# A Model of Supplier Finance*

Bo Hu† Makoto Watanabe‡ Jun Zhang§

May 16, 2025

**Abstract**

We develop a theoretical model of supplier finance where an intermediary (e.g., a large buyer) pools trade credit and allocates liquidity across heterogeneous suppliers. Optimal supplier finance creates profit-driven liquidity cross-subsidization and explains selective supplier inclusion as an equilibrium outcome. Surprisingly, higher funding costs can *increase* welfare by expanding supplier participation. The model also uncovers a novel non-monotonic relationship between operational efficiency and supplier finance adoption. Depending on financing costs, intermediation and liquidity provision act as either complements or substitutes, offering new insights into why some buyer firms adopt supplier finance while others do not.

**Keywords**: *Supplier Finance*, *Liquidity Pooling*, *Trade Credit*, *Liquidity Cross-subsidization*, *Intermediary*

*An earlier version of this paper circulated under the title "A Model of Supply Chain Finance." We are benefited from comments, discussions and suggestions by Briana Chang, Jay Choi, Piero Gottardi, Yoske Igarashi, Bruno Jullien, Todd Keister, Peter Kondor, Noritaka Kudoh, Nan Li, Cyril Monnet, Volker Nocke, Yoshiaki Ogura, Jean Tirole, Hirofumi Uchida, Yu Zhu, and participants in Kyoto Workshop 2023 on Digitalization and Macroprudential Policies, Workshop on Platform Economics at Yonsei University in Korea, the 2023 Summer Workshop Money, Banking, Payment, and Finance in Switzerland, the 2023 Autumn meeting of the Japanese Applied Economics Association, the 2024 Spring meeting of the Japanese Economic Association, the Essex-UCL Workshop on Decentralized Financial Markets, the 2024 Asian Meeting of the Econometric Society in Ho Chi Minh City. The remaining errors are ours. Hu acknowledges the financial support from the Natural Science Foundation of China (Grant 72003041), the Shanghai Pujiang Program (Grant 21PJC011), and the Shanghai Institute of International Finance and Economics. Watanabe acknowledges the financial support from JSPS KAKENHI (Grants JP23H00054, JP22K20161, and JP23K17286) and the Murata Science Foundation. Zhang acknowledges the financial support from the Ministry of Education of China (Grants #IRT_17R24 and #2023JZDZ018).

†Institute of World Economy, Fudan University; hu_bo@fudan.edu.cn.

‡Institute of Economic Research, Kyoto University; watanabe.makoto.2d@kier.kyoto-u.ac.jp.

§China Center for Economic Studies, Fudan University; junzh_2000@fudan.edu.cn.

# 1 Introduction

The advent of digital finance, e.g., digital invoice platforms, multilateral trade credit, and electronic payments, has significantly altered the landscape of business-to-business supply chain financing. These innovations have modernized long-standing trade finance practices, giving rise to a new form known as "supplier finance" (also called "supply chain finance" or "reverse factoring"). At its core, supplier finance enables a large buyer firm to offer suppliers the option of *early payment* on approved invoices while the buyer benefits from *extended payment terms*.[1] This model has gained significant traction among major retailers such as Walmart, Alibaba, and JD.com, as well as manufacturers including GE, Nestlé, Siemens, and Samsung. A 2019 PwC survey reports that 68% of companies in Europe and North America use supplier finance. Industry forecasts from McKinsey & Company and PwC project continued double-digit annual growth.[2]

The widespread adoption of supplier finance by major buyer firms underscores the growing importance of digital finance in securing supplier liquidity and enhancing business performance. Yet, despite these apparent advantages, other leading firms have pursued strikingly different strategies. Amazon's recent termination of its in-house lending program, alongside the persistent avoidance of supplier finance by major retailers such as Aldi, IKEA, and Costco, presents a notable divergence.[3] This gives rise to the first puzzle: why is a seemingly beneficial digital financing tool embraced by some major buyer firms while others deliberately avoid it?

A deeper examination of supplier finance mechanisms reveals a second, perhaps more paradoxical, puzzle. While designed to enhance supplier liquidity through accelerated payments, these programs typically require participating suppliers to extend trade credit to the buyer, effectively making them creditors (Tunca and Zhu, 2017). This structure—often promoted by digital finance providers as a benefit to buyers—raises a fundamental question: why would a financial tool intended to improve supplier cash flow require those same suppliers to finance large, capital-rich buyers? This question reflects a central debate in the trade credit literature (see, e.g., Klapper, Laeven and Rajan, 2012).

Finally, a third puzzle arises from the typically selective nature of supplier finance participation. Access is usually by invitation, with buyers curating a limited subset of suppliers from their broader supply base. This selectivity—which excludes many suppliers from financing benefits—raises the third puzzle: why and how do buyer firms act as stringent gatekeepers, offering

[1] Digital finance technologies are crucial in facilitating supplier finance for a broader spectrum of companies. For instance, digital invoices allow for real-time monitoring of trade credit; Fintech platforms such as PrimeRevenue simplify the process of early payments and enhance the efficiency of liquidity management; data analytics plays a key role in improving supplier selection by providing precise assessments of credit risk and liquidity requirements; and the integration of cloud-based Enterprise Resource Planning (ERP) systems ensures seamless synchronization of supplier finance activities with existing business operations.

[2] The global corporate supplier finance market was valued at \$1.8 trillion in 2021 (BCR Publishing Ltd.), comparable to the global private equity market (\$4.1 trillion, Preqin) and the global securitization market (\$2.3 trillion, SIFMA).

[3] Amazon Lending, launched in 2011, provided short-term financing to third-party sellers on its marketplace. Loans were invitation-only, based on sales history, inventory, and customer service metrics, with repayments automatically deducted from sellers' accounts. Amazon shut down the program in March 2024.

supplier finance to only a restricted group rather than broadly across their supply base?[4]

To answer these puzzles, we consider a simple model of supplier finance. There is a retail market that operates across two sequential sub-periods: early and late. All production must occur in the early sub-period, and it requires suppliers to pay costs $c$ upfront using a numeraire (e.g., cash payments for raw materials). However, suppliers begin with no numeraire endowment. This creates a time gap between when costs must be paid and when revenue is received. If a supplier matches with consumers in the early sub-period, he receives revenues in time to cover production costs, enabling trade as in standard frictionless markets. However, if he only matches with consumers in the late sub-period, he lacks the funds to finance production costs, and no trade occurs. We denote the probability of this late-period matching — effectively a liquidity shock — by $\lambda$.

This setup reflects real-world scenarios where suppliers' liquidity depends critically on their retail technologies. The timing of consumer payments is influenced by suppliers' technological capabilities, and limited technologies often prevent suppliers from securing early payments. For instance, a supplier lacking advanced display technologies may only be able to showcase products late in the season, missing early-paying customers who purchase based on visibility. Similarly, suppliers with limited inventory capabilities may only manage late-period deliveries, failing to serve customers who pay upfront for prompt delivery. Likewise, suppliers without efficient production-to-order systems must rely on inventory-based production, unable to leverage advance payments from customers. In each case, better retail technologies—whether in product display, delivery systems, or production flexibility—would increase the probability of early-period transactions, enabling trade that would otherwise be impossible.

Suppliers vary in their production costs $c$ and the probability of liquidity shock $\lambda$. We assume that the pair $(\lambda, c)$ of each supplier is publicly observable.

In this economy, we introduce an intermediary who cannot produce goods, but possesses superior retail and enforcement technologies. The intermediary's retail advantage manifests in her enhanced ability to match with consumers, represented by a lower probability of experiencing liquidity shocks, $m\lambda$ with $m < 1$. This operational efficiency arises from advanced retail technologies that include superior product display advertising, improved inventory management to deliver on time, and improved communication for efficient production-to-order.

The intermediary also possesses enforcement technologies for credit arrangements with suppliers, powered by, e.g., digital finance solutions. This finance service incurs costs, and it enables two crucial functions. First, it allows for the pooling of trade credit, where the intermediary collects funds from early retail revenues. Second, it facilitates early payments, allowing the in-

[4] For example, Co-op, a major UK supermarket chain, launched a supplier finance program in 2020 in partnership with the fintech platform PrimeRevenue. Co-op selected eligible suppliers, extended payment terms, and allowed suppliers to receive early payments from PrimeRevenue at competitive rates. The program reportedly improved both supplier access to financing and Co-op's cash flow.

termediary to allocate funds to suppliers who need liquidity to produce.

Given its retail technological advantages, the intermediary optimally offers retail intermediation to all available suppliers. Her key decision then becomes whether to extend the costly finance service to specific suppliers. This decision requires careful evaluation: the intermediary must ensure that her total liquidity—comprising her own balance and trade credit from early-trading suppliers—sufficiently covers early payment obligations to participating suppliers. Thus, the intermediary evaluates potential finance service recipients based on both their profitability and their contribution to the overall liquidity pool.

Our model delivers three central results.

First, supplier finance enables profit-driven liquidity cross-subsidization among suppliers. Suppliers facing a higher likelihood of liquidity shocks (higher $\lambda$) and lower production costs (lower $c$) tend to generate higher profits, while those with lower liquidity risk and similarly low costs contribute to the liquidity pool. The intermediary optimally reallocates trade credit from less profitable suppliers to support more profitable but liquidity-constrained ones. This selective strategy—redistributing liquidity to maximize profit—outperforms both universal inclusion and selection based solely on profitability. This mechanism resolves the second and third puzzles: selective supplier inclusion and the requirement to extend trade credit are the instruments through which buyers orchestrate liquidity cross-subsidization.

Second, the model identifies a novel, non-monotonic relationship between operational efficiency and supplier finance adoption. Supplier finance is not a simple synergy of trade and finance. As matching efficiency improves (lower $m$), the pool of funds available for redistribution grows, reducing financing costs. At the same time, improved efficiency lowers suppliers' liquidity risk, reducing demand for financing. The net effect depends on the external funding cost. At low interest rates, reduced demand dominates, making retail efficiency and liquidity provision *substitutes*. At high rates, efficient liquidity pooling prevails, turning them into *complements*. Crucially, beyond a certain threshold of matching efficiency, supplier finance becomes unprofitable as rapid inventory turnover eliminates the need for liquidity support. This explains the first puzzle: why some buyer firms forgo supplier finance.

Third, rising funding costs can, counterintuitively, enhance welfare. When funding costs are low, the intermediary selects suppliers based on individual profitability. As costs rise, the intermediary broadens supplier participation to reinforce the liquidity pool. This expansion increases trading volume and welfare. Welfare gains are especially pronounced when the intermediary operates efficiently, as lower liquidity shock risk and stronger supplier contributions enable wider inclusion and offset higher external funding costs.

From a policy standpoint, concerns among regulators and practitioners center on the risk that rising funding costs—whether driven by financial crises, credit contractions, or changes in monetary policy—could destabilize supplier finance programs. Such disruptions may trigger widespread supplier bankruptcies and substantial declines in overall economic output. However, our model identifies a mechanism by which intermediaries mitigate the impact of rising external funding costs: they increase reliance on trade credit provided by liquidity-abundant suppliers. This strategic adjustment reduces dependence on external financing, with the internal liquidity pool serving as a direct buffer against cost pressures.[5]

The internal liquidity pool also elucidates a further critical concern in supplier finance: window-dressing. This moral hazard arises when buyer firms record payment obligations as accounts payable rather than debt, thereby potentially understating leverage. As window-dressing tends to be more prevalent when supplier finance is heavily reliant on external liquidity, our findings indicate that it is not an intrinsic feature of all supplier finance programs. Instead, the degree of external liquidity utilization is contingent upon the characteristics of the supplier pool. Consequently, promoting transparency in supplier finance arrangements is essential to allow investors to evaluate resilience to potential window-dressing risks. This emphasis on transparency is consistent with recent accounting standards requiring disclosure of such programs.[6]

In Section 4, we demonstrate the robustness of our model through several extensions. Our findings remain valid when suppliers have equal or lower liquidity costs than the intermediary, when the retail market exhibits standard downward-sloping demand, and when suppliers face alternative outside options. We complement our theoretical analysis with real-world cases supporting our model's implications and discuss its policy relevance.

In Section 5, we extend our analysis to the manufacturing setting, where a manufacturer sources intermediate goods from suppliers to produce final goods for consumers. Here, we attempt to cover the fact that supplier finance is adopted not only by retailers but also by manufacturers. The manufacturing setting differs from retail intermediation: the timing of consumer purchases (early or late) affects only the final goods and is independent of intermediate goods sourcing. Our main insight—that supplier finance is an equilibrium feature and that the aforementioned liquidity cross-subsidization emerges—remains valid in this setting and holds across a range of production functions, including linear, CES, and more general specifications.

The remainder of this section is dedicated to a literature review. All proofs are included in the Appendix.

[5]Nonetheless, higher external funding costs may necessitate a more selective supplier pool. This increased selectivity, shaped by the liquidity cross-subsidization mechanism discussed above, can lead to the exclusion of even the most efficient suppliers, thereby diminishing the overall profitability of the program.

[6]The FASB's ASU No. 2022-04, effective for fiscal years beginning after December 15, 2022, requires companies to disclose details of their supplier finance programs.

## Related Literature

Our contributions can be summarized as follows. First, our study advances the growing body of literature on digital finance within supply chains (see, for instance, works in operations management such as Tunca and Zhu, 2017; Devalkar and Krishnan, 2019; Kouvelis and Xu, 2021; Dong, Qiu and Xu, 2022; and Yan, Chen and Ding, 2024). In contrast to these existing research works which predominantly focus on bilateral buyer-supplier relationships and their financial arrangements, our work introduces a novel perspective by modeling the buyer as a multi-product intermediary. We argue that this intermediation is a critical catalyst for supplier finance, as digital finance can significantly alleviate financing constraints in this context due to the scale and extensive network of suppliers accessible through such intermediaries.[7] This intermediation framework allows us to analyze supplier finance as a multilateral arrangement, revealing insights into liquidity cross-subsidization among diverse suppliers and the selective nature of financial inclusion.[8]

Second, the liquidity cross-subsidization mechanism developed in this paper sheds light on a long-standing puzzle in the trade credit literature: why small suppliers extend credit to large buyers who do not seem to face a shortage of working capital (e.g., Klapper, Laeven and Rajan, 2012), a paradox that is further accentuated in the age of digital finance. We argue that, rather than mere credit extraction, the intermediary's liquidity redistribution enables otherwise impossible production and trade, yielding efficiency gains.[9] Additionally, this mechanism provides a novel rationale for the coexistence of deposit-taking and lending. While supplier finance is not banking in the traditional sense, our model complements the analysis of Kashyap, Rajan and Stein (2002), who argue that there are synergies between deposit-taking and lending. In our model, "deposit-taking" corresponds to the buyer firm's collection of funds from suppliers receiving early retail revenues, while "lending" represents the allocation of these funds to suppliers requiring liquidity for production.

Third, a key feature of supplier finance is the intermediary's selection of suppliers based on their profitability and liquidity needs. While this selection resembles liquidity pooling in the classical banking and liquidity creation literature, supplier finance differs in an important aspect:

[7] He and Li (2025) highlight the applicability of credit chains in shadow banking for reducing liquidation costs, a relevant insight for supplier finance networks.

[8] Our approach also contributes to the market-making intermediary and platform literature by identifying a novel role in the digital era: liquidity provision. Our model of a multi-product intermediary complements Rhodes, Watanabe and Zhou (2021) by adding this liquidity dimension. Previous work has explored how intermediaries enhance market efficiency through channels such as market presence (Rubinstein and Wolinsky, 1987; Nosal, Wong and Wright, 2015), inventory management (Watanabe, 2010, 2018, 2020; Li, Murry, Tian and Zhou, 2024), product variety (Camera, 2001; Johri and Leach, 2002; Shevchenko, 2004; Smith, 2004; Dong, 2010), and quality assurance (Biglaiser, 1993; Li, 1998). Our paper also relates to the growing literature on hybrid or dual-mode platform economies (e.g., Tirole and Bisceglia (2023), Madsen and Vellodi (2023), Gautier, Hu and Watanabe (2023), Etro (2023), Etro (2024), Shopova (2023), Hagiu, Teh and Wright (2022), Anderson and Bedre-Defolie (2022), Kang and Muir (2022), Padilla, Perkins and Piccolo (2022), Zennyo (2022), Etro (2021a), and Etro (2021b)). These studies focus on platforms that act as intermediaries between consumers and third-party sellers while also offering their own first-party products.

[9] For the trade credit literature, see, e.g., Petersen and Rajan (1997), Burkart and Ellingsen (2004), Cuñat (2007), Giannetti, Burkart and Ellingsen (2011), Garcia-Appendini and Montoriol-Garriga (2013), Nocke and Thanassoulis (2014), Bottazzi, Gopalakrishna and Tebaldi (2023).

financial inclusion is peripheral in the banking models following Diamond and Dybvig (1983). The selection mechanism enables targeted liquidity provision, reflecting a trade-off between each supplier's contributions in profits and liquidity. Notably, we show that this critical trade-off between profitability and liquidity needs can lead to the optimal exclusion of certain suppliers. Moreover, unlike traditional banking with demand deposits, supplier finance avoids bank runs through its limited liquidity commitments.

Finally, our model contributes to the literature linking the advantages of entities in the goods market to liquidity provision (e.g., Donaldson, Piacentino and Thakor 2018; Huang 2021; Boualam and Yoo 2022; Li and Pegoraro 2022). Donaldson, Piacentino and Thakor (2018) show that superior storage technology in warehouses enables a monotonic increase in credit extension through unbacked receipts with greater technological advantage. Supplier finance also shares parallels with platform-provided credit, where merchants pledge a portion of their marketplace sales for loan repayment (Li and Pegoraro 2022). Crucially, applying the logic of these theories to supplier finance would suggest that firms with greater success in the goods market should be more inclined to adopt it. However, our first puzzle highlights the contrasting reality: the divergent adoption decisions of seemingly equally successful buyer firms. Our analysis reveals a non-monotonic interplay between the intermediary's operational efficiency (retail technologies in our context) and liquidity provision. The adoption of supplier finance is influenced by broader financial conditions, specifically the external funding cost.

