretic themes. *International Economic Review*, 55(2):375–397.

Yavaş, A. (1994). Middlemen in bilateral search markets. *Journal of Labor Economics*, 12(3):406–429.

Yavaş, A. (1996). Search and trading in intermediated markets. *Journal of Economics & Management Strategy*, 5(2):195–216.

Zhu, F. and Liu, Q. (2018). Competing with complementors: An empirical look at amazon. com. *Strategic Management Journal*, 39(10):2618–2642.