A Model of Supply Chain Finance

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December 14, 2023

What is Supply Chain Finance (SCF)?

Consider a large buyer firm, e.g., Walmart, Siemens, etc.

- a large number of heterogeneous suppliers
- suppliers face shortage of working capital from time to time

Supply chain finance:

- a program offered by the buyer firm (possibly with financiers)
- select among its suppliers to join
- suppliers are given extended payment terms
- suppliers can request immediate payment at a small discount

The Co-op Partners with PrimeRevenue to Protect Suppliers Amid Economic Volatility

UK's sixth largest food retailer makes strategic transition to PrimeRevenue platform

Atlanta, GA – Manchester, UK, August 11, 2020 – PrimeRevenue, the leading platform for working capital finance solutions, and The Co-operative Group, today announce a new supply chain finance partnership. Barclays Bank PLC, who introduced The Co-op to PrimeRevenue, will be providing funding on the supply chain finance programme followed by other financial institutions as the programme grows.

Co-op has made the strategic decision to partner with PrimeRevenue for its new supply chain finance offering. Fueled by a highly challenging business climate heightened by the pandemic, the company aims to offer suppliers a simple method of early payment to help with their cash flow without having a detrimental impact to Co-op's own cash position. This is particularly relevant in the current environment where the old adage "cash is king" has never been truer.

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Why do we care?

SCF is happening and on the rise

- SCF has been widely adopted by large corporations
- The size of the SCF market is $1.8 trillion globally in 2021

SCF can be a "SLEEPING RISK" that "MASKS EPISODES OF FINANCIAL STRESS." (S&P Global Inc.)

- The buyer firm (and associated supply chain) may face difficulties if financing cost increases
- FASB: Starting in 2023, corporations will have to disclose the terms and size of the SCF programs in the financial statement.

Preview of the model

- A simple model of a middleman funding suppliers.
- Heterogeneous suppliers: productivity and liquidity needs.
- The middleman selects suppliers into the SCF program
- We then integrate this model into a standard monetary framework (Lagos and Wright, 2005).

Preview of key results

Liquidity cross-subsidization

- use liquidity from suppliers with negative profits
- to fund suppliers with positive profits
- links to the cost of market liquidity

Friedman rule can be suboptimal

- when *market liquidity* is more costly, SCF makes more use of suppliers' liquidity
- more suppliers included, more trade created under certain conditions

Related literature

- Multi-product intermediaries:
  - Rhodes et al. (2021), Spulber (1996).
  - Liquidity issues are not addressed in previous models.
- Bankings and Money search
  - Berentsen et al. (2007), Gu et al. (2013), Andolfatto et al. (2019)
  - Our model emphasizes the ex-ante section of depositors
  - Unlike in Diamond and Dybvig (1983), the late-type withdrawers in our model do not have the incentive to run
- Supply chain finance:
  - In econ and finance, closely related is trade credit.
  - In management science, e.g., Kouvelis and Xu (2021)
  - Our model: one big buyer firm with many suppliers.

This talk

1. Benchmark model
   - an endowment economy
   - a subperiod (DM) in Lagos and Wright (2015) framework
2. Endogenous liquidity holdings
3. Welfare analysis
4. Extension

1. The Benchmark Model

Agents

- A mass of suppliers:
  - each produces a unique and indivisible good
  - constant marginal costs, $c \in [\underline{c}, \bar{c}]$, differ among suppliers
  - c is publicly observable
- A mass of consumers:
  - unit demand for each good with *common* utility $u > \bar{c}$
- One middleman:
  - purchases from suppliers and resells to consumers
  - operates an SCF program (specified later)
  - fixed cost $k > 0$ to handle each supplier

Endowments/Liquidity

- There is a *numeraire* good
- Consumers have an endowment of the numeraire good
- The middleman has an endowment of measure $L \geq 0$
- Suppliers have *no* endowment, however, production cost c needs to be paid using the numeraire good.

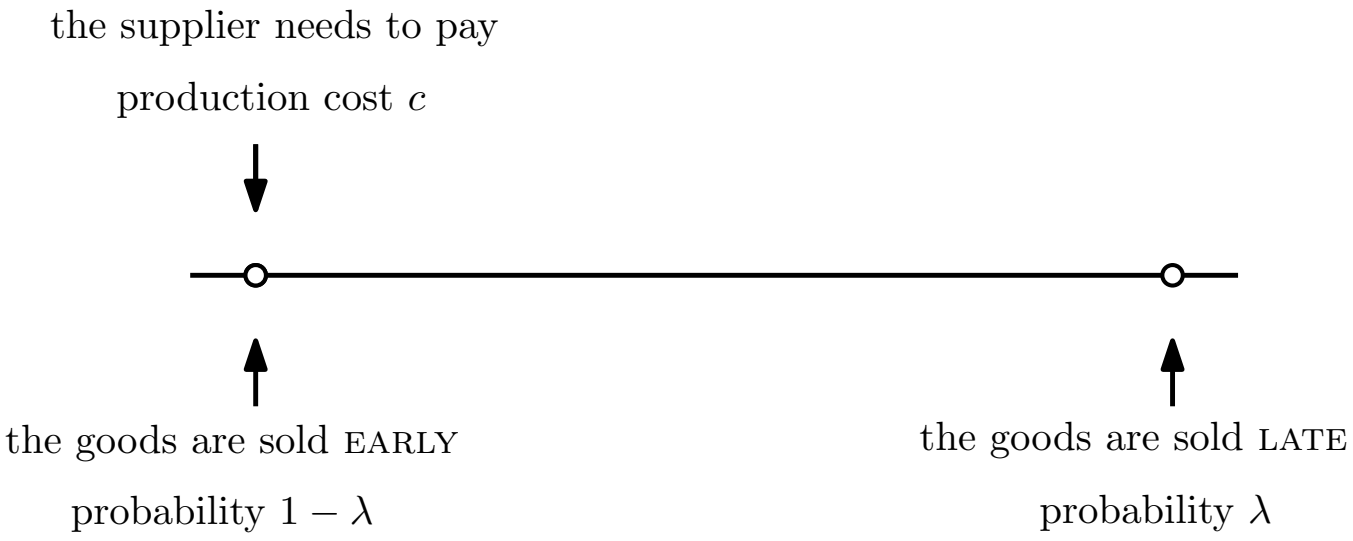
Retail market

- Without the middleman, suppliers trade directly with consumers.
- Suppliers can meet ALL consumers, trade bilaterally:
  - if a trade occurs, the retail-trade surplus is split equally:

$$p - c = (u - c)/2$$

  - however, trade may not occur due to liquidity frictions.

Liquidity shocks



- A liquidity shock is realized at beginning of period
- With prob $1-\lambda$, a supplier encounters no liquidity issue, c can be covered by using retail revenue
- With prob λ, a supplier encounters a liquidity issue, the supplier cannot produce since he has no numeraire

Ex ante heterogeneity of suppliers

- Each supplier is indexed by

$$(\lambda, c) \in \Omega = [0,1] \times [\underline{c}, \bar{c}],$$

  where λ is prob liquidity shock, c is the const marginal cost

- (λ, c) is publicly observable, following a distribution C.D.F. G, P.D.F. $g > 0$ on Ω

- The realization of the liquidity shock can be public or private information.

Middleman and SCF program

- Commitment: Individual suppliers are not able to use credit deals due to the limited commitment. HOWEVER, middleman can commit to the deals with suppliers
- Middleman observes (λ, c), and selects suppliers into SCF program.
- Selection policy:

$$q(\lambda, c) = \begin{cases} 1 & \text{if } (\lambda, c) \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

Middleman and SCF program (cont.)

Given a supplier is invited $q(\lambda, c) = 1$, the middleman gives a TIOLI offer based on (λ, c):

- The middleman sells the goods on behalf of the supplier
  - bilateral trade / suppliers quite the market.
- The middleman transfers a revenue $f(\lambda, c)$ at end of period.
- The middleman pays c to the supplier at beginning of period.