## 2 The model

Consider a one-period economy with three types of agents: a mass of consumers, a mass of suppliers (he), and one intermediary (she). Each supplier produces a unique and indivisible good at a constant marginal cost $c$. Suppliers differ in $c \in [\underline{c}, \bar{c}]$, where $\bar{c} > \underline{c} > 0$, and $c$ is publicly observable. Consumers are homogeneous and have unit demand for each good, with a common utility $u \geq \bar{c}$. The intermediary neither produces nor consumes. Instead, she can act as a middleman, buying goods from suppliers and reselling them to consumers. She also has access to a costly financing technology that allows her to delay payments to suppliers and use the funds to support suppliers needing liquidity. The details of these technologies will be specified below.

A numeraire good, which we refer to as money or liquidity, facilitates retail payments in this economy. We assume that consumers have sufficient money endowments to make retail purchases. In contrast, suppliers begin with no money endowment (we will relax this assumption in Section 4). The intermediary can choose to hold money, denoted by $L \geq 0$, which she obtains from banks or money markets at a nominal interest rate $i \geq 0$. These external funding sources are not explicitly modeled.

Agents trade in a retail market where each good is sold either by its supplier or by the intermediary. If the intermediary sells a good, she purchases it from the supplier, who then exits the retail market (if suppliers are indifferent, they join the intermediary rather than selling directly in the market). Regardless of whether the supplier or the intermediary sells the good, the seller can reach all consumers.

When a consumer and a seller (supplier or intermediary) meet, trade occurs bilaterally, and the trade surplus is split equally. The equilibrium retail price is thus given by

$$p - c = \frac{u - c}{2}. \tag{1}$$

While we use the Nash bargaining solution in the benchmark model, our conclusions are robust to alternative pricing mechanisms (see Section 4).

**Liquidity shocks.** Suppliers need the numeraire to cover production costs $c$ to produce a good. In a Walrasian market, this is not an issue because suppliers can do so using their retail revenue. However, suppliers' finance matters when there exists a disparity in the timing between production and trade, and a liquidity shock prevents them from receiving revenue before production.

To be more precise, retail trade occurs in two sequential subperiods: *early* and *late*. All production must take place in the early subperiod. Before production, an idiosyncratic shock is realized, indicating whether a given supplier's good will match with consumers in the early or late subperiod. With probability $1 - \lambda$, consumers match with the good early and thus they pay in the early subperiod. In this case, the supplier can use this immediate revenue to cover production costs, enabling production and trade even without initial money holdings. However, with probability $\lambda$, the good matches late, meaning payment occurs only in the late subperiod. Since production costs must be paid early but revenue arrives late, the supplier faces a liquidity shortage and cannot produce. Thus, the timing mismatch between required early production and potential late revenue creates liquidity risk, with late consumer arrival effectively generating a liquidity shock.

This setup captures real-world scenarios where suppliers' liquidity depends on their retail technologies. No trade occurs because of limited retail technologies possessed by suppliers to convince consumers to pay early rather than late. For instance,

- Display/advertisement: A supplier can display his good to consumers in the early subperiod with probability $1 - \lambda$ and in the late subperiod with probability $\lambda$. If consumers buy only after inspection, then it is only in the former case that the supplier can produce and trade. Better advertising technologies increase the chance of early display.

- Delivery/inventory: A supplier can deliver his good to consumers in the early subperiod with probability $1 - \lambda$, and in the late subperiod with probability $\lambda$. If consumers pay only after delivery, then it is only in the former case that the supplier can produce and trade. Better inventory technologies increase the chance of early delivery.

- Production-to-order: A supplier has access to "production-to-order" technology with probability $1 - \lambda$ and can only "produce to inventory" with probability $\lambda$. Production-to-order allows suppliers to produce goods after receiving an order and payment from consumers. Then it is only when this technology is accessible that the supplier can produce and trade. Better promotion or communication with consumers, facilitated by competent sales persons, increases the chance of production to order.

We assume that the probability of a liquidity shock varies among different goods and is publicly observable. Suppliers' ex-ante heterogeneity can be indexed by a pair $(\lambda, c)$. Denote the two-dimensional space where $(\lambda, c)$ belongs to by $\Omega \equiv [0, 1] \times [\underline{c}, \bar{c}]$ with $0 < \underline{c} < \bar{c} < u$. $(\lambda, c)$ follows a continuous distribution that has a cumulative distribution function $G$, and a density function $g$ that is everywhere positive in $\Omega$.

Finally, due to a lack of enforcement technologies, there is no credit market among suppliers. Consequently, individual suppliers are unable to hedge against liquidity shocks or combine liquidity resources by themselves.

**The retail technology.** The intermediary can sell goods on behalf of suppliers with superior retail technologies, giving her a matching advantage over suppliers. This is in the same spirit as Rubinstein and Wolinsky (1987) where the very reason for the emergence of intermediaries is due to their relative advantage in matching efficiency over suppliers. When the intermediary sells a good of $\lambda$, her advanced retail technologies reduce the probability of late matching from $\lambda$ to $m\lambda$, where $m \in (0, 1)$ measures her matching advantage. Thus, the probability of early matching increases from $1 - \lambda$ to $1 - m\lambda$ ($> 1 - \lambda$). For example, the intermediary has:

- better advertisement technologies that increase the chance of early display;
- better inventory technologies that increase the chance of early delivery;
- better promotion/communication technologies with consumers that increase the chance of production to order.

**The finance technology.** The intermediary can also act as a financier, providing liquidity to suppliers through two functions: (1) extending credit to suppliers who need early payment, and (2) pooling retail revenue from early-paying consumers to support suppliers who need liquidity. The second function works by delaying payments to suppliers who receive early revenue, effectively obtaining trade credit from them (detailed in the contracts below). Using this finance technology incurs a fixed cost $k$ per financed supplier, where $k \in (0, \bar{k})$ with $\bar{k} \equiv \frac{(u-\underline{c})^2}{2(u+\underline{c})}$ (see footnote 11).

**The contracts offered by the intermediary.** Observing $(\lambda, c)$, the intermediary offers a contract to each supplier. Given her matching advantage in the retail market, the intermediary will offer intermediation services to every supplier through one of two contracts.

A pure *middleman contract* stipulates that: (1) The intermediary sells the good on behalf of the supplier; (2) The intermediary pays the supplier a reward $f_M(\lambda, c) \geq 0$ *immediately after receiving payment from consumers*. If no payment is received, the supplier receives nothing. Under this contract, suppliers can produce only if the intermediary matches with consumers early (probability $1 - m\lambda$), as they need the revenue to cover production costs.

A *middleman-finance contract* (hereafter, the *finance contract*) stipulates that: (1) The intermediary sells the good on behalf of the supplier; (2) The intermediary pays the supplier a reward $f_F(\lambda, c) \geq 0$ *at the end of the period*; (3) The intermediary advances the production cost $c$ to the supplier in the early subperiod.

The finance contract differs from the middleman contract in two aspects. First, in the finance contract, payments to suppliers are postponed to the end of the period. With finance technology, this deferral allows the intermediary to leverage delayed payments as a liquidity source to fund suppliers that need liquidity. Second, the finance contract extends liquidity support of $c$ at the time of production, which ensures that the supplier can always produce and trade even if the supplier has no money in hand. Given that utility is transferable and there is no asymmetric information, these two contract types are sufficient to capture the optimal contract the intermediary can offer.

Suppliers' outside values matter for the contract offer. A supplier who does not accept an intermediary's offer sells directly to consumers, in which case the supplier can produce and trade only if he is not hit by the shock (i.e., if he is matched with consumers early). Let $q(\lambda, c) \in \{0, 1\}$ be the selection function, where $q(\lambda, c) = 1$ if a $(\lambda, c)$-supplier is offered a finance contract, and $q(\lambda, c) = 0$ if he is offered a middleman contract. With this, we can summarize the set of the intermediary's offers by a triple:

$$\{q(\lambda, c), f_F(\lambda, c), f_M(\lambda, c)\}_{(\lambda,c)\in\Omega}.$$

**Timing.** The intermediary first determines the numeraire holding $L$. She then announces offers to each supplier based on $(\lambda, c)$, indicating selection and contract type (middleman or finance). Selected suppliers decide whether to accept. Second, liquidity shocks are realized and trade occurs. Finance contract suppliers may request early payment of $c$. Middleman contract suppliers receive $f_M(\cdot)$ after consumer payment. Finally, the intermediary settles all outstanding supplier payments $f_F(\cdot)$ at the end of the period.

# 3 The Equilibrium

We now solve the equilibrium. Suppose that the intermediary has a real balance of $L$, we follow backward induction and work on the supplier selection problem first.

**Suppliers' participation constraints.** The intermediary makes offers subject to the participation constraints of the suppliers. If a supplier of $(\lambda, c)$ chooses not to participate in the intermediary, then he can produce and trade only if he matches with consumers early. Thus, the expected profits are given by

$$(1-\lambda)(p-c) = (1-\lambda)(u-c)/2,$$

where we have inserted $p$ from (1). Since the intermediary can observe $(\lambda, c)$, she can make the rewards $f_F$ and $f_M$ dependent on $(\lambda, c)$. To entice the supplier to participate, it is sufficient for the intermediary to offer him the value of his outside option so that

$$f_F(\lambda, c) = \frac{(1-\lambda)(u-c)}{2}, \tag{2}$$

and

$$f_M(\lambda, c) = \frac{(1-\lambda)(u-c)/2}{1-m\lambda} + c. \tag{3}$$

$f_M$ differs from $f_F$ because, in a middleman contract, production costs $c$ are covered by the supplier, not the intermediary. The reward $f_M$ is given to the supplier only when the intermediary successfully trades, which happens with probability $1 - m\lambda$ rather than $1 - \lambda$. With these fees, all active suppliers are induced to accept the offered contract.

**Suppliers' profit and liquidity contributions.** Next, we derive the profit and liquidity that the intermediary obtains by having a supplier in a middleman or finance contract. With a middleman contract, the intermediary's expected profit from a supplier $(\lambda, c)$ is

$$\pi_M(\lambda, c) = (1-m\lambda)\left(p - f_M(\lambda, c)\right) = (1-m)\lambda(u-c)/2, \tag{4}$$

which is positive since $m < 1$. The second equality follows from (1) and (3). The source of the profit is the intermediary's matching advantage: the supplier does not receive a liquidity shock (and can trade successfully by himself) with probability $1-\lambda$, whereas the intermediary can do so with probability $1 - m\lambda$; the difference is given by $1 - \lambda - (1 - m\lambda) = (1-m)\lambda$. Note that without incurring the cost of finance technology $k$, the intermediary cannot enforce any credit deals that allow her to fund suppliers. Instead, she only provides intermediation services.

If financed by the intermediary, participating suppliers can produce even if matches occur in the late subperiod. Thus, with a finance contract, the intermediary's expected profit from a supplier $(\lambda, c)$ is

$$\pi_F(\lambda, c) = p - c - f_F(\lambda, c) - k = \lambda(u - c)/2 - k, \tag{5}$$

where the intermediary receives payment $p$ from consumers, covers the supplier's production costs $c$ in the form of advanced payment, and rewards the supplier by $f_F$ at the end. The second equality follows from (1) and (2). The expected profit is higher with a higher $\lambda$ (as the supplier is less likely to trade if he chooses to operate independently) and a lower $c$ (as the good has a higher profit margin).

When providing the finance service, liquidity is actually an issue because the intermediary needs to cover the production cost $c$ of all participating suppliers at the time of production. The source of this funding is the revenue $p$ from early matches which occur with probability $1 - m\lambda$ for a supplier (or good) of type $\lambda$. Hence, the net expected amount of money that a supplier $(\lambda, c)$ contributes to the intermediary at the time of production is

$$\theta_F(\lambda, c) = (1 - m\lambda)p - c = (1 - m\lambda)(u + c)/2 - c. \tag{6}$$

**The intermediary's supplier selection problem.** Let $\Theta$ be the total liquidity contributed by all the suppliers that are financed by the intermediary:

$$\Theta = \int_\Omega \Big[q(\lambda, c)\theta_F(\lambda, c)\Big] dG.$$

Then the liquidity constraint that the intermediary faces can be written as:

$$\Theta + L \geq 0. \tag{7}$$

The liquidity constraint states that the total liquidity contribution of financed suppliers, plus the available liquidity $L \geq 0$ that is held by the intermediary herself should be non-negative.[10]

Using (4) and (5), and defining $\Delta\pi$ as the incremental change in profits when a supplier $(\lambda, c)$ is financed compared to not being financed:

$$\Delta\pi(\lambda, c) \equiv \pi_F(\lambda, c) - \pi_M(\lambda, c) = m\lambda(u - c)/2 - k,$$

the intermediary's problem of selecting suppliers into either middleman or finance contracts can be formulated as

$$V^m(L) \equiv \max_{\{q(\lambda,c)\}_{(\lambda,c)\in\Omega}} \int_\Omega [q(\lambda, c)\Delta\pi(\lambda, c)]\, dG, \text{ s.t. (7)}. \tag{8}$$

The problem can be understood as the intermediary obtaining $\pi_M(\lambda, c)$ for all the active suppli-

[10]We have a continuum of suppliers, each facing independent liquidity shocks. This allows us to leverage the Law of Large Numbers. As such, we interpret the requirement for the liquidity constraint to hold "almost surely" in a probabilistic sense. For a continuum, a literal "worst case" of every supplier defaulting simultaneously is measure-theoretically irrelevant.

ers and additionally deciding whether to finance suppliers to earn $\Delta\pi(\lambda, c)$ subject to liquidity constraint (7).

The intermediary's problem defined above is an optimization of functionals, and the optimal solution can be derived using the following Lagrange method (see e.g., Rhodes, Watanabe and Zhou 2021). Let $\mu \geq 0$ be the multiplier associated with the liquidity constraint (7). We can construct the Lagrangian:

$$\mathcal{L} = \int_{\Omega} \Big[ q(\lambda, c) \Big( \Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \Big) \Big] dG(\lambda, c).$$

Note that $\Delta\pi(\lambda, c)$ and $\theta_F(\lambda, c)$ can be positive or negative depending on the parameters. In particular, given the cost $k$ of using finance technology, it is not profitable to fund all suppliers, that is, there exist suppliers with negative $\Delta\pi(\cdot)$.

Using this Lagrangian, the solution to the intermediary's problem can be obtained as an optimal selection policy that depends not only on $(\lambda, c)$ but also on $\mu$. With a slight abuse of notation, we shall refer to this optimal policy to finance a supplier as $q(\lambda, c, \mu)$, which is given by:

$$q(\lambda, c, \mu) = \begin{cases} 1 & \text{if } \Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0; \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Condition (9) indicates that $q(\lambda, c, \mu) = 1$ consists of three possible scenarios:

$$\Delta\pi(\lambda, c) \geq 0,\ \theta_F(\lambda, c) \geq 0, \tag{10a}$$

$$\Delta\pi(\lambda, c) > 0,\ \theta_F(\lambda, c) < 0,\ -\Delta\pi/\theta_F \geq \mu, \tag{10b}$$

$$\Delta\pi(\lambda, c) < 0,\ \theta_F(\lambda, c) > 0,\ -\Delta\pi/\theta_F \leq \mu. \tag{10c}$$

In scenario (10a), the intermediary selects suppliers with positive increments in profits $\Delta\pi$ and positive liquidity contribution $\theta_F$ to finance. In scenario (10b), the intermediary selects suppliers with positive increments in profits $\Delta\pi$ and negative liquidity contribution $\theta_F$ to finance, provided the gross return of liquidity, measured by $-\Delta\pi/\theta_F$, is higher than the shadow value of liquidity $\mu$. In the last scenario (10c), the intermediary selects suppliers with negative $\Delta\pi$ and positive $\theta_F$ to finance, as these suppliers contribute to the aggregate liquidity of the intermediary. The cost of getting one unit of liquidity from these suppliers is $-\Delta\pi/\theta_F$, and the intermediary should absorb the liquidity from these suppliers if $-\Delta\pi/\theta_F \leq \mu$.

To illustrate the three scenarios in a figure, we insert $\Delta\pi(\cdot)$ and $\theta_F(\cdot)$ and obtain three bound-

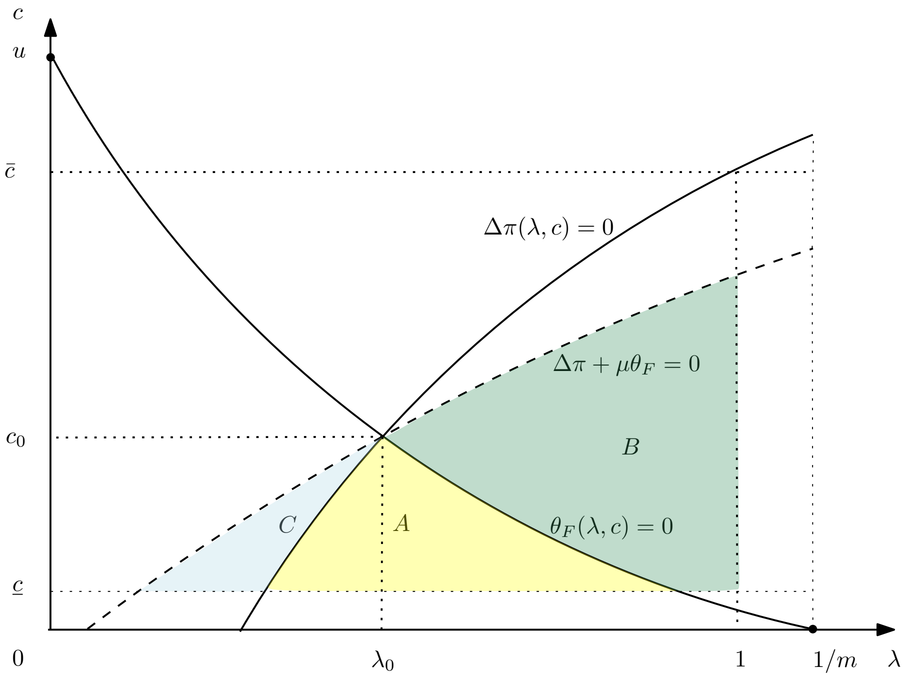



Figure 1: The intermediary's selection of suppliers

aries that lie in $\Omega$:

$$\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\theta_F}(\lambda) \equiv \frac{1 - m\lambda}{1 + m\lambda} u, \tag{11a}$$

$$\Delta\pi(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\Delta\pi}(\lambda) \equiv u - \frac{2k}{m\lambda}, \tag{11b}$$

$$\Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq b(\lambda, \mu) \equiv \frac{m\lambda u - 2k + \mu(1 - m\lambda)u}{m\lambda + \mu(1 + m\lambda)}. \tag{11c}$$

Note that the right-hand side of (11c) is a "weighted average" of the right-hand sides of the first two. We plot these three boundaries in Figure 1, annotated by $\theta_F(\lambda, c) = 0$, $\Delta\pi(\lambda, c) = 0$, and $\Delta\pi + \mu\theta_F = 0$, respectively. The intersection is denoted by $(\lambda_0, c_0)$. Any suppliers below $\theta_F(\lambda, c) = 0$ contribute to the liquidity pool, and any suppliers below $\Delta\pi(\lambda, c) = 0$ contribute to the intermediary's profits.[11]

The overlapping region $A$ represents suppliers in scenario (10a), which are financed by the intermediary because they contribute to both profits $\Delta\pi$ and liquidity $\theta_F$. Suppliers in region $B$, corresponding to scenario (10b), have net liquidity needs, $\theta_F < 0$, while contributing to profits $\Delta\pi > 0$. Suppliers in region $C$, corresponding to scenario (10c), when included in a finance contract, give the intermediary lower profits $\Delta\pi < 0$, but contribute to the liquidity pool. Suppliers outside $A$, $B$ or $C$ are not financed.

