

A Model of Supply Chain Finance*

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Abstract

This article develops a model where an intermediary, benefiting from a matching advantage, operates both as a conventional middleman and participates in a finance program to support suppliers. The finance program pools suppliers' liquidity, enabling prompt payment to those facing urgent liquidity needs. We demonstrate how the intermediary selects suppliers for both modes. In the finance model, the intermediary strategically chooses suppliers based not only on their positive profitability but also considers those with negative profitability who still contribute to the liquidity pool. By integrating the model into a standard monetary framework, we find that higher nominal interest rates prompt adjustments in the middleman's mode allocation, with the finance mode prioritizing liquidity more over profitability. We identify non-monotonic effects of funding costs on the liquidity provision of the finance program and overall welfare.

Keywords: Supply Chain Finance, Liquidity Pooling, Liquidity Cross-subsidization, Money Search, Intermediary

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1 Introduction

Trade finance has a long history that dates back to at least the 14th century. In recent years, the rise of fintech innovation and increased financial accessibility have led to the popularity of a new type of trade finance known as “supply chain finance” (SCF, also called “supplier finance”, “reverse factoring”, or “payable finance”).¹ According to the estimates from BCR Publishing Ltd, the size of the corporate supply-chain finance market has increased to \$1.8 trillion globally in the year 2021, with a 38% growth compared to the previous year.² Despite the huge success in the industry, SCF has received little attention from the economics academia.

We develop a theoretical framework of supply chain finance to capture the following four features that are commonly adopted by SCF programs: (1) a big buyer firm, e.g., a retailer like Walmart or a manufacturer like Siemens, initiates the SCF program, choosing from a vast array of diverse suppliers; (2) the selection is aimed at enhancing not only the profitability but also the financial health of the entire supply chain; (3) suppliers are often required to give extended trade credit, allowing the buyer firm to delay payment for a significant period; and (4) participating suppliers can access early payment options.

In the benchmark outlined in Section 2, we consider a one-period model with a mass of suppliers, each producing a distinct consumption good, and a mass of consumers, each endowed with a numeraire good and having unit demand for all the consumption goods. Suppliers and consumers engage in bilateral trade. With probability $1 - \lambda$, a supplier can use his retail revenue, i.e., the numeraire received from a consumer, to cover production costs. However, with probability λ the supplier is unable to commit to trade. This creates a liquidity shock to the supplier, because this time he cannot use the forthcoming retail revenue to cover production costs. Consequently, without his own endowment, the supplier cannot produce nor trade.

We assume that suppliers differ in marginal production cost, c , and the probability of liquidity shocks, λ . The pair (λ, c) summarizes all relevant information about the heterogeneity of suppliers. We assume that (λ, c) is publicly observable.

In this economy, we introduce an intermediary who does not produce nor consume but is specialized in operating an SCF program. The intermediary could be a retailer, a manufacturer, or a commercial bank partnered with key players in the supply chain. We could add other roles

¹The *accounting standards update 2022-04* issued by the Financial Accounting Standards Board (FASB) precisely describes the operation of a supply chain finance program: “Typically, a buyer in a program (1) enters into an agreement with a finance provider or an intermediary to establish the program, (2) purchases goods and services from suppliers with a promise to pay at a later date, and (3) notifies the finance provider or intermediary of the supplier invoices that it has confirmed as valid. Suppliers may then request early payment from the finance provider or intermediary for those confirmed invoices.”

²Supply chain finance has been widely adopted by many large companies, including retail intermediaries such as Walmart, Amazon, Alibaba, JD.com, and large manufacturers such as GE, Nestle, Siemens, and Samsung. The trend has been accompanied by the emergence of digital platforms such as Taulia, C2FO, PrimeRevenue, and Tradeshift, offering a range of financial solutions with SCF as their main service. According to a 2019 PwC survey, 68% of companies in Europe and North America use supply chain finance, with 27% using reverse factoring specifically. The 2020 McKinsey Global Payments Report estimates that \$400 billion of assets were financed through reverse factoring in 2018, and the report also projects an expected growth rate of 15% – 20% between 2019 and 2024.

of intermediation, which are considered to be important, e.g. providing a platform or holding inventory, but it turns out that those roles are not very crucial at least in our model. The intermediary selects suppliers to invite to the SCF program and makes each of them a take-it-or-leave-it offer. Those who are not selected operate as an independent entity subject to liquidity shocks. The participating suppliers are required to give the intermediary their retail revenue. In return, the suppliers receive an early payment of c to cover their production costs, which frees them from liquidity shocks, and a fixed reward at the end of the period, which could depend on (λ, c) . In the benchmark model, we assume that the intermediary has an initial endowment of the numeraire good that can be contributed to the program. We endogenize her endowment in later sections. Handling each participating supplier requires a fixed cost for the intermediary.

Each participating supplier can make a positive or negative contribution to the program in terms of profits and liquidity. Suppliers with a higher likelihood of liquidity shocks, i.e. a higher λ (who appreciate the program more) and a lower marginal cost c (who have a higher profit margin) are more likely to make a positive contribution in profits. Suppliers with a lower λ (who are less likely to request early payment) and a lower c (who need a smaller amount of liquidity) are more likely to make a positive contribution to the liquidity pool. We show that the profit-maximizing SCF program involves “liquidity cross-subsidization” among suppliers, i.e., subsidizing the liquidity needs of suppliers who have positive profitability by using the liquidity contributed by suppliers who have negative profitability. We show this strategy is more profitable than selecting only suppliers with positive profits.

The significance of liquidity cross-subsidization is determined by the shadow value of liquidity in the SCF program. This value equals the multiplier of the middleman’s liquidity constraint, which is higher as the liquidity constraint becomes more stringent or the middleman’s initial endowment decreases. In situations where the endowment is sufficiently large, liquidity ceases to be a concern for the overall pool of suppliers, and the SCF program only invites suppliers with positive profitability.

To endogenize the middleman’s initial endowment, and also to make the medium of exchange explicit, in Section 3, we insert the baseline model to a standard monetary framework of Lagos and Wright (2005) as the day market, and the intermediary and consumers can access liquidity in the night (Walrasian) market. We maintain the assumption that suppliers are liquidity-constrained when entering the day market. We show that at the optimal (interior) liquidity holding, the shadow value of liquidity in the SCF program equals the liquidity price in the market, i.e., the nominal interest rate. With higher nominal interest rates, the SCF program emphasizes the liquidity contribution more and the profitability contribution less. When the nominal interest rate is sufficiently high, the intermediary does not hold money but the SCF program remains to be profitable thanks to the liquidity cross-subsidization among suppliers.

In our economy, supply chain finance is welfare improving. This is because the program

enables suppliers experiencing a liquidity shock to continue their production. Moreover, the more suppliers participating in the program, the higher the aggregate trading volume, and the higher the aggregate welfare. At the Friedman rule, liquidity is not a concern for the selection of suppliers, and so the SCF program only invites suppliers with positive profitability. At a positive nominal interest rate, liquidity becomes costly for the intermediary and so she must compare the non-zero cost of her own liquidity holding versus the cost of using the liquidity contribution by participating suppliers. Due to the liquidity cross-subsidization, the latter cost is derived taking into account the participants' profit contribution. When the intermediary is more efficient, i.e., with a lower cost of handling suppliers, the profit contribution of every supplier is uniformly higher and so the cost of using the suppliers' liquidity pool is relatively lower. Therefore, with a sufficiently efficient intermediary, more suppliers are invited to the program as the liquidity cost increases. Then, the aggregate trading volume is higher. The suboptimality of the Friedman rule follows.

In Section 4, we show that an SCF program can be active for intermediate nominal-interest rates, even when suppliers' access to the money market is allowed. For low nominal interest rates, suppliers with positive profit contributions choose to hold onto their money, rather than using the SCF, to prepare for the liquidity shock. For high nominal interest rates, consumers trade only with those suppliers with low prices, namely low c 's, to avoid inflation costs. In either case, the intermediary becomes unprofitable and so the SCF ceases to be active. For intermediate nominal-interest rates, supply chain finance and suppliers' money holdings coexist. Namely, suppliers with high λ and low c choose to hold money by themselves, and a subset of the rest of the suppliers are selected for the SCF program.

In Section 5, we offer anecdotal evidence that supports the implications of our model and demonstrate its relevance to other financial arrangements, such as keiretsu in Japan and rural credit cooperatives in 19th-century Germany. All proofs are included in the Appendix. The rest of this section is a literature review.

Related Literature

Among the New Monetarist models based on Lagos and Wright (2005), our paper is broadly related to the banking models, e.g., Berentsen, Camera and Waller (2007), Gu, Mattesini, Monnet and Wright (2013), and Andolfatto, Berentsen and Martin (2019), and the financial intermediation models, e.g. Bethune, Sultanum and Trachter (2022). A distinct feature of our model is the ex-ante section of heterogeneous depositors (suppliers in our model). This feature is also absent in the nonmonetary banking literature following Diamond and Dybvig (1983). An intermediary in supply chain finance, which could be a bank collaborating with key players in the supply chain, is endowed with the information advantage in observing the type of each supplier. Unlike

demand deposits, the SCF contract only promises to advance a limited amount of liquidity to suppliers. Thus, runs can be avoided.

Closely related to ours is the growing literature on money and corporate finance.³ Rocheteau, Wright and Zhang (2018) emphasize the strategic role of firms' liquidity holdings. A lower nominal interest rate prompts firms to hold more cash, which helps to negotiate a favourable loan term with the bank. Bethune, Rocheteau, Wong and Zhang (2021) highlight a monetary channel through which a lower nominal interest rate decreases the banks' incentive to create lending relationships with firms who have a stronger bargaining position. Our model uncovers a novel channel of monetary policy transmission to corporate finance — the provision of trade credit. In the context of supply chain finance, a lower nominal interest rate induces the intermediary to use their own money holdings more, and use trade credit of suppliers less. Thus, suppliers of higher profits, rather than higher liquidity, are more likely to be included in the supply chain of the intermediary.

In the literature on the coexistence of money and credit, Gu, Mattesini and Wright (2016) show that changes in credit limit have no impact on allocations and welfare in a general setup. In their model, this occurs because money and credit are perfect substitutes and so real money balance can adjust perfectly to changes in credit conditions. Trade credit in our model is also a perfect substitute for money. However, since our credit is very different from theirs, it is not clear how to define the credit limit that is comparable to their model. One key difference would be that in our model, an extension of the size of intermediation always improves welfare because it makes trade/consumption happen even when suppliers are hit by a liquidity shock.

Finally, our model of the intermediary is closely related to Rhodes, Watanabe and Zhou (2021) who study the product assortment problem of a multi-product intermediary. They show that the middleman's problem can be described as the choice of a set of points in a simple two-dimensional statistic, just like ours. The middleman's optimal product assortment includes high-value products with low profitability, which make a direct loss to the intermediary, and low-value products with high profitability, which recoups those losses. In our baseline model, this mechanism creates the liquidity cross-subsidization that the intermediary optimally induces when selecting among heterogeneous suppliers. Further, we endogenize the middleman's liquidity-holding decision, and link it to the extent to which liquidity cross-subsidization occurs in a standard monetary equilibrium.

³There is a literature on supply chain finance in operations management, e.g., Tunca and Zhu (2017), Devalkar and Krishnan (2019), Kouvelis and Xu (2021), with the focus on comparing supply chain finance with other types of financial arrangements for various stakeholders in the supply chain. Our paper differs from the literature in many major aspects, e.g., we study supply chain finance as a contract between an intermediary and multiple small suppliers, rather than a single contract between one buyer firm and one supplier. Our study is also related to the trade credit literature in finance, e.g., Petersen and Rajan (1997), Burkart and Ellingsen (2004), Cuñat (2007), Giannetti, Burkart and Ellingsen (2011), Garcia-Appendini and Montoriol-Garriga (2013), Nocke and Thanassoulis (2014), and Bottazzi, Gopalakrishna and Tebaldi (2023), etc. This literature argues that suppliers have a monitoring advantage over banks, which motivates the provision of trade credit despite high implicit interest rates. We consider supply chain finance as a type of financing that enables early payment to suppliers based on the trade credit provided by them, which eventually leads to liquidity reallocation among suppliers.

2 The benchmark model

2.1 Set-ups

Consider a one-period economy with three types of agents: a mass one of consumers, a mass one of suppliers (*he*), and *one* intermediary (*she*). Each supplier produces a unique and indivisible good at a constant marginal cost c . Suppliers differ in $c \in [\underline{c}, \bar{c}]$ with $\bar{c} > \underline{c} \geq 0$, and c is publicly observable. Consumers are homogeneous and have unit demand for each good with a common utility $u \geq \bar{c}$.

The intermediary does not produce nor consume, instead she can opt to work as a middleman that buys goods from suppliers and resells to consumers. Besides, she also has access to a costly finance technology that enables her to delay payments to suppliers and then make use of the funds to finance suppliers that are in need of liquidity support. The details of these technologies will be specified below.

There exists a numeraire good that can be used as the means of payment in retail trade. In the benchmark, we assume buyers have an endowment of the numeraire good sufficient for retail trade. The intermediary has an endowment denoted by $L \geq 0$, whereas suppliers have no endowment. These endowments are exogenously given. We will endogenize the endowment of all agents in the latter sections.

Agents trade in a retail market where each good is sold either by its supplier or by the intermediary. If the intermediary chooses to sell a good, she purchases the goods from the supplier, and the supplier exits the retail market.⁴ No matter who sells the goods, whether the supplier or the intermediary, the seller can reach all consumers. Once a consumer and a supplier (or intermediary) meet, trade takes place bilaterally, and the trade surplus is split equally between the two parties. The equilibrium retail price is given by

$$p - c = \frac{u - c}{2}.$$

Liquidity shocks. Suppliers need immediate funds to produce. In particular, they need to pay production costs c using the numeraire good. However, since they possess no endowment of the numeraire, c can only be covered by using sales revenue or by external financing.

Suppliers face an idiosyncratically liquidity shock. With probability $1 - \lambda$, the supplier can commit to trade. Then, consumers are willing to pay p and the supplier can use the retail revenue to cover production costs. In this case, production (payment of production cost c) and trade (payment of retail price p) take place simultaneously, just like in the frictionless trade. The supplier can engage in production and trade without possessing the numeraire good. With probability

⁴The tie-breaking assumption is that if suppliers are indifferent, they join the intermediary rather than selling directly in the market.

λ , the supplier is unable to commit to trade and consumers are unwilling to pay p until the goods are delivered to them. As a result, the production cost c cannot be covered using the retail revenue, and the supplier cannot produce because he has no numeraire goods.

We can interpret λ as the probability of being hit by a liquidity shock. We assume that it differs across suppliers and is publicly observable. For later discussion, we label suppliers encountering a liquidity shock as *late* suppliers, because their retail revenue is obtained only after production is completed. We label suppliers experiencing no liquidity shock as the *early* suppliers.⁵

Suppliers' ex-ante heterogeneity can be indexed by a pair (λ, c) . Denote the two-dimensional space where (λ, c) belongs to by $\Omega \equiv [0, \bar{\lambda}] \times [\underline{c}, \bar{c}]$ with $\bar{\lambda} < 1, 0 < \underline{c} < \bar{c} < u$. The pair (λ, c) follows a continuous distribution which has a cumulative distribution function G , and a density function g that is everywhere positive in Ω .

Finally, we assume that, due to a lack of enforcement technologies, there's no credit market among suppliers. Consequently, individual suppliers are unable to hedge against liquidity shocks or combine liquidity resources independently.

Middleman and Finance modes of the intermediary. The intermediary can opt to operate two modes, a *middleman* mode and a *finance* mode. In the middleman mode (also referred to as the *M* mode), the intermediary sells the goods on behalf of suppliers. Relative to individual suppliers, the intermediary may have a stronger or a weaker commitment to provide the goods to consumers. We assume that when intermediary sells goods for a supplier of λ , the probability that the intermediary cannot commit to trade is $m\lambda$. If $m < 1$, the intermediary has more commitment power than an individual supplier since the probability of not commit to trade is lower. If $1 < m < 1/\bar{\lambda}$, the intermediary has a lower commitment power than an individual supplier.

Nevertheless, the commitment power of the intermediary is positively associated with the commitment power of the supplier. We believe that this assumption is realistic. In the real world, when a supplier possesses strong commitment power, perhaps through long-standing partnerships or market dominance, the intermediary is likely to ensure product availability and exposure to consumers. Conversely, if a supplier lacks the ability to fully commit to trade, possibly due to limited market presence or an uncertain supply relationship, the intermediary faces similar constraints in ensuring product promotion and availability. Therefore, the commitment power of the intermediary reflects that of the original suppliers with whom she engages with the possibility that commitment power is further strengthened or weakened.⁶

⁵An alternative interpretation of the liquidity shock is as follows. Suppose a period is divided into two parts: an early subperiod and a late subperiod. Production is only possible in the early subperiod, while the supplier meets consumers either early or late but not both. At the beginning of the period, a shock is realized, indicating whether the supplier's consumers will arrive early or late. If the supplier meets consumers early, he can use the retail revenue to cover production costs. In this case, production (paying the cost c) and trade (receiving the retail price p) occur simultaneously. If the consumers arrive late, he faces a liquidity shortage because he cannot use future revenue to meet the immediate need to cover c . In essence, when consumers arrive late, it's akin to experiencing a liquidity shock, as it highlights the need for external financing to bridge the gap between production costs and revenue.