An SCF program can be represented by:

$$\{q(\lambda, c), f(\lambda, c)\}_{(\lambda,c)\in\Omega} \in \{0, 1\} \times \mathbb{R}_+.$$

SCF program (alternative setting)

Intermediary, instead of a middleman

Given $q(\lambda, c) = 1$, SCF gives a TIOLI offer based on (λ, c):

- Supplier gives his retail revenue p to the intermediary
- The intermediary transfers to the supplier a reward at end of period
  - $f^E(\lambda, c)$ if revenue transferred at beginning of period
  - $f^L(\lambda, c)$ if revenue transferred at end of period
- The intermediary always pays c to supplier at beginning of period

An SCF program can be represented by:

$$\{q(\lambda, c), f^E(\lambda, c), f^L(\lambda, c)\}_{(\lambda,c)\in\Omega} \in \{0, 1\} \times \mathbb{R}_+.$$

Timing

1. The middleman announces the SCF program and invites selected suppliers, and suppliers decide whether to accept or not.
2. The liquidity shock for each supplier is realized. The middleman pays c to all participating suppliers. Meanwhile, trade occurs in the retail market.
3. The middleman pays each supplier $f(\lambda, c)$.

Analysis

Solution concept

- Complete information game
- Subgame perfection

Suppliers' participation decision

- Supplier (λ, c) joins SCF program if

$$\underbrace{f(\lambda, c)}_{\textit{join SCF}} \geq \underbrace{(1-\lambda)(u-c)/2}_{\textit{not join SCF}}$$

$$\Rightarrow f(\lambda, c) = (1-\lambda)(u-c)/2$$

Profits and liquidity contributions to SCF

- A supplier contributes to SCF in PROFIT and LIQUIDITY.
- Profit contribution:

$$\pi(\lambda, c) = p - c - f - k = \lambda(u - c)/2 - k.$$

- Liquidity contribution at the time of production:

$$\theta(\lambda, c) = (1 - \lambda)p - c = (1 - \lambda)(u + c)/2 - c.$$

- π and θ can be positive or negative depending on (λ, c)