Overall, the intermediary adopts what we call a *profit-based liquidity cross-subsidization* strategy. This involves using the positive net liquidity contributions of the suppliers in regions $A$ and

[11] Note that $k < \bar{k}$ ensures that $c_0 > \underline{c}$. Also, Figure 1 is drawn with $\lambda_0 < 1$ and $\bar{c} > c_0$. The complete analysis, including cases where $\lambda_0 \geq 1$ or $\bar{c} < c_0$, is provided in the proof.

$C$ to address the liquidity needs of the suppliers in region $B$. In particular, when the intermediary uses liquidity contributions from region $C$, it incurs a cost in the form of reduced (or negative) profits to these suppliers. However, when providing liquidity support to suppliers in region $B$, the intermediary expects a return in the form of positive profits from these suppliers.

In the standard liquidity pooling (a la Diamond and Dybvig 1983), agents are homogeneous and so, translated into our context, only those who make a positive profit contribution would be selected. However, with heterogeneous agents, we show that this is suboptimal; whether a supplier will be included in the intermediary's liquidity risk sharing program depends not only on its liquidity contribution, but also on its profit contribution.

In a nutshell, the selective approach to participating suppliers—the second puzzle raised in the Introduction—matters because it is driven by the buyer firm's pursuit of profit maximization. This profit-driven strategy results in liquidity cross-subsidization, thereby addressing the third puzzle: why suppliers, who are ostensibly meant to receive funding, instead extend trade credit to the buyer firm.

It remains to determine $\mu$, the shadow value of liquidity for the intermediary. If (7) is binding, then $\mu$ is determined by

$$L = -\Theta(\mu) \equiv -\int_{\Omega} q(\lambda, c, \mu)\theta_F(\lambda, c)dG. \tag{12}$$

If (7) is slack, $\mu = 0$. In this case, the intermediary selects suppliers irrespective of liquidity concerns, i.e. the optimal selection policy is to select suppliers solely based on $\Delta\pi$.

**Lemma 1** *If $\Theta(0) + L < 0$, then there exists a unique $\mu > 0$ that satisfies (12); and otherwise $\mu = 0$.*

The liquidity value $\mu$ can be zero if participating suppliers provide a sufficiently large amount of liquidity, $-\Theta(0) \leq L$. Note that this can be with either a positive liquidity pool $\Theta(0) \geq 0$ or a negative liquidity pool $\Theta(0) < 0$. Otherwise, the intermediary's endowment has a positive liquidity value, $\mu > 0$.

For finance contracts to be active, the set of suppliers with $\Delta\pi > 0$ need to be nonempty. A necessary and sufficient condition is $\Delta\pi(1, \underline{c}) > 0$, or equivalently, $\frac{k}{m} < \frac{u-\underline{c}}{2}$. This, along with $c_0 > \underline{c}$ (implied by $k < \bar{k}$), ensures that region $A$ in Figure 1 always exists. Selecting suppliers in region $A$ (those with both a positive $\Delta\pi$ and a positive $\theta_F$) to finance is feasible with regard to the liquidity constraint and results in positive profits.

**Lemma 2** *There exist suppliers that are financed by the intermediary if and only if $k/m < (u - \underline{c})/2$.*

The intuition of this lemma is as follows. Whether supplier finance is activated or not hinges on two parameters. First, the finance service requires a per-supplier cost $k$. When $k$ is large, the incremental profit $\Delta\pi$ decreases, making it less attractive to finance suppliers. Second, when $m$

is small, indicating that the intermediary has a high matching advantage, consumers are likely to match in the early subperiod. This reduces the benefit of financing suppliers. In the extreme case, if $m \to 0$, the liquidity issue is eliminated altogether, making the finance contracts unnecessary.

The first puzzle introduced in the Introduction—why seemingly equally successful buyer firms might simultaneously adopt or forgo supplier finance—is accounted for by the notion that sufficiently high matching efficiency diminishes its necessity.

We summarize the results so far in the following theorem.

**Theorem 1 (Selection of suppliers)** *Taking $L \geq 0$ as given and assuming $k/m < (u - \underline{c})/2$, the intermediary's profit-maximizing strategy exists uniquely with the selection policies to finance suppliers $q(\lambda, c, \mu)$, satisfying (9), the reward to suppliers $f_F(\lambda, c)$ and $f_M(\lambda, c)$ satisfying (2) and (3), and the shadow value of liquidity $\mu \geq 0$ uniquely determined in Lemma 1.*

The intermediary's available liquidity $L$ shapes the feasibility of the finance contracts via the liquidity constraint and especially $\mu$. It is intuitive that $\mu$ is strictly decreasing in $L \in [0, -\Theta(0)]$: an additional unit of the intermediary's money holding is appreciated more when her money holdings are relatively low. If $L$ is higher, the curve $\Delta\pi + \mu\theta_F = 0$ is closer to $\Delta\pi = 0$, and the intermediary selects suppliers primarily based on profits. If $L$ is lower (which leads to a larger $\mu$), $\Delta\pi + \mu\theta_F = 0$ is closer to $\theta_F = 0$. Then liquidity becomes more important when selecting suppliers, and the intermediary relies more on liquidity cross-subsidization among suppliers. It is important to note that even a supplier that has high profits may not be chosen for the finance contract if he contributes little to the liquidity pool.

**Corollary 1** *$\mu(L) > 0$ is strictly decreasing in $L$ if $\Theta(0) + L < 0$.*

**The intermediary's liquidity holding.** Next, we turn to the intermediary's numeraire holding, which is determined by the following profit-maximization problem:

$$\max_{L \geq 0} \; V^m(L) - i \cdot L,$$

where $V^m(L)$ is the intermediary's profits if she holds a numerical of measure $L$ as is defined in (8) and the second term is the corresponding funding cost.

Applying the Envelop condition $V^{m\prime}(L) = \mu(L)$, the first order condition for $L$ is:

$$i \geq \mu(L). \tag{13}$$

Recall that $\mu(0)$ is the shadow value of liquidity to the intermediary if her liquidity holding $L = 0$. From Lemma 1, we have $\mu(0) > 0$ if and only if $\Theta(0) < 0$, and $\mu(0) = 0$ otherwise. Then we can characterize the intermediary's optimal liquidity holdings by comparing $i$ and $\mu(0)$.

There are two scenarios to consider. In the first scenario, $\Theta(0) < 0$, which implies $\mu(0) > 0$

(see Corollary 1). If the nominal interest rate is relatively high, namely $i \geq \mu(0)$, the optimal money holding is $L(i) = 0$, indicating that the funding source is entirely given by the pooled liquidity of suppliers. If the nominal interest rate is relatively low, namely $i < \mu(0)$, then (13) holds with equality, and the intermediary holds a positive amount of money $L(i) = -\Theta(i) > 0$. In the second scenario, $\Theta(0) > 0$, which implies $i > \mu(0) = 0$ (see Lemma 1), and the intermediary can finance all suppliers with positive profit contributions $\Delta\pi$, without holding liquidity.

**Proposition 1 (Intermediary's liquidity holdings)** *The optimal liquidity holdings of the intermediary follow $L(i) = -\Theta(i) > 0$ if $i < \mu(0)$, and $L(i) = 0$ otherwise. The value of liquidity with the intermediary's optimal liquidity holdings is given by*

$$\mu^*(i) = \min\left(i, \mu(0)\right). \tag{14}$$

With the intermediary's optimal selection policy (Theorem 1) and the numeraire holding rule (Lemma 1) now established, we proceed to the equilibrium.

**Theorem 2 (The Equilibrium)** *An equilibrium exists and is unique where the intermediary operates with $(q(\lambda, c, \mu^*(i)), f_M(\lambda, c), f_F(\lambda, c), L(i), \mu^*(i))$, as characterized by Theorem 1 and Lemma 1.*

### 3.1 The impact of funding cost $i$

We now examine how changes in the nominal interest rate affect both the intermediary's optimal liquidity holdings and her selection of suppliers for financing. Our analysis reveals that suppliers' internal liquidity pool serves as an effective buffer against external funding cost shocks, yielding important implications for policy makers.

Following Proposition 1, when $i < \mu(0)$, the shadow value of liquidity $\mu^*(i) = i$ is strictly increasing in the funding cost $i$. This means that as the cost of external funds increases, the value of each unit of liquidity held by the intermediary also increases. Consequently, the intermediary's liquidity holdings, $L(i) = -\Theta(\cdot)$, are positive and strictly decreasing in $i$. In other words, the intermediary holds less liquidity as the cost of that liquidity increases. This effect is depicted in Figure 1 by the selection curve $\Delta\pi + \mu\theta_F = 0$ rotating clockwise around the point $(\lambda_0, c_0)$.

The threshold $\mu(0)$ represents the inherent liquidity richness of the supplier pool. A small $\mu(0)$ indicates abundant liquidity among suppliers, making the program less sensitive to changes in external funding costs. In contrast, a large $\mu(0)$ suggests a less liquid supplier pool, making the program more vulnerable to increases in $i$. When $i > \mu(0)$, the intermediary no longer holds the numeraire and relies entirely on cross-subsidizing liquidity among suppliers.

**Sudden increase in financing costs.** Regulators and practitioners worry that a sudden increase in funding costs (e.g., due to a financial crisis, a credit crunch, or driven by monetary policy) could disrupt supplier finance programs, potentially leading to widespread supplier failures and substantial aggregate output losses. This vulnerability to fluctuations in market funding costs becomes a "sleeping risk" associated with the growing use of supplier finance.[12]

Our model demonstrates that this concern, while valid, may be overstated in certain circumstances. Importantly, the possibility of shrinking coverage or liquidity provision upon the financing cost increase should be evaluated program by program. Supplier finance programs, as modeled here, draw liquidity from *both* the market (external liquidity with cost $i$) and the liquidity pool among suppliers (internal liquidity with cost $\mu(0)$). As $i$ increases, the intermediary optimally shifts its reliance toward the internal liquidity pool, effectively reducing its dependence on external financing. When external funding costs $i$ exceed the threshold $\mu(0)$, the intermediary ceases to hold numeraire and relies entirely on the internal liquidity pool, which acts as a direct buffer, absorbing the impact of changes in $i$.

Our model therefore suggests that regulators and investors should pay attention to the *liquidity characteristics* of the supplier pools. Specifically, they should evaluate the capacity of the internal liquidity pool, which depends on factors such as the number and financial health of the participating suppliers. A larger and more diversified pool of suppliers, particularly those with strong financial health and a high likelihood of early revenue, enhances the program's resilience to external funding shocks. In this context, policies that improve the transparency of supplier finance programs are highly valued. For example, the Financial Accounting Standards Board (FASB), starting in 2023, requires corporations to disclose the terms and size of their supplier finance programs in their financial statements. These disclosures help investors understand the internal liquidity capacity of the supplier pool, which is a crucial factor in evaluating a program's resilience to increases in external funding costs and thus its overall risk.

### 3.2 Comparative statics of matching efficiency $m$

We now turn to how matching efficiency $m$ affects the operation of the intermediary. We start by showing how the curves $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ change as $m$ decreases, which captures an improvement in matching efficiency. Figure 2 illustrates that as $m$ decreases from $m_1$ to $m_2$, the incremental profits $\Delta\pi(\lambda, c)$ decrease, causing the curve $\Delta\pi(\lambda, c) = 0$ to shift downwards. At the same time, since the intermediary is more likely to match with consumers early, the liquidity contribution of suppliers improves. As a result, the $\theta_F(\lambda, c) = 0$ curve rotates upwards.

Notably, $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ intersect at $(c_0, \lambda_0) = (k + u - \sqrt{k^2 + 4uk}, \frac{k+\sqrt{k^2+4ku}}{2mu})$. Since $c_0$ does not depend on $m$, as $m$ decreases, the two curves intersect along the horizontal line

[12]For more on this, see the Wall Street Journal report titled "Supply-Chain Finance Is New Risk in Crisis": https://www.wsj.com/articles/supply-chain-finance-is-new-risk-in-crisis-11585992601. Accessed on Jul 17, 2023.

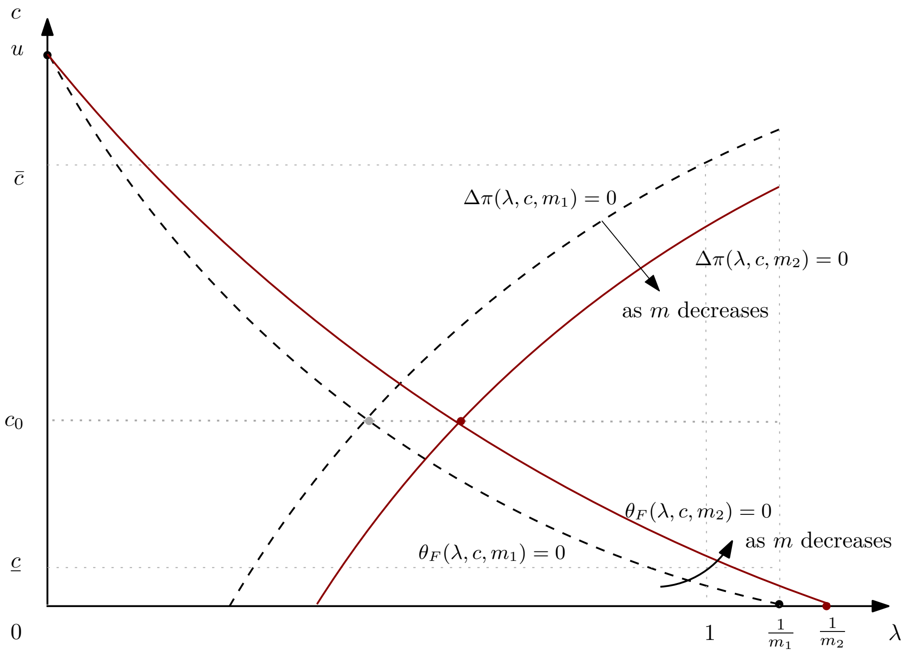



Figure 2: Effects of a decrease in $m$ on $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$

of $c = c_0$, and the intersection point moves to the right. The intersection point lies within the set $\Omega$ as long as $\lambda_0 \leq 1$, or equivalently,

$$m \geq \tilde{m} \equiv \frac{k + \sqrt{k^2 + 4uk}}{2u}. \tag{15}$$

When $m > \tilde{m}$, all three regions of (10) are nonempty, as illustrated in Figure 1, that is, there are suppliers in $\Omega$ with positive $\Delta\pi(\cdot)$ and negative $\theta_F(\cdot)$. Thus, liquidity cross-subsidization is plausible in equilibrium.

In contrast, when $m \leq \tilde{m}$, the $\Delta\pi(\cdot) = 0$ curve lies entirely below the $\theta_F(\cdot) = 0$ curve. Figure 3 illustrates the case of $m = \tilde{m}$ where $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ intersect at $\lambda = 1$. In this case, all suppliers who bring a positive $\Delta\pi$ (the shaded region) also give the intermediary a positive liquidity contribution, leading to $\mu = 0$. Indeed, a necessary condition for a binding liquidity constraint for the intermediary is $m > \tilde{m}$.

**Matching efficiency and finance inclusion.** Improved matching efficiency influences the trade-off between the finance mode and the middleman mode. We are interested in how this trade-off shapes the optimal set of financed suppliers. We focus on the scenario where the intermediary's liquidity holding is interior ($L > 0$), implying an effective funding cost of $\mu^*(i) = i$.[13]

To analyze the marginal suppliers, Figure 4 illustrates four different supplier groups, cate-

[13]This requires $\mu(0) > 0$ and thus $m > \tilde{m}$. Lemma 2 establishes that the finance mode is active if and only if $m > \frac{k}{(u-\underline{c})/2}$. When $m \in \left(\frac{k}{(u-\underline{c})/2}, \tilde{m}\right)$, all suppliers selected for finance contracts contribute positive liquidity and no cross-subsidization occurs. Since the liquidity constraint is not binding, the selection rule for financing suppliers is simply $\Delta\pi(\lambda, c) \geq 0$. As $m$ decreases, the set of financed suppliers contracts.