⁶To see intuitively the link of commitment power between suppliers and the intermediary, imagine a small, local

In the finance mode (also referred to as the F mode), the intermediary offers liquidity to suppliers. The *finance technology* provides two key advantages. Firstly, it gives the intermediary the enforcement capability to make credit deals with suppliers. This stands in contrast to the suppliers, who lack such enforcement technologies. Secondly, it allows the intermediary to use the numeraire that she has not yet paid to suppliers to support other suppliers in need of liquidity. This makes the delayed payments to suppliers, essentially the “trade credit”, a crucial source of liquidity for the intermediary’s finance mode. Using this finance technology is costly. We assume that for each supplier are that included in the F mode, there is a lump-sum cost $k \in (0, \bar{k})$ with $\bar{k} < u \frac{\bar{\lambda}^2}{1+\bar{\lambda}} < u/2$.

It’s possible for the intermediary to both sell goods on behalf of the supplier and provide liquidity support simultaneously. This mode is known as the hybrid mode (the H mode). In this mode, the probability of not committing to trade is $m\lambda$, reflecting the intermediary’s involvement in selling the goods. Additionally, the intermediary utilizes financial technology to postpone payment and allocate funds to support suppliers’ production costs. To further distinguish between the three modes, let’s examine the contracts offered by the intermediary.

The intermediary observes (λ, c) of all suppliers and selects a subset of them into one of three modes. Let $q_j(\lambda, c) \in \{0, 1\}$, $j \in \mathcal{J} = \{M, F, H\}$ be the selection function which equals 1 indicating the (λ, c) -suppliers are selected into the j mode. A supplier who does not join the intermediary sells directly to consumers in the retail market, in which case the supplier can produce and trade only if he is not hit by the shock (i.e., his consumers arrive early).

Middleman contracts. Suppose a supplier is invited to the middleman mode. Then, he is offered a take-it-or-leave-it contract which stipulates that: (1) The intermediary sells the goods on the supplier’s behalf (and the supplier exits the market); (2) The Intermediary pays $f_M(\lambda, c) \geq 0$ to the supplier *immediately after she receives payment from consumers*, and pays nothing to the supplier if she receives no payment from consumers. Essentially, suppliers in the middleman mode can proceed with production and delivery only if the intermediary successfully commits to consumers the delivery of goods (which occurs with probability $1 - m\lambda$). And only in that case can the supplier use the revenue to cover production costs.

Middleman finance contracts. Suppose a supplier is invited to the middleman finance mode (or hybrid mode), $q_H(\lambda, c) = 1$, then he is offered a take-it-or-leave-it contract which stipulates that: (1) The intermediary sells the goods on behalf of the supplier; (2) In return, the supplier is

artisanal cheese producer supplies their products to Walmart. Due to constraints such as limited production capacity or inconsistent quality control, the cheese producer struggles to reliably meet the market demand. In this situation, the cheese producer’s inability to fully commit to trade directly affects Walmart’s ability to ensure product availability and consistency on its shelves. Despite Walmart’s efforts to promote the cheese products, irregular supply from the producer leads to stock shortages and inconsistent availability for consumers. As a result, Walmart’s commitment power is clearly related to that of the cheese producer.

awarded a payment $f_H(\lambda, c) \geq 0$ at the end of the period; (3) The intermediary pays the cost c to the supplier at the time of production.

The hybrid mode contract differs from the middleman contract in two aspects. First, in a hybrid contract, payments to suppliers are postponed to the end of the period. With the finance technology, this deferral allows the intermediary to leverage the delayed payments as a liquidity source to fund suppliers that are in need of liquidity. Second, the hybrid contract extends liquidity support of c at the time of production, which ensures the supplier can produce and trade despite the supplier may not have the numeraire to cover production costs.

Finance contracts. Suppose a supplier is invited to the finance mode, then he is offered a take-it-or-leave-it contract which stipulates that: (1) The supplier sells the good directly to consumers, and transfer his retail revenue to the intermediary; (2) In return, the supplier receives a reward at the end of the period. The amount of the reward depends on the timing of the revenue transfer. If the supplier transfers his revenue to the intermediary at the time of production, he receives $f^E(\lambda, c)$, and if he transfers after production, he receives $f^L(\lambda, c)$. (3) The intermediary pays the cost c to the supplier at the time of production, irrespective of whether she has received the transferred revenue from the supplier.

If a supplier accepts this contract, he participates in the finance program which enables him to always produce and trade, irrespective of whether he is hit by a liquidity shock or not. It is clear that a participating supplier transfers the revenue after production if he experiences a liquidity shock, because his consumers pay only after the goods have been produced. On the other hand, a participating supplier will transfer the revenue at the time of production if he encounters no liquidity shock. This is incentive compatible as long as $f^E(\lambda, c) \geq f^L(\lambda, c)$.⁷ Obviously, a profit-maximizing intermediary should choose $f^E(\lambda, c) = f^L(\lambda, c) \equiv f_F(\lambda, c)$.

We summarize the set of the intermediary's offers by a pair of functions for $j \in \mathcal{J} \equiv \{M, H, F\}$:

$$\{q_j(\lambda, c), f_j(\lambda, c)\}_{(\lambda, c) \in \Omega}.$$

Timing. First, observing (λ, c) of all suppliers, the intermediary announces her offers, and selects which suppliers to invite and to which mode. The selected suppliers decide whether or not to accept the intermediary's offer. Second, the liquidity shock is realized for each supplier and for each goods that intermediary sells, and trade occurs in the retail market. Suppliers that join the finance or the hybrid mode can request early payment of c . Finally, the intermediary settles any outstanding payments due to the suppliers, i.e., $f_M(\cdot)$ immediately after consumers pay, and

⁷A potential incentive concern is that suppliers may request early payment even when they don't need liquidity. This issue doesn't arise in the current setup, as suppliers must always pay c , which is known by the intermediary. However, consider a scenario where a supplier has a probability of requiring liquidity to meet production costs, but the realized need isn't observed by the intermediary. In such a case, a small additional cost for requesting early payment would ensure that the finance offer is incentive-compatible.

$f_H(\cdot)$ as well as $f_F(\cdot)$ by the end of the period.

2.2 The intermediary

The intermediary makes offers subject to suppliers' participation constraints. If a supplier of (λ, c) directly sells in the retail market, he produces and sells only if he turns out to be an early type. The expected profit is

$$(1 - \lambda)(p - c).$$

Since the intermediary can observe (λ, c) , she can make the rewards f_j , $j \in \mathcal{J}$, dependent on (λ, c) . To entice the supplier to participate, the reward must be no less than what he would make by selling directly to consumers. This implies that for the middleman mode:

$$(1 - m\lambda)f_M \geq (1 - \lambda)(p - c) + (1 - m\lambda)c,$$

f_M is multiplied by probability $(1 - m\lambda)$ because the supplier receives f_M only if the middleman receives payment from consumers. Also, in the middleman mode, the intermediary has to compensate for the production cost c using f_M . This is not the case for the finance and hybrid modes since in those modes c is covered through the finance support:

$$f_F \geq (1 - \lambda)(p - c), \quad f_H \geq (1 - \lambda)(p - c).$$

Profit-maximization of the intermediary then leads to all these individual rationality constraints binding:

$$f_M(\lambda, c) = \frac{(1 - \lambda)(u - c)/2}{1 - m\lambda} + c, \quad f_H(\lambda, c) = f_F(\lambda, c) = (1 - \lambda)(u - c)/2, \quad (1)$$

where we have used that trade surplus is split equally.

Next, consider the profit and liquidity the intermediary obtains by having a supplier. If a supplier is selected into the middleman mode, the intermediary can make a profit of $\pi_M(\lambda, c)$:

$$\pi_M(\lambda, c) = (1 - m\lambda)(p - f_M(\lambda, c)) = (1 - m)\lambda(u - c)/2. \quad (2)$$

Intuitively, the middleman mode can make a positive profit only if $m < 1$. The intermediary only contributes if individual suppliers cannot commit to trade but the intermediary can commit to trade, which happens with probability $(1 - m)\lambda$. Thus, $(1 - m)\lambda(u - c)/2$ represents the contribution of the intermediary. Of course, without the use of financial technology, the middleman cannot utilize the numeraire goods from the profits π_M for other purposes, such as lending to suppliers. Then the intermediary obtains no liquidity from the middleman mode that are relevant for her profit-maximization problem:

$$\theta_M(\lambda, c) = 0.$$

If a supplier is selected into the finance mode, then the intermediary can make a profit of $\pi_F(\lambda, c)$:

$$\pi_F(\lambda, c) = p - c - f_F - k = \lambda(u - c)/2 - k, \quad (3)$$

since the intermediary receives a transfer p , covers the supplier's production costs c , and rewards him by f_F , given a fixed cost $k > 0$. It is clear from (3) that the benefit of the finance mode is to make production possible even for late suppliers, as is captured by the term $\lambda(u - c)/2$. This expected benefit is higher with a higher λ (as the supplier is less likely to trade if he chooses to operate independently) and a lower c (as the good has a higher profit margin).

Turn to the liquidity side of the F mode. The intermediary obtains a retail revenue when the supplier can commit to the trade, and at the same time, she must cover the production costs of all the participating suppliers. Hence, the net expected amount of the numeraire good that a supplier indexed by (λ, c) brings to the intermediary before f_F is paid is

$$\theta_F(\lambda, c) = (1 - \lambda)p - c = (1 - \lambda)(u + c)/2 - c. \quad (4)$$

Finally, if a supplier is selected into the hybrid mode, then the intermediary can make a profit of $\pi_H(\lambda, c)$:

$$\pi_H(\lambda, c) = p - c - f_H - k = \lambda(u - c)/2 - k, \quad (5)$$

which is the same as in the finance mode. However, the liquidity side differs. The intermediary obtains a retail revenue when herself (*not the supplier*) can commit to the trade which occurs with probability $1 - m\lambda$, and at the same time, she covers the production costs. This give the following liquidity contribution of a supplier in the hybrid mode:

$$\theta_H(\lambda, c) = (1 - m\lambda)p - c = (1 - m\lambda)(u + c)/2 - c. \quad (6)$$

Now, the intermediary's problem of selecting which subset of suppliers to invite and to which mode can be written as:

$$\max_{\{q_j(\lambda, c)\}_{j \in \mathcal{J}} \in \{0,1\}^3} \int_{\Omega} \left[\sum_{j \in \mathcal{J}} q_j(\lambda, c) \pi_j(\lambda, c) \right] dG, \quad (7)$$

subject to the liquidity constraint:

$$\Theta + L \geq 0, \quad (8)$$

where Θ is the total liquidity contributed by all suppliers participating in the intermediary:

$$\Theta = \int_{\Omega} \left[\sum_{j \in \mathcal{J}} q_j(\lambda, c) \theta_j(\lambda, c) \right] dG.$$

The constraint states that the total liquidity contribution of participating suppliers plus the avail-

able liquidity $L \geq 0$ supplied by the intermediary's endowment should be non-negative.

The intermediary's problem defined above is an optimization of functionals, and the optimal solution can be derived by using the following Lagrange method (see e.g., Rhodes et al. 2021). Let $\mu \geq 0$ be the multiplier associated with the liquidity constraint (8). Combining (8) with the objective function, we can construct the Lagrangian:

$$\mathcal{L} = \int_{\Omega} \left[\sum_{j \in \mathcal{J}} q_j(\lambda, c) \left(\pi_j(\lambda, c) + \mu \theta_j(\lambda, c) \right) \right] dG(\lambda, c).$$

Notice that π_j and θ_j can be positive or negative depending on parameters (note that $\theta_M(\lambda, c)$ is always zero.) Using this Lagrangian, the solution to the intermediary's problem can be obtained as the optimal selection policies $q_j(\lambda, c, \mu)$, which depend not only on (λ, c) but also on μ :

$$q_j(\lambda, c, \mu) = \begin{cases} 1 & \text{if } \pi_j(\lambda, c) + \mu \theta_j(\lambda, c) \geq \max_{h \neq j} \{ \pi_h(\lambda, c) + \mu \theta_h(\lambda, c), 0 \}; \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

The optimal selection policies can be further simplified by considering if the intermediary has a stronger commitment power, $m < 1$, or a weaker commitment power, $m > 1$, compared to individual suppliers.

Weaker commitment power of the intermediary: $m > 1$. In this case, the middleman mode is never profitable: $\pi_M(\lambda, c) < 0$ for all $(\lambda, c) \in \Omega$, and the finance mode dominates the hybrid mode as it delivers a higher liquidity contribution $\theta_F(\lambda, c) > \theta_H(\lambda, c)$ for all $(\lambda, c) \in \Omega$.⁸ This pins down the problem of the intermediary as whether or not to select a supplier into the finance mode. The optimal selection rule is simplified to $q_M = q_H = 0$ and

$$q_F(\lambda, c, \mu) = \begin{cases} 1 & \text{if } \pi_F(\lambda, c) + \mu \theta_F(\lambda, c) \geq 0; \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

Note that $q_F(\lambda, c, \mu) = 1$ consists of three possible scenarios:

$$\pi_F(\lambda, c) \geq 0, \theta_F(\lambda, c) \geq 0, \quad (11a)$$

$$\pi_F(\lambda, c) > 0, \theta_F(\lambda, c) < 0, -\pi_F/\theta_F \geq \mu, \quad (11b)$$

$$\pi_F(\lambda, c) < 0, \theta_F(\lambda, c) > 0, -\pi_F/\theta_F \leq \mu. \quad (11c)$$

In scenario (11a) the intermediary selects suppliers with positive π_F and positive θ_F , who contribute to both profits and liquidity. In scenario (11b), the intermediary selects suppliers with positive π_F and negative θ_F , provided the gross return of liquidity, measured by $-\pi_F/\theta_F$, is

⁸When liquidity constraint (8) is not binding, it makes no difference for the intermediary to use finance mode and hybrid mode because $\mu = 0$ and $\pi_F(\lambda, c) = \pi_H(\lambda, c)$ for all $(\lambda, c) \in \Omega$. There is also no difference in allocations because participating suppliers sell to all consumers given the liquidity support in either mode. So we let the intermediary use hybrid mode in this case.

higher than the shadow value of liquidity μ . In the last scenario (11c), the intermediary selects suppliers with negative π_F and positive θ_F as these suppliers contribute to aggregate liquidity. The cost of getting one unit of liquidity from these suppliers is $-\pi_F/\theta_F$, and the intermediary should absorb liquidity from these suppliers if $-\pi_F/\theta_F \leq \mu$.

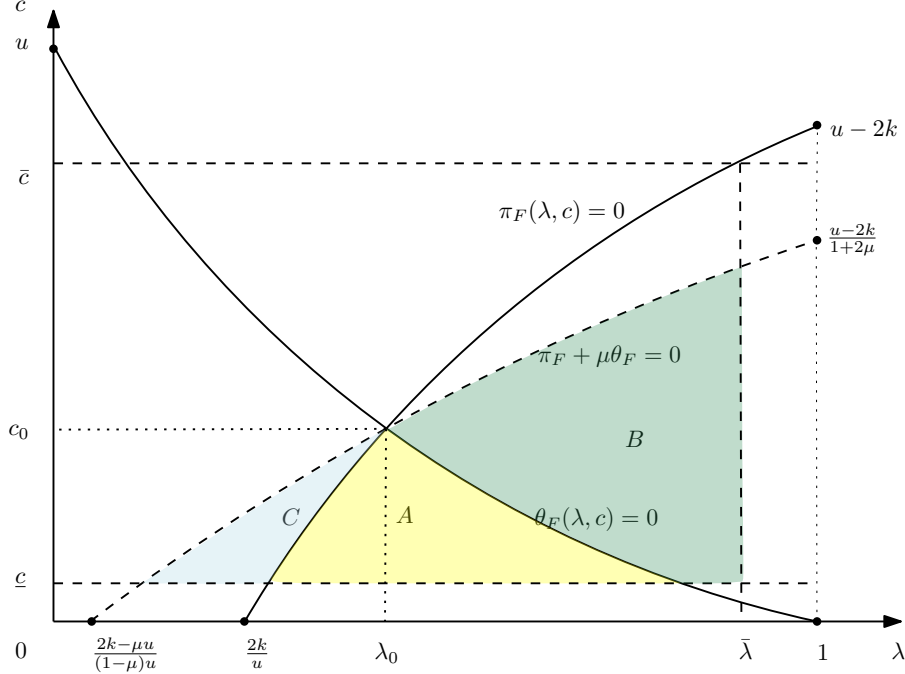


Figure 1: Middleman's selection of suppliers ($m < 1$)

To illustrate the three scenarios in a figure, we insert π_F and θ_F from (3) and (4) and obtain three boundaries that lie in Ω :

$$\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\theta_F}(\lambda) \equiv \frac{1-\lambda}{1+\lambda}u \quad (12a)$$

$$\pi_F(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\pi_F}(\lambda) \equiv u - \frac{2k}{\lambda} \quad (12b)$$

$$\pi_F(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq b_F(\lambda, \mu) \equiv \frac{\lambda u - 2k + \mu(1-\lambda)u}{\lambda + \mu(1+\lambda)} \quad (12c)$$

Note that the right-hand side of (12c) is a “weighted average” of the right-hand sides of the first two. We plot these three boundaries in Figure 1, annotated by $\theta_F(\lambda, c) = 0$, $\pi_F(\lambda, c) = 0$, and $\pi_F + \mu\theta_F = 0$, respectively. Any suppliers below $\theta_F(\lambda, c) = 0$ contribute to the liquidity pool of the program, and any suppliers below $\pi_F(\lambda, c) = 0$ contribute to the intermediary's profits. The overlapping set A represents suppliers in scenario 12a, which are selected in the finance mode because they contribute to both profits and liquidity. Suppliers in set B (corresponding to scenario 12b) have net liquidity needs, but the intermediary earns positive profits. Suppliers in set C give the intermediary negative profits but contribute to the liquidity pool as in scenario 12c.