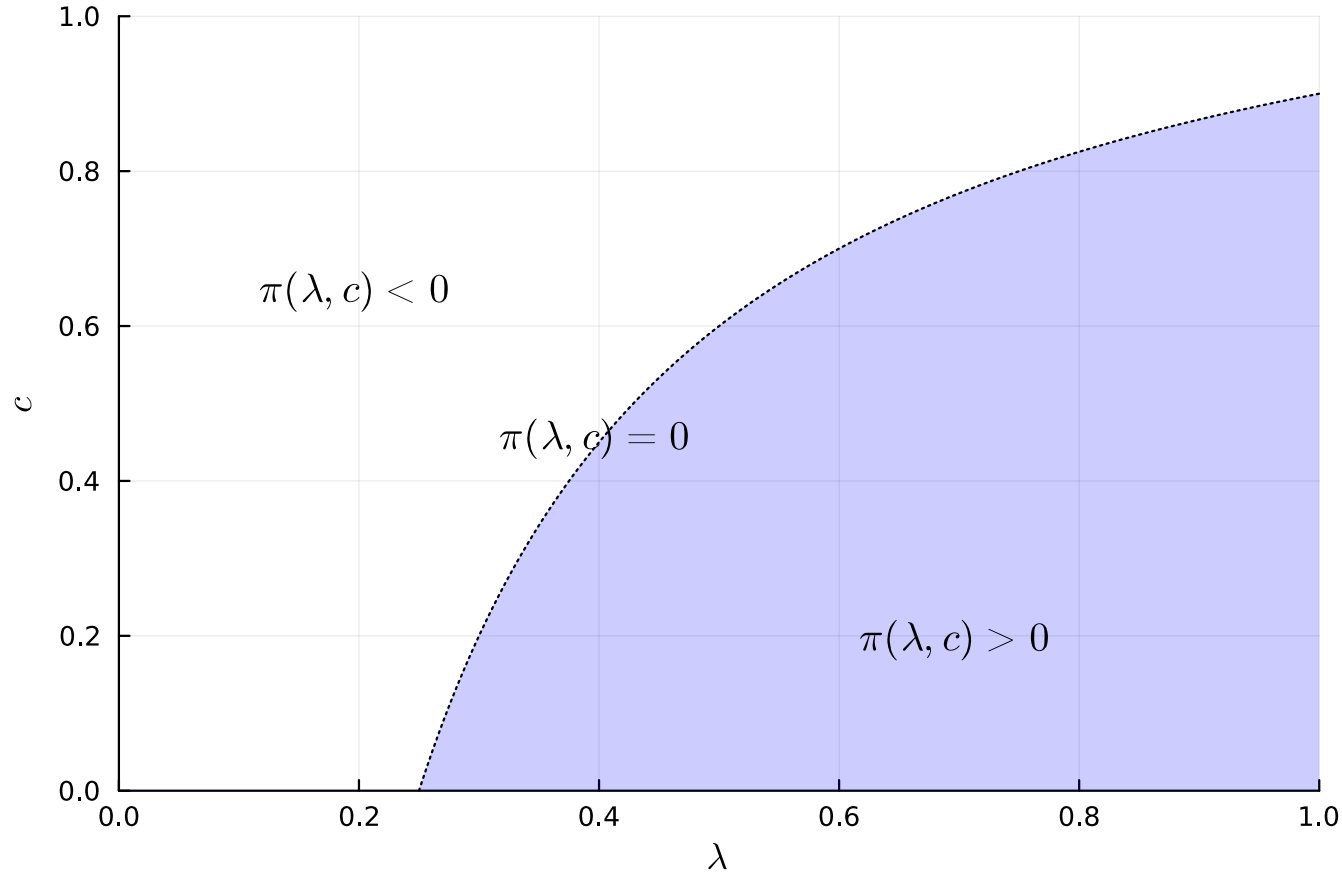


Figure: profit contributions in (λ, c) space

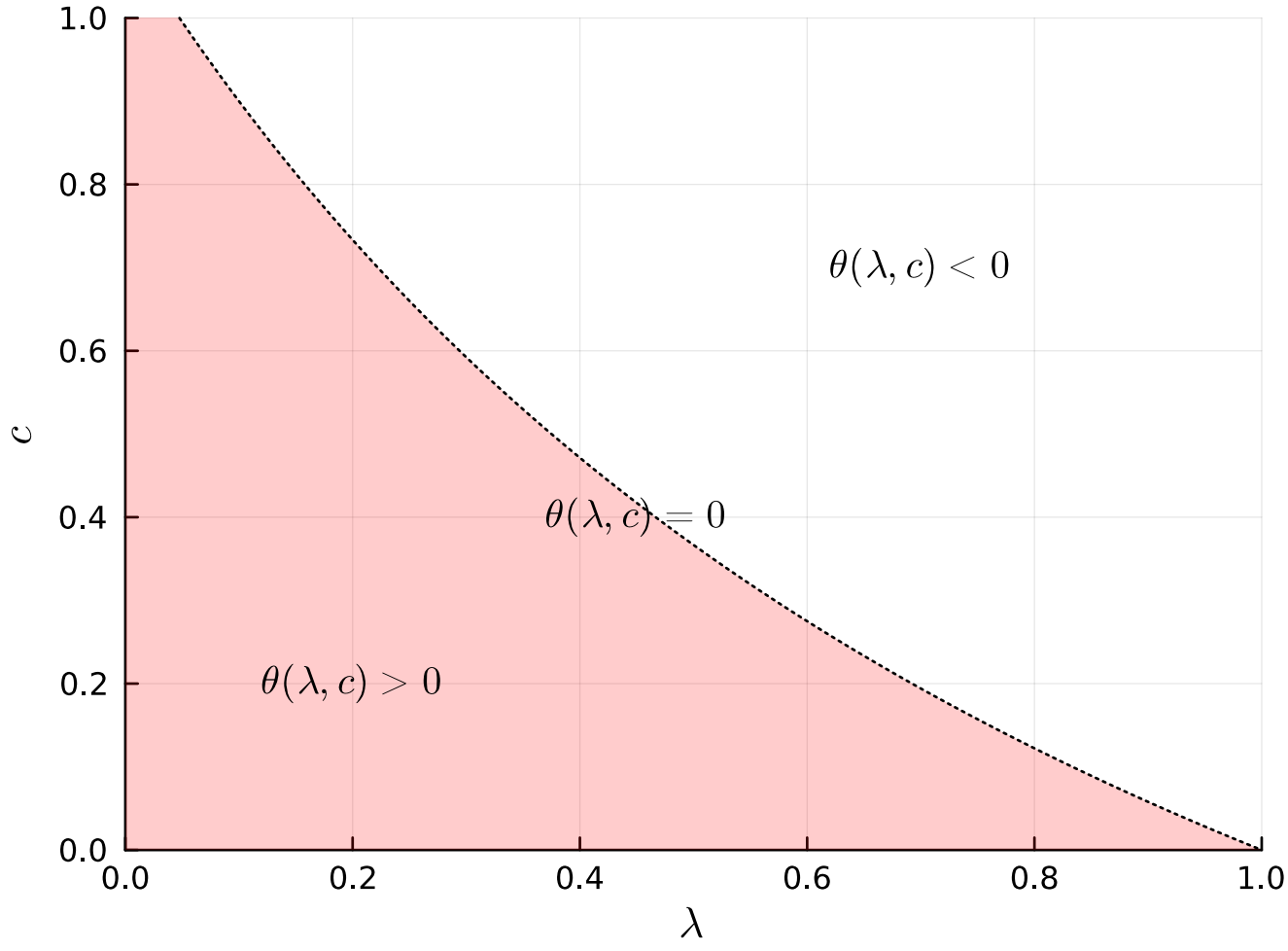


Figure: liquidity contributions in (λ, c) space

- The middleman's profit maximization problem:

$$\max_{q(\lambda,c)\in\{0,1\}} \int_{\Omega} q(\lambda,c)\pi(\lambda,c)dG$$

$$\text{s.t.} \quad \Theta + L \geq 0.$$

- Aggregate liquidity contribution from suppliers:

$$\Theta = \int_{\Omega} q(\lambda,c)\theta(\lambda,c)dG.$$

- Initial liquidity holdings (exogenous for now):

$$L \geq 0.$$

Profit-maximizing selection policy

- The middleman's problem can be solved using the Lagrangian:

$$\mathcal{L} = \int_{\Omega} q(\lambda, c) \Big[\pi(\lambda, c) + \mu\theta(\lambda, c) \Big] dG(\lambda, c).$$

- $\mu \geq 0$: the Lagrangian multiplier associated with the liquidity constraint; the shadow value of liquidity.

- The optimal selection policy:

$$q(\lambda, c, \mu) = \begin{cases} 1 & \text{if } \pi(\lambda, c) + \mu\theta(\lambda, c) \geq 0 \\ 0 & \text{if otherwise.} \end{cases}$$

Proposition (Liquidity cross-subsidization)

The middleman optimally selects suppliers from

- *Region A: positive profit and positive liquidity contributions*

$$\pi(\lambda, c) \geq 0, \quad \theta(\lambda, c) \geq 0$$

- *Region B: positive profit and negative liquidity*

$$\pi(\lambda, c) > 0, \quad \theta(\lambda, c) < 0, \quad \underbrace{-\pi/\theta}_{\textit{returns}} \geq \mu$$

- *Region C: negative profit and positive liquidity*

$$\pi(\lambda, c) < 0, \quad \theta(\lambda, c) > 0, \quad \underbrace{-\pi/\theta}_{\textit{costs}} \leq \mu$$