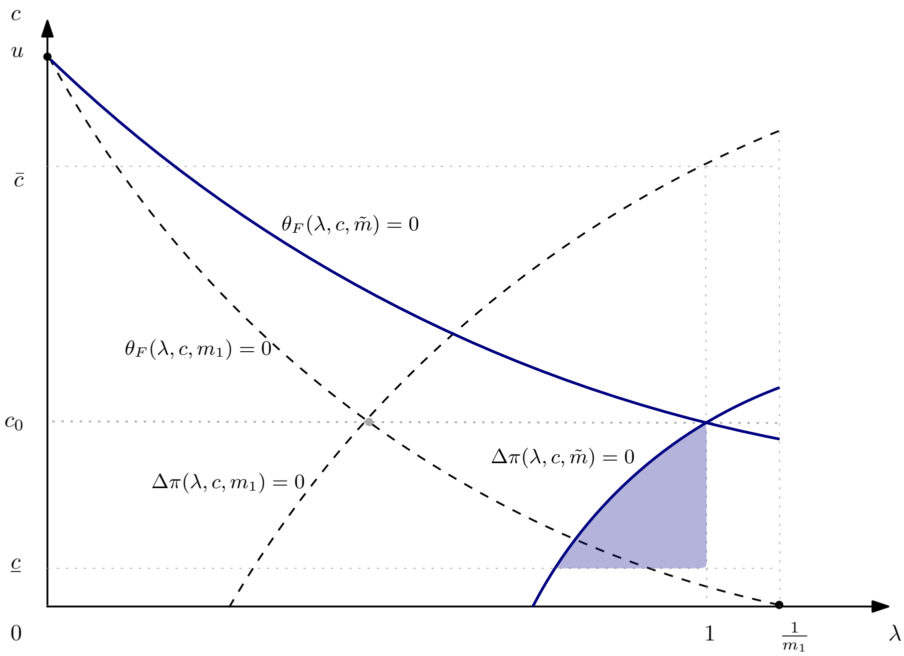



Figure 3: The curves of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$ when $m = \tilde{m}$ and $m = m_1 > \tilde{m}$

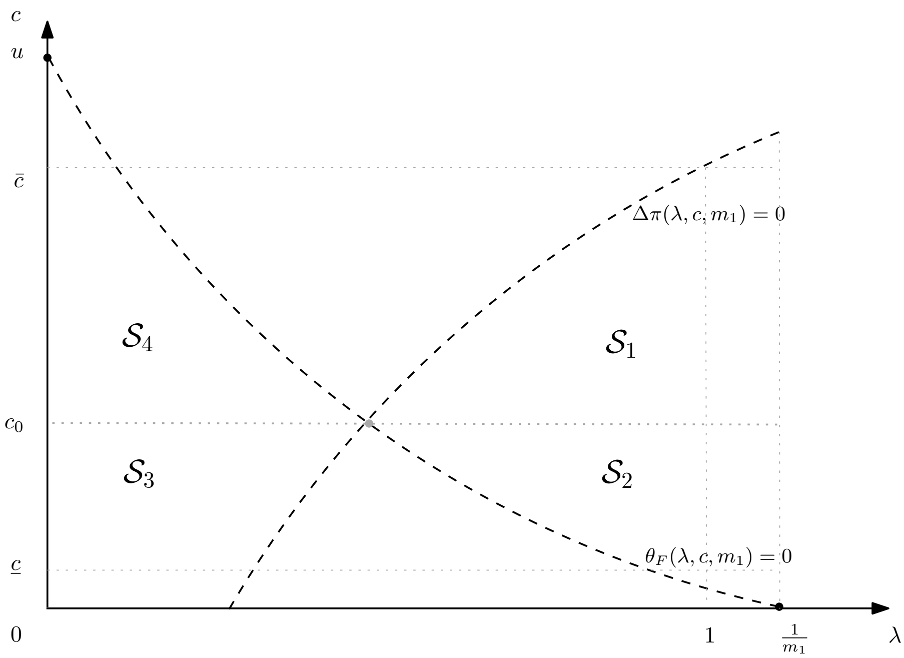



Figure 4: Four sets of suppliers by profit and liquidity contributions

gorized by their contributions to profit and liquidity. Suppliers in $S1$ and $S2$ generate positive profits but require liquidity ($\Delta\pi(\cdot) > 0, \theta_F(\cdot) < 0$), while suppliers in $S3$ and $S4$ provide liquidity but generate negative profits ($\Delta\pi(\cdot) < 0, \theta_F(\cdot) > 0$).

The following lemma then introduces a threshold rate $i_0$ that determines the slope of the selection curve. When $i < i_0$, the selection curve is upward-sloping (see Figure 1), and marginally financed suppliers —- those suppliers who lie exactly on the selection curve $\Delta\pi + \mu\theta_F = 0$ and are thus indifferent between being financed or not —-are in regions $S1$ and $S3$. In contrast, when $i > i_0$, the selection curve is downward-sloping and these marginally financed suppliers are in regions $S2$ and $S4$.

**Lemma 3** *Let* $i_0 = \left(k + \sqrt{k^2 + 4uk}\right) / (2u)$. *Given* $\mu^*(i) = i$, *if* $\max\{i, \mu(0)\} < i_0$, *then* $b'_\lambda(\lambda, i) > 0$; *if* $\min\{i, \mu(0)\} > i_0$, *then* $b'_\lambda(\lambda, i) < 0$.

To analyze how improved matching efficiency affects the set of financed suppliers, we examine the profit-liquidity ratio $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$ of suppliers. This ratio measures the profit gained (or cost incurred) per unit of liquidity for each supplier. Since $\mu^*$ is fixed, the change in this ratio as $m$ decreases determines whether more or fewer suppliers will be financed.

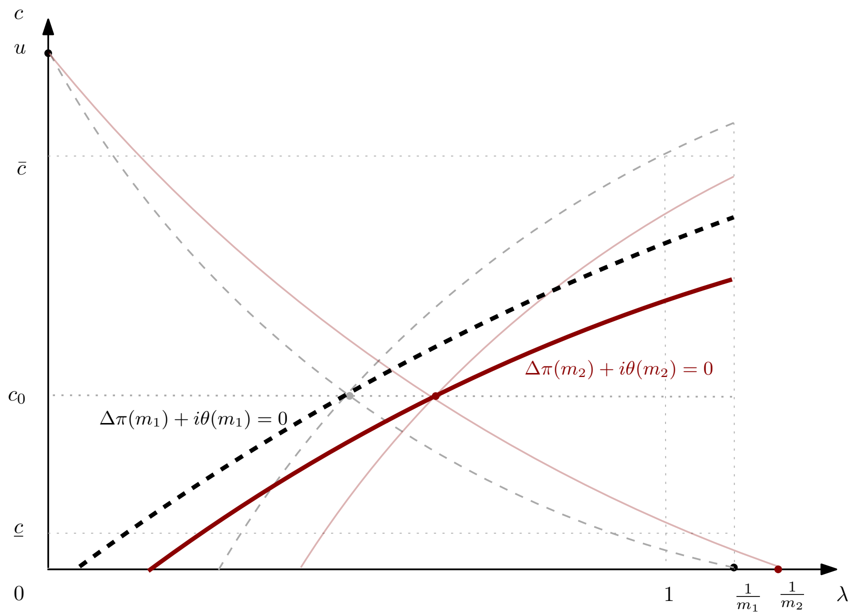
For suppliers in regions $S1$ and $S2$ that require liquidity ($\theta_F < 0$), this ratio is:

$$\frac{\Delta\pi(\cdot)}{-\theta_F(\cdot)} = \frac{m\lambda(u-c) - 2k}{m\lambda(u+c) + c - u}.$$

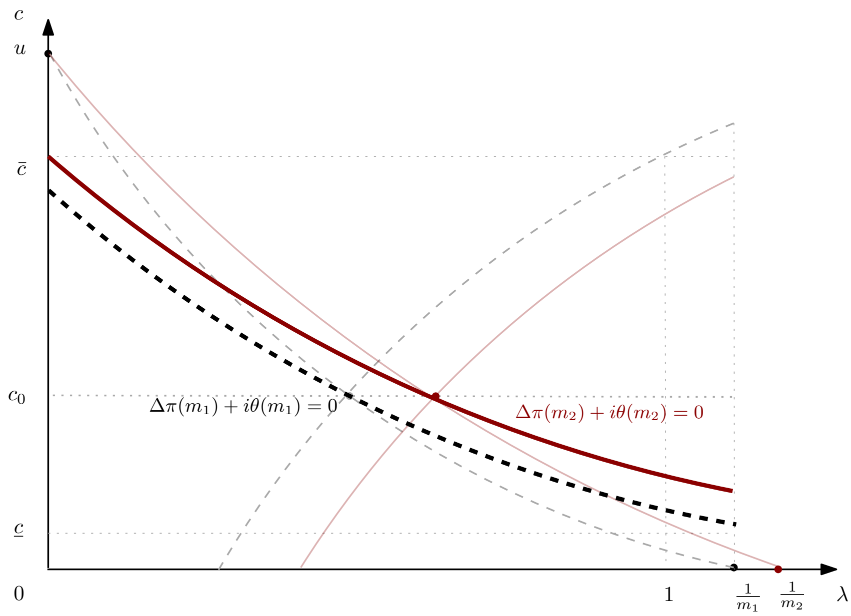
As $m$ increases, both the profit and required liquidity increase, but at different rates. Profit increases at rate $\lambda(u-c)$, while required liquidity increases at rate $\lambda(u+c)$. Since $\lambda(u+c) > \lambda(u-c)$, the required liquidity generally grows faster than profit. However, $c$ plays a crucial role through its effect on the denominator. When $c$ is high, the higher liquidity requirement dampens the relative increase in required liquidity as $m$ increases. This causes the profit effect to dominate, making the profit-liquidity ratio increase with $m$. When $c$ is low, the effect of liquidity requirements dominates, causing the ratio to decrease with $m$. When $c = c_0$, the two effects strike a balance. A parallel analysis applies to suppliers in regions $S3$ and $S4$. Lemma 4 summarizes how the profit-liquidity ratio varies with $m$:

**Lemma 4** *Given* $m > \tilde{m}$, *as the matching efficiency improves (that is,* $m$ *decreases), the profit-liquidity ratio* $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$ *decreases for suppliers in* $S1$ *and* $S4$ *and increases for suppliers in* $S2$ *and* $S3$.

We now connect this result to the impact of matching efficiency on the scope of supplier finance. If $i < i_0$, the selection curve is upward-sloping and the marginal suppliers are in $S1$ and $S3$ (see Figure 5(a)). The improved matching efficiency (lower $m$) reduces the return on financing suppliers in $S1$ and increases the cost of acquiring liquidity from $S3$. This leads to a contraction of supplier finance, suggesting that retail matching efficiency and supplier finance act as substitutes.




(a) $\mu^*(i) = i < i_0$, and the selection curve is upward-sloping




(b) $\mu^*(i) = i > i_0$, and the selection curve is downward-sloping

Figure 5: Change in supplier finance selection curve as matching efficiency increases ($m$ decreases)

Conversely, if $i > i_0$, the selection curve is downwards sloping and the marginal suppliers are in $S2$ and $S4$ (see Figure 5(b)). As $m$ decreases, the return on financing suppliers in $S2$ increases, and the cost of acquiring liquidity from $S4$ decreases. This leads to an expansion of supplier finance, indicating a complementary relationship between retail matching efficiency and supplier finance.

**Proposition 2** *Suppose that the effective funding cost satisfies $\mu^*(i) = i$. If $i < i_0$, a decrease in matching frictions (i.e., improved matching efficiency, represented by a lower $m$) reduces the optimal scope of supplier finance, suggesting a substitution relationship between supplier finance and retail matching efficiency. Conversely, if $i > i_0$, a decrease in matching frictions expands the optimal scope of supplier finance, indicating a complementary relationship with retail matching efficiency.*

It is worth mentioning that a declined demand can be equivalently modeled as an increase in $m$, reversing the effect described above. Specifically, when $i < i_0$, the model predicts that as demand shrinks, the scope of supplier finance expands (see the discussions in Section 4).

### 3.3 Welfare

Next, we examine social welfare in our economy. Since a profit-maximizing intermediary does not internalize consumer surplus, the equilibrium outcome differs from the social optimum even with supplier finance. Surprisingly, we find that increasing the funding cost can sometimes improve social welfare. We provide conditions under which welfare exhibits a nonmonotonic relationship with the nominal interest rate.

**Social optimum.** Consider a planner who takes $i$ as given, selects suppliers from $\Omega$ and decides whether to finance them.[14] Let $s(\lambda, c)$ be a binary function equal to one if a supplier of $(\lambda, c)$ is financed, and let $L \geq 0$ be the liquidity chosen by the planner. The total welfare, excluding funding costs, is given by

$$\mathcal{W} = \max_{s(\cdot)\in\{0,1\}} \int_\Omega \left\{ s(\lambda,c)(u-c-k) + \Big(1 - s(\lambda,c)\Big)(1-m\lambda)(u-c) \right\} dG,$$
$$\text{s.t.} \int_\Omega s(\lambda,c)\theta_F(\lambda,c)dG + L \geq 0.$$

The total surplus for the goods is $u - c - k$ if the supplier is financed and is $(1 - m\lambda)(u - c)$ if not. Following the procedure in the previous sections, let $\mu^s(L)$ be the multiplier associated with the

[14]Generally, the planner assigns suppliers to direct selling, middleman contracts, or middleman-finance contracts. Since $m < 1$ makes intermediary technology more efficient than individual suppliers in matching goods with demand, all suppliers are allocated to either middleman or middleman-finance contracts. Furthermore, if the planner could choose the funding cost, setting $i = 0$ would dominate by removing the liquidity constraint.

liquidity constraint. The planner's optimal selection rule into the finance mode can be written as

$$s(\lambda, c, \mu^s) = \begin{cases} 1 & \text{if } \Delta v(\lambda, c) + \mu^s \theta(\lambda, c) \geq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

where $\Delta v(\lambda, c) \equiv m\lambda(u - c) - k > \Delta\pi(\lambda, c)$ is the gain of surplus, and crucially, it is larger than the gain of intermediary profits since the latter ignore the surplus of the consumer side. Furthermore, $\mu^s = 0$ if $\Theta(0) + L \geq 0$, and otherwise $\mu^s$ is pinned down by $\Theta(\mu^s) + L = 0$. Then the planner's liquidity holding is determined by comparing $\mu^s(0)$ and $i$ where $L > 0$ only if $i < \mu(0)$.

Like the intermediary's optimal choice, the planner's solution also features liquidity cross-subsidization but is based on surplus gains. Throughout the analysis, we assume $i < \min\{\mu^s(0), \mu(0)\}$ so that the planner has the same liquidity value as the intermediary. The planner then chooses a positive amount of liquidity holding $L > 0$ and the liquidity value $\mu^s = i$. According to (16), the planner selects suppliers with positive surplus gain and liquidity ($\Delta v > 0, \theta > 0$), as well as those with negative surplus gain but positive liquidity ($\Delta v < 0, \theta > 0$ with $-\Delta v/\theta \leq i$), and uses the pooled liquidity and her own endowment $L$ to fund suppliers with liquidity need ($\Delta v > 0, \theta < 0$) if $-\Delta v/\theta \geq i$.

Since the social surplus gain exceeds the profit gain ($\Delta v(\lambda, c) > \Delta\pi(\lambda, c)$), the planner's financing decision encompasses a broader set of suppliers than the intermediary. Figure 6 illustrates this difference: the orange region represents suppliers who are financed by the planner but not by the intermediary. These suppliers fall into two categories: those who contribute positively to the liquidity pool and those who require liquidity support. Although the intermediary's profit-maximizing selection never achieves the social optimum, the supplier finance program still enhances welfare by facilitating liquidity flows from early-revenue suppliers to those facing liquidity constraints.

**Funding cost and welfare.** The liquidity cross-subsidization mechanism has important implications for social welfare. We demonstrate that increasing the funding cost can, counterintuitively, improve welfare through enhanced liquidity cross-subsidization.

To establish this result, we focus on cases where $\mu(0) > 0$ for $i \in [0, \varepsilon)$, with $\varepsilon$ being a small positive number. When this condition does not hold, the outcome becomes trivial: the intermediary's financing decision remains unchanged with interest rate, only financing suppliers with $\Delta\pi > 0$. Given $\mu(0) > 0$, a marginal increase in $i$ from zero implies $\mu(i) = i$ (see Equation (14)). Figure 7 illustrates this scenario, where the gray region represents the set $\Omega$ of all suppliers with different subsets labeled in capital letters.

When $i = 0$, the intermediary finances all suppliers with positive profitability, covering regions $A$, $B$, and $D$. As $i$ increases, the higher funding cost induces the intermediary to engage

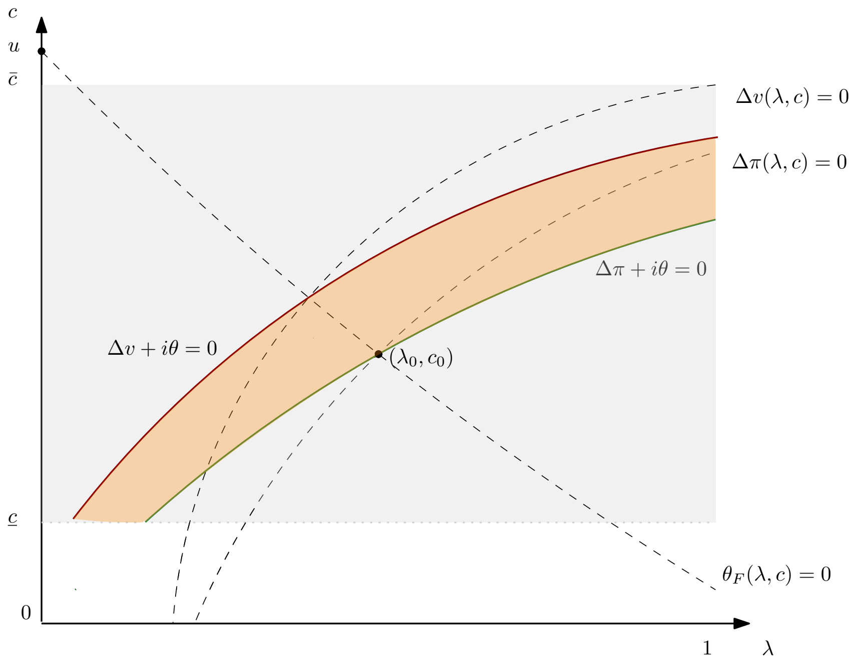



Figure 6: Compare the planner and the intermediary's selection rules

in liquidity cross-subsidization. This process involves excluding suppliers with positive profitability but negative liquidity (region $D$), while including suppliers with negative profitability but positive liquidity (region $C$). Consequently, the intermediary's profits decrease. Suppliers' profits do not change because they are indifferent between being funded and not funded by the intermediary. However, there is a potential for an increase in consumer surplus if the total trading volume increases. Ultimately, whether social welfare is improved or not depends on the dominance of the consumer surplus effect.