Overall, the profit-maximizing intermediary adopts what we called a *profit-based liquidity*

cross-subsidization strategy. This approach involves using the positive net liquidity contributions from suppliers in regions A and C to address the liquidity needs of suppliers in region B .

What sets this strategy apart from standard liquidity pooling across homogeneous agents is that utilizing liquidity from suppliers comes at a cost, and providing liquidity support to suppliers requires a sufficiently high profit return. In essence, when the intermediary uses liquidity contributions from region C , it incurs a cost in the form of reduced (or negative) profits to these suppliers. However, when providing liquidity support to suppliers in region B , the intermediary expects a return in the form of positive profits from these suppliers.

Balancing the cost and benefit of obtaining liquidity and providing liquidity is crucial, and it's done based on the profit-liquidity ratio. In other words, the intermediary carefully evaluates whether the profit gained from providing liquidity support outweighs the cost incurred from using liquidity contributions from other suppliers.⁹

The intermediary's available liquidity L shapes the feasibility of the program via the liquidity constraint and especially μ . If L is higher (which leads to a smaller μ as shown in Corollary 1), the curve $\pi_F + \mu\theta_F = 0$ is closer to $\pi_F = 0$, and the intermediary selects suppliers primarily based on profits. If L is lower (which leads to a larger μ), $\pi_F + \mu\theta_F = 0$ is closer to $\theta_F = 0$. Then liquidity becomes more important when selecting suppliers, and the intermediary relies more on the profit liquidity cross-subsidization among suppliers. It is important to note that even a supplier that has high profits may not be chosen by the intermediary if he contributes little to the liquidity pool.

It remains to determine μ , the shadow value of liquidity for the intermediary. If (8) is binding, then μ is determined by

$$L = -\Theta(\mu) \equiv - \int_{\Omega} q(\lambda, c, \mu) \theta_F(\lambda, c) dG. \quad (13)$$

If (8) is not binding, then $\mu = 0$. In this case, the intermediary selects suppliers irrespective of liquidity concerns, i.e. the optimal selection rules select supplier solely based on π_F .

Proposition 1 *If $\Theta(0) + L < 0$, then there exists a unique $\mu > 0$ that satisfies (13); and otherwise, $\mu = 0$.*

Corollary 1 *Provide that the finance mode is active, $\mu(L) > 0$ is strictly decreasing in L if $\Theta(0) + L < 0$.*

The liquidity value μ can be zero if the participating suppliers provide a sufficiently large amount of liquidity, $-\Theta(0) \leq L$. Note that this can be either with a positive liquidity pool $\Theta(0) \geq 0$ or a negative liquidity pool $\Theta(0) < 0$. Otherwise, the intermediary's endowment has a positive liquidity value, $\mu > 0$. It is intuitive that μ is strictly decreasing in $L \in [0, -\Theta(0)]$: an ad-

⁹It is a dual problem of the original problem (7) to maximize the total liquidity contribution Θ subject to the total profit contributions to be no less than a certain level.

ditional unit of the intermediary's endowment is appreciated more when her initial endowment is relatively low.

As can be seen from Figure 1, if one moves from the case $\mu = 0$ (i.e. high L) to the case $\mu > 0$ (i.e. low L), the selection curve $\pi_F + \mu\theta_F = 0$ moves clockwise centered at the point (λ_0, c_0) . That is, in the region with $\theta_F < 0$, π_F has to be increased from 0 to a positive as μ increases from 0 to a positive, deleting the suppliers with relatively high λ and c . On the other hand, in the region with $\theta_F > 0$, π_F has to be decreased from 0 to a negative as μ increases from 0 to a positive, adding the suppliers with relatively low λ and c . Thus, when her own liquidity endowment L gets smaller, the intermediary's selection makes more use of the liquidity from suppliers.

Under what conditions that the finance mode is profitable? To see this, recall that (λ_0, c_0) is the interaction point of two curves $\pi_F(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$, as is shown in Figure 1. Intuitively, as k increases, the $\pi_F(\lambda, c) = 0$ curve moves downward. Thus, c_0 is strictly decreasing in k . Moreover, selecting all the suppliers with both a positive π_F and a positive θ_F (the set of suppliers in region A) satisfies the liquidity constraint for any $L \geq 0$, and gives a positive profit to the intermediary. Thus, the finance mode must be activated as long as $c_0 > \underline{c}$. The following lemma states that $c_0 > \underline{c}$ is both necessary and sufficient condition for a profitable F mode under $L = 0$.

Lemma 1 *Suppose $L = 0$ and $m > 1$, the intermediary opts for the finance mode if and only if $c_0 \geq \underline{c}$, or equivalently, the cost of the financial technology k is sufficiently low:*

$$k \leq \frac{u - \underline{c}}{2} \frac{u - \underline{c}}{u + \underline{c}}. \quad (14)$$

Proof. With $c_{\pi_F}(\lambda) = c_{\theta_F}(\lambda)$, we can solve for $c_0(k) = k + u - \sqrt{k(k + 4u)}$. Thus, $c_0 \geq \underline{c}$ iff (14) holds. If $c_0 > \underline{c}$, region A in Figure 1 is non-empty, F mode then makes positive profits under any $L \geq 0$. Next we argue that if $L = 0$ and $c_0 \leq \underline{c}$, using the F mode is not profitable. Note $b_F(\lambda, \mu)$ is either upward sloping or downward sloping with respect to λ (it holds that $b_\lambda(\lambda, \mu) \geq 0$ if and only if $\mu \leq \frac{k + \sqrt{k^2 + 4uk}}{2u}$.) If $b(\cdot)$ is upward sloping with λ , only suppliers with negative θ_F is selected and the liquidity constraint is violated. If $b(\cdot)$ is downward sloping with respect to λ , then only suppliers with negative π_F is selected leading to a negative profits to the intermediary. ■

When L is positive, there is no need to reply on the delayed payment to suppliers to operate the finance mode. The finance mode is always profitable as long as there are suppliers that can give a positive $\pi_F(\lambda, c)$.

Lemma 2 *Under $L > 0$ and $m > 1$, the finance mode is activated if and only if $c_{\pi_F}(\bar{\lambda}) \geq \underline{c}$, or equivalently,*

$$k \leq \frac{u - \underline{c}}{2} \bar{\lambda}. \quad (15)$$

If $c_{\pi_F}(\bar{\lambda}) \geq \underline{c}$, the intermediary can always use her endowment to fund some suppliers with

positive π_F , e.g., the supplier indexed by $(\bar{\lambda}, \underline{c})$. Otherwise, all suppliers generate a negative profits to the intermediary and the finance mode makes only negative profits.

Stronger commitment power of the intermediary: $m < 1$. In this case, the finance mode always brings lower liquidity contribution than the hybrid mode: $\theta_F(\lambda, c) < \theta_H(\lambda, c)$ for all $(\lambda, c) \in \Omega$. Thus, the hybrid mode dominates the finance mode. Moreover, the middleman profit is always positive: $\pi_M(\lambda, c) > 0$ for all $(\lambda, c) \in \Omega$. Thus, the intermediary's problem is to select the supplier either to the middleman mode or to the hybrid mode. The optimal section rule can be simplified to

$$q_H(\lambda, c, \mu) = 1 \text{ iff } \Delta\pi(\lambda, c) + \mu\theta_H(\lambda, c) \geq 0; \quad (16)$$

$$q_M(\lambda, c, \mu) = 1 \text{ otherwise,} \quad (17)$$

where $\Delta\pi$ is the incremental change in profits when a supplier is selected to the hybrid mode compared to being selected to the middleman mode:

$$\Delta\pi(\lambda, c) \equiv \pi_H(\lambda, c) - \pi_M(\lambda, c) = m\lambda(u - c)/2 - k.$$

$\Delta\pi(\lambda, c)$ increases in m . It should be emphasized that $\Delta\pi(\lambda, c)$ can be positive or negative depending on (λ, c) , and parameters m and k .

Now we focus on the selection rule of the hybrid mode, with the remaining suppliers being subsequently invited to participate in the middleman mode. Condition (16) indicates that $q_H(\lambda, c, \mu) = 1$ consists of three possible scenarios:

$$\begin{aligned} \Delta\pi(\lambda, c) &\geq 0, \theta_H(\lambda, c) \geq 0, \\ \Delta\pi(\lambda, c) &> 0, \theta_H(\lambda, c) < 0, -\Delta\pi/\theta_H \geq \mu, \\ \Delta\pi(\lambda, c) &< 0, \theta_H(\lambda, c) > 0, -\Delta\pi/\theta_H \leq \mu. \end{aligned}$$

And the liquidity value μ is still determined according to Proposition 1. These conditions illustrate how the *profit-based liquidity cross-subsidization strategy* works in the case of $m < 1$. The expressions are the same as in (11) except that $\Delta\pi$ replaces π_F and θ_H replaces θ_F . The intermediary will select suppliers with increments in profits $\Delta\pi > 0$ and positive liquidity contributions θ_H to the hybrid mode (the first case); suppliers with negative liquidity contribution can still be selected into the hybrid mode if $\Delta\pi$ is sufficiently high (the second case); and finally even if $\Delta\pi$ is negative so that the middleman mode will make a higher profit to the intermediary, still such supplier can be selected to the hybrid mode if he contribute sufficient liquidity to the program. Suppliers that are not selected into the hybrid mode will then be selected into the middleman mode.

Inserting $\Delta\pi(\cdot)$ and $\theta_F(\cdot)$, we obtain three boundaries that lie in Ω as is illustrated in Figure

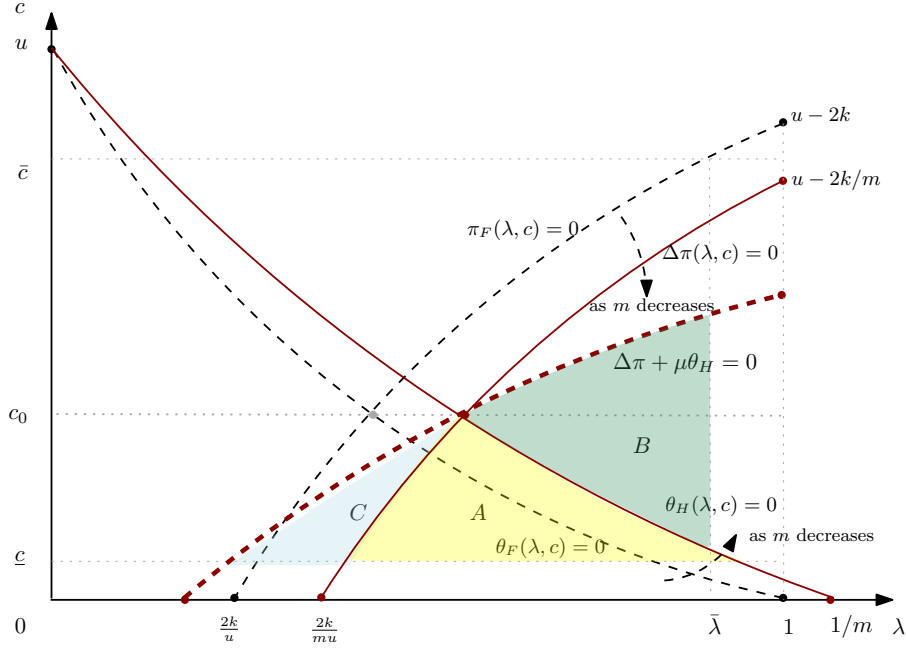


Figure 2: Middleman's selection of suppliers ($m < 1$)

2 (we also plot $\pi_F(\cdot) = 0$ and $\theta_F(\cdot) = 0$ for comparison):

$$\theta_H(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\theta_H}(\lambda) \equiv \frac{1 - m\lambda}{1 + m\lambda}u, \quad (18a)$$

$$\Delta\pi(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\Delta\pi}(\lambda) \equiv u - \frac{2k}{m\lambda}, \quad (18b)$$

$$\Delta\pi(\lambda, c) + \mu\theta_H(\lambda, c) \geq 0 \Leftrightarrow c \leq b_H(\lambda, \mu) \equiv \frac{m\lambda u - 2k + \mu(1 - m\lambda)u}{m\lambda + \mu(1 + m\lambda)}. \quad (18c)$$

As $m \rightarrow 1$, all three curves converge to the corresponding curves in (12). As in (12), the right-hand side of (18c) is a "weighted average" of the right-hand sides of the first two. Any suppliers below $\theta_H(\lambda, c) = 0$ contribute to the liquidity pool of the program, and any suppliers below $\Delta\pi(\lambda, c) = 0$ gives a higher profit in hybrid mode than in the middleman mode. The overlapping set A is selected in the hybrid mode because they contribute to both profits and liquidity. Suppliers in set B have net liquidity needs, but the intermediary earns higher profits by funding them. Suppliers in set C give the intermediary negative profits but contribute to the liquidity pool.

The figure also shows how the two curves $\Delta\pi(\cdot) = 0$ and $\theta_H(\cdot) = 0$ change as m decreases, namely, the intermediary's commitment power becomes even stronger. For a given (λ, c) , using the hybrid mode results in lower incremental profits $\Delta\pi(\lambda, c)$ compared to the middleman mode. Consequently, the $\Delta\pi(\lambda, c) = 0$ curve shifts downward. At the same time, the intermediary becomes more effective at persuading consumers to pay before goods are delivered, leading to an increase in liquidity contribution $\theta_H(\lambda, c)$. Consequently, the $\theta_H(\lambda, c) = 0$ curve shifts upward.

$\Delta\pi(\cdot) = 0$ and $\theta_H(\cdot) = 0$ intersect at $(c_0, \tilde{\lambda}_0) = (k + u - \sqrt{k^2 + 4uk}, \frac{k + \sqrt{k^2 + 4uk}}{2mu})$. Importantly, c_0 does not depend on m . That is to say, as m decreases, the two curves intersect along the

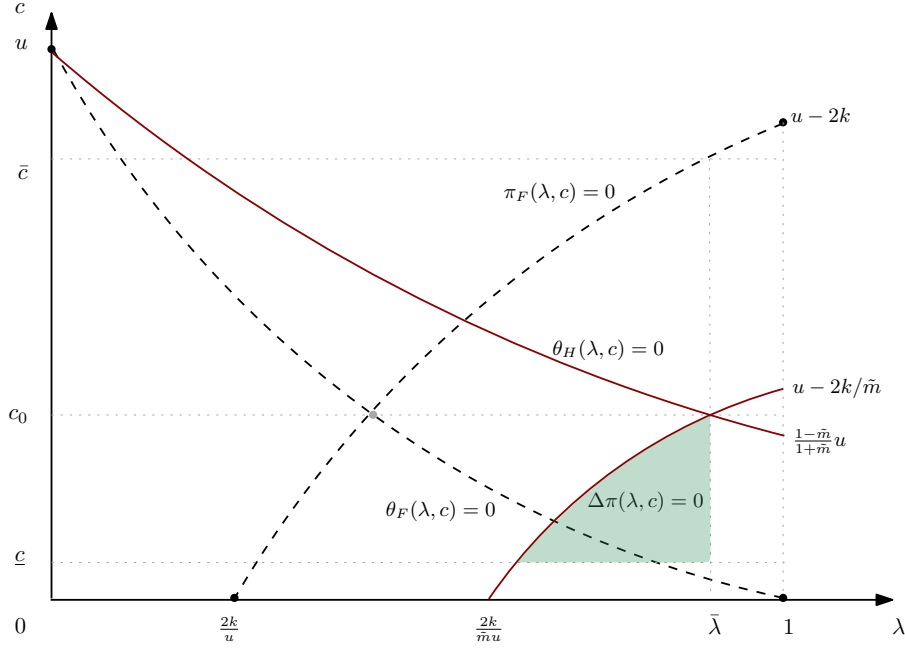


Figure 3: Intermediary's selection of suppliers ($m = \tilde{m}$)

horizontal line of $c = c_0$ and both curves move to the right. They intersect in the region Ω as long as $\lambda_0 \leq \bar{\lambda}$, or equivalently, $m \geq \tilde{m} \equiv \frac{k + \sqrt{k^2 + 4uk}}{2\lambda u}$.

When $m \leq \tilde{m}$, the $\Delta\pi = 0$ curve lies entirely below the $\theta_H = 0$ curve. Figure 3 illustrate the case when $m = \tilde{m}$. That is, $\Delta\pi(\cdot) = 0$ and $\theta_H(\cdot) = 0$ intersects at $(c_0, \bar{\lambda})$. In this case, all suppliers that bring a positive $\Delta\pi$ (the shaded region) also give the intermediary a positive liquidity contribution. Thus, the liquidity constraint of the hybrid mode is never binding, leading to $\mu = 0$, and the intermediary selects all the suppliers that have a positive $\Delta\pi$ into the hybrid mode and the rest of the suppliers into the pure middleman mode.

The following lemma summarizes the conditions under which the hybrid mode is active with $m < 1$.

Lemma 3 Suppose $m < 1$. The pure middleman mode is always activated. When $m \leq \tilde{m}$ or $L > 0$, the hybrid mode is activated if and only if $c_{\Delta\pi}(\bar{\lambda}) \geq \underline{c}$, or equivalently,

$$k \leq \frac{u - \underline{c}}{2} \bar{\lambda} m; \quad (19)$$

when $m > \tilde{m}$ and $L = 0$, hybrid mode is activated if and only if $c_0 \geq \underline{c}$ or equivalent, condition (14).

Intermediary's active modes We illustrate the active modes of a profit-maximizing intermediary in Figure 4(a) and Figure 4(b). Figure 4(a) shows the profit-maximizing choice of modes under $L = 0$. When $m > 1$, only the finance mode is active if (14) holds, which is shown as a horizontal line that separate the *inactive* and the *finance* regions. When $m < 1$, the middleman

mode is always profitable, and Lemma 3 then tells under which condition that the hybrid mode is used. For $m > \tilde{m}$, the condition is $c_0 \geq \underline{c}$, and for $m < \tilde{m}$, it is $c_{\Delta\pi}(\bar{\lambda}) \geq \underline{c}$.