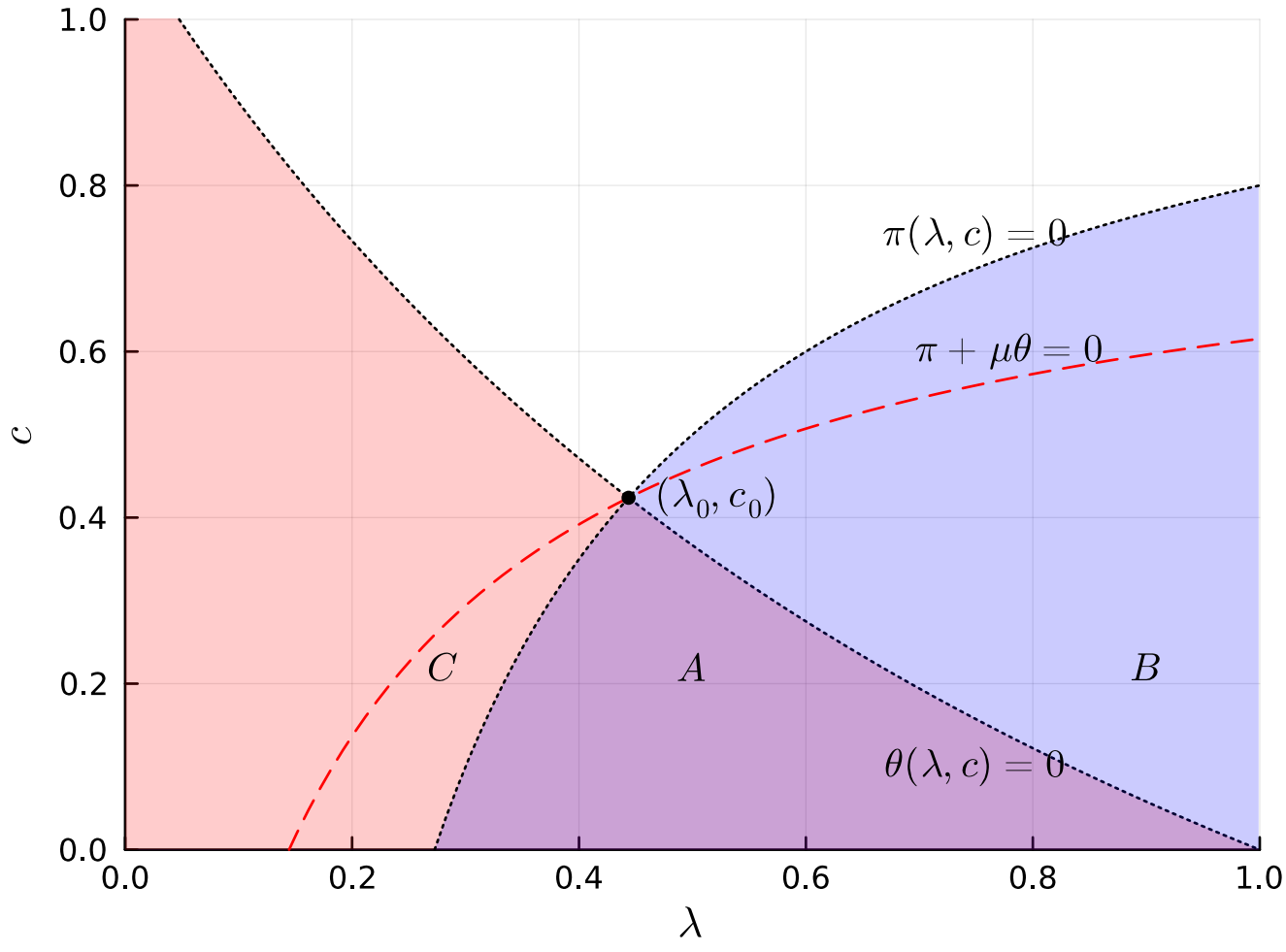


Figure: Liquidity cross-subsidization

Determination of μ

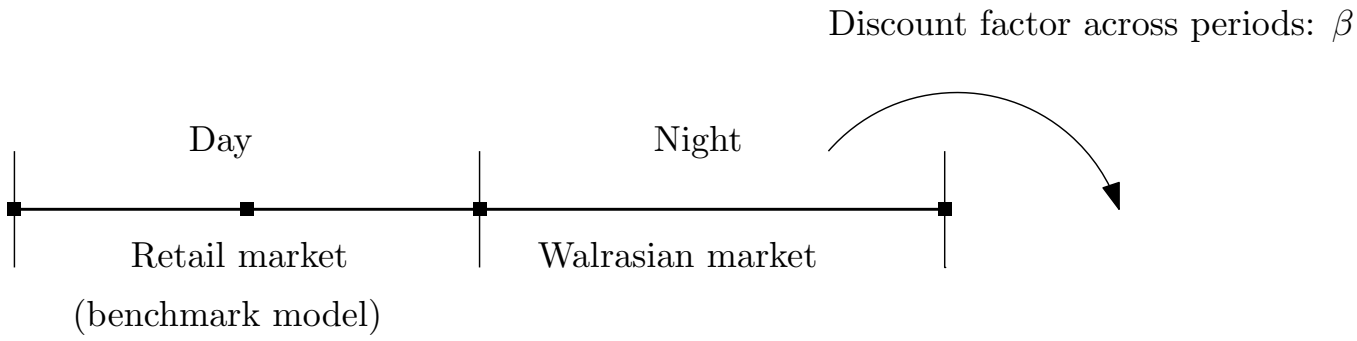
The liquidity constraint determines

$$\mu = \mu(L)$$

- $\mu(L) = 0$: liquidity does not matter for selecting suppliers; selection is solely based on $\pi(\lambda, c)$
- $\mu(L) > 0$: liquidity cross-subsidization, strictly decreases in L
- $\mu(0)$: the liquidity value at $L = 0$, or shadow price of the first marginal unit of liquidity
  - $\mu(0) = 0$ is possible, then suppliers' liquidity is self-sustained.

2. Endogenous liquidity holdings

Standard monetary approach (Lagos and Wright, 2005)



- Day market (the benchmark model)
  - the numeraire good is a medium of exchange, e.g., fiat money
  - suppliers must pay for production costs using fiat money
- Night market (Walrasian)
  - all other markets, where the middleman and consumers can "earn" fiat money by producing a "general good"
  - 1 unit of fiat money worth ϕ_t units of general good: $L_t = \phi_t l_t$.

Liquidity holdings of the middleman

- The middleman chooses $l(\equiv L/\phi)$ units fiat money

$$\max_{l\geq 0} \quad \Big\{ -\phi_{t-1} l + \beta V_t(l) \Big\} \quad \Rightarrow \quad \phi_{t-1} \geq \beta V_t'(l).$$

- middleman's value of carrying l units of fiat money:

$$V_t(l) = \Big\{ \phi_t l + \max_{q(\lambda,c)} \int_\Omega q(\lambda,c)\pi(\lambda,c) dG, \quad \text{s.t.} \quad \Theta + \phi_t l \geq 0. \Big\}$$

$$\Rightarrow \quad V_t'(l) = \phi_t \Big(1 + \mu(L) \Big)$$

- Euler equation: $\phi_{t+1} \geq \beta \phi_t (1 + \mu(L))$, or equivalently

$$i \geq \mu(L).$$

Definition

A stationary monetary equilibrium consists of a tuple

$$\{f(\lambda, c), q(\lambda, c, \mu), \mu, L\}.$$

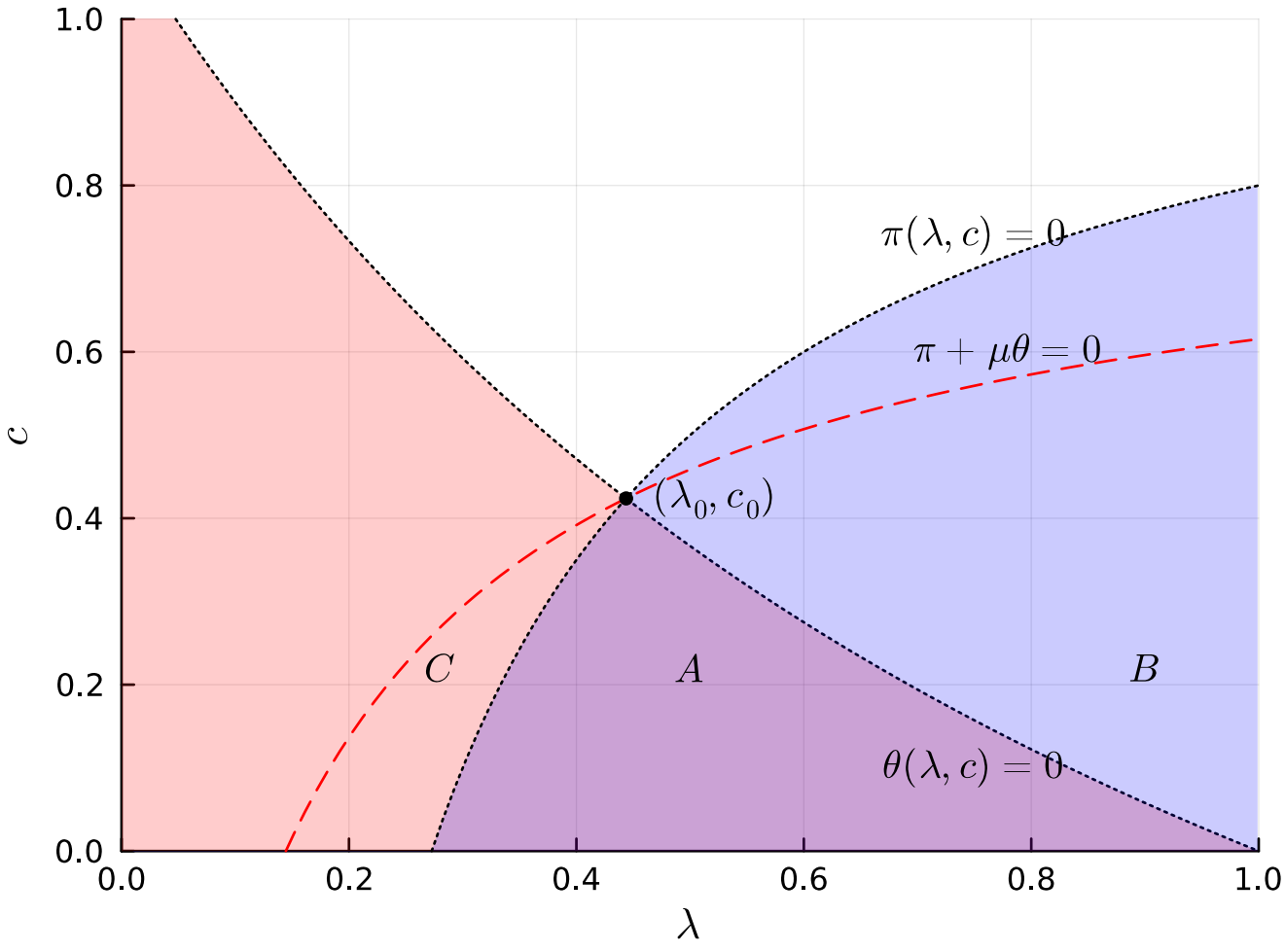
Proposition

For $i \leq \bar{i}$, there exists a unique monetary equilibrium with SCF program described by $q(\lambda, c, \mu)$, $f(\lambda, c)$, shadow value of liquidity:

$$\mu = \min\{\mu(0), i\},$$

and middleman's liquidity holding:

$$\begin{cases} \mu(L^*) = i & \text{if } i < \mu(0); \\ L^* = 0 & \text{if } i \geq \mu(0). \end{cases}$$



In equilibrium, $\mu = \min\{\mu(0), i\}$.