To analyze the impact on trading volume, let $\lambda^\pi(c)$ represent the combinations of $(\lambda, c)$ for which $\Delta\pi(\lambda, c) = 0$, and let $\lambda^\mu(c)$ represent the combinations of $(\lambda, c)$ such that $\Delta\pi(\lambda, c) + \mu\theta(\lambda, c) = 0$. Excluding suppliers in region $D$ leads to a decreased trading volume given by:

$$m \int_{c_0}^{\bar{c}} \int_{\lambda^\pi(c)}^{\lambda^\mu(c)} \lambda g(\lambda, c) d\lambda dc,$$

while including suppliers in region C leads to an increased trading volume given by:

$$m \int_{\underline{c}}^{c_0} \int_{\lambda^\mu(c)}^{\lambda^\pi(c)} \lambda g(\lambda, c) d\lambda dc.$$

This is because, for instance, each of the newly added suppliers, measured by $\int_{c_0}^{\bar{c}} \int_{\lambda^\pi(c)}^{\lambda^\mu(c)} g(\lambda, c) d\lambda dc$, will become available to consumers even when he is hit by a liquidity shock, which occurs with probability $m\lambda$.

Comparing the above two volumes, we can see that when $c_0$ is large enough, the former volume can be made arbitrarily small, while when $c_0$ is small enough, the latter volume can be

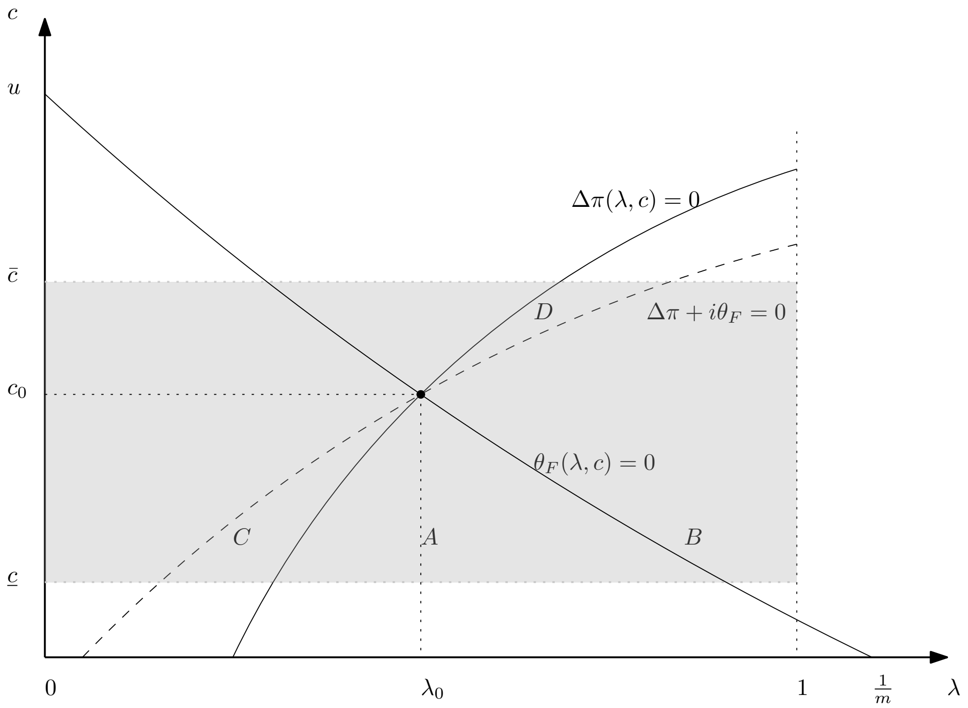



Figure 7: Welfare can increase as interest rate increases

made arbitrarily small. Thus, for a sufficiently large $c_0$, the consumer surplus effect dominates, and $i$ deviating from $i = 0$ (known as the Friedman rule) improves welfare. In other words, $c_0$ crucially determines the number of suppliers to exclude (those who contribute to negative liquidity) and the number of suppliers to include (those who contribute to positive liquidity). $c_0$ is determined as the intersection of $\theta_F(\lambda, c) = 0$ with $\Delta\pi(\lambda, c) = 0$. As $k$ decreases, $c_0$ increases accordingly: for $k \to 0$, $c_0 \to u > \bar{c}$, and for $k \to \bar{k}$, $c_0 \to \underline{c}$. Therefore, with $k$ sufficiently small, increased trading volume outweighs decreased trading volume.

A similar analysis applies to $\lambda$. If $\lambda_0$ is sufficiently large, according to the liquidity cross-subsidization strategy, as $i$ marginally increases from $i = 0$, the intermediary excludes fewer suppliers and includes more suppliers in finance contracts. Exactly like before, $m$ determines $\lambda_0$. With $m$ sufficiently small, e.g., close to $\tilde{m}$, $\lambda_0$ is larger and closer to $\lambda = 1$. As a result, the increase in trading volume outweighs the decrease in trading volume.

The following proposition provides sufficient conditions for critical values of $k$ and $m$ such that the decreased trading volume is smaller than the increased trading volume if $k$ or $m$ is smaller than the critical value. Social welfare increases because there is a significant improvement in consumer surplus that outweighs the decrease in the intermediary's profits. Hence, deviating from the Friedman rule is welfare-improving.

**Proposition 3** *Let $\kappa \equiv \frac{k}{u} \in (0, \frac{\bar{k}}{u})$ and define $\tilde{m}(\kappa) = \frac{1}{2}\left(\kappa + \sqrt{\kappa^2 + 4\kappa}\right)$. Suppose that $(\lambda, c)$ follows a uniform distribution and $\mu(0,0) > 0$. There exists a critical value $\kappa^* \in (0, \frac{\bar{k}}{u}]$ and $m^*(\kappa) \in (\tilde{m}(\kappa), 1]$,* *such that under $m < m^*(\kappa)$ or $\kappa < \kappa^*$ that marginally increases $i$ from $i = 0$ improves welfare.*

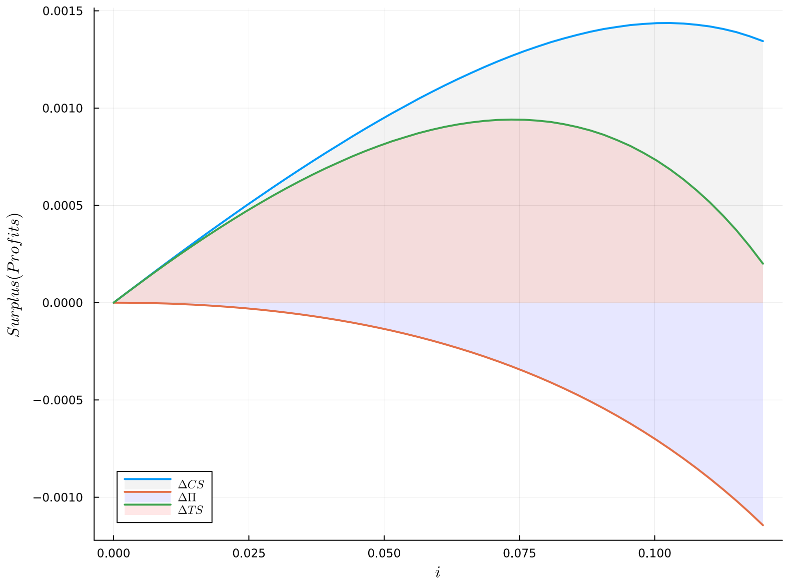
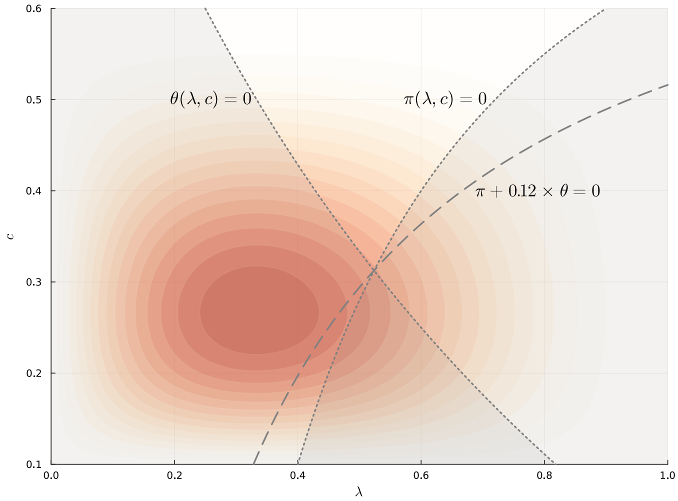


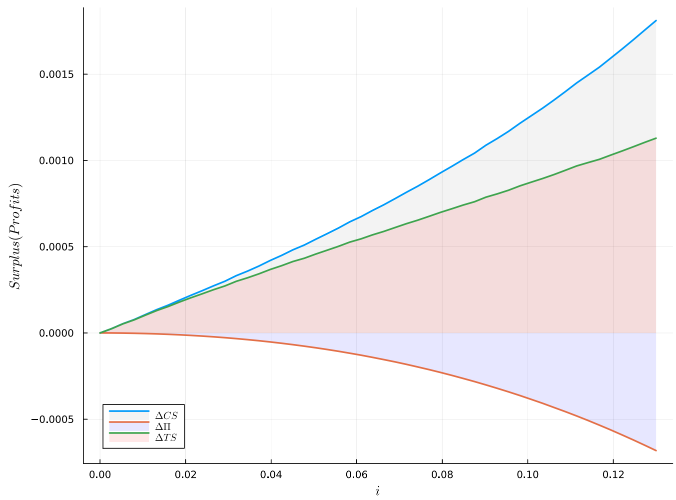

Figure 8: Welfare is non-monotonic in $i$ under uniform distribution of $(\lambda, c)$

The proposition also establishes sufficient conditions for a nonmonotonic impact of $i$ on welfare. Figure 8 illustrate how welfare changes with $i$ using a numerical example, with $u = 1, k = 0.1, m = 1, \underline{c} = 0.1, \bar{c} = 0.6$, and a uniform distribution of $(\lambda, c)$. Under these values, $\mu(0) = 0.26$. The figure shows that (1) aggregate profits ($\Delta\Pi$, red curve) exhibit a monotonic decrease in $i$ due to the exclusion of suppliers with positive $\pi$ and the inclusion of suppliers with negative $\pi$; and (2) total consumer surplus (blue curve $\Delta CS$ follows an inverted U-shaped because total trading volume first increases and then decreases. The effect of consumer surplus dominates. Consequently, the total surplus (green curve $\Delta TS$) first increases and then decreases at relatively higher levels of $i$.

Of course, the suboptimality of the Friedman rule can occur with non-uniform distributions. Figure 9 provides a numerical exercise with $u = 1, k = 0.18, m = 1, \underline{c} = 0.1, \bar{c} = 0.6$, and both $\lambda$, and $c$ follow a $Beta(2, 3)$ distribution. Under these values, $\mu(0) = 0.137$. Panel (a) shows the implied densities (contour graph in red) and a particular selection rule of $\pi + 0.12 \times \theta = 0$. Panel (b) shows that, as $i$ increases, the blue curve representing the total consumer surplus ($\Delta CS$) increases monotonically, which outweighs the decrease in total profits ($\Delta\Pi$, the red curve), resulting in a monotonically increasing total surplus ($\Delta TS$, the green curve) for the shown range of nominal rate.




(a) Density and selection rule ($\mu = 0.12$)




(b) Welfare change

Figure 9: Welfare increases in $i$ if $(\lambda, c) \sim Beta(2, 3)$

# 4 Discussions and extensions

Our benchmark model provides a framework for understanding key features of real-world supplier financing programs. This section proceeds as follows. First, we connect the model's core mechanisms to three prominent characteristics observed in supplier finance practice. Second, we extend the benchmark model in several ways, relaxing the assumptions made for tractability and exploring the robustness of our findings.

## 4.1 Linking theory to practice in supplier finance

We examine three key features prevalent in supplier finance practices: the selective participation of suppliers, the cross-subsidization of liquidity across suppliers, and the role of the intermediary's matching efficiency in shaping the demand for supplier finance. These characteristics, observed in various real-world programs, anchor our theoretical model.

**Selective supplier participation.** Real-world supplier finance programs typically involve a selective process. For example, when Co-op launched its program in 2020, it selected fewer than 100 suppliers from its network of thousands. Similarly, Amazon Lending, offered to third-party merchants, is an invitation-only program with customized credit terms.[15] This selectivity reflects a careful weighing of the benefits and costs of including each supplier, echoing the best practices emphasized in the *Supply Chain Finance Knowledge Guide* published by the International Finance Corporation. The guide highlights the importance of prioritizing suppliers based on their relationship with the buyer company and their financial needs. Our model captures these considerations: suppliers with stronger relationships are more crucial to the buyer's value creation (profit contributions), while the intermediary's liquidity constraint reflects the consideration of suppliers' financial needs. The selection mechanism in our model thus provides an economic rationale for the observed selectivity in supplier finance programs.

**Liquidity cross-subsidization.** A key feature of many supplier finance programs is liquidity cross-subsidization, where larger, more financially stable suppliers effectively provide liquidity that is used to finance smaller, more liquidity-constrained suppliers. The JingBaoBei program operated by JD.com, the largest e-commerce platform in China, illustrates this. JingBaoBei allows suppliers to request advance payment based on their receivables from JD.com, providing funding to over 200,000 vendors with a total amount of more than 730 billion RMB. Crucially, the program is primarily funded by pooled liquidity, much of which originates from the suppliers themselves, particularly through trade credit. JD.com's reliance on supplier trade credit is evident in their financial reporting. For instance, on 2021, the increase in accounts payable constituted over 77% of JD's net cash inflow from operating activities.[16] While JD.com has introduced asset-backed securities to further finance JingBaoBei, supplier trade credit remains a substantial funding source. Large suppliers like Lenovo, Philips, and Bosch, who rarely request advance payments, effectively subsidize liquidity to smaller suppliers who need it more. Our model captures this feature through the heterogeneity in supplier liquidity needs, with some suppliers acting as net providers of liquidity, while others are net users.

[15]For details of Co-op's supplier finance program, see `https://scfcommunity.org/briefing/news/2020-retail-and-apparel-winner-co-operative-group/`. Accessed on Jul 17, 2023. For details on Amazon lending, see `https://www.junglescout.com/blog/amazon-lending-program`. Accessed on Jul 17, 2023.

[16]From 2015 to 2018, JD's accounts payable turnover days have gone up from 41.9 to 58.1 days. This means, for example, in 2018, it took more than 58 days on average for JD to pay its suppliers. On the other hand, JD's accounts receivable turnover is quite short, with payments being received from customers within five days of a sale. Combining these numbers with a 30-day inventory turnover, JD can efficiently use supplier trade credit for about 23 ($= 58 - 5 - 30$) days before having to pay it off. Notably, this strategy has proven successful for JD, as its cash position has improved consistently alongside its total revenue.

**Matching Efficiency, liquidity, and supplier finance growth.** Our model highlights the interplay between the intermediary's matching efficiency and suppliers' liquidity needs in driving the growth of supplier finance. This is evident in recent market trends. For example, disruptions during the pandemic led to increased inventories and extended payment terms, prompting retailers to increasingly utilize supplier financing to support their suppliers' cash flow. These disruptions can be modeled as a decrease in $m$, reflecting reduced matching efficiency. Our model predicts that, when funding cost $i$ is relatively low (to be precise, $i < i_0$), a decrease in $m$ expands the scope of supplier finance, consistent with the observed market behavior.[17]

The pandemic is not the sole driver of growing supplier liquidity demand. For example, inventory turnover days for sellers on Taobao, China's leading e-commerce platform, increased significantly from 6 days in 2017 to more than 20 days in 2020, indicating a reduced likelihood of suppliers matching with consumers. This trend, captured by a decrease in $m$, predates the pandemic and may be driven by increased competition on the platform. Consistent with our model predictions, Ant Finance's supplier finance program (Alibaba's financial subsidiary) expanded substantially, with credit balances rising from 647.5 billion RMB in 2017 to 2,153.6 billion RMB in 2020.[18]Our model therefore provides a theoretical framework to understand the growing demand for supplier finance in a world of evolving retail technologies and market dynamics.

## 4.2 Suppliers' access to money market

We first show that supplier finance can still exist even when suppliers can borrow from the money market. Suppose that suppliers can access the money market or bank credit and hold liquidity for their needs. Assume $\bar{c} > c_0 > \underline{c}$ and $\lambda_0 < 1$. Let $i^s > 0$ be the interest rate suppliers face, which may be higher or lower than the intermediary's rate $i$. Let $z^s = z^s(c)$ be the real balance held by a supplier with cost $c$. A supplier $(\lambda, c)$ with $z^s(c)$ has a retail market value of:

$$z^s + \left((1-\lambda) + \lambda \min\left\{\frac{z^s}{c}, 1\right\}\right) \frac{u-c}{2}.$$

Here, $\lambda \min\left\{\frac{z^s}{c}, 1\right\}$ shows that, with a liquidity shock, the supplier can use money holdings to produce and sell to $\min\{z^s/c, 1\}$ consumers. The supplier's money-holding problem is:

$$\max_{z^s} \left\{ \left[ z^s + \left((1-\lambda) + \lambda \min\left\{\frac{z^s}{c}, 1\right\}\right) \frac{u-c}{2} \right] - (1+i^s) z^s \right\}.$$

[17]For example, Constellation Brands Inc., a New York-based producer of Corona beer and Svedka vodka—launched a supplier finance program in 2022 in response to significant inventory growth and extended days of payables outstanding. Similarly, VF Corp., the parent company of popular brands such as Vans, North Face, and Supreme, initiated a supplier finance program in 2022 under similar circumstances. For further information, see the Wall Street Journal report at *https://www.wsj.com/articles/companies-offer-supply-chain-financing-to-vendors-as-they-bulk-up-on-inventory-push-out-payment-terms-11658316600?*, accessed on Jul 17, 2023.