Figure 4(b) shows the profit-maximizing choice of modes under $L > 0$. Still, m bigger or smaller than *one* determines whether the middleman mode is active and whether the intermediary will use a hybrid mode (for $m < 1$) or a finance mode (for $m > 1$) to provide liquidity to suppliers. What differs this case from the case $L = 0$ is that it is always $c_{\Delta\pi}(\lambda) \geq \underline{c}$ (or $c_{\pi_F}(\lambda) \geq \underline{c}$ for $m > 1$) that determines whether to support suppliers' liquidity needs. This is because when the intermediary has endowment, it does not need to rely on suppliers' liquidity to use the finance technology.

Overall, the intermediary employs two approaches to address the commitment (liquidity) issue. Firstly, when the intermediary has more commitment power ($m < 1$), they sell the goods on behalf of suppliers. Secondly, the intermediary can utilize financial technology to provide liquidity support. If the cost of this financial technology is sufficiently low, the intermediary engages in profit-based liquidity cross-subsidization. This means using liquidity from certain suppliers to support the liquidity needs of others. Both approaches can be active simultaneously when both k and m are small.

We summarize the results so far in the following theorem.

Theorem 1 (Profit-maximizing Intermediary) *The intermediary's profit-maximizing mode exists uniquely with the selection policies $q_j(\lambda, c, \mu)$, $j \in \mathcal{J}$ satisfying (10), (16), and (17), the reward to suppliers $f_j(\lambda, c)$ satisfying (1), and the shadow value of liquidity $\mu \geq 0$ uniquely determined in Proposition 1. The intermediary uses the middleman mode iff $m \leq 1$, and provides liquidity support to suppliers if k is sufficiently low. She uses the finance mode when $m > 1$ and k satisfies conditions in Lemma 1 and Lemma 2, and uses the hybrid mode when $m \leq 1$ and k satisfies conditions in Lemma 3.*

2.3 Socially optimal intermediation

We now compare the social planner's and the intermediary's selection of suppliers. A social planner seeks to maximize total surplus. Let the planner's selection rule be $I_j(\lambda, c)$, $j \in \mathcal{J}$, where $I_j = 1$ represents selecting the supplier into the j mode. Compared to the intermediary, a planner's objective for each selection is the total surplus

$$v_M(\lambda, c) = (1 - m)\lambda(u - c)$$

instead of π_M for the middleman mode,

$$v_H(\lambda, c) = v_F(\lambda, c) = \lambda(u - c)$$

instead of π_H and π_F for the hybrid and finance mode, while the liquidity contributions remain the same. The planner's problem is as follows:

$$\begin{aligned} \max_{\{I_j(\lambda, c)\}_{j \in \mathcal{J} \in \{0,1\}^3}} & \int_{\Omega} \left[\sum_{j \in \mathcal{J}} I_j(\lambda, c) v_j(\lambda, c) \right] dG, \\ \text{s.t.} & \int_{\Omega} \left[\sum_{j \in \mathcal{J}} I_j(\lambda, c) \theta_j(\lambda, c) \right] dG + L \geq 0, \end{aligned} \quad (20)$$

where $L \geq 0$ is taken as given.

First, while both the planner and the intermediary activate the middleman mode if and only if $m \leq 1$, compared to the intermediary, the planner is more like to provide liquidity support to suppliers. This is because planner cares about the total surplus rather than profits. When $m > 1$, the planner cares about $v_F(\lambda, c) > \pi_F(\lambda, c)$. When $m < 1$, the planner cares about $\Delta v(\lambda, c) \equiv v_H(\lambda, c) - v_M(\lambda, c) = m\lambda(u - c) > \Delta\pi(\lambda, c)$. In both cases, there's a range of k where the intermediary either operates solely as a middleman or remains inactive, while the planner opts to provide liquidity support. For example, in Figure 5, the intersection point between the $\Delta v(\lambda, c) = 0$ curve and the $\theta_H(\lambda, c) = 0$ curve, denoted by (λ'_0, c'_0) lies above (λ_0, c_0) , namely, $c'_0 > c_0$. Suppose $L = 0$, then the planner would activate the hybrid mode only if $c'_0 \geq \underline{c}$, while the intermediary would do so if $c_0 \geq \underline{c}$.

Second, provided that the planner provides liquidity support to suppliers, like in the intermediary's selection, the planner's selection rule involves liquidity cross-subsidization based on trade surplus (rather than profits).

Consider the case of $m < 1$, and let μ^s be the associated multiplier to the liquidity constrain. The social optimal selection rule into the hybrid mode can be written as

$$I_H(\lambda, c, \mu^s) = \begin{cases} 1 & \text{if } \Delta v(\lambda, c) + \mu^s \theta_H(\lambda, c) \geq 0, \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

where $\mu^s = 0$ if $\Theta(0) + L \geq 0$, and otherwise μ^s is pinned down by $\Theta(\mu^s) + L = 0$. Based on (21), the planner selects with a positive surplus increment compared to the pure middleman mode and positive liquidity ($\Delta v > 0, \theta_H > 0$), as well as suppliers with negative surplus increment but positive liquidity ($\Delta v < 0, \theta_H > 0$), and uses the pooled liquidity and the intermediary's endowment L to fund suppliers with a liquidity need ($\Delta v > 0, \theta_H < 0$) if $-\Delta v / \theta_H \geq \mu^s$.

Moreover, given \bar{c} is not too low, we observe that there always exist suppliers with positive surplus increment Δv and negative profit increment $\Delta\pi$ that will be selected into the hybrid mode by the social planner but not by the intermediary. The orange region in Figure 5 illustrates the set of such suppliers. In the figure, the intermediary's selection rule is denoted by $\Delta\pi + \mu\theta_F = 0$ (the green curve), and the planner's selection rule is denoted by $\Delta v + \mu^s\theta_F = 0$ (the red curve). The set of the orange region always exists since μ and μ^s must be finite. Figure 5 also illustrates

that it is possible that some suppliers (the blue region) are selected into the hybrid mode by the intermediary but not by the planner. This happens when $\mu^s > \mu$. In this case, the benefit of funding these suppliers is lower than μ^s (too expensive for the planner) and is higher than μ (profitable for the intermediary).

Finally, the intermediary's choice is never the same as the planner's solution. Despite of the inefficiency, the intermediary is welfare-improving.¹⁰ This improvement stems from two sources: the intermediary's matching advantage $m < 1$ and the cross-subsidization facilitated by the finance/hybrid mode. It's noteworthy that even when $m > 1$, negating any matching advantage, cross-subsidization alone still enhances welfare. This is because late suppliers are liquidity-constrained in the absence of the supplier finance program, while the idle liquidity of early suppliers remains underutilized.

3 Supply chain finance in a monetary equilibrium

To endogenize the middleman's liquidity holdings L and be explicit about the medium of exchange, we integrate the benchmark model into a monetary model of Lagos and Wright (2005). Time is discrete and continues forever.

Day and night markets. Each period consists of two subperiods: day and night. A retail market is open during the day for the indivisible goods, just like the one in the benchmark model. We assume all indivisible consumption goods are perishable. The night market is Walrasian. In night market, all agents can consume and produce a divisible general good with a price normalized to one. The general good is produced one-to-one using labor h . There exists another divisible good, called fiat money, that can be used as a medium of exchange. The fiat money can be traded for the general good in night market at a price ϕ per unit. The utility function of consuming x units of the general goods, denoted by $U(x)$, is strictly increasing, concave, and twice continuously differentiable. We normalize $U(x^*) = x^*$ where x^* solves $U'(x^*) = 1$.

Agents. The intermediary and the consumers live infinitely with a common discount factor $\beta \in (0, 1)$. We assume that suppliers only live for one period. Therefore, suppliers are new-born without holding any money at the beginning of the day. This assumption aligns with our benchmark and reflects the liquidity constraints faced by small enterprises in the real world. In the next section, we will relax this assumption and allow suppliers to hold fiat money that can be used to meet their liquidity needs. No agents discount within a period.

¹⁰See equation (30) and discussions in section 3.3.

Fiat Money. Meetings in the day market are anonymous, and effort in the night market is non-contractible. Therefore, fiat money has the essential role as a medium of exchange. Consumers must prepare fiat money in the night market so as to trade during the day. The intermediary must prepare fiat money, namely L , if contributing liquidity to the SCF program increases her profits. Suppliers need to use fiat money to pay for production costs, but since they hold no money at the beginning of the day, they must receive them from consumers via retail revenue. As such, the liquidity needs of suppliers characterized in the benchmark naturally arise.

The supply of fiat money is controlled by the government. Let M and M_{-1} be the money supply of current and the previous periods, respectively, with $M = \gamma M_{-1}$ where γ is the growth rate of money. Changes in M occur during the night via lump-sum transfers to (taxes from) consumers if $\gamma > 1$ ($\gamma < 1$) by an amount of real value T . The nominal interest rate is given by the Fisher equation $1 + i = \gamma/\beta$, and we assume $\gamma > \beta$. The Friedman rule is the limiting case $i \rightarrow 0$. We focus on the symmetric steady-state monetary equilibrium where agents of identical types choose identical strategies, and all real variables are constant over time. In particular, $\frac{\phi_{-1}}{\phi} = \gamma$.

3.1 The monetary equilibrium

As in the benchmark model, p represents the real price and is determined to satisfy $p - c = (u - c)/2$. Let z be the real value of money holdings. We index the consumers' value by superscript b , the suppliers' value by superscript s , and the middleman's value by superscript m .

We work backward and begin with the night market. At the beginning of night, a consumer who holds z^b money has an expected value $W^b(z^b)$ given by

$$W^b(z^b) = \max_{x, h, z_+^b} \{U(x) - h + \beta V_+^b(z_+^b)\},$$

$$\text{s.t. } x = z^b + T + h - \frac{\phi}{\phi_+} z_+^b,$$

where V_+^b denotes the expected value of entering into the next day market. Inserting the budget constraint and $U(x^*) = x^*$, we have

$$W^b(z^b) = z^b + T + \max_{z_+^b} \left\{ -\frac{\phi}{\phi_+} z_+^b + \beta V_+^b(z_+^b) \right\}. \quad (22)$$

As standard in the literature, z_+^b is determined independently of current money holding z^b .

Likewise, the intermediary who holds a real value of L fiat money has an expected value given by

$$W^m(L) = \max_{x, h, L_+} \{U(x) - h + \beta V_+^m(L_+)\}, \text{ s.t. } x = L + h - \frac{\phi}{\phi_+} L_+,$$

where V_+^m is the middleman's expected value of entering the next day market. Then,

$$W^m(L) = L + \max_{L_+} \left\{ -\frac{\phi}{\phi_+} L_+ + \beta V_+^m(L_+) \right\}. \quad (23)$$

A supplier who holds m^s money entering the night market faces the following static problem:

$$W^s(z^s) = \max_{x,h} \{U(x) - h\}, \text{ s.t. } x = z^s + h.$$

Since he lives only for one period, the supplier will use up all his money to purchase the general good, yielding $W^s(z^s) = z^s$.

Consumers' money holdings. In the day market, consumers purchase indivisible goods from available suppliers using fiat money. Available suppliers in the market are those who have either experienced no liquidity shock, or joined the SCF program. Denote the set of available suppliers in the market by $\hat{\Omega} \subset \Omega$, which is realized after a liquidity shock happens but before consumers make a purchase decision. Suppose now that $\hat{\Omega}$ is non-empty. The production costs of suppliers in $\hat{\Omega}$ follow a (marginal) distribution with density of

$$\hat{g}(c) = \int_0^1 [q(\lambda, c) + (1 - q(\lambda, c))(1 - \lambda)] g(\lambda, c) d\lambda. \quad (24)$$

$\hat{g}(c)$ is an object to be determined in equilibrium. Let $\omega(c, i) \in \{0, 1\}$ be consumers' purchase decision, where $\omega(c, i) = 1$ represents that consumers purchase the good with production cost c , and $\omega(c, i) = 0$ represents that consumers do not purchase it. The dependence of ω on the nominal interest rate i will become clear shortly below.

A consumer who holds a real balance of z^b has the day market value given by

$$\begin{aligned} V^b(z^b) &= \max_{\omega(c,i)} \int_{\underline{c}}^{\bar{c}} [\omega(c, i)u] \hat{g}(c, i) dc + W^b \left(z^b - \int_{\underline{c}}^{\bar{c}} [\omega(c, i)p(c)] \hat{g}(c, i) dc \right) \\ \text{s.t. } &\int_{\underline{c}}^{\bar{c}} [\omega(c, i)p(c)] \hat{g}(c, i) dc \leq z^b, \end{aligned}$$

where we denote $p(c) = (u + c)/2$ to clarify the dependence of p on suppliers' type c . Consumers obtain a common utility u for each indivisible good and pay the price $p(c)$. Using $W^b(z^b) = z^b + W^b(0)$, we have $V^b(z^b) = \int_{\underline{c}}^{\bar{c}} \omega(c, i) \frac{u-c}{2} \hat{g}(c, i) dc + z^b + W^b(0)$. Since holding money is costly, the budget constraint is always binding. Inserting z^b into the objective function, we can see that the consumer's problem is to choose $\omega(c, i)$ to maximize

$$\int_{\underline{c}}^{\bar{c}} \omega(c, i) \left[-\frac{\phi}{\phi_+} p(c) + \beta \left(\frac{u-c}{2} + p(c) \right) \right] \hat{g}(c) dc.$$

The objective function is linear in $\omega(c, i)$, thus, $\omega(c, i) = 1$ iff $-\frac{\phi}{\phi_+} p(c) + \beta \left(\frac{u-c}{2} + p(c) \right) \geq 0$. Inserting $p(c)$, the condition becomes $1 + \frac{u-c}{u+c} \geq \frac{\phi}{\phi_+} \frac{1}{\beta}$. In steady state, $\frac{\phi}{\phi_+} \frac{1}{\beta} = \frac{\gamma}{\beta} = 1 + i$. Using

this, we have that for suppliers in $\hat{\Omega}$:

$$\omega(c, i) = \begin{cases} 1 & \text{if } \frac{u-c}{u+c} \geq i, \\ 0 & \text{otherwise.} \end{cases} \quad (25)$$

Consumers' consumption decision depends on i , which follows directly from (25). Define $i_1 \equiv \frac{u-\bar{c}}{u+\bar{c}}$ and $i_2 \equiv \frac{u-c}{u+c}$. Note that $i_2 > i_1 > 0$. For $i \leq i_1$, consumers hold enough money to buy all the available goods in the retail market. As i increases above the critical value i_1 , some goods become too costly for consumers to include in their consumption basket, given that their money holding becomes smaller. In other words, goods with costs $c > \bar{c}(i) \equiv \frac{1-i}{1+i}u$ are not purchased by consumers and so these suppliers drop out of the market one by one as i increases. Eventually, when i reaches the critical value i_2 , consumers cannot afford to buy any goods available in the retail market, and so for $i > i_2$ no suppliers can make a sale.

To summarize, the consumers' money-holding decision imposes a new constraint on the middleman's SCF selection problem in monetary equilibrium. It appears as an effective space of suppliers, defined by

$$\Omega(i) \equiv [0, 1] \times [\underline{c}, \bar{c}(i)] \subset \Omega \quad (26)$$

where now $\bar{c}(i) = \bar{c}$ for $i \leq i_1$ and $\bar{c}'(i) < 0$ for $i \in (i_1, i_2)$. $\Omega(i)$ is nonempty for $i < i_2$.

Suppliers' participation decision. A newborn supplier (λ, c) has zero money holdings. If $c \geq \bar{c}(i)$, the supplier's retail market value is zero. If $c < \bar{c}(i)$, his retail market value is

$$\max \left\{ W^s \left((1-\lambda)(p-c) \right), W^s \left(f(\lambda, c) \right) \right\}.$$

The first term is the expected value if not joining the SCF program, and in that case, he trades only with probability $1 - \lambda$. The second term is the value of participating in the SCF program, and in that case, he produces for sure and obtains a pay of real value $f(\lambda, c)$ from the intermediary. The supplier participate in the SCF program if $W^s \left(f(\lambda, c) \right) \geq W^s \left((1-\lambda)(p-c) \right)$. Since $W^s(z^s) = z^s$, it is equivalent to $f \geq (1-\lambda)(p-c)$. A profit-maximizing intermediary always chooses f such that the condition is binding, yielding a reward of

$$f(\lambda, c) = (1-\lambda)(u-c)/2.$$

Therefore, the supplier's expected value of entering the day market is

$$W^s \left((1-\lambda)(u-c)/2 \right) = (1-\lambda)(u-c)/2.$$

The middleman's money holdings. The intermediary must decide in the night market how much money to carry to the next day, and on the next day, which suppliers to select into the SCF

program. Suppose the intermediary holds a real balance of $L \geq 0$ and enters the day market. Her value is given by

$$V^m(L) = \max_{\{q(\lambda, c)\}_{(\lambda, c) \in \Omega(i)}} \left\{ -k \int_{\Omega(i)} q(\lambda, c) dG + W^m \left(L + \int_{\Omega(i)} q(\lambda, c) [p(c) - f(\lambda, c) - c] dG \right) \right\},$$

subject to the liquidity constraint (8), provided that $\Omega(i)$ is non-empty.

Using the linearity of W^m and ignoring a constant term $W^m(L)$, we can see that the middleman's selection problem is the same as in the benchmark model, except that the set of suppliers now is $\Omega(i) \subset \Omega$.