Characterization of the monetary equilibrium

- The SCF program $\mu = \min\{\mu(0), i\}$ is jointly shaped by
  - the inherent richness of suppliers' liquidity: $\mu(0)$
  - cost of market liquidity: i
- As $i \to 0$, SCF features $\mu \to 0$
  - all suppliers with positive π are selected
- As i increases from 0 to $\mu(0)$, L decreases.
- For $i \geq \mu(0)$, middleman holds $L = 0$
  - SCF solely relies on liquidity from suppliers
  - resilient to rising funding costs

3. Welfare and Inflation

Planner's problem

- Trade surplus:

$$v(\lambda, c) = \lambda(u - c) - k.$$

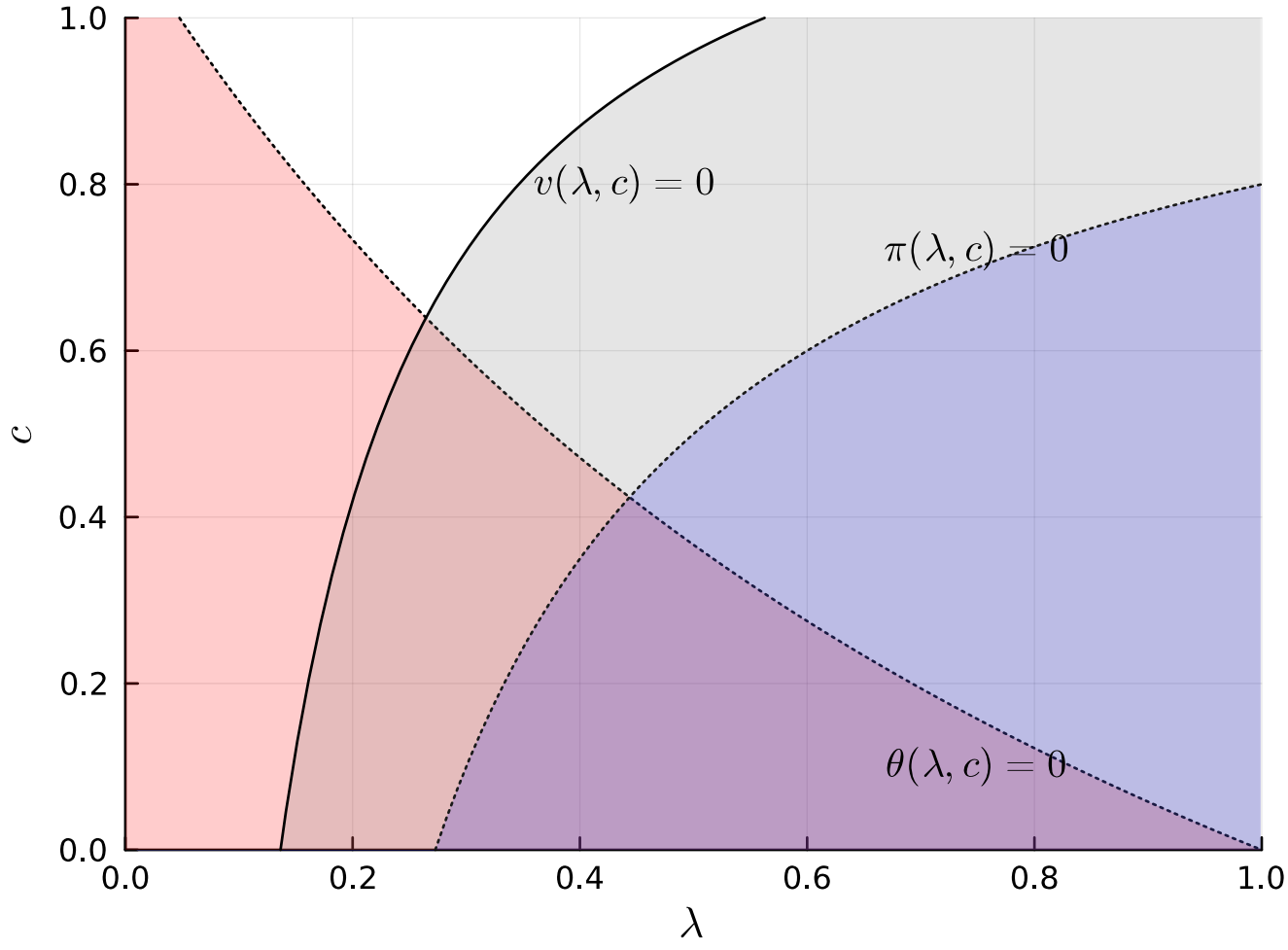
  Rather than profits $\pi(\lambda, c) = \lambda(u - c)/2 - k$.

- Planner's problem:

$$\max_{I(\lambda,c)} \int_{\Omega} I(\lambda, c) v(\lambda, c) dG.$$

- The efficient allocation:

$$I(\lambda, c) = 1 \text{ if } v(\lambda, c) \geq 0$$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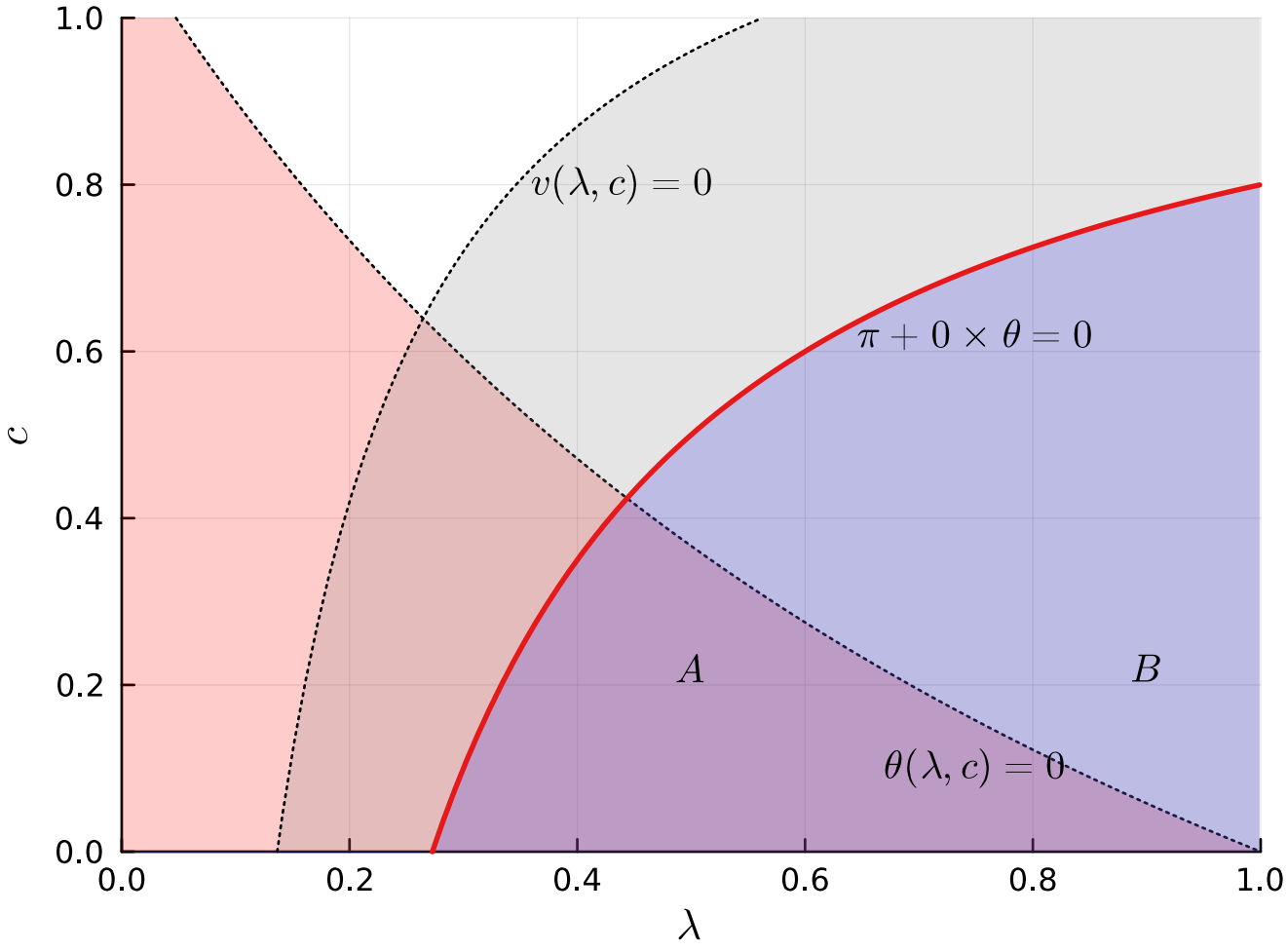
$\pi(\lambda, c) = 0$
$\theta(\lambda, c) = 0$