[18]The inventory turnover days are obtained from *https://www.gurufocus.com/*. The credit balances of the supplier finance program are obtained from the *Ant Group Co., Ltd. Initial Public Offering and Listing on the STAR Market Prospectus*.

Suppliers never hold $z^s > c$, as it is inefficient. The first-order condition shows that suppliers with $(\lambda, c)$ satisfying $\frac{\lambda(u-c)}{2} > i^s c$ hold money. This simplifies to:

$$c < c^s(\lambda, i^s) \equiv \frac{\lambda}{\lambda + 2i^s} u. \tag{17}$$

Thus, suppliers with $c < c^s(\lambda, i^s)$ hold $z^s(c) = c$, while those with $c \geq c^s(\lambda, i^s)$ hold $z^s(c) = 0$.

Next, we consider the intermediary's problem. She can only offer finance contracts to suppliers who don't hold money. The feasible set of suppliers is:

$$\tilde{\Omega}(i^s) = \{(\lambda, c) \in \Omega | c \geq c^s(\lambda, i^s)\},$$

which is nonempty. Her supplier selection problem is:

$$\max_{\{q(\cdot)\}_{(\lambda,c)\in\tilde{\Omega}(i^s)}} \int_{\tilde{\Omega}(i^s)} q(\lambda, c)\Delta\pi(\lambda, c) dG,$$

subject to the liquidity constraint:

$$\int_{\tilde{\Omega}(i^s)} q(\lambda, c)\theta_F(\lambda, c) dG + L \geq 0,$$

where $i^s$ and $L$ are given.

In earlier sections, we showed that finance contracts are profitable when $\lambda_0 < 1$ because region $A$ in Figure 1 is nonempty (see Lemma 2 for details). But when suppliers can access the money market, finance contracts may not always be activated under the same conditions.

**Proposition 4** *Suppose $\lambda_0 < 1$, $\underline{c} > 0$, $i < \frac{k\bar{\lambda}}{mu\bar{\lambda} - 2k}$, and suppliers face money market rate $i^s$. There exist thresholds $i < \underline{i}^s < \bar{i}^s \equiv \frac{(u-\underline{c})\bar{\lambda}}{2\underline{c}}$ such that:*

- *If $i^s \leq \underline{i}^s$, suppliers with $c \leq c^s(\lambda, i^s)$ hold money for liquidity, and supplier finance stays inactive.*
- *If $i^s \geq \bar{i}^s$, no supplier holds money, and supplier finance is activated for some suppliers.*
- *If $i^s \in (\underline{i}^s, \bar{i}^s)$, suppliers with $c \leq c^s(\lambda, i^s)$ have money, while supplier finance activates for other suppliers.*

Figure 10 shows the third case. Suppliers with costs below $c^s(\lambda, i^s)$ (region $E$, dark blue) hold money and skip the intermediary's finance. Those with costs above $c^s(\lambda, i^s)$ do not hold money. Among them, suppliers in regions $A$, $B$, and $C$ join the finance contract. This can happen even if $i^s < i$, meaning suppliers borrow cheaper than the intermediary. Supplier finance still works because the intermediary uses liquidity more efficiently, leveraging the law of large numbers. This requires $c^s(\lambda, i^s)$ to cross the selection curve $\Delta\pi + \mu\theta_F = 0$ below $c_0$, ensuring region $A$ exists, as shown in Figure 10.

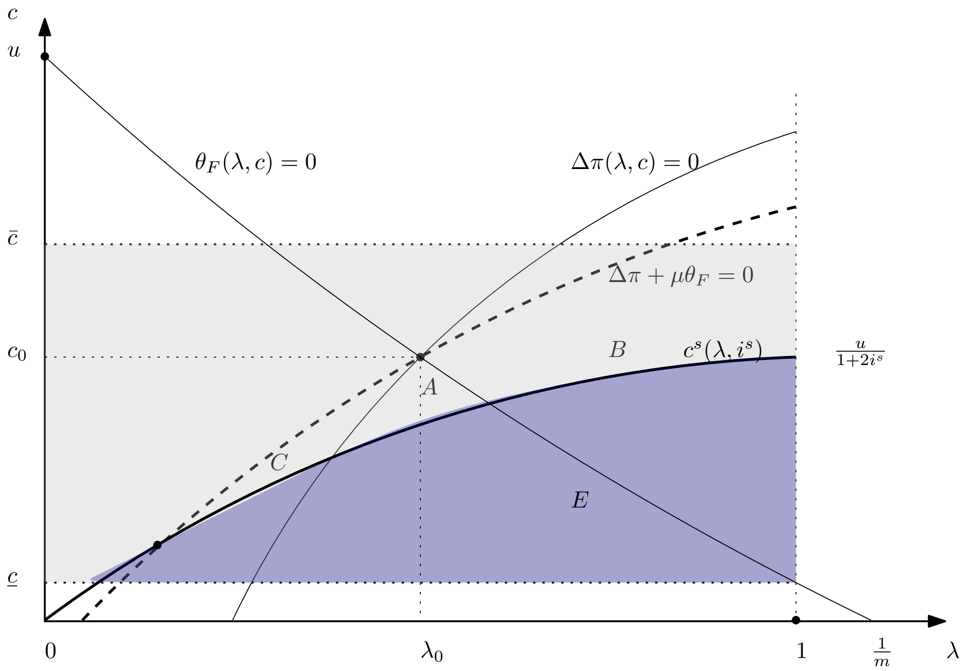



Figure 10: Suppliers' money holdings and intermediary finance coexist

## 4.3 General consumer demand

In the benchmark model, we assumed unit demand for simplicity. Here, we extend our results to a general setting where each supplier is a monopolist facing a downward-sloping demand function $Q(p)$. A seller (supplier or intermediary) sets the price to maximize profit:

$$\max_{p \in \mathbb{R}_+} (p - c)Q(p),$$

where $c$ is the constant production cost, as before. This problem is the same whether consumers arrive early or late. Assume $Q(p)$ yields a single-peaked profit function with a unique optimal price $p^m(c)$ and maximized profit $\tilde{\pi}(c)$. Under standard conditions, $\tilde{\pi}(c)$ is continuously differentiable, strictly decreasing, and convex. As in the benchmark, each good has a probability $1 - \lambda$ of early consumer arrivals and $\lambda$ of late arrivals.

When a supplier $(\lambda, c)$ joins a middleman contract, the profit contribution is $\pi_M(\lambda, c) = (1 - m)\lambda\tilde{\pi}(c)$. For a finance contract, it is $\pi_F(\lambda, c) = \tilde{\pi}(c) - k$. The difference is:

$$\Delta\pi(\lambda, c) = \pi_F(\lambda, c) - \pi_M(\lambda, c) = \tilde{\pi}(c) - k - (1 - m)\lambda\tilde{\pi}(c) = m\lambda\tilde{\pi}(c) - k,$$

which rises with $\lambda$ and falls with $c$. The liquidity contribution from a finance contract is:

$$\theta_F(\lambda, c) = (1 - m\lambda)(p^m(c) - c) - c,$$

which decreases in both $\lambda$ and $c$. Under these conditions, the existence of the selection curve and the existence of supplier finance (provided that $k$ is sufficiently large and $m$ is not too small, as in the benchmark model) can be established.

As a parametric example, suppose that $Q(p) = u - p$. Then $\tilde{\pi}(c) = (u-c)^2/4$, so $\Delta\pi(\lambda, c) = m\lambda(u-c)^2/4 - k$ and $\theta_F(\lambda, c) = (1-m\lambda)(u-c)/2 - c = (1-m\lambda)(u+c)/2 - c$. Note that the functional form of $\theta_F$ is exactly the same as that in the benchmark model. The relevant boundary conditions now become:

$$\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\theta_F}(\lambda) \equiv \frac{1-m\lambda}{1+m\lambda}u,$$
$$\Delta\pi(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\Delta\pi}(\lambda) \equiv u - \frac{2\sqrt{k}}{\sqrt{m\lambda}}.$$

### 4.4 Generalizing supplier outside options

In the benchmark model, suppliers' outside option value is $(1-\lambda)(u-c)/2$, reflecting their ability to sell directly to consumers when matched early. We now examine the robustness of the intermediary's problem to a general outside option denoted by $\Phi(\lambda, c)$, which may or may not depend on $\lambda$ and $c$, subject to a condition ensuring that all suppliers participate with the intermediary, at least under the middleman contract. Specifically, we assume $\Phi(\lambda, c) \leq (1 - m\lambda)\frac{u-c}{2}$ for all $(\lambda, c) \in \Omega$. This condition guarantees that the middleman contract is attractive relative to suppliers' outside options for all suppliers.

Under the *finance contract*, the intermediary pays $c$ upfront, ensures production with certainty, and sets $f_F = \Phi(\lambda, c)$ to match the outside option, yielding profit $\pi_F = \frac{u-c}{2} - \Phi(\lambda, c) - k$, with liquidity contribution unchanged at $\theta_F = (1-m\lambda)\frac{u+c}{2} - c$.

For the *middleman contract*, production and the cost $c$ occur only if the match is early (probability $1 - m\lambda$). The supplier receives $f_M$ when trade occurs, with expected profit $(1-m\lambda)(f_M - c)$. Setting this equal to $\Phi(\lambda, c)$, we have $f_M = c + \frac{\Phi(\lambda,c)}{1-m\lambda}$, and the intermediary's profit becomes $\pi_M = (1-m(\lambda)\frac{u-c}{2} - \Phi(\lambda, c)$.

The profit difference is $\Delta\pi = \pi_F - \pi_M = m\lambda\frac{u-c}{2} - k$, which matches the benchmark exactly. Since $\Delta\pi$ and $\theta_F$ remain identical to the benchmark model, the intermediary's selection problem (8) and all subsequent results, including Theorems 1 and 2, remain unchanged. This invariance demonstrates that the intermediary's optimal strategy depends solely on her matching advantage ($m$) and financing cost ($k$), not on the specific form of $\Phi(\lambda, c)$.

## 5 Manufacturing supplier finance

We now shift our focus to model supplier finance for manufacturers, who procure intermediate goods from suppliers to produce final goods for consumers. This combination of various intermediate goods into a unified output alters the operation of supplier finance, necessitating a tailored framework. We demonstrate that supplier finance persists in equilibrium and retains liquidity cross-subsidization—where financially stable suppliers support those with liquidity needs—as a central feature, consistent with the benchmark model.

Consider a manufacturer ($M$) that produces final goods using intermediate goods sourced from suppliers and sells these goods to consumers in a retail market in two sub-periods: early and late. A fraction $\alpha$ of consumers purchase in the early subperiod, and the remaining $1-\alpha$ purchases in the late subperiod. The retail price is normalized to 1 per unit. Consumers buy all final goods produced by $M$ at this fixed price, with a split demand between early (fraction $\alpha$) and late (fraction $1-\alpha$) subperiods, and the total quantity demanded equals the manufacturer's output.

There is a measure one of the suppliers indexed by the pair $(\lambda, c)$. Each supplier can produce one unit of an intermediate good with cost $c \in [\underline{c}, \bar{c}]$. With probability $\lambda$, a supplier cannot initiate production due to insufficient funds to purchase required inputs from input producers (who are passive in the model). The joint distribution of $(\lambda, c)$ is described by an atomless C.D.F. $G(\lambda, c)$ defined on $\Omega = [0,1] \times [\underline{c}, \bar{c}]$.

The manufacturer sources intermediate goods through two channels. The first channel is supplier financing contracts: $M$ offers financing contracts to a selected set of suppliers, providing them with working capital of $c$ if they are hit by liquidity shocks (observable to $M$). In return, these suppliers commit to the delivery of measure one of intermediate goods in the early subperiod and receive a reward $f(\lambda, c)$ in the late subperiod. The second channel is a competitive wholesale market that operates in the early subperiod following the realization of liquidity shocks. Only suppliers with adequate working capital (those not hit by liquidity shocks) can participate in this market, since production must occur before market entry. We do not restrict the pricing mechanism in the wholesale market. The only restriction being that the wholesale price denoted by $w(c)$ lies in $[c, 1]$ and is non-decreasing in $c$.

The timing is as follows: (1) At the beginning of the period, $M$ announces the set of suppliers selected for financing contracts and the set that is targeted for wholesale market purchases. This announcement includes $M$'s commitment to provide liquidity support to financed suppliers. (2) In the early subperiod, a fraction $\alpha$ of consumers (early consumers) arrive. Their payments generate immediate revenue that $M$ uses for supplier financing and wholesale market transactions. (3) Upon realization of liquidity shocks, $M$ provides working capital of $c$ to financed suppliers who experience shocks. Meanwhile, unshocked non-financed suppliers enter the wholesale market, where $M$ immediately pays $w(c)$ for their output. Using intermediate goods from both the financing and the wholesale channels, $M$ produces final goods. A portion $\alpha$ of these goods serves the demand of the early consumers, while the remainder is stored for the late consumers. (4) In the late subperiod, the remaining measure $(1-\alpha)$ of late consumers arrives and purchases from $M$. Finally, $M$ settles the precommitted payments $f(\lambda, c)$ with the financed suppliers.

## 5.1 Linear production function

To establish connections with our benchmark model, we first consider a linear production function with homogeneous intermediate goods. Since the wholesale price satisfies $w(c) < 1$, the manufacturer finds it profitable to source from all suppliers in $\Omega$. We maintain the assumption that the manufacturer has sufficient liquidity to do so.[19] Under these conditions, the manufacturer's decision for each supplier is binary: either provide financing ($q(\cdot) = 1$) or purchase through the wholesale market ($q(\cdot) = 0$). The total effective input $I$ is the sum of the intermediate goods:

$$I = \int_{\Omega} \Big( q(\lambda, c) + (1 - q(\lambda, c))(1 - \lambda) \Big) dG,$$

and the production function is $Q(I) = I$.

Each supplier can sell in the wholesale market at $w(c)$. Thus, the expected outside value is $(1 - \lambda)(w(c) - c)$. To ensure supplier participation in the financing contract, the manufacturer must give a reward $f(\lambda, c)$ at least the value of the outside option. With the supplier participation constraint, it follows that a financed supplier of $(\lambda, c)$ contributes to $M$ a profit of $\pi_F(\lambda, c) = 1 - c - (1 - \lambda)(w(c) - c) - k$. We assume the operating cost per supplier $k \in (0, 1 - \underline{c})$. The financed supplier also contributes liquidity of $\theta_F(\lambda, c) = \alpha - \lambda c$. Here, $\alpha$ captures the expected early-period revenue, and $\lambda c$ captures the expected funding request. For suppliers from whom $M$ purchases in the wholesale market, the contributions to profit and liquidity of $M$ are scaled by the probability of market participation $(1 - \lambda)$. The profit contribution is $\pi_W(\lambda, c) = (1 - \lambda)(1 - w(c))$, and the liquidity contribution is $\theta_W(\lambda, c) = (1 - \lambda)(\alpha - w(c))$.

The manufacturer's problem is to choose $q(\lambda, c)$ to maximize:

$$\int_{\Omega} \Big( q(\lambda, c)\pi_F(\lambda, c) + (1 - q(\lambda, c))\pi_W(\lambda, c) \Big) dG,$$

subject to the liquidity constraint:

$$\int_{\Omega} \Big( q(\lambda, c)\theta_F(\lambda, c) + (1 - q(\lambda, c))\theta_W(\lambda, c) \Big) dG + L \geq 0.$$

With $\mu$ denoting the multiplier on the liquidity constraint and defining $\Delta\pi \equiv \pi_F - \pi_W = \lambda(1 - c) - k$ and $\Delta\theta \equiv \theta_F - \theta_W = \lambda(\alpha - c) + (1 - \lambda)w(c)$, the optimal financing decision follows that $q(\cdot) = 1$ if and only if

$$\Delta\pi + \mu\Delta\theta \geq 0, \tag{18}$$

or $\Big(\lambda(1 - c) - k\Big) + \mu\Big(\lambda(\alpha - c) + (1 - \lambda)w(c)\Big) \geq 0$.

Let $c_{\Delta\pi}(\lambda) = 1 - k/\lambda$ be the cost threshold where $\Delta\pi(\lambda, c) = 0$, which is concave and strictly increasing in $\lambda$. Let $c_{\Delta\theta}(\lambda)$ be the threshold of $c$ such that $\Delta\theta(\lambda, c) = 0$. While $c_{\Delta\theta}(\lambda)$ depends on

[19] Formally, we assume $\int_{\Omega}(1 - \lambda)(w(c) - c)dG + L \geq 0$.

the unspecified wholesale price function $w(c)$, it is bounded by:

$$\frac{\alpha\lambda}{2\lambda-1} \equiv c^L_{\Delta\theta}(\lambda) \leq c_{\Delta\theta}(\lambda) \leq c^H_{\Delta\theta}(\lambda) \equiv \frac{1}{\lambda} - (1-\alpha),$$

where the upper and lower bounds, $c^H_{\Delta\theta}(\lambda)$ and $c^L_{\Delta\theta}(\lambda)$, correspond to $w(c) = 1$ and $w(c) = c$, respectively. And for both cases, $c_{\Delta\theta}(\lambda)$ is strictly decreasing in $\lambda$. For $\lambda \in [\frac{1}{2-\alpha}, 1]$, both bounds are non-negative, and they intersect at the interval endpoints of $\lambda = \frac{1}{2-\alpha}$ and $\lambda = 1$. In Figure 11, we plot the three bounds $c = c_{\Delta\theta}(\lambda)$ (solid blue), $c = c^L_{\Delta\theta}(\lambda)$, and $c = c^H_{\Delta\theta}(\lambda)$ (dash blue). $c_{\Delta\theta}(\lambda)$ is bounded by $c^L_{\Delta\theta}(\lambda)$ and $c^H_{\Delta\theta}(\lambda)$, and coincide at the end points.