Theorem 1 characterizes the profit-maximizing SCF program, but since $\Omega(i)$ depends on i , we need to take into account that the multiplier μ depends not only on L , but also depends on i . With a slight abuse of notations, we write $\mu(L, i)$, rather than $\mu(L)$, to clarify this. As in Lemma ??, $\mu(L, i)$ is generically unique for given i and L . And as in Corollary 1, $\mu(L, i)$ is strictly decreasing in L if $\mu(L, i) > 0$.

We derive the middleman's optimal money holdings in the night market, as stated in (23). The first-order condition is

$$\phi_{-1} \geq \beta \phi V^{m'}(L/\phi),$$

with equality if and only if $L > 0$. Applying the Envelop condition $V^{m'}(L/\phi) = 1 + \mu(L, i)$, we can write the first order condition as:

$$\phi_{-1} \geq \beta \phi (1 + \mu).$$

Applying $\frac{\phi_{-1}}{\phi} \frac{1}{\beta} = \frac{\gamma}{\beta} = 1 + i$ in steady state, the first order condition can be simplified to

$$i \geq \mu. \tag{27}$$

This is essentially the Euler equation that determines the middleman's money holdings as a function of the nominal interest rate. Note that the liquidity constraint (13) should be modified to reflect the dependence of Θ on i . We write $\Theta(\mu, i)$, rather than $\Theta(\mu)$, to clarify this. Recall that $\mu(0, i)$ is the shadow value of liquidity to the intermediary if her liquidity holding $L = 0$. From Lemma ??, we have that $\mu(0, i) > 0$ if and only if $\Theta(0, i) < 0$, and $\mu(0, i) = 0$ otherwise. Then we can characterize the middleman's optimal money holdings by comparing i and $\mu(0, i)$.

There are two scenarios to consider. In the first scenario, $\Theta(0, i) < 0$, which implies $\mu(0, i) > 0$ (see Corollary 1). If the nominal interest rate is relatively high, namely $i \geq \mu(0, i)$, the optimal money holding of the intermediary is $L(i) = 0$, indicating that the profit-maximizing SCF program is entirely financed by the pooled liquidity of suppliers. If the nominal interest rate is relatively low, namely $i < \mu(0, i)$, then (27) holds with equality, and the intermediary holds a positive amount of money $L(i) = -\Theta(i, i) > 0$.

In the second scenario, $\Theta(0, i) > 0$, which implies $i > \mu(0, i) = 0$ (see Lemma ??), and the intermediary can invite all the suppliers with positive profit contributions π , and the intermediary does not hold money, $L(i) = 0$.

Lemma 4 (Intermediary's Money Holdings) *The optimal money holding of the intermediary follows $L(i) = -\Theta(i, i) > 0$ if $i < \mu(0, i)$, and $L(i) = 0$ otherwise.*

To summarize, the optimal liquidity weight of the middleman's selection is given by

$$\mu(i) = \mu(L(i), i) = \begin{cases} i & \text{if } i \leq \mu(0, i), \\ \mu(0, i) & \text{otherwise,} \end{cases} \quad (28)$$

and the middleman's profit-maximizing SCF program in a monetary equilibrium is described by the selection policy $q(\lambda, c, \mu(i))$ satisfying (??), the pay to suppliers $f(\lambda, c)$ satisfying (1), and the optimal money holdings $L(i)$ determined in Lemma 4.

The monetary equilibrium. For a monetary equilibrium to exist, consumers must hold a positive amount of money during the day, i.e., $i < i_2$.

Theorem 2 *A monetary equilibrium exists if and only if $i \in (0, i_2)$, and if it exists it is unique, satisfying:*

- the real balance of consumers is given by $z^b = \int_{\underline{c}}^{\bar{c}} \omega(c, i) p(c) \hat{g}(c) dc$, where $\omega(c, i)$ represents consumers' choice of which suppliers' good to buy as is given by (25);
- the set of effective suppliers is given by (26);
- the SCF program operates with $\{f(\lambda, c), q(\lambda, c, \mu(i)), L\}$ as is characterized by Theorem 1 and Lemma 4.

The following corollary says that at the Friedman rule, the intermediary selects suppliers solely based on profits, so the liquidity constraint is not an issue.

Corollary 2 *As $i \rightarrow 0$, the profit-maximizing SCF program features that $\mu(i) \rightarrow 0$.*

3.2 Changes in the nominal interest rate

Next, we study how the nominal interest rate i , which is the measure of liquidity cost in our model, affects the shadow value of liquidity $\mu(i)$, and the middleman's liquidity holdings $L(i)$. In general, there are potentially two effects. First, there is a direct effect when $\mu(i) = i$, in which case the middleman's liquidity holding L decreases in i . In Figure ??, this is shown as the selection curve $\pi + \mu\theta = 0$ rotating clockwise around (λ_0, c_0) . Second, there can be an indirect effect since i

increases the consumers' money-holding cost. That is, i can constrain the feasible set of suppliers via the upper bound $\bar{c}(i)$, which eventually affects the selection of suppliers in the SCF program. The indirect effect of i has an ambiguous effect on $L(i)$ and $\mu(i)$, which is determined by how the pooled liquidity from suppliers changes.

Depending on the level of i and the liquidity holding of the intermediary, there are several possible cases. We start with the simple case of $i < i_1$, then, $\Omega(i) = \Omega$, and $\mu(0, i)$ is independent of i . In this case, i only affects the SCF program via the direct effect. It follows immediately that whenever $i < \mu(0, i)$, the shadow value of liquidity $\mu(i) = i$ is strictly increasing in i , and $L(i) = -\Theta(\cdot)$ is positive and strictly decreasing in i .

When i surpasses i_1 and continues to increase, $\bar{c}(i)$ decreases, and eventually, it will intersect with the middleman's selection curve $b(\lambda, \mu(i))$ defined in (??). At this point, the indirect effect of i manifests itself. To simplify the exposition and without loss of generality, we assume that $\bar{c} > c_0$ in the following analysis. There are two cases depending on whether $L > 0$, where $\mu(0, i) > i$ holds, or $L = 0$, where $\mu(0, i) \leq i$ holds.

Consider first the scenario of $\mu(0, i) > i$, namely, $\mu(i) = i$ and $L(i) > 0$. Let i_0 represent the interest rate such that $\bar{c}(i_0) = c_0$. Given $\mu(i) = i$, the following lemma characterizes how $\bar{c}(i)$ restricts the set of chosen suppliers, contingent upon whether $i < i_0$ or not.

Lemma 5 *Suppose that $\mu(i) = i$. If $i < i_0$, then $b'_\lambda(\lambda, i) > 0$ and $b(\lambda, i)$ lies entirely below $\bar{c}(i)$. If $i > i_0$, then $b'_\lambda(\lambda, i) < 0$ and $b(\lambda, i)$ lies entirely above $\bar{c}(i)$.*

According to Lemma 5, when $i < i_0$, $b(\cdot)$ lies below $\bar{c}(i)$, meaning that all the suppliers selected by the intermediary satisfy the condition $c \leq \bar{c}(i)$. This relationship is depicted in Figure ?? panel (a). Notably, since $\bar{c}(i)$ has no influence on the middleman's optimal choice, i impacts the middleman's liquidity holding solely through the direct effect.¹¹ Therefore, like in the scenario where $i \leq i_1$, here as well, $L(i)$ is strictly decreasing in i .

When $i > i_0$, $b(\cdot)$ lies above $\bar{c}(i)$, meaning that all suppliers in $\Omega(i)$ are selected into the SCF program, see panel (b) of Figure ?. In this case, it is $\bar{c}(i)$ that determines the selection of suppliers and money holdings of the intermediary. Since the liquidity constraint of the intermediary must be binding (holding money is costly), $L(i)$ is given by

$$L(i) = -\Theta(i, i) = - \int_{\Omega(i)} \theta(\lambda, c) dG = - \int_0^1 \int_{\underline{c}}^{\bar{c}(i)} \theta(\lambda, c) g(\lambda, c) dc d\lambda > 0,$$

A lower $\bar{c}(i)$ may increase or decrease the total liquidity from available suppliers. Thus, $L(i)$ can increase or decrease in i . For example, if the total liquidity decreases in i ,

$$\frac{d\Theta(i, i)}{di} = \int_0^1 \bar{c}'(i) \theta(\lambda, \bar{c}(i)) g(\lambda, \bar{c}(i)) d\lambda < 0,$$

then $L(i)$ increases, despite of a higher cost of liquidity.

¹¹At i_0 , $c = b(\lambda, i_0)$ becomes a horizontal line, coinciding with $c = c_0$.

Consider the second scenario where $\mu(0, i) < i$, namely, $\mu(i) = \mu(0, i)$ and $L(i) = 0$. In this case, Lemma 5 does not apply, and i affects the selection of suppliers only indirectly through $c \leq \bar{c}(i)$. If $i < i_0$, which is illustrated in Figure ?? panel (a), then as i increases, $\bar{c}(i)$ crosses $b(\cdot)$, and excludes suppliers of negative liquidity contributions. Thus, the total liquidity from suppliers would increase. As a result, $\mu(0, i)$ must decrease (provided it is positive). That is, the intermediary relies less on the liquidity cross-subsidization among suppliers.

If $i > i_0$, the shadow value liquidity $\mu(0, i)$ may increase or decrease in i (see Figure ?? panel (b)). $\mu(0, i)$ increases in i if a lower $\bar{c}(i)$ reduces the pooled liquidity from suppliers, namely, $\partial\Theta(\mu, i)/\partial i < 0$. Then, $\mu(0, i)$ must increase, leading to more liquidity cross-subsidization among suppliers. We summarize these results in the following proposition.

Proposition 2 *Suppose $\mu(0, i) > i$, then $\mu(i) = i$, and $L(i) > 0$. $L(i)$ strictly decreases in i if $i < i_0$; and may decrease or increase in i if $i > i_0$. Suppose $\mu(0, i) < i$, then $\mu(i) = \mu(0, i)$, and $L(i) = 0$. $\mu(i)$ strictly decreases in i if $i < i_0$; and may increase or decrease in i if $i > i_0$.*

3.3 Efficiency and inflation

In this section, we first examine the social optimum in the economy and demonstrate that a profit-maximizing supply chain finance cannot achieve this optimum. Nevertheless, it improves welfare regardless of the nominal interest rate that the intermediary faces. Then we illustrate that, somehow surprisingly, the social welfare attained by supply chain finance may even increase in nominal interest rate. We provide a sufficient condition under which deviating from the Friedman rule (zero nominal interest rate) improves social welfare.

Social optimum. Like in the endowment economy, let the planner choose whether or not to include a supplier into the SCF program. Additionally, let the planner choose a nominal interest rate. In this new planner's problem (with the extended choice set of i), setting the nominal interest rate $i = 0$ is dominant as it eliminates the liquidity constraint of the planner's problem (20). The total welfare per period is given by

$$\mathcal{W} = \int_{\Omega} \left\{ I(\lambda, c)(u - c - k) + (1 - I(\lambda, c))(1 - \lambda)(u - c) \right\} dG.$$

The total surplus for the goods is $u - c - k$ if the supplier is included in the SCF program and is $(1 - \lambda)(u - c)$ otherwise. The constrained efficient allocation is

$$I(\lambda, c) = \begin{cases} 1 & \text{if } \lambda(u - c) - k \geq 0, \\ 0 & \text{otherwise.} \end{cases} \quad (29)$$

The SCF program does not achieve the welfare maximum because the intermediary ignores the positive externality to consumers when a supplier joins the SCF program. For example, at $i = 0$, the profit-maximizing intermediary only invites a supplier if $\lambda(u - c)/2 - k \geq 0$, disregarding the potential total surplus of $\lambda(u - c) - k$ that could be achieved.

Nevertheless, supply chain finance is welfare improving. For any $i < i_2$, the welfare change with an active SCF program is given by

$$\begin{aligned}\Delta\mathcal{W}(i) &\equiv \int_{\Omega(i)} \left\{ q(\lambda, c)(u - c - k) + (1 - q(\lambda, c))(1 - \lambda)(u - c) \right\} dG - \int_{\Omega(i)} (1 - \lambda)(u - c) dG \\ &= \int_{\Omega(i)} q(\lambda, c)[\lambda(u - c) - k] dG.\end{aligned}\tag{30}$$

Since $\Delta\mathcal{W}(i) > \int_{\Omega(i)} q(\lambda, c)[\lambda(u - c)/2 - k] dG > 0$, supply chain finance is always welfare improving.

Inflation and welfare. Deviating from the Friedman rule may increase social welfare through liquidity cross-subsidization. We shall focus on the case that $\mu(0, 0) > 0$, because otherwise the outcome is trivial, i.e., independent of the nominal interest rate, the intermediary always and only selects those with $\pi > 0$. We consider a marginal increase in i from $i = 0$, which together with $\mu(0, 0) > 0$ implies $\mu^*(i) = i$ (see (28)). This situation is depicted in Figure 6. The grey region represents the set of suppliers Ω , and other relevant regions of suppliers are denoted by capital letters.

When the nominal interest rate is zero, the intermediary selects all suppliers with positive profitability, covering regions A , B , and D in the figure. As the interest rate rises, the intermediary faces higher funding costs, then liquidity cross-subsidization among suppliers becomes profitable. This process involves excluding suppliers with positive profitability but negative liquidity (region D), while including suppliers with negative profitability but positive liquidity (region C). Consequently, the middleman's profits decrease. Suppliers' profits do not change since they are indifferent between participating and not in the SCF program. However, there is a potential for an increase in consumer surplus if the total trading volume increases. Ultimately, whether social welfare is improved depends on the dominance of the consumer surplus effect.

To analyze the impact on trading volume, let $\lambda^\pi(c)$ represent the combinations of (λ, c) for which $\pi(\lambda, c) = 0$, and let $\lambda^\mu(c)$ represent the combinations of (λ, c) such that $\pi(\lambda, c) + \mu\theta(\lambda, c) = 0$. Excluding suppliers in region D leads to a decreased trading volume given by:

$$\int_{c_0}^{\bar{c}} \int_{\lambda^\pi(c)}^{\lambda^\mu(c)} \lambda g(\lambda, c) d\lambda dc,$$

while including suppliers in region C leads to an increased trading volume given by:

$$\int_{\underline{c}}^{c_0} \int_{\lambda^\mu(c)}^{\lambda^\pi(c)} \lambda g(\lambda, c) d\lambda dc.$$

This is because, for instance, each of the newly added suppliers, measured by $\int_{c_0}^{\bar{c}} \int_{\lambda^\pi(c)}^{\lambda^\mu(c)} g(\lambda, c) d\lambda dc$, will become available to consumers even when he is hit by a liquidity shock, which occurs with probability λ . Comparing the above two volumes, we can see that when c_0 is large enough, the former volume can be made arbitrarily small, while when c_0 is small enough, the latter volume can be made arbitrarily small. Thus, for a sufficiently large c_0 , the consumer surplus effect dominates, and deviating from the Friedman rule improves welfare.

c_0 is determined crucially by k . Indeed, c_0 decreases with k , satisfying as $k \rightarrow 0$, we have $c_0 \rightarrow u > \bar{c}$ while as $k \rightarrow \bar{k}$, we have $c_0 \rightarrow \underline{c}$. We can see the effect of decreasing k as an upward shift of $\pi(\lambda, c) = 0$ curve. Intuitively, the more efficient the intermediary is, the more suppliers she wishes to include in the SCF program. This point can be seen from the middleman's selection of suppliers as is given by (??) with $\mu = i$. That is, suppliers in region D are excluded because they require liquidity ($\theta < 0, \pi > 0$) but give a low return to the middleman's funding:

$$\pi/(-\theta) < i. \quad (31)$$

Suppliers in region C are included because they contribute liquidity ($\theta > 0, \pi < 0$), and from the middleman's perspective, the costs of extracting liquidity from them are sufficiently low:

$$-\pi/\theta < i. \quad (32)$$

Now, as k becomes smaller, π becomes larger for all suppliers since $\pi = \lambda(u - c)/2 - k$. With a larger π , the L.H.S. of (31) increases, leading to fewer suppliers being excluded from the SCF program. Namely, region D becomes smaller. On the other hand, using (32), and noting that in this case, π is negative; thus, with a larger π , the L.H.S. becomes smaller, and more suppliers are included in the SCF program.

All in all, if k is sufficiently small, the intermediary is sufficiently efficient so that she has a relatively high incentive to include (rather than exclude) new suppliers. This explains why the overall trading volume increases in response to a marginal deviation from the Friedman rule.

In general, we can prove that there exists a unique critical value of k such that the decreased trading volume is smaller than the increased trading volume if k is lower than this critical value (see the proof of Proposition 3). Ultimately, it results in a significant improvement in consumer surplus that outweighs the decrease in the middleman's profits. Hence, deviating from the Friedman rule is welfare-improving.

Proposition 3 *Let $\kappa \equiv k/u \in (0, \frac{\bar{k}}{u})$. Suppose that (λ, c) follows a uniform distribution and $\mu(0, 0) > 0$. Consider a marginal increase in i from $i = 0$. There exists a critical value $\kappa^* \in (0, \frac{\bar{k}}{u}]$ such that the*

Friedman rule is suboptimal if and only if $\kappa < \kappa^$.*

Given $\mu(0,0) > 0$, the proposition establishes the potential suboptimality of the Friedman rule, considering a marginal deviation, with uniform distribution where only the value of c_0 (and hence κ) matters. There are of course other cases, which are not covered by Proposition 3 but induces the suboptimality of Friedman rule. We show two such cases below.