SCF is welfare improving

- At any given $i \leq \bar{i}$, SCF leads to an increase in welfare:

$$\begin{aligned}\Delta\mathcal{W}(i) &= \int_{\Omega} q(\lambda, c, \mu(i))v(\lambda, c)dG \\ &\geq \int_{\Omega} q(\lambda, c, \mu(i))\pi(\lambda, c)dG > 0.\end{aligned}$$

- $\Delta\mathcal{W}(i)$ may increase in i:
  - $k > 0$; not all suppliers are in SCF under $i = 0$
  - higher i induces more cross-subsidization and more trade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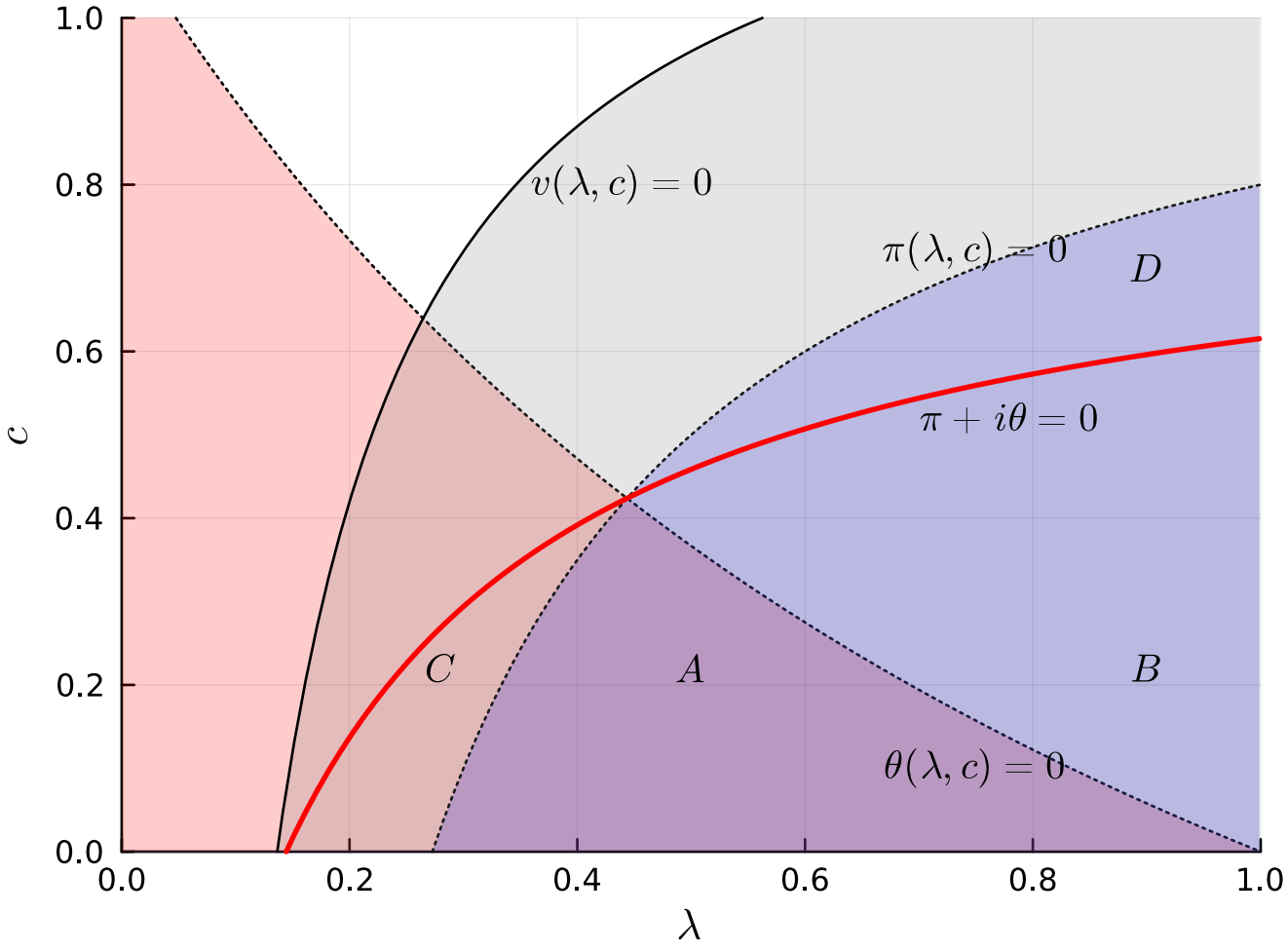
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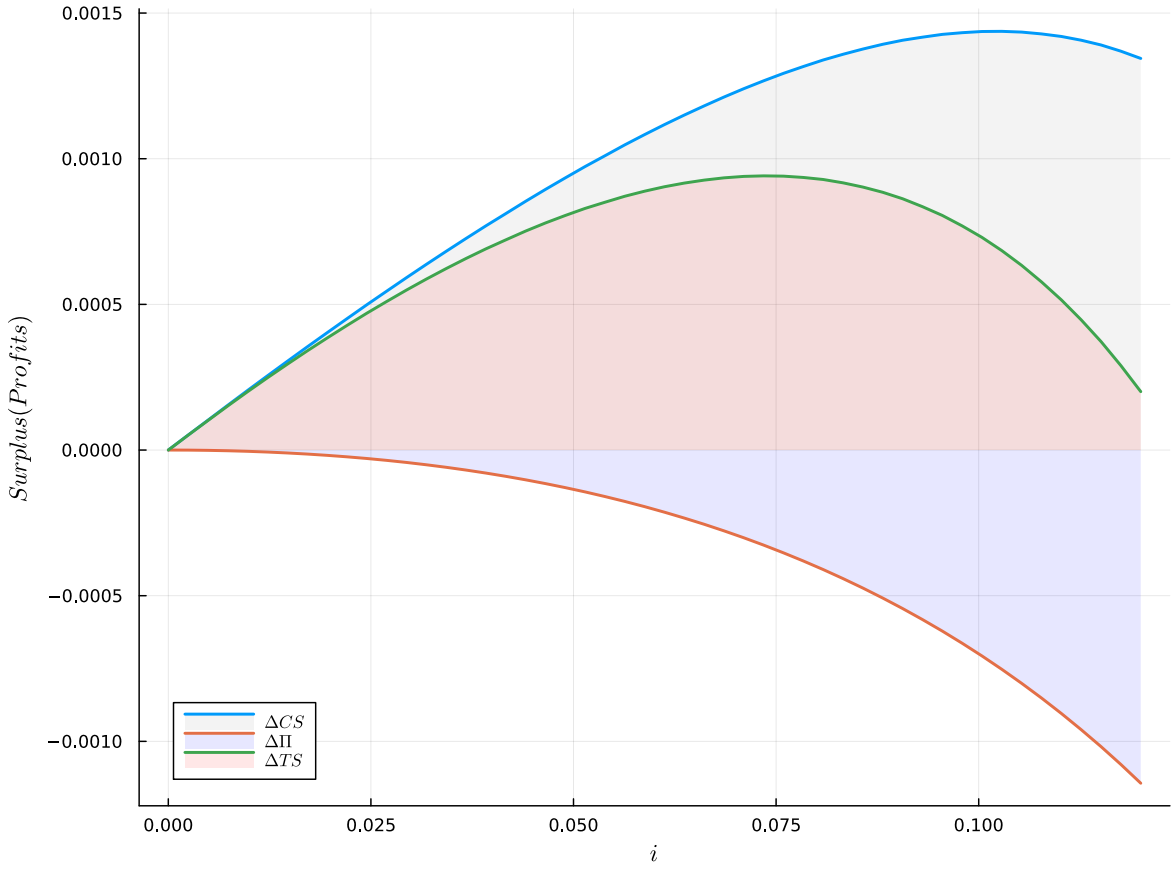


Figure: Welfare is non-monotonic in i under uniform distribution of (λ, c)