The curves $c_{\Delta\pi}(\lambda)$ and $c_{\Delta\theta}(\lambda)$ intersect if and only if $k < 1-\alpha$, in which case there can be liquidity cross-subsidization among suppliers. Figure 11 illustrates this by plotting $c_{\Delta\pi}(\lambda)$ along with the bounds $c^L_{\Delta\theta}(\lambda)$ and $c^H_{\Delta\theta}(\lambda)$. And if we let $M$ choose the liquidity $L$ to hold, then at optimality, $\mu^* = \max\{i, \mu(0)\}$. Conversely, when $k \geq 1-\alpha$, any supplier that contributes positively to profits also contributes positively to liquidity, eliminating the need for cross-subsidization. In this case, the supplier finance program still exists and is profitable, but with $\mu^* = 0$.

**Proposition 5 (Liquidity cross-subsidization with linear production technologies)** *Given a liquidity holding $L \geq 0$, the manufacturer has a unique profit maximizing strategy: the supplier financing selection rule is $q(\lambda, c, \mu) = 1$ if and only if condition (18) holds; the reward for suppliers is $f(\lambda, c) = (1-\lambda)(w(c) - c)$; and the shadow value of liquidity, $\mu(L) \geq 0$, is uniquely determined by the binding liquidity constraint. Liquidity cross-subsidization emerges when $k < 1-\alpha$ and $\max\{i, \mu(0)\} > 0$.*

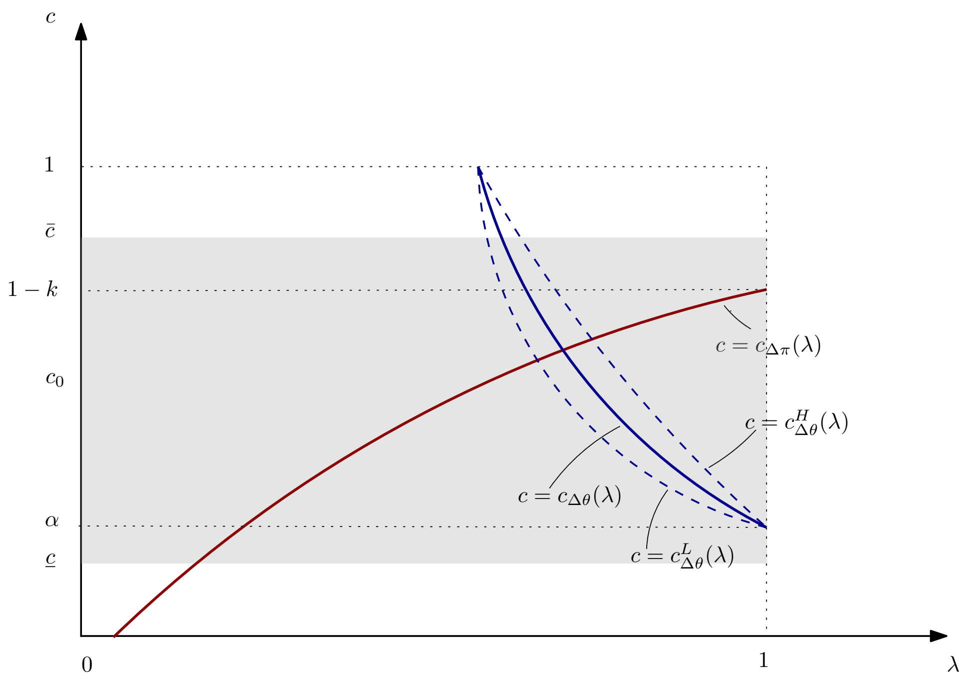



Figure 11: Manufacturer financing selection under linear production function

**Discussions: Retail demand and production efficiency.** In this model, production efficiency equals manufacturer's profit, $\int_\Omega [q(\lambda,c)\pi_F(\lambda,c) + (1-q(\lambda,c))\pi_W(\lambda,c)]dG$, since consumers are passive and suppliers earn fixed outside values. The retail market demand timing, captured by $\alpha$ - the share of early consumers - alters the efficiency of production through the supplier financing channel, determining which suppliers contribute to production.

For $\alpha < 1-k$, scarce early revenue tightens liquidity, raising $\mu$. The manufacturer then prioritizes suppliers with low liquidity needs, despite higher costs and lower profits, over efficient, low-$c$, high-$\lambda$ suppliers whose liquidity drain prevails when $\mu$ is high. As $\alpha$ nears $1-k$, liquidity eases, $\mu$ drops, and more low-$c$ suppliers are financed, boosting efficiency. Above $1-k$, with $\mu = 0$, financing targets all $c \leq c_{\Delta\theta}$, maximizing profit unconstrained. Thus, $\alpha$ tunes efficiency by steering the supplier mix.

This demand-side mechanism reveals how the retail market matching affects production efficiency. The share of early consumers ($\alpha$) shapes the manufacturer's liquidity position, which determines its supplier selection strategy. When early revenue is high, manufacturers can engage more cost-efficient suppliers despite their higher liquidity risks, thereby achieving greater production efficiency. Our explicit modeling of supplier heterogeneity is crucial for understanding this novel efficiency channel.[20]

## 5.2 General production functions

Our analysis extends to general production functions. We define $Q(x(\lambda,c))$ as a function dependent on the input strategy $x(\lambda,c) = q(\lambda,c) + (1-q(\lambda,c))(1-\lambda)$, where $q : \Omega \to \{0,1\}$ is the financing policy function, $\Omega = [0,1] \times [\underline{c}, \bar{c}]$ with $|\Omega| = 1$, and $G$ is a continuous, atomless probability measure. Let $X$ be the space of measurable $q$, and $Q : X \to \mathbb{R}_+$ be bounded and continuous in the $L^1(\Omega, G)$ topology.[21] We assume $Q$ is Fréchet differentiable, so the marginal effect of changing $x(\lambda,c)$, denoted $\frac{\delta Q}{\delta x(\lambda,c)}$, exists for almost every $(\lambda,c)$ in $\Omega$.[22] We also assume that the procurement of intermediate goods from all suppliers through the wholesale market is profitable and liquidity-feasible for the manufacturer. This simplifies our analysis, as we only need to determine whether each supplier should be sourced through the wholesale market or the financing channel.[23]

[20]For example, fast-fashion retailers like Zara leverage high early-season demand (high $\alpha$) to generate immediate revenue, enabling them to finance agile but liquidity-constrained suppliers in developing economies. These suppliers often combine low production costs with higher liquidity risk, a trade-off that becomes optimal when strong early sales provide sufficient liquidity.

[21]In the $L^1(\Omega, G)$ topology, the distance between two strategies $q$ and $q'$ is $\|q - q'\|_1 = \int_\Omega |q(\lambda,c) - q'(\lambda,c)|dG(\lambda,c)$, which measures the "size" of the supplier set where financing choices differ, weighted by $G$. Continuity ensures that if this distance is small, $Q(q)$ and $Q(q')$ are close in value.

[22]The Fréchet derivative captures the rate of change in $Q$. For a small perturbation $h(\lambda,c)$ in $x$, we have $Q(x+\epsilon h) = Q(x) + \epsilon \int_\Omega \frac{\delta Q}{\delta x(\lambda,c)} h(\lambda,c)dG + o(\epsilon)$, where $o(\epsilon)$ is a remainder that vanishes faster than $\epsilon$. Since $q$ is binary, $\frac{\delta Q}{\delta q} = \frac{\delta Q}{\delta x} \cdot \lambda$, reflecting the change from $1-\lambda$ to 1.

[23]This assumption holds if the marginal product of the intermediate good (evaluated when all suppliers contribute $x(\lambda,c) = 1-\lambda$) exceeds the highest wholesale cost $w(\bar{c})$ for almost every supplier $(\lambda,c)$. Formally,

The manufacturer seeks to maximize profits, defined as:

$$Q(x) - \int_\Omega \left[q\left(c + (1-\lambda)(w(c) - c) + k\right) + (1-q)(1-\lambda)w(c)\right] dG,$$

by optimally choosing the supplier financing selection rule $q(\lambda, c)$, subject to the liquidity constraint:

$$\Theta(x) + L = \alpha Q(x) - \int_\Omega \left[q\lambda c + (1-q)(1-\lambda)w(c)\right] dG + L \geq 0.$$

The associated Lagrangian is:

$$\mathcal{L} = (1+\mu\alpha)Q(x) - \int_\Omega q\left[\lambda c(1+\mu) + k - \mu(1-\lambda)w(c)\right] dG - \int_\Omega (1-q)(1-\lambda)w(c)(1+\mu)dG + \mu L.$$

A supplier with characteristics $(\lambda, c)$ is financed, i.e., $q(\lambda, c, \mu) = 1$, whenever:

$$\lambda\left(\frac{\delta Q}{\delta x} - c\right) - k + \mu\left[\lambda\left(\alpha\frac{\delta Q}{\delta x} - c\right) + (1-\lambda)w(c)\right] \geq 0,$$

where $\mu \geq 0$, $\Theta \geq 0$, and the complementary slackness condition $\mu\Theta = 0$ holds. Under the assumptions on the production function $Q$ outlined earlier, we demonstrate in the appendix that there exists a solution $q(\cdot) \in X$ to this profit maximization problem, satisfying the liquidity constraint.

A key insight is the emergence of liquidity cross-subsidization across suppliers. For example, if $w(c) = c$, a financed supplier contributes positively to profit when $c < c_{\Delta\pi} \equiv \frac{\delta Q}{\delta x} - \frac{k}{\lambda}$ and to liquidity when $c < c_{\Delta\theta} \equiv \frac{\alpha\lambda}{2\lambda-1}\frac{\delta Q}{\delta x}$. Extending the analysis akin to the linear production function case, liquidity cross-subsidization arises when $\alpha < 1 - \frac{k}{\frac{\delta Q}{\delta x}}$, mirroring the linear benchmark.

Notably, this general production function framework encompasses two important cases: (i) homogeneous intermediate goods with a nonlinear production technology, and (ii) heterogeneous intermediate goods aggregated via a constant elasticity of substitution (CES) production function.

**Example 3 (Nonlinear production function with homogeneous intermediate goods)** *Let $I = \int_\Omega [q + (1-q)(1-\lambda)]dG$ represent the measure of intermediate goods supplied to M. The final output, $Q(I)$, is a smooth, concave function of I, with $Q'(|\Omega|) > w(\bar{c})$ ensuring a sufficiently large marginal product even when all suppliers are included. The manufacturer solves:*

$$\max_{q(\cdot)\in\{0,1\}} Q(I) - \int_\Omega \left[q(\lambda,c)\left(c + (1-\lambda)(w(c) - c) + k\right) + (1-q(\lambda,c))(1-\lambda)w(c)\right] dG,$$

*subject to $\Theta(I, q) + L \geq 0$, where the pooled liquidity is:*

$$\Theta(I, q) = \alpha Q(I) - \int_\Omega [q(\lambda,c)\lambda c + (1-q(\lambda,c))(1-\lambda)w(c)]dG.$$

*By forming the Lagrangian and applying pointwise optimization, we obtain that the optimal financing*

---

$\left.\frac{\delta Q}{\delta x(\lambda,c)}\right|_{x(\lambda,c)=1-\lambda \text{ for all } (\lambda,c)} > w(\bar{c}).$

*decision follows $q(\lambda, c) = 1$ if and only if:*

$$\lambda\Big(Q'(I) - c\Big) - k + \mu\Big[\lambda\Big(\alpha Q'(I) - c\Big) + (1-\lambda)w(c)\Big] \geq 0.$$

*Suppose $w(c) = c$, a financed supplier contributes positively to profit when $c < c_{\Delta\pi} \equiv Q'(I) - k/\lambda$ and to liquidity when $c < c_{\Delta\theta} \equiv Q'(I)\alpha\lambda/(2\lambda - 1)$. Thus, liquidity cross-subsidization can occur if $\alpha < 1 - k/Q'(I)$.*

**Example 4 (CES production function with heterogeneous intermediate goods)** *The manufacturer aggregates effective inputs $x(\lambda, c)$ from a continuum of suppliers via*

$$Q = \left(\int_\Omega x(\lambda, c)^\rho dG\right)^{\frac{1}{\rho}}, \quad \rho < 1,$$

*where $x(\lambda, c) = q(\lambda, c) + (1 - q(\lambda, c))(1 - \lambda)$. Suppose it is profitable to include all suppliers.*[24] *The manufacturer solves the problem of choosing $q(\cdot) \in \{0, 1\}$ to maximize profits:*

$$\Pi = Q - \int_\Omega \Big[q(\lambda, c)\Big(c + (1-\lambda)(w(c) - c) + k\Big) + \Big(1 - q(\lambda, c)\Big)(1-\lambda)w(c)\Big]dG,$$

*subject to $\Theta + L \geq 0$, where the pooled liquidity is: $\Theta = \alpha Q - \int_\Omega [q(\lambda, c)\lambda c + (1 - q(\lambda, c))(1 - \lambda)w(c)]dG$. Forming the Lagrangian:*

$$\mathcal{L} = Q - \int_\Omega q(\lambda, c)\Big(\lambda c + k\Big)dG + \mu\Big[\alpha Q - \int_\Omega \Big[q(\lambda, c)\Big(\lambda c - (1-\lambda)w(c)\Big)\Big]dG.$$

*Noting $x(\lambda, c) = 1$ if $q = 1$, or $1 - \lambda$ if $q = 0$, the change of $Q$ when $q(\lambda, c)$ switches from $0$ to $1$ is $\lambda Q^{1-\rho} g(\lambda, c)$, where $Q^{1-\rho} g(\lambda, c) = \partial Q / \partial x(\lambda, c)$ at $x = 1$. Thus, the marginal impact of switching from $q = 0$ to $q = 1$ on profits is:*

$$\lambda\Big(Q^{1-\rho} - c\Big) - k + \mu\Big[\lambda\Big(\alpha Q^{1-\rho} - c\Big) + (1-\lambda)w(c)\Big].$$

*And $q(\lambda, c) = 1$ if the impact above is non-negative. Liquidity cross-subsidization mirrors the general case, occurring when $\alpha < 1 - k/Q^{1-\rho}$ for some suppliers.*

# 6 Conclusion

This paper develops a tractable model of supplier finance, capturing its key features: supplier selection, liquidity pooling, and advance payments to liquidity-constrained suppliers, all enabled by digital finance innovations. Our analysis highlights the crucial role of liquidity cross-subsidization for the viability and welfare implications of supplier finance programs. We demon-

[24] A sufficient condition for the manufacturer to profitably source all suppliers in $\Omega = [0, 1] \times [\underline{c}, \bar{c}]$ via the wholesale market is $Q_0^{1-\rho} > w(\bar{c})$, where $Q_0 = \left(\int_\Omega (1-\lambda)^\rho g(\lambda, c) d\lambda dc\right)^{\frac{1}{\rho}}$ is the output when all $q(\lambda, c) = 0$. The marginal product per unit of $x(\lambda, c) = 1 - \lambda$ is $Q^{1-\rho}(1-\lambda)^{\rho-1}$, exceeding the marginal cost $w(c)$ if $Q^{1-\rho}(1-\lambda)^{\rho-1} > w(c)$. Since $Q \geq Q_0$, $w(c) \leq w(\bar{c})$, and $(1-\lambda)^{\rho-1} \geq 1$ for $\rho < 1$, $Q_0^{1-\rho} > w(\bar{c})$ ensures this holds for all $(\lambda, c)$. Furthermore, for wholesale market sourcing to be liquidity feasible, we require $\alpha Q_0 - \int_\Omega (1-\lambda)w(c)dG + L \geq 0$.

strate how the intermediary's funding costs influence the trade-off between liquidity provision and profitability, showing that surprisingly, higher funding costs can even enhance social welfare. Furthermore, we examine the interplay between supplier finance and suppliers' direct access to money markets, illustrating how the financing costs faced by suppliers shape the equilibrium allocation of liquidity. Our results are robust to a wide range of extensions, and the model also applies to supplier finance in the manufacturing industry.

Intermediaries operating supplier finance programs often possess an informational advantage over traditional financial institutions, such as commercial banks, due to their established industry presence and long-term relationships with suppliers. Our model, based on the assumption of perfect information, provides a tractable framework for understanding supplier finance. Future research could fruitfully extend this model to incorporate information asymmetry.

## References


ANDERSON, S., AND BEDRE-DEFOLIE, Ö. "Online Trade Platforms: Hosting, Selling, or Both?" *International Journal of Industrial Organization*, Vol. 84 (2022), 102–861.

BIGLAISER, G. "Middlemen as Experts." *The RAND Journal of Economics*, Vol. 24 (1993), 212–223.

BOTTAZZI, L., GOPALAKRISHNA, G., AND TEBALDI, C. "Supply Chain Finance and Firm Captial Structure." CEPR Discussion Paper DP18468, 2023.

BOUALAM, Y., AND YOO, P. "Fintech Disruption, Banks, and Credit (Dis-) Intermediation: When Do Foes Become Friends?", Working paper, 2022.

BURKART, M., AND ELLINGSEN, T. "In-Kind Finance: A Theory of Trade Credit." *American Economic Review*, Vol. 94 (2004), 569–590.

CAMERA, G. "Search, Dealers, and the Terms of Trade." *Review of Economic Dynamics*, Vol. 4 (2001), 680–694.

CUÑAT, V. "Trade Credit: Suppliers as Debt Collectors and Insurance Providers." *The Review of Financial Studies*, Vol. 20 (2007), 491–527.

DEVALKAR, S. K., AND KRISHNAN, H. "The Impact of Working Capital Financing Costs on the Efficiency of Trade Credit." *Production and Operations Management*, Vol. 28 (2019), 878–889.

DIAMOND, D. W., AND DYBVIG, P. H. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, Vol. 91 (1983), 401–419.

DONALDSON, J. R., PIACENTINO, G., AND THAKOR, A. "Warehouse Banking." *Journal of Financial Economics*, Vol. 129 (2018), 250–267.

DONG, L., QIU, Y., AND XU, F. "Blockchain-Enabled Deep-Tier Supply Chain Finance." *Manufacturing  Service Operations Management*, Vol. 24, jun (2022), 2871–2887.

DONG, M. "Inflation and Variety." *International Economic Review*, Vol. 51 (2010), 401–420.

ETRO, F. "Device-funded vs Ad-funded Platforms." *International Journal of Industrial Organization*, Vol. 75 (2021a), 102–711.

——— "Product Selection in Online Marketplaces." *Journal of Economics & Management Strategy*, Vol. 30 (2021b), 614–637.