First, the suboptimality can occur for non-marginal deviations from $i = 0$. Figure 7 provides a numerical example with $u = 1.0, k = 0.1, \underline{c} = 0.1, \bar{c} = 0.6$, and a uniform distribution of (λ, c) . Under these values, $\mu(0, i) = 0.26$ for $i \leq i_1 = 0.25$. The figure shows that (1) the aggregate profits ($\Delta\Pi$, red curve) exhibit a monotonic decrease in i due to the exclusion of suppliers with positive π and the inclusion of suppliers with negative π ; and (2) the total consumer surplus (ΔCS blue curve) follows an inverted U-shape because total trading volume first increases and then decreases. The effect of consumer surplus dominates. Consequently, the total surplus (ΔTS green curve) first increases and then decreases at relatively higher levels of i .

Second, the suboptimality can occur with a non-uniform distribution. Figure 8 provides a numerical exercise with $u = 1.0, k = 0.18, \underline{c} = 0.1, \bar{c} = 0.6$, and both λ , and c follow a $Beta(2, 3)$ distribution. Under these values, $\mu(0, i) = 0.137$ for $i < i_1 = 0.25$. Panel (a) shows the implied densities (contour graph in red) and a particular selection rule of $\pi + 0.12 \times \theta = 0$. Panel (b) shows that, as i increases, the blue curve representing total consumer surplus (ΔCS) increases monotonically, which outweighs the decrease in total profits ($\Delta\Pi$, red curve), resulting in a monotonically increasing total surplus (ΔTS , green curve).

4 Suppliers' access to money market

In this section, we present an extension of the model where suppliers have access to the money market and can hold money to prepare for liquidity needs. Suppose that suppliers are infinitely lived, and have a discount factor $\beta^s \in (0, \beta]$. The nominal interest rate is $i = \frac{\gamma}{\beta} - 1$, while the effective nominal interest rate faced by suppliers is $i^s = \frac{\gamma}{\beta^s} - 1$. Let $\Delta i^s = i^s - i \geq 0$ be the premium. To isolate our point, away from the influence of $\bar{c}(i)$, we shall focus our attention on $i \leq i_1$. We outline the main results, reserving the detailed proofs for Appendix A.8.

Denote by $z^s = z^s(c)$ a real balance held by a supplier with c . If $z^s \geq c$, he has enough money to pay for production costs and will not join the SCF program. If $z^s < c$, he is able to produce only if he has joined the SCF program or if it turns out that he faces no liquidity issue. The intermediary observes the supplier's real balance z^s and sets $f(\lambda, c)$ satisfying (1). Hence, the real balance of the supplier c by the end of the day are:

$$\tilde{z}^s = z^s + \mathbb{I} \times \frac{u - c}{2} + (1 - \mathbb{I}) \times (1 - \lambda) \frac{u - c}{2},$$

where $\mathbb{I} \in \{0, 1\}$ with $\mathbb{I} = 1$ if $z^s \geq c$, and 0 otherwise.

Obviously, since suppliers need money only for buying production inputs (which costs c), they will never hold $z^s > c$. Hence, we shall consider whether holding $z^s = c$ is profitable. It is profitable if

$$\beta^s \left[\frac{\lambda(u - c)}{2} + c \right] \geq \frac{\phi}{\phi_+} c,$$

because c money today is worth $\beta^s c$ tomorrow, and will allow the supplier to produce even when hit by a liquidity shock (with probability λ), generating $\beta^s(u - c)/2$. Using $i^s = \gamma/\beta^s - 1$ and $\gamma = \phi/\phi_{+1}$, this condition can be written as

$$c < c^s(\lambda, i^s) \equiv \frac{\lambda}{\lambda + 2i^s} u. \quad (33)$$

Turn to the intermediary. She can only invite suppliers who choose not to hold money themselves. Given that $i < i_1$, the feasible set of suppliers only depends on i^s :

$$\tilde{\Omega}(i^s) = \{(\lambda, c) \in \Omega \mid c \geq c^s(\lambda, i^s)\}$$

which is nonempty. Then, the middleman's supplier selection problem becomes:

$$\max_{q(\lambda, c) \in \{0, 1\}} \int_{\tilde{\Omega}(i^s)} q(\lambda, c) \pi(\lambda, c) dG,$$

subject to the liquidity constraint:

$$\int_{\tilde{\Omega}(i^s)} q(\lambda, c) \theta(\lambda, c) dG + L \geq 0,$$

where i^s and L are taken as given.

In the previous sections, we showed that the intermediary is always profitable in a money equilibrium when $i \leq i_1$ ($< i_2$). However, with suppliers having access to market liquidity, the intermediation may not be always profitable even when $i \leq i_1$.

Proposition 4 Suppose $\underline{c} > 0$, $i < \min\{i_1, \frac{k}{u-2k}\}$, and suppliers can access the money market at an effective interest of $i^s \geq i$. Then there exists $i < \underline{i}^s < \bar{i}^s \equiv \frac{u-\underline{c}}{2\underline{c}}$ such that:

- If $i^s \leq \underline{i}^s$, suppliers with $c \leq c^s(\lambda, i^s)$ hold money for liquidity needs, and the SCF program is inactive.
- If $i^s \geq \bar{i}^s$, no supplier holds money, and the SCF program is active.
- If $i^s \in (\underline{i}^s, \bar{i}^s)$, suppliers with $c \leq c^s(\lambda, i^s)$ holds money while the SCF program is active.

In the first case where $i^s \leq \underline{i}^s$, all suppliers with a positive π opt to hold money individually, resulting in a zero profit for the intermediary. In the second case where $i^s \geq \bar{i}^s$, even the supplier with $\lambda = 1$ and $c = \underline{c}$, the one who possesses the highest benefit from holding money due to

requiring the smallest amount \underline{c} and facing the greatest likelihood of a liquidity shock $\lambda = 1$, chooses not to hold money.

In the third case, suppliers holding money individually coexist with suppliers utilizing the supply chain finance. Figure 9 depicts the third scenario, wherein suppliers falling below $c^s(\lambda, i^s)$ opt for individual money holding (region E in dark blue), abstaining from participating in the SCF program. Conversely, suppliers exceeding c^s choose not to hold money. Notably, suppliers located within regions A , B , and C enroll in the SCF program. Finally, the Friedman rule is not optimal obviously because different agents have different values of the discount factor.

Of particular interest is the scenario where $i^s = i$, namely, suppliers face the same nominal interest rate as the intermediary. The main concern is whether the SCF is still active in equilibrium. To address this issue, we need to extend the range of i to $[0, i_2]$. We have two results. First, in a monetary equilibrium, there always exists a set of suppliers who choose to hold money and prepare for liquidity needs by themselves. These suppliers have (λ, c) that satisfies $c < c^s(\lambda, i)$. Note that for all $i < i_2$, it holds that $\underline{c} < c^s(1, i)$, indicating that such a set of suppliers is non-empty. Second, the SCF liquidity coexists with suppliers' individual liquidity when the intermediary is efficient, i.e., with a small k , and the nominal interest rate takes some intermediate values. A profitable SCF requires a small k because otherwise, all suppliers in the feasible set make negative profits. A profitable SCF also requires i not to be too low because otherwise, all the suppliers with a positive profit π find it cheaper to hold money by themselves than using the SCF, which makes the intermediary non-profitable. On the other hand, if i is too high, consumers opt only for those suppliers with $c < \bar{c}(i)$. And among these suppliers, those with a positive profit π choose to hold money individually rather than using the SCF, which again makes the intermediary non-profitable.

Proposition 5 *For intermediate values of $i = i^s < i_2$, an active intermediary with $k < u/6$ can coexist with suppliers who hold money by themselves.*

5 Discussions

In this section, we provide anecdotal evidence demonstrating how our analysis captures the essential features of supply chain finance. We also illustrate how our model sheds light on other financial arrangements.

5.1 Anecdotal evidence

Selecting suppliers. Highly selected participants are a common feature of many SCF programs in the real world. For instance, Co-op Food, a major player in the UK food retail market, care-

fully handpicked fewer than 100 suppliers when launching their SCF program in 2020, despite having almost 2400 stores in the UK and thousands of suppliers. Similarly, Amazon Lending, an SCF program offered to third-party merchants on the Amazon platform, follows an invitation-only approach, providing customized credit amounts and terms tailored to the specific needs and situations of each seller. Some other SCF programs adopt an open approach, allowing all suppliers to participate. However, these programs still attract targeted suppliers through their designed terms of trade. In 2017, Richards Bay Minerals, South Africa’s largest mineral sand producer, launched an SCF program accessible to all suppliers, and around 30% of suppliers have registered since its inception, indicating that the designed terms effectively attract targeted suppliers.¹²

Among various factors to be considered in selecting suppliers, we have specifically modelled each supplier’s profit and liquidity contribution. This aligns with the common practice of supplier segmentation and prioritization in supply chain optimization. The *Supply Chain Finance Knowledge Guide* published by the International Finance Corporation states that to implement supply chain finance, strategic suppliers should be prioritized based on their *relationships with the buyer firm* and the *financial needs*. Suppliers with strong, stable, and long-term relationships with the buyer firm tend to be crucial to the buyer firm’s value creation, corresponding to a large and positive π in the model. The likelihood of financial needs is captured directly by λ in the model. For instance, in the Amazon Lending program mentioned above, merchants with a proven track record of growing sales and high customer satisfaction are more likely to be invited.

Liquidity cross-subsidization. We use JD to illustrate liquidity cross-subsidization in the real world. JD is the leading e-commerce platform in China and has been publicly listed on NASDAQ since 2014. In 2013, JD initiated a supply chain finance program called “JingBaoBei”. The operation of JingBaoBei closely resembles the key mechanisms revealed in our model.

JingBaoBei targets all suppliers of JD, including those of the direct selling channel as well as the third-party merchants on the platform. JingBaoBei allows these suppliers to request advance payment based on their accounts receivable from JD. From 2013 to 2021, JingBaoBei provided funding to over 200,000 vendors with a total amount of more than \$100 billion.

JingBaoBei is mainly funded by pooled liquidity from suppliers. Prior to 2016, JingBaoBei relied solely on JD’s self-funding, and in particular, on suppliers’ trade credit, which can be traced by JD’s financial reports. For instance, in 2021 the increase in accounts payable alone constituted more than 77% of the net cash inflow of JD’s operating activities. JD’s cash conversion cycle also confirms that JD sources significant cash inflows through the use of suppliers’ trade credit. A

¹²For more details of Co-op’s SCF program, see <https://scfcommunity.org/briefing/news/2020-retail-and-apparel-winner-co-operative-group/>. For model details about Amazon Lending, see <https://www.junglescout.com/blog/amazon-lending-program>. For more details about Richards Bay Minerals SCF program, see <https://scfcommunity.org/briefing/news/2020-manufacturing-and-industrial-winner-richards-bay-minerals/>. All links were accessed on Jul 17, 2023.

simple calculation reveals that JD can freely use suppliers' trade credit for more than 20 days.¹³

In 2016, JD introduced partial funding for JingBaoBei through asset-backed securities, akin to the liquidity holdings L in our model, with the underlying assets being suppliers' accounts receivable. Despite this, JD's self-funding continues to be the main funding source for JingBaoBei.

Note that there exists a group of suppliers that offer trade credit to JD, but almost never ask for funds from JingBaoBei. Indeed, while JingBaoBei can be an important source of liquidity for small and medium-sized suppliers that are constantly under the pressure of liquidity needs (these suppliers correspond to those of large λ in the model), large manufacturing firms like Lenovo, Philips, and Bosch that supply directly to JD rarely use JingBaoBei if any. These suppliers correspond to those with small λ in the model and essentially subsidize liquidity to other suppliers in JD's supply chain.

Other types of supply chain finance. Our model captures the essential features of various financial arrangements within the broader definition of supply chain finance, including pre-shipment finance, distributor finance, and dynamic discounting.¹⁴ In pre-shipment finance, suppliers have the option to receive an upfront payment for verified purchase orders, enabling them to access liquidity before the goods are shipped. In distributor finance, distributors of large corporations receive funding to cover inventory holding costs and bridge the liquidity gap until they receive sales revenue. In dynamic discounting, the buyer and supplier negotiate a discount rate based on payment timing. If the supplier accepts the early payment offer, the receivable is reduced. In all these arrangements, the intermediary works with a diverse group of suppliers/distributors and adjusts payment terms strategically to pool liquidity. The intermediary then takes advantage of the liquidity pool to fund suppliers/distributors requiring immediate funding.

Growing demand for liquidity and the popularity of supply chain finance. Our model aligns with the broader evidence indicating that the liquidity needs of small suppliers are a key factor driving the rise of supply chain finance. This trend has become even more pronounced after the pandemic. As companies increase their inventory and extend payment terms, they turn to supply-chain financing to ensure suppliers have the necessary cash flow to maintain timely delivery of goods and services. An example is Constellation Brands Inc., a New York-based company that produces beer, wine, and spirits, including Corona beer and Svedka vodka. Constellation

¹³From 2015 to 2018, JD's accounts payable turnover days have gone up from 41.9 to 58.1 days. This means, for instance, in 2018, it took more than on average 58 days for JD to pay off its suppliers. On the other hand, JD's accounts receivable turnover is quite short, with payments being received from customers within five days of a sale. Combining these numbers with a 30-day inventory turnover, JD can efficiently use supplier trade credit for about 23 ($= 58 - 5 - 30$) days before having to pay it off. Notably, this strategy has proven successful for JD, as its cash position has consistently improved alongside its total revenue.

¹⁴One may categorize our narrowly defined supply chain finance as post-shipment finance. For more details about these arrangements, see for instance <https://www.adb.org/what-we-do/trade-supply-chain-finance-program/scf>, accessed on Jul 17, 2023.

Brands launched a supply-chain financing program in 2022 due to significant inventory growth and an increase in days of payables outstanding. VF Corp., the parent company of popular brands such as Vans, North Face, and Supreme, faced similar circumstances and launched an SCF program in 2022. Indeed, according to a report by the Wall Street Journal, the corporate supply-chain finance market has witnessed significant growth since the beginning of the pandemic. The report also suggests a close link between this growth and the prevalence of liquidity constraints faced by small suppliers.¹⁵

5.2 Other related financial arrangements

Keiretsu. Our model can also speak to keiretsu, a dominant organizational structure in the Japanese economy. Keiretsu consists of a group of companies with interlocking business relationships and shareholdings, and is characterized by its inherent liquidity-sharing nature.¹⁶ A crucial aspect of keiretsu is the establishment of joint financing initiatives, wherein member companies create shared financing vehicles such as joint venture funds, investment funds, or specialized financing entities. These vehicles pool funds contributed by participating companies to create a collective source of financing. Through the shared financing vehicle, loans, equity investments, or other financial instruments can be provided to member companies within the keiretsu. For this reason, the concept of liquidity cross-subsidization still applies to keiretsu.¹⁷

German Rural Credit Cooperatives. Credit cooperatives and microcredit institutions play a pivotal role as financial intermediaries in growing economies. A standout historical example is the German rural credit cooperatives of the 19th century. These credit cooperatives accepted deposits from members and made loans to members. By 1914, there were 19,000 credit cooperatives, accounting for approximately 7% of all German banking liabilities. The cooperatives exhibit several characteristics that align with our supply chain finance model.¹⁸

Like suppliers in our model, potential members of the German credit cooperatives faced high borrowing costs. The nation in the nineteenth century had a liberated yet undercapitalized peasantry. Before the advent of credit cooperatives, smallholders relied heavily on costly credit from informal lenders, often facing annual interest rates exceeding 30%. The emergence of cooperatives provided a much-needed alternative, offering more affordable credit options.

While modern supply chain finance leverages information advantage due to buyer firms'

¹⁵See the WSJ report at <https://www.wsj.com/articles/companies-offer-supply-chain-financing-to-vendors-as-they-bulk-up-on-inventory-push-out-payment-terms-11658316600?>, accessed on Jul 17, 2023.

¹⁶In horizontal keiretsu, a bank serves as the central entity, providing financial services to member companies. The "Big Six" horizontal keiretsu in Japan include Fuyo, Sanwa, Sumitomo, Mitsubishi, Mitsui, and DKB Group. Vertical keiretsu, on the other hand, connects suppliers, manufacturers, and distributors within a specific industry, with less influence from banks. Examples of vertical keiretsu include Toyota, Toshiba, and Nissan.

¹⁷It is important to note that the distribution of value in joint financing initiatives can be distinct from that in supply chain finance. In supply chain finance, the intermediary typically holds more power than suppliers, whereas in joint financing initiatives, the allocation of value is determined through negotiations among member companies.

¹⁸The facts and evidence described below come from Guinnane (2001).

close ties with suppliers, the German rural cooperatives relied on intimate community knowledge among members. Cooperatives deliberately limited their operations to compact geographic regions, often just one or two villages, and excluded residents from outside their designated area. This gives the cooperatives an in-depth understanding of the members' habits, character, and abilities, allowing a highly selective membership process based on this information. Not all applicants were granted membership, and even among members, not all were approved for loans. Any member exhibiting behaviours, such as excessive drinking, deemed detrimental to the cooperative's ethos could face expulsion.

Our model underscores significant heterogeneity among participants of SCF programs, which also holds among German cooperative members. As documented by Guinnane (2001), a cooperative called Diestedde, which operated for two villages, Diestedde and Stunnighausen, had 282 members. These members had different land types and farm sizes. For example, 61 members are large farmers, while 115 are small farmers. The Diestedde cooperative tailored the provision of credit, including loan sizes and terms, to these specific member profiles. Likewise, the liquidity needs of members vary. In the Diestedde cooperative, 56% of the members hadn't borrowed even six months after joining. Yet, in stark contrast, many were granted loans on the very day they became members.

The prevalence of liquidity cross-subsidization is evident. For instance, in the Diestedde cooperative, half of its members did not borrow during their initial five years of membership. This implies that these members essentially contributed funds to meet the liquidity needs of other members. In a similar vein, another cooperative called Hatzfeld exhibited a lower but still significant proportion, with one-fifth of its members being non-borrowers and serving as pure fund contributors.