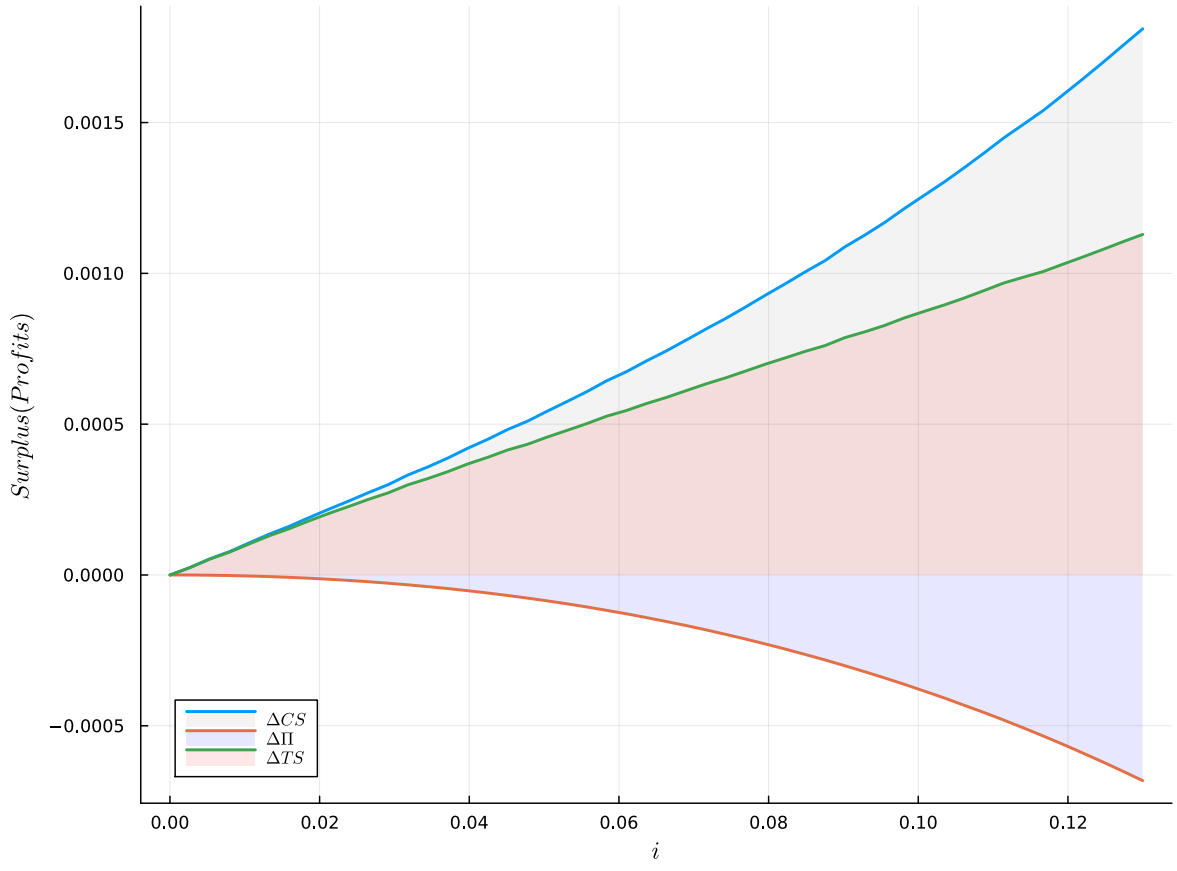


Figure: Welfare increases in i under Beta distributions of λ and c

4. Extension: If suppliers access to market liquidity?

Suppliers' money holding

- A supplier needs to hold $\hat{m} = \frac{c}{\phi_{+1}}$ in the previous night market:

$$\text{cost}: \ \phi\hat{m} \ \text{ v.s. } \ \text{benefit}: \ \beta^s\left[\phi_{+1}\hat{m} + \lambda(p-c)\right].$$

- Suppliers purchase money in previous night market if

$$c < c^s(\lambda, i) \equiv \frac{\lambda}{i^s + \lambda} p.$$

- The *updated* selection rule:

$$q(\lambda, c, \mu) = 1 \ \text{ if } \ \pi(\lambda, c) + \mu\theta(\lambda, c) \geq 0 \ \text{ and } \ c \geq c^s(\lambda, i).$$

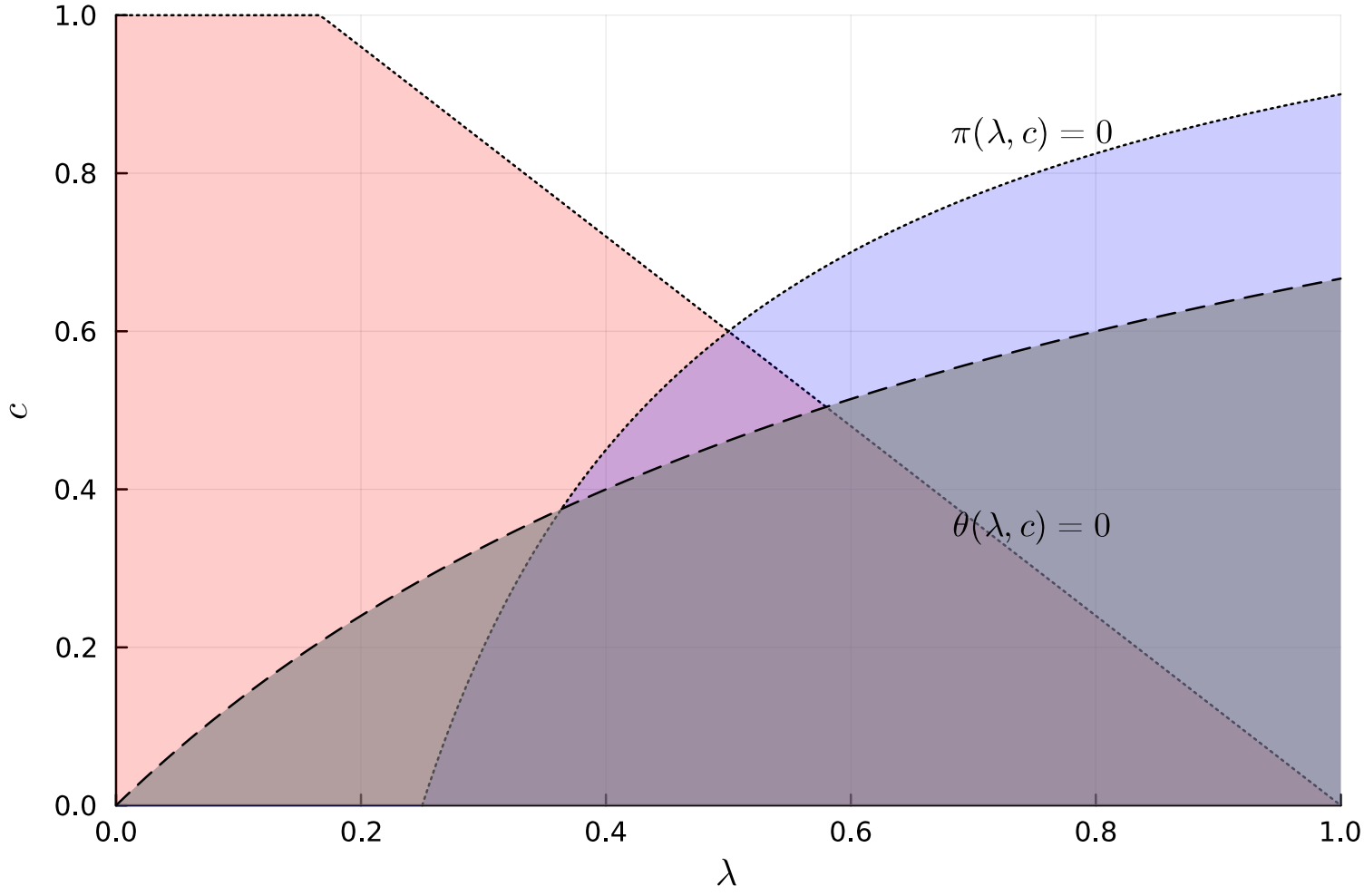


Figure: Suppliers access to liquidity (high i)

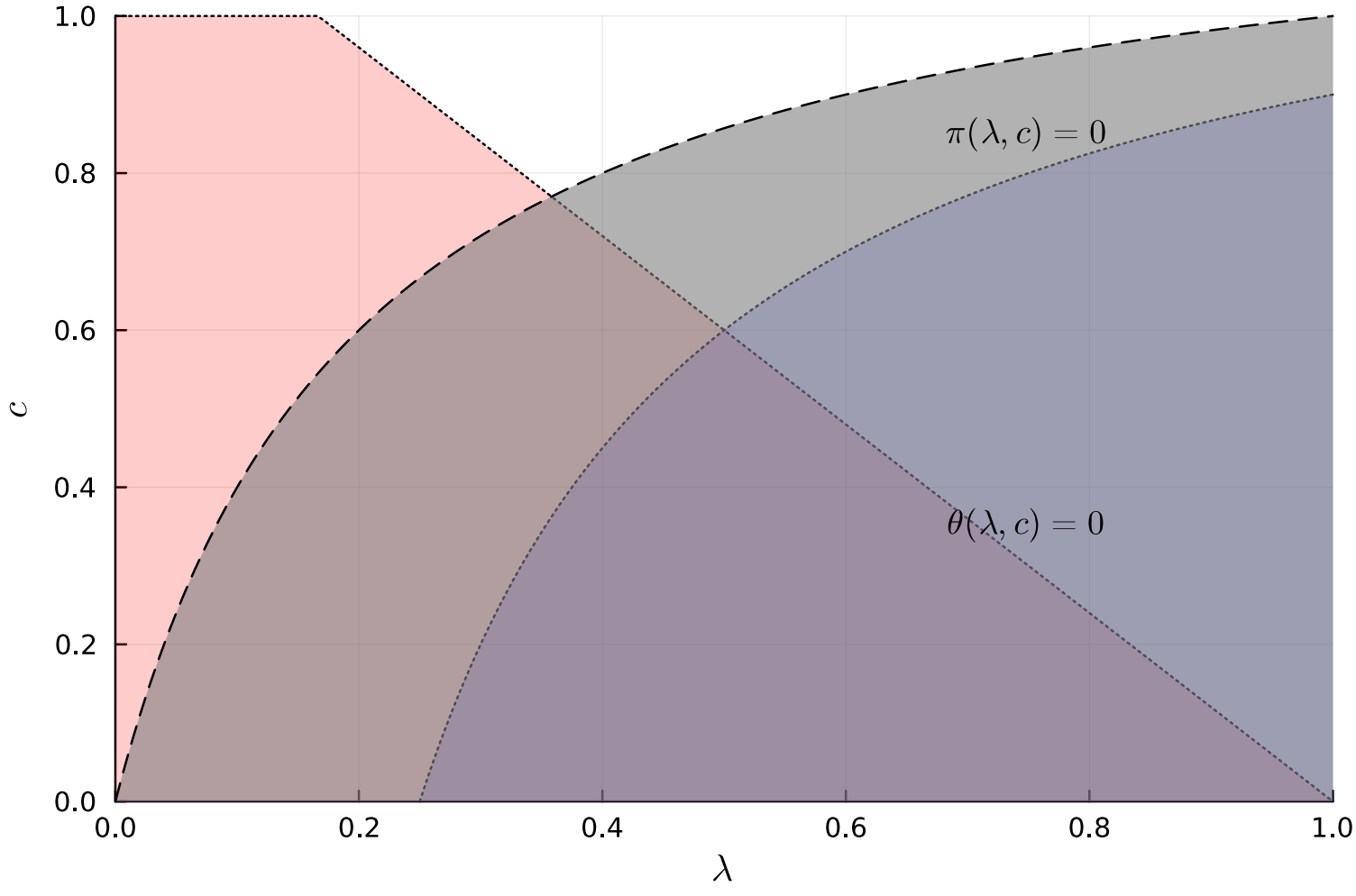
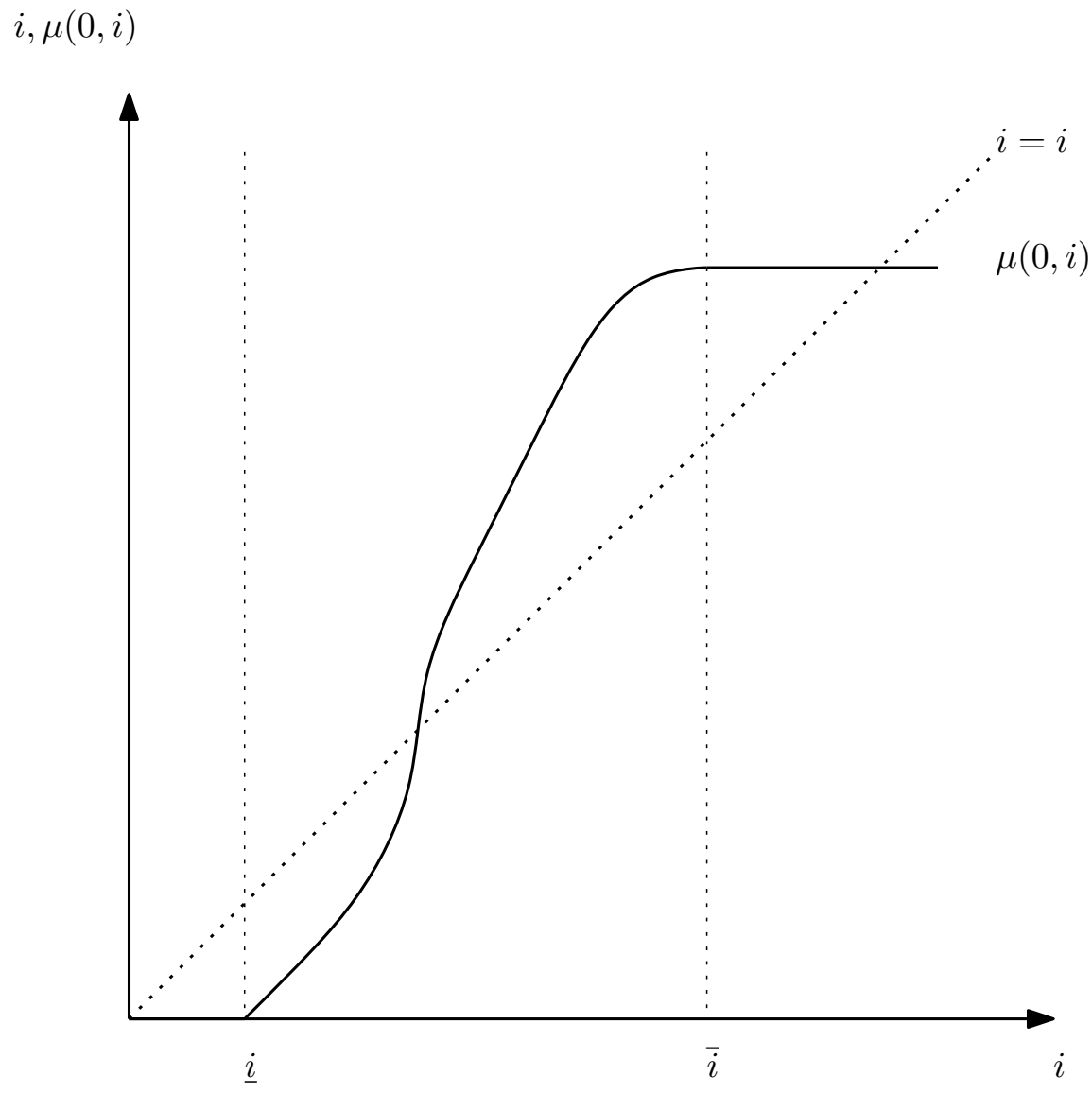


Figure: Suppliers access to liquidity (low i)


$i, \mu(0,i)$
$i = i$
$\mu(0,i)$
$\underline{i}$
$\bar{i}$
i

Takeaways

- SCF: a middleman (buyer-firm and associated financiers as a consolidated agent) pools liquidity from (early) suppliers, and funds suppliers for liquidity needs
- SCF features LIQUIDITY CROSS-SUBSIDIZATION
- SCF helps mitigate the high cost of market liquidity
- Deviating from Friedman rule can be welfare-enhancing