A deeper exploration of these fund contributors reveals a striking resemblance to our model. In our model, the intermediary acquires liquidity from suppliers who operate at a negative profit, effectively subsidizing them through retail revenue. Guinnane (2001) suggested that those who primarily contributed funds to cooperatives often had businesses dependent on the prosperity of their local community, such as shopkeepers or local artisans. As a result, these members' funding contributions are also "subsidized" by other community members who purchase goods or services from them.

6 Conclusion

We developed a simple model of supply chain finance. The model incorporates essential features of SCF, including selection among heterogeneous suppliers, pooling liquidity from suppliers, and giving advance payment to those in need of liquidity. Our findings highlight the significance of liquidity cross-subsidization for the effective functioning of supply chain finance and its overall

welfare effect. We show that the nominal interest rate affects the trade-off between liquidity and profitability for the operation of supply chain finance. We demonstrate that deviating from the Friedman rule may lead to welfare gains. When suppliers also have access to the money market, we investigate the coexistence of supply chain finance and suppliers' holdings of liquidity, illustrating how the interest rate premium faced by the suppliers shapes the equilibrium.

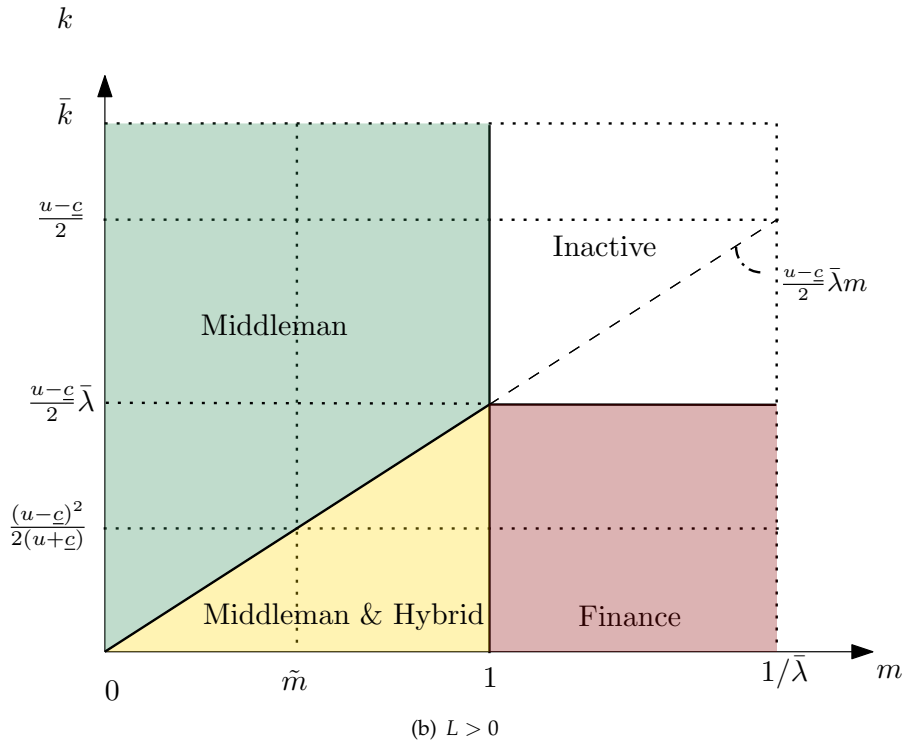
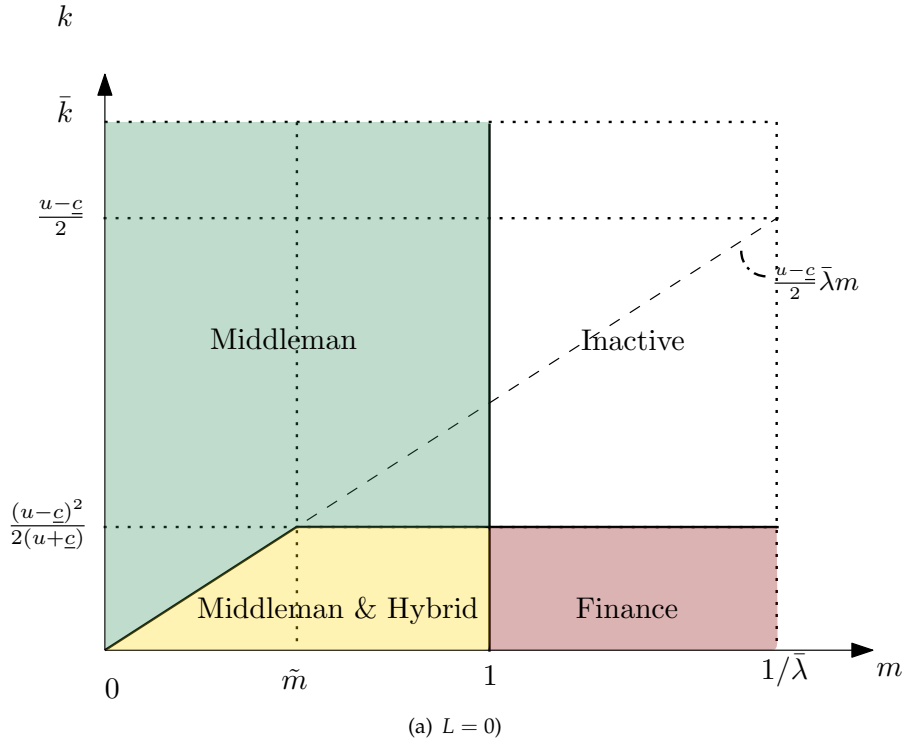


Figure 4: Intermediary's choice of modes

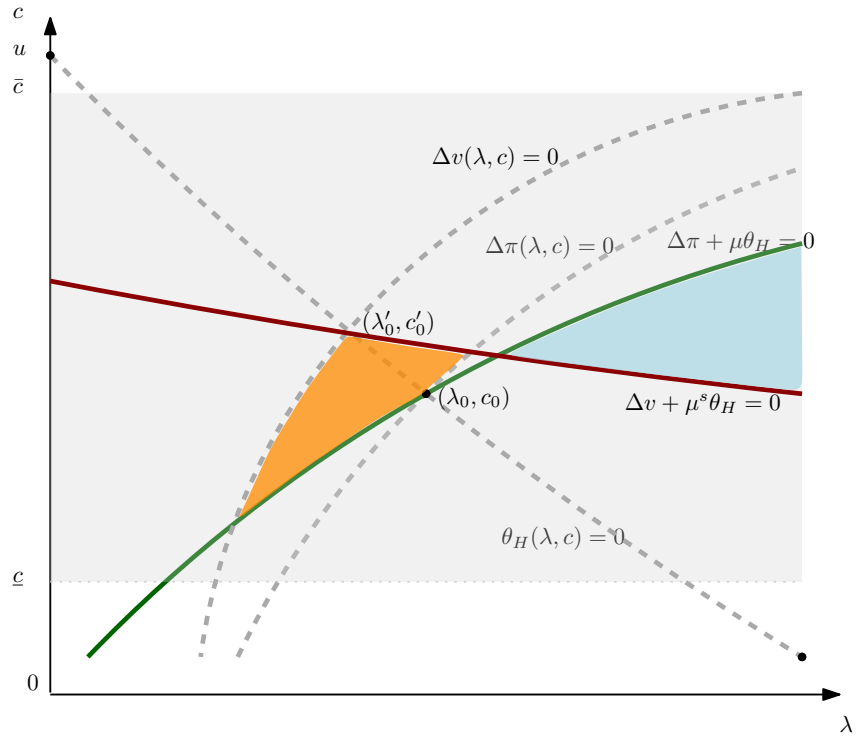


Figure 5: Compare the planner and the intermediary's selection rules

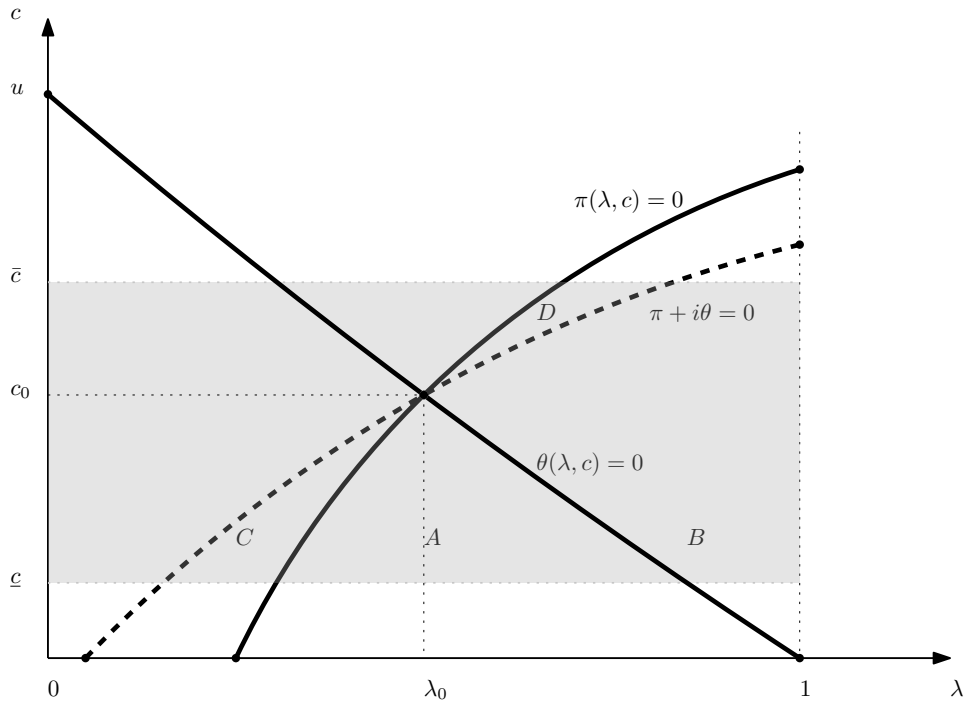


Figure 6: Friedman rule and Welfare

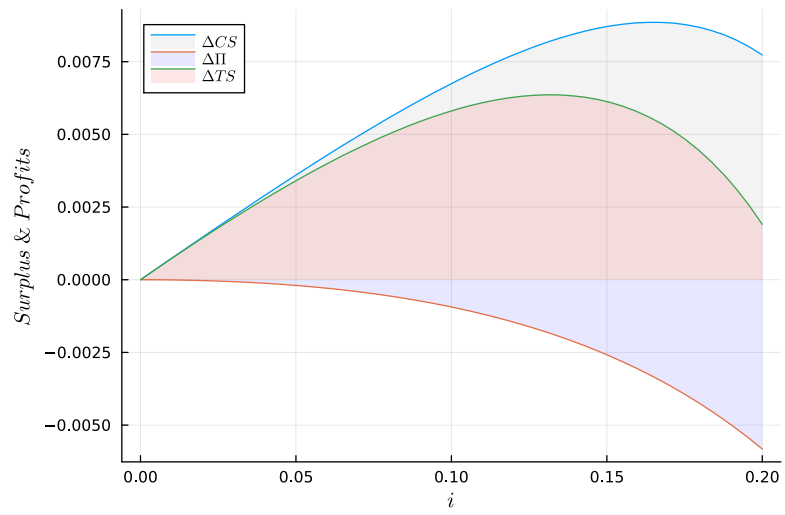
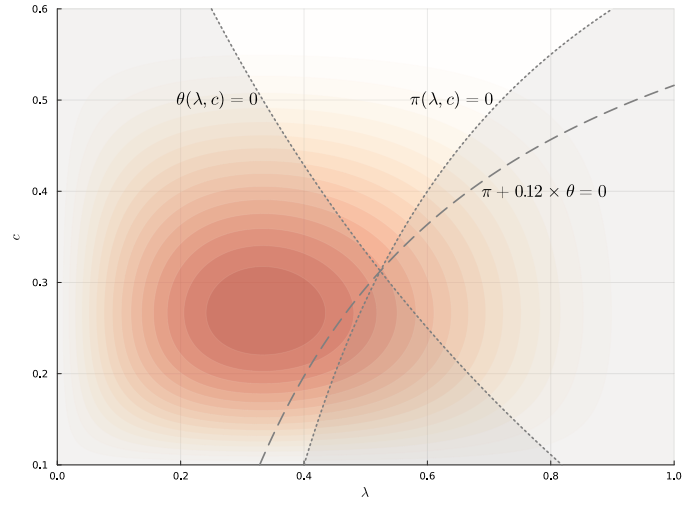
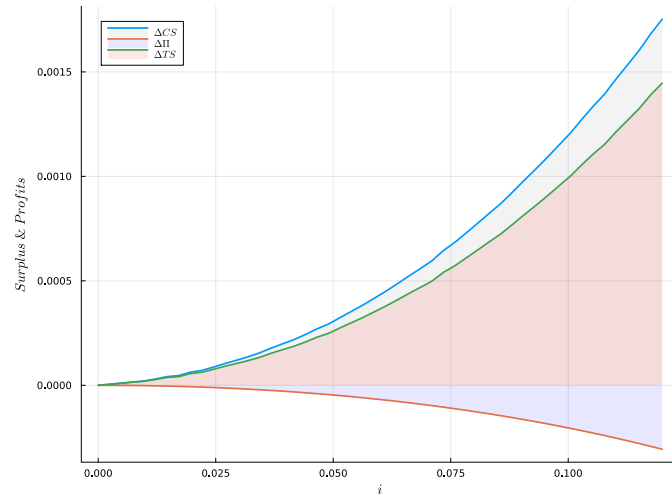


Figure 7: Welfare is non-monotonic in i under uniform distribution of (λ, c)



(a) Density and selection rule ($\mu = 0.12$)



(b) Welfare change

Figure 8: Welfare increases in i if $(\lambda, c) \sim \text{Beta}(2, 3)$

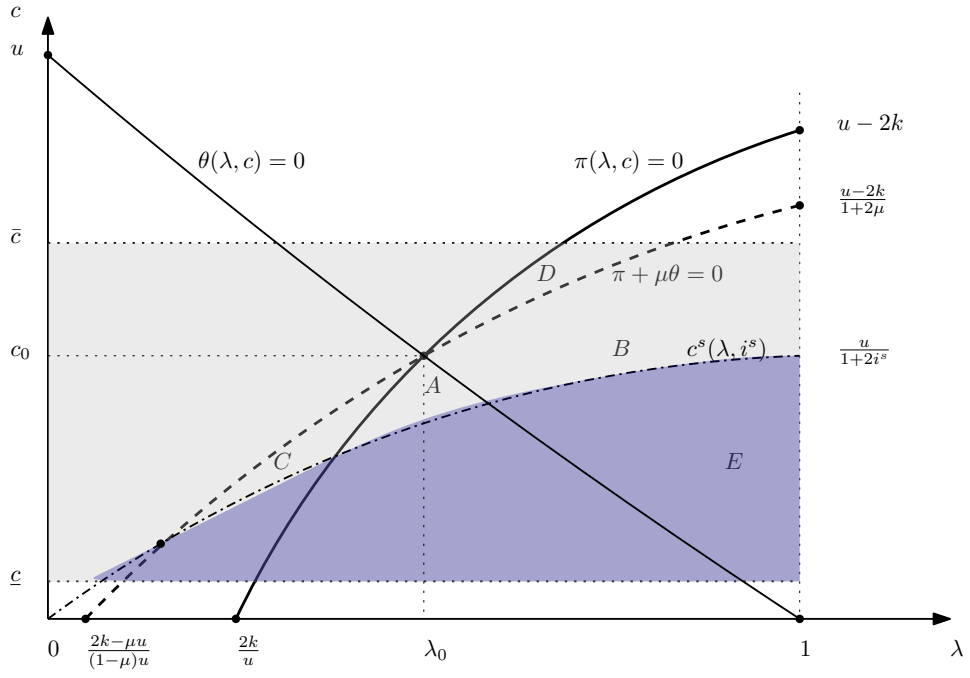


Figure 9: Suppliers' money holdings coexist with supply chain finance

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Now we show that μ is generically unique. Since $\Theta(\mu)$ is monotonically increasing in μ , if $\Theta(0) + L \geq 0$, then $\mu = 0$. If $\Theta(0) + L < 0$, then the liquidity constraint is binding at some $\mu \in (0, \infty)$, which is uniquely pinned down by $\Theta(\mu) + L = 0$. Note that when $L = -\int_{\Omega} \theta(\lambda, c) dG > 0$ and $c_0 > \bar{c}$, any $\mu \in [\underline{\mu}, \bar{\mu}]$ satisfies $\Theta(\mu) + L = 0$. When $L = 0$ and $c_0 < \underline{c}$, there is also a range of μ that $\Theta(\mu) = 0$.

The case for $m \in (\tilde{m}, 1)$ is similar except that $(\lambda_0, c_0) = \left(\frac{k + \sqrt{k^2 + 4ku}}{2mu}, u + k - \sqrt{k^2 + 4ku} \right)$. When $m \leq \tilde{m}$, the liquidity constraint is never binding, thus $\mu = 0$. ■

A.2 Proof of Theorem 1, Theorem 2 and Proposition 2

In text. ■

A.3 Proof of Corollary 1

Given that $\Theta(0) + L < 0$, $\mu(L)$ is determined by (13). Since $\Theta(\mu)$ is strictly increasing in μ , the statement follows. ■

A.4 Proof of Lemma 4

By the Euler equation (27), there are two cases. First, if $i \geq \mu(0, i)$, then $L = 0$. This case is valid either if $\mu(0, i) = 0$ (then $i > \mu = 0$ follows), or if $\mu(0, i) > 0$. Second, $i = \mu(L, i) > 0$ and $L > 0$, which requires that $\Theta(0, i) < 0$ and $i \leq \mu(0, i)$. ■

A.5 Proof of Corollary 2

It follows immediately from (28). ■

A.6 Proof of Lemma 5

Given that $b'_\lambda(\lambda, i) = \frac{2(k + ik - i^2u)}{(i + \lambda + i\lambda)^2}$, it is straightforward to verify the sign of $b'_\lambda(\cdot)$. The relationship between $c = b(\lambda, i)$ and $c = \bar{c}(i)$ can be obtained by comparing $b(1, i) = \frac{u - 2k}{1 + 2i}$ with $\bar{c}(i) = \frac{1 - i}{1 + i}u$. ■

A.7 Proof of Proposition 3

For all $i \leq i_1$, $\mu(0, i) = \mu(0, 0) > 0$, thus $\mu(i) = i$. If $c_0 \geq \bar{c}$, i.e., $k \leq \frac{(u - \bar{c})^2}{2(u + \bar{c})}$, then a marginal increase of i at $i = \mu = 0$ would lead to an increase in the number of suppliers joining the SCF program, thus social welfare must be improved. Now consider $c_0 < \bar{c}$, i.e., $k > \frac{(u - \bar{c})^2}{2(u + \bar{c})}$. The

middleman's selection rule (??) is $q(\lambda, c, \mu) = 1$ iff $c \in [\underline{c}, b(\lambda, \mu)]$ whenever $b(\lambda, \mu) \geq \underline{c}$. The welfare gain of having an active intermediary, compared to not having an intermediary, is

$$\Delta \mathcal{W}(\mu) = \int_{\underline{\lambda}(\mu)}^{\bar{\lambda}(\mu)} \int_{\underline{c}}^{b(\lambda, \mu)} (\lambda(u - c) - k) g(\lambda, c) dc d\lambda + \int_{\bar{\lambda}(\mu)}^1 \int_{\underline{c}}^{\bar{c}} (\lambda(u - c) - k) g(\lambda, c) dc d\lambda,$$

when $b(\lambda, \mu)$ is upward sloping with respect to λ , which is always the case when μ is in the neighborhood of $\mu = 0$. Here, $\bar{\lambda}(\mu) = \min\{1, \frac{2k - \mu(u - \bar{c})}{(u - \bar{c}) - \mu(u + \bar{c})}\}$, and $\underline{\lambda}(\mu) = \max\{0, \frac{2k - \mu(u - \underline{c})}{u - \underline{c} - \mu(u + \underline{c})}\}$.

Observe that $\frac{\partial \Delta \mathcal{W}(\mu)}{\partial \mu} = \int_{\underline{\lambda}(\mu)}^{\bar{\lambda}(\mu)} (\lambda(u - b(\mu)) - k) g(\lambda, b(\mu)) b'_\mu(\lambda, \mu) d\lambda$. Since (λ, c) follows a uniform distribution, g is a constant. Letting \propto represent "proportional to", and inserting $b'(0) = 2\frac{k + k\lambda - u\lambda^2}{\lambda^2}$, we have

$$\frac{\partial \Delta \mathcal{W}(\mu)}{\partial \mu} \Big|_{\mu=0} \propto \int_{\underline{\lambda}}^{\bar{\lambda}} \left[\frac{\kappa}{\lambda^2} + \frac{\kappa}{\lambda} - 1 \right] d\lambda, \quad (34)$$

where $\kappa \equiv k/u$, $\bar{\lambda} \equiv \bar{\lambda}(0) = \min\{1, \frac{k}{(u - \bar{c})/2}\}$, and $\underline{\lambda} \equiv \underline{\lambda}(0) = \frac{k}{(u - \underline{c})/2} < 1$ (guaranteed by $k \leq \bar{k}$). Further define $\bar{\varepsilon} \equiv \frac{u}{(u - \bar{c})/2}$, $\underline{\varepsilon} \equiv \frac{u}{(u - \underline{c})/2}$. It holds that $\bar{\varepsilon} > \underline{\varepsilon} > 2$. Then $\bar{\lambda} = \min\{1, \kappa \bar{\varepsilon}\}$, and $\underline{\lambda} = \kappa \underline{\varepsilon}$. Since $\underline{\lambda} < 1$, we must have $\kappa \in (0, \frac{1}{\underline{\varepsilon}})$.

To continue, we clarify the range of κ to be examined for the rest of the proof. Since $k < \bar{k}$, we have $\kappa < \bar{\kappa} \equiv \bar{k}/u = \frac{(u - \underline{c})^2}{2u(u + \underline{c})}$. On the other hand, given that we are considering the case that k is larger than $\frac{(u - \bar{c})^2}{2(u + \bar{c})}$ (namely, $c_0 < \bar{c}$), we have $\kappa > \underline{\kappa} \equiv \frac{(u - \bar{c})^2}{2u(u + \bar{c})}$. Note that $\underline{\kappa} < \frac{1}{\bar{\varepsilon}}$, and $\bar{\kappa} < \frac{1}{\underline{\varepsilon}}$. Thus, $\kappa \in [\underline{\kappa}, \bar{\kappa}]$. There are two cases to be considered depending on whether $\bar{\lambda} = 1$ or not.

Suppose $\kappa \in [\frac{1}{\bar{\varepsilon}}, \bar{\kappa}]$. Then $\bar{\lambda} = 1$. By (34), we have

$$\frac{\partial \Delta \mathcal{W}(\mu)}{\partial \mu} \Big|_{\mu=0} \propto \left[-\lambda + \kappa \left(\log(\lambda) - 1/\lambda \right) \right]_{\underline{\lambda}}^1 = (\underline{\varepsilon} - 1) \left(\kappa - \frac{1}{\underline{\varepsilon}} \right) - \kappa \log(\kappa \underline{\varepsilon}) \equiv h(\kappa).$$

Note that $h'(\kappa) = \underline{\varepsilon} - 2 - \log(\underline{\varepsilon} \kappa) > 0$ since $\underline{\varepsilon} > 2$ and $\kappa \underline{\varepsilon} = \underline{\lambda} < 1$. Then $h(\kappa) < h(\bar{\kappa}) < h(1/\underline{\varepsilon}) = 0$ (since $\bar{\kappa} < \frac{1}{\underline{\varepsilon}}$). Thus, $\frac{\partial \Delta \mathcal{W}(\mu)}{\partial \mu} \Big|_{\mu=0} < 0$ for all $\kappa \in [\frac{1}{\bar{\varepsilon}}, \bar{\kappa}]$.

Suppose that $\kappa \in (0, \min\{\bar{\kappa}, \frac{1}{\bar{\varepsilon}}\})$. Then $\bar{\lambda} < 1$. By (34) we have

$$\frac{\partial \Delta \mathcal{W}(\mu)}{\partial \mu} \Big|_{\mu=0} \propto -\kappa(\bar{\varepsilon} - \underline{\varepsilon}) + \frac{\bar{\varepsilon} - \underline{\varepsilon}}{\bar{\varepsilon} \underline{\varepsilon}} + \kappa \left(\log(\bar{\varepsilon}) - \log(\underline{\varepsilon}) \right),$$

which is positive iff $\kappa < \frac{1}{\bar{\varepsilon} \underline{\varepsilon} \left(1 - \frac{\log(\bar{\varepsilon}) - \log(\underline{\varepsilon})}{\bar{\varepsilon} - \underline{\varepsilon}} \right)} \equiv \tilde{\kappa} > 0$. Next we show $\tilde{\kappa} < 1/\bar{\varepsilon}$, which is equivalent to $\frac{\log(\bar{\varepsilon}) - \log(\underline{\varepsilon})}{\bar{\varepsilon} - \underline{\varepsilon}} < 1 - \frac{1}{\underline{\varepsilon}}$. Define an auxiliary function $z(x) \equiv \log(\bar{\varepsilon}) - \log(x)$, and note that $z'(x) = -1/x$. By the mean value theorem, we have that for some $x_0 \in (\underline{\varepsilon}, \bar{\varepsilon})$

$$\frac{\log(\bar{\varepsilon}) - \log(\underline{\varepsilon})}{\bar{\varepsilon} - \underline{\varepsilon}} = -\frac{z(\bar{\varepsilon}) - z(\underline{\varepsilon})}{\bar{\varepsilon} - \underline{\varepsilon}} = -z'(x_0) = \frac{1}{x_0} \in \left(\frac{1}{\bar{\varepsilon}}, \frac{1}{\underline{\varepsilon}} \right).$$

Then it follows that $\frac{\log(\bar{\varepsilon}) - \log(\underline{\varepsilon})}{\bar{\varepsilon} - \underline{\varepsilon}} < \frac{1}{\underline{\varepsilon}} < 1 - \frac{1}{\underline{\varepsilon}}$, where the last inequality is due to $\underline{\varepsilon} > 2$. Defining $\kappa^* \equiv \min\{\tilde{\kappa}, \bar{\kappa}\}$, we obtain the claim. ■

A.8 Proof of Proposition 4

Let $\Pi(i^s, i)$ be the maximized profits of the intermediary. Let $c^\pi(\lambda) = u - 2k/\lambda$ denote the curve of (λ, c) such that $\pi(\lambda, c) = 0$. $c^s(\lambda, i^s)$ and $c^\pi(\lambda)$ cross each other only if $i^s \geq \frac{k}{u-2k}$, and if they intersect, they only intersect once. If $c^s(1, i^s) > c^\pi(1)$, or equivalently, $i^s < \frac{k}{u-2k}$, then $c^s(\lambda, i^s) > c^\pi(\lambda)$ for all $\lambda \in [0, 1]$, meaning that all suppliers with positive profits $\pi(\lambda, c)$ are excluded from $\tilde{\Omega}(i^s)$. Thus, we must have $\Pi(i^s, i) = 0$. On the other hand, if $i^s \geq \bar{i}^s \equiv \frac{u-c}{2c}$, then $\tilde{\Omega}(i^s) = \Omega$, resulting in $\Pi(i^s, i) > 0$. Finally, $\Pi(\cdot)$ is weakly increasing in i^s , given the optimal selection of suppliers, because as i^s increases, the set of feasible suppliers $\tilde{\Omega}(i^s)$ becomes larger. Therefore, $i^s \in [\frac{k}{u-2k}, \bar{i}^s)$ must exist. Combined with the suppliers' money-holding decision rule (see condition (33) in the main text), this proves the claims in the proposition. ■

A.9 Proof of Proposition 5

For $i \in (0, i_2)$, it holds that $c = c^s(\lambda, i)$ must cross $c = \bar{c}(i)$ since $c^s(0, i) < \bar{c}(i) < c^s(1, i)$. Thus, there exist suppliers with (λ, c) satisfying $c \in [\underline{c}, \min\{\bar{c}(i), c^s(\lambda, i)\}]$. In equilibrium, these suppliers choose to hold money by themselves.

Turn to the middleman's problem. The feasible set of suppliers is given by $\{(\lambda, c) \in \Omega \mid c^s(\lambda, i) \leq c \leq \bar{c}(i)\}$. Note that $\bar{c}(i)$ and $c^s(\lambda, i)$ intersect at $(\lambda_1, c_1) = \left(1 - i, \frac{1-i}{1+i}u\right)$. In Figure 11, the feasible set of suppliers is represented by the green region between $\bar{c}(i)$ and $c^s(\lambda, i)$ and to the left of (λ_1, c_1) . A necessary condition for an active intermediary in equilibrium is that (λ_1, c_1) locates

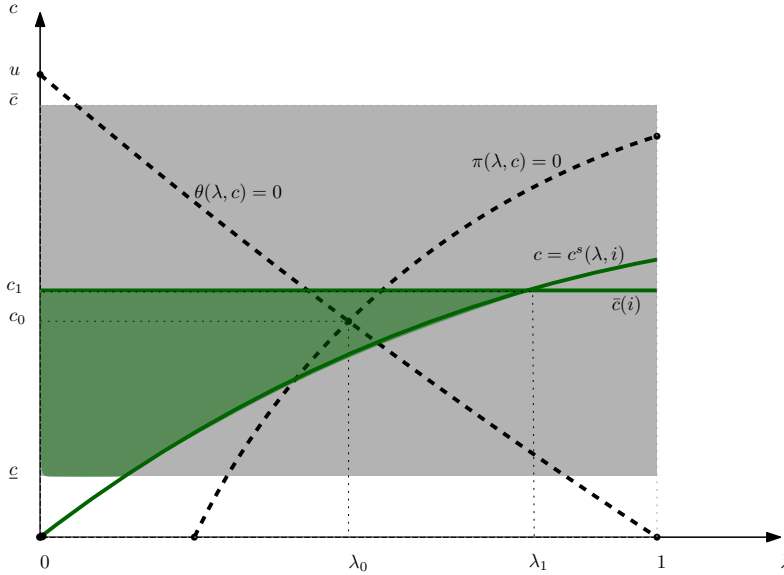


Figure 11: Illustration for $\pi(\lambda_1, c_1) < 0$

below the curve of $\pi(\lambda, c) = 0$, i.e., there exists some set of i such that

$$\pi(\lambda_1, c_1) = \pi\left(1 - i, \frac{1-i}{1+i}u\right) > 0. \quad (35)$$

Otherwise, all suppliers in the feasible set give a negative profit to the intermediary. (35) gives

$i \in (i_-, i_+)$, where $i_- \equiv \frac{-\sqrt{k^2-6ku+u^2}-k+u}{2u}$ and $i_+ \equiv \frac{\sqrt{k^2-6ku+u^2}-k+u}{2u}$. In Figure 12, we illustrate

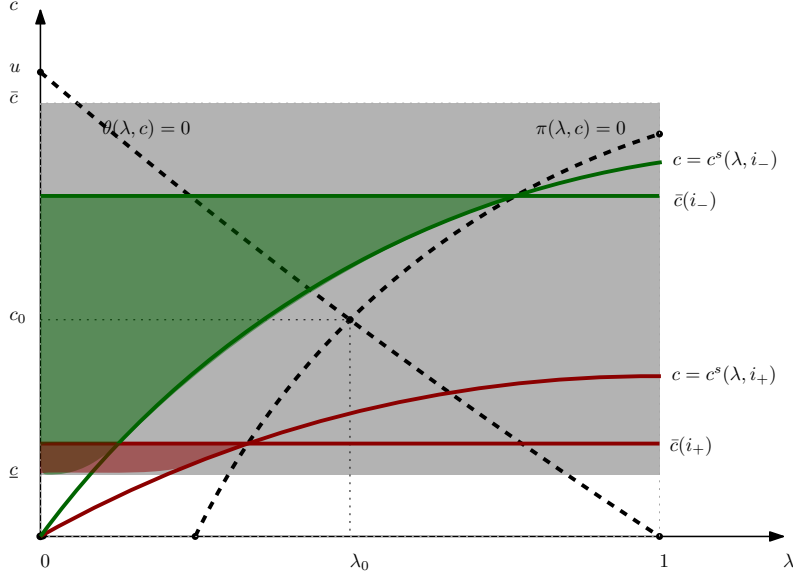


Figure 12: Illustration for i_- and i_+

$c = c^s(\lambda, i)$ and $c = \bar{c}(i)$ at the two levels of nominal interest rates, i_- and i_+ , as well as the feasible set of suppliers in each case, in green and red, respectively. For i_- and i_+ to exist, the intermediary needs to be sufficiently efficient, i.e., $k < (3 - 2\sqrt{2})u$. Define i_0 by $\bar{c}(i_0) = c_0$. Note that $i_- < i_0$ always holds, and $i_+ > i_0$ if and only if $k < u/6$ ($< (3 - 2\sqrt{2})u$). Also note that $c = c^s(\lambda, i)$ crosses $c = c^\pi(\lambda)$ at $c = k/i$. Suppose $i > k/c_0$ so that $c = c^s(\lambda, i)$ crosses $c = c^\pi(\lambda)$ below c_0 . For such i to be in (i_-, i_+) we require $i_+ > i_0$, or equivalently $k < u/6$. Then, there exists a set of suppliers who contribute positive profits and liquidity, represented by the green region in Figure 13.

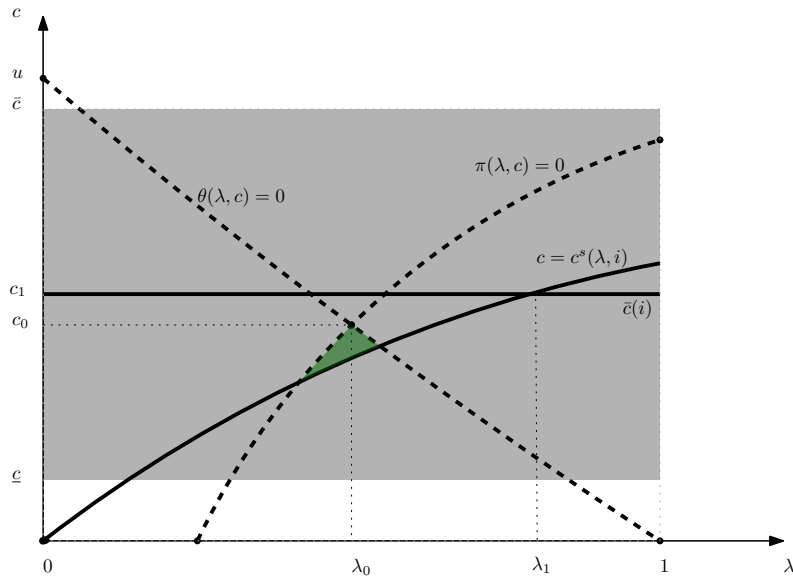


Figure 13: Illustration for $k/i < c_0$

As a result, the intermediary must be profitable. ■