——— "Hybrid Marketplaces with Free Entry of Sellers." *Review of Industrial Organization*, Vol. 62 (2023), 119–148.

——— "E-commerce Platforms and Self-preferencing." *Journal of Economic Surveys*, Vol. 38 (2024), 1516–1543.

GARCIA-APPENDINI, E., AND MONTORIOL-GARRIGA, J. "Firms as Liquidity Providers: Evidence from the 2007–2008 Financial Crisis." *Journal of Financial Economics*, Vol. 109 (2013), 272–291.

GAUTIER, P., HU, B., AND WATANABE, M. "Marketmaking Middlemen." *The RAND Journal of Economics*, Vol. 54 (2023), 83–103.

GIANNETTI, M., BURKART, M., AND ELLINGSEN, T. "What You Sell Is What You Lend? Explaining Trade Credit Contracts." *The Review of Financial Studies*, Vol. 24 (2011), 1261–1298.

HAGIU, A., TEH, T.-H., AND WRIGHT, J. "Should Platforms be Allowed to Sell on Their Own Marketplaces?" *The RAND Journal of Economics*, Vol. 53 (2022), 297–327.

HE, Z., AND LI, J. "Intermediation via Credit Chains." Working paper, 2025.

HUANG, J. "The Economics of Consumer Credit on an E-Commerce Platform." Working paper, 2021.

JOHRI, A., AND LEACH, J. "Middlemen and the Allocation of Heterogeneous Goods." *International Economic Review*, Vol. 43 (2002), 347–361.

KANG, Z. Y., AND MUIR, E. V. "Contracting and Vertical Control by a Dominant Platform." Working paper, 2022.

KASHYAP, A. K., RAJAN, R., AND STEIN, J. C. "Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-taking." *The Journal of Finance*, Vol. 57 (2002), 33–73.

KLAPPER, L., LAEVEN, L., AND RAJAN, R. "Trade Credit Contracts." *The Review of Financial Studies*, Vol. 25 (2012), 838–867.

KOUVELIS, P., AND XU, F. "A Supply Chain Theory of Factoring and Reverse Factoring." *Management Science*, Vol. 67 (2021), 6071–6088.

LI, F., MURRY, C., TIAN, C., AND ZHOU, Y. "Inventory Management in Decentralized Markets." *International Economic Review*, Vol. 65 (2024), 431–470.

LI, J., AND PEGORARO, S. "Borrowing from a Bigtech Platform." Working paper, 2022.

LI, Y. "Middlemen and Private Information." *Journal of Monetary Economics*, Vol. 42 (1998), 131–159.

MADSEN, E., AND VELLODI, N. "Insider Imitation." SSRN working paper 3832712, 2023.

NOCKE, V., AND THANASSOULIS, J. "Vertical Relations under Credit Constraints." *Journal of the European Economic Association*, Vol. 12 (2014), 337–367.

NOSAL, E., WONG, Y.-Y., AND WRIGHT, R. "More on Middlemen: Equilibrium Entry and Efficiency in Intermediated Markets." *Journal of Money, Credit and Banking*, Vol. 47 (2015), 7–37.

PADILLA, J., PERKINS, J., AND PICCOLO, S. "Self-Preferencing in Markets with Vertically Integrated Gatekeeper Platforms." *The Journal of Industrial Economics*, Vol. 70 (2022), 371–395.

PETERSEN, M. A., AND RAJAN, R. G. "Trade Credit: Theories and Evidence." *The Review of Financial Studies*, Vol. 10 (1997), 661–691.

RHODES, A., WATANABE, M., AND ZHOU, J. "Multiproduct Intermediaries." *Journal of Political Economy*, Vol. 129 (2021), 421–464.

RUBINSTEIN, A., AND WOLINSKY, A. "Middlemen." *Quarterly Journal of Economics*, Vol. 102 (1987), 581–593.

SHEVCHENKO, A. "Middlemen." *International Economic Review*, Vol. 45 (2004), 1–24.

SHOPOVA, R. "Private Labels in Marketplaces." *International Journal of Industrial Organization*, Vol. 89 (2023), 102–949.

SMITH, E. "Intermediated Search." *Economica*, Vol. 71 (2004), 619–636.

TIROLE, J., AND BISCEGLIA, M. "Fair Gatekeeping in Digital Ecosystems." TSE Working Paper, 2023.

TUNCA, T. I., AND ZHU, W. "Buyer Intermediation in Supplier Finance." *Management Science*, Vol. 64 (2017), 5631–5650.

WATANABE, M. "A Model of Merchants." *Journal of Economic Theory*, Vol. 145 (2010), 1865–1889.

——— "Middlemen: The Visible Market-Makers." *The Japanese Economic Review*, Vol. 69 (2018), 156–170.

——— "Middlemen: A Directed Search Equilibrium Approach." *The BE Journal of Macroeconomics (Advanced)*, Vol. 20 (2020), 1–37.

YAN, X., CHEN, L., AND DING, X. "Optimal Cash Management with Payables Finance." *Operations Research*, jun (2024).

ZENNYO, Y. "Platform Encroachment and Own-content Bias." *The Journal of Industrial Economics*, Vol. 70 (2022), 684–710.

# A Appendix

## A.1 Proof of Lemma 1

The intersection point of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$ is $(\lambda_0, c_0) = \left(\frac{k+\sqrt{k^2+4ku}}{2mu},\ u + k - \sqrt{k^2 + 4ku}\right)$. We can derive that

$$\frac{\partial b(\lambda, \mu)}{\partial \mu} = \frac{2\left(km\lambda + k - u(m\lambda)^2\right)}{(m\lambda\mu + m\lambda + \mu)^2},$$

which is positive if $\lambda < \lambda_0$ and negative if $\lambda > \lambda_0$. That is, as $\mu$ increases, $c = b(\lambda, \mu)$ rotates around $(\lambda_0, c_0)$ clockwise, which implies that more suppliers with positive $\theta_F$ are selected (i.e. $q(\lambda, c, \mu)$ is increasing in $\mu$ for $(\lambda, c)$ such that $\theta_F(\lambda, c) > 0$) and fewer suppliers with negative $\theta_F$ are selected (i.e. $q(\lambda, c, \mu)$ is decreasing in $\mu$ for $(\lambda, c)$ such that $\theta_F(\lambda, c) < 0$).

Taking $\bar{c}$ as given. If $c_0 \in [\underline{c}, \bar{c}]$, since $g(\cdot)$ is everywhere positive in $\Omega$, it holds that $\Theta(\mu, i) = \int_\Omega q(\lambda, c, \mu)\theta_F(\lambda, c)dG$ is strictly increasing in $\mu$.

If $c_0 > \bar{c}$, and suppose $\lambda_0 < 1$, then there exist unique threshold values, denoted by $\underline{\mu} > 0$ and $\bar{\mu} \in (\underline{\mu}, \infty)$, such that the curve of $c = b(\lambda, \mu)$ lies entirely above $c = \bar{c}$ for $\mu \in (\underline{\mu}, \bar{\mu})$, see Figure 12. For $\mu \in (\underline{\mu}, \bar{\mu})$, $\Theta = \int_\Omega \theta_F(\lambda, c)dG$ is independent of $\mu$, which means that $\mu$ does not influence the selection of suppliers. For $\mu \in (0, \underline{\mu}) \cup (\bar{\mu}, \infty)$, by the same logic as shown above, $\Theta(\mu)$ is strictly increasing in $\mu$. Suppse $\lambda_0 \geq 1$, then $\Theta(\mu)$ is strictly increasing in $\mu$ for $\mu \in (0, \underline{\mu})$ and keeps constant for $\mu > \underline{\mu}$.

Common to all cases is that when $\mu$ approaches infinity, only suppliers with positive $\theta_F$ are selected, thus $\Theta(\infty) > 0$.

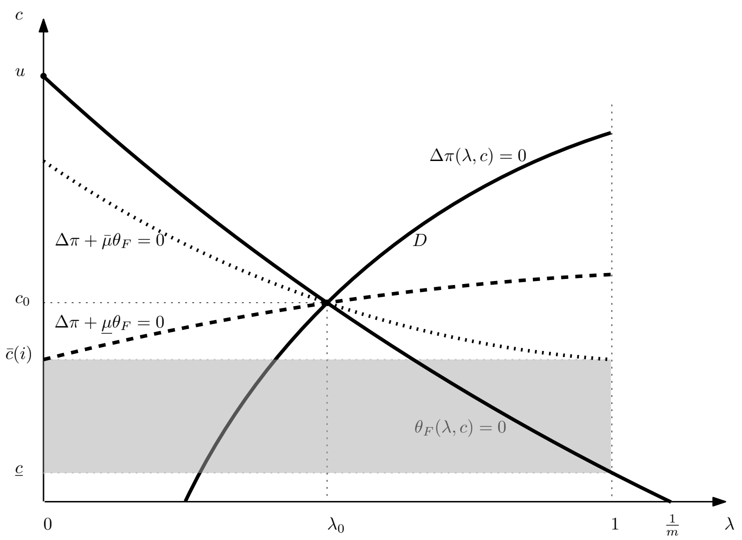



Figure 12: For $\mu \in (\underline{\mu}, \bar{\mu})$, $c = b(\lambda, \mu)$ lies above $\bar{c}$.

Now we show that $\mu$ is generically unique. Since $\Theta(\mu)$ is monotonically increasing in $\mu$, if $\Theta(0) + L \geq 0$, then $\mu = 0$. If $\Theta(0) + L < 0$, then the liquidity constraint is binding at some $\mu \in (0, \infty)$, which is uniquely pinned down by $\Theta(\mu) + L = 0$. Note that when $L = -\int_\Omega \theta_F(\lambda, c)dG \geq 0$ and $c_0 > \bar{c}$, any $\mu \in [\underline{\mu}, \bar{\mu}]$ satisfies $\Theta(\mu) + L = 0$. In this case, we define the solution $\mu$ as $\underline{\mu}$. ■

## A.2 Proof of Corollary 1

Given that $\Theta(0) + L < 0$, $\mu(L)$ is determined by (12). Since $\Theta(\mu)$ is strictly increasing in $\mu$ outside the interval $[\underline{\mu}, \bar{\mu}]$ provided the range exists and also note that we have selected $\mu = \underline{\mu}$ if $\Theta(\underline{\mu}) = \Theta(\bar{\mu}, i) = L$, the statement follows. ■

## A.3 Proof of Proposition 1

By the Euler equation (13), there are two cases. First, if $i \geq \mu(0)$, then $L = 0$. This case is valid either if $\mu(0) = 0$ (then $i > \mu = 0$ follows), or if $\mu(0) > 0$. Second, $i = \mu(L) > 0$ and $L > 0$, which requires that $\Theta(0) < 0$ and $i \leq \mu(0, i)$. ■

## A.4 Proof of Lemma 3

Given that $b'_\lambda(\lambda, i) = \frac{2m(k+ik-i^2u)}{(i+m\lambda+im\lambda)^2}$, it is straightforward to verify that $b'_\lambda(\cdot) > 0$ if $i < i_0 \equiv \frac{k+\sqrt{k^2+4uk}}{2u}$, and $b'_\lambda(\cdot) < 0$ if $i > i_0$.■

## A.5 Proof of Lemma 4

Let $Ratio(m) \equiv -\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)} = -\frac{m\lambda(u-c)/2-k}{(1-m\lambda)(u+c)/2-c}$. Then the derivative $Ratio'(m)$ has the same sign as $c^2 - 2(k+u)c + u(u-2k)$. It follows that $Ratio'(m) < 0$ if $c < c_0$, and $Ratio'(m) > 0$ if $c > c_0$.■

## A.6 Proof of Proposition 3

Note that $k < \bar{k}$ implies $c_0 > \underline{c}$. Let $\kappa \equiv \frac{k}{u}$ and $\bar{\kappa} \equiv \frac{\bar{k}}{u} \equiv \frac{(u-\underline{c})^2}{2u(u+\underline{c})}$. For all $i \leq i_1$, the multiplier $\mu$ at $L = 0$ does not depend on $i$: $\mu(0, i) = \mu(0, 0)$. We suppose the latter is positive throughout the proof, thus $\mu(i) = i$. A necessary condition of $\mu(0, 0) > 0$ is $m > \tilde{m}(\kappa)$. We focus on the parameter space where $\kappa \in (0, \bar{\kappa})$, and $m \in (\tilde{m}(\kappa), 1]$ for each value of $\kappa$.

There are two cases for $\kappa < \bar{\kappa}$. One case is that $\kappa \leq \underline{\kappa} \equiv \frac{(u-\bar{c})^2}{2u(u+\bar{c})} (< \bar{\kappa})$, then $c_0 \geq \bar{c}$. A marginal increase of $i$ from $i = 0$ will select more suppliers into the finance contract and drop no suppliers from it. Thus, social welfare must be improved.

The other case is that $\kappa \in (\underline{\kappa}, \bar{\kappa})$, namely, $c_0 \in (\underline{c}, \bar{c})$. Then the middleman's selection rule for the finance contract is $q(\lambda, c, \mu) = 1$ iff $c \in [\underline{c}, b(\lambda, \mu)]$ whenever $b(\lambda, \mu) \geq \underline{c}$. Here, $b(\lambda, \mu) = \frac{m\lambda u - 2k + \mu(1-m\lambda)u}{m\lambda + \mu(1+m\lambda)}$. The welfare gain by having active middleman finance is

$$\Delta\mathcal{W}(\mu) = \int_{\lambda_l(\mu)}^{\lambda_h(\mu)} \int_{\underline{c}}^{b(\lambda,\mu)} \Big(m\lambda(u-c) - k\Big) g(\lambda, c) dc d\lambda + \int_{\lambda_h(\mu)}^{1} \int_{\underline{c}}^{\bar{c}} \Big(m\lambda(u-c) - k\Big) g(\lambda, c) dc d\lambda,$$

where we have imposed that $b(\lambda, \mu)$ is upward sloping with respect to $\lambda$, which is always the case when $\mu$ is in the neighborhood of $\mu = 0$. Here, $\lambda_h(\mu) = \min\{1, \frac{1}{m} \frac{2k-\mu(u-\bar{c})}{(u-\bar{c})-\mu(u+\bar{c})}\}$, and $\lambda_l(\mu) = \max\{0, \frac{1}{m} \frac{2k-\mu(u-\underline{c})}{u-\underline{c}-\mu(u+\underline{c})}\}$.

Observe that $\frac{\partial \Delta \mathcal{W}(\mu)}{\partial \mu} = \int_{\lambda_l(\mu)}^{\lambda_h(\mu)} \Big(m\lambda(u-b(\lambda,\mu))-k\Big) g(\lambda, b(\mu)) b'_\mu(\lambda,\mu) d\lambda$. Since $(\lambda, c)$ follows a uniform distribution, $g$ is a constant. Let $\propto$ represent "proportional to". Inserting $b(\lambda, 0) = u - \frac{2k}{m\lambda}$ and $b'(0) = 2u\left(\frac{\kappa}{m^2\lambda^2} + \frac{\kappa}{m\lambda} - 1\right)$, we have

$$\frac{\partial \Delta \mathcal{W}(\mu)}{\partial \mu}|_{\mu=0} \propto \int_{\lambda_l}^{\lambda_h} \Big[\frac{\kappa}{m^2\lambda^2} + \frac{\kappa}{m\lambda} - 1\Big] d\lambda, \tag{19}$$

where $\lambda_h \equiv \lambda_h(0) = \min\{1, \frac{1}{m}\frac{u\kappa}{(u-\bar{c})/2}\}$, and $\lambda_l \equiv \lambda_l(0) = \frac{1}{m}\frac{u\kappa}{(u-\underline{c})/2}$. Note that $\lambda_l < 1$. This is because with $m > \tilde{m}$, $c_\pi(1) > c_0 > \underline{c}$ where the second inequality is given by $\kappa < \bar{\kappa}$. And $c_\pi(1) > \underline{c}$ is equivalent to $\lambda_l < 1$. Define $\bar{\varepsilon} = \frac{u}{(u-\bar{c})/2}$ and $\underline{\varepsilon} = \frac{u}{(u-\underline{c})/2}$. It holds that $\bar{\varepsilon} > \underline{\varepsilon} > 2$. Furthermore, it is straightforward to check that $\underline{\kappa} < \frac{1}{\bar{\varepsilon}} < \bar{\kappa} < \frac{1}{\underline{\varepsilon}}$. With this, $\lambda_h = \min\{1, \frac{1}{m}\kappa\bar{\varepsilon}\}$, $\lambda_l = \frac{1}{m}\kappa\underline{\varepsilon}$.

Substitute for $\lambda$ by $x \equiv m\lambda$. Then the upper and lower bounds become $x_h = \min\{m, \kappa\bar{\varepsilon}\}$, and $x_l = \kappa\underline{\varepsilon}$, and we have

$$\frac{\partial \Delta \mathcal{W}(\mu)}{\partial \mu}|_{\mu=0} \propto \int_{\kappa\underline{\varepsilon}}^{\min\{m,\kappa\bar{\varepsilon}\}} \Big[\frac{\kappa}{x^2} + \frac{\kappa}{x} - 1\Big] dx \equiv H(m, \kappa). \tag{20}$$

In the following, we look for the parameter space for $H(m,\kappa) > 0$.

Suppose $\bar{\varepsilon}\kappa < m$. Then, by (20) $H(m,\kappa)$ does not depend on $m$ directly:

$$H(m,\kappa) = -\kappa(\bar{\varepsilon} - \underline{\varepsilon}) + \frac{\bar{\varepsilon} - \underline{\varepsilon}}{\bar{\varepsilon}\underline{\varepsilon}} + \kappa\Big(\log(\bar{\varepsilon}) - \log(\underline{\varepsilon})\Big),$$

which is positive iff $\kappa < \frac{1}{\bar{\varepsilon}\underline{\varepsilon}} \frac{1}{1 - \frac{\log(\bar{\varepsilon}) - \log(\underline{\varepsilon})}{\bar{\varepsilon} - \underline{\varepsilon}}} \equiv \kappa^*$.

Suppose $\bar{\varepsilon}\kappa \geq m$, then

$$H(m,\kappa) = \int_{\kappa\underline{\varepsilon}}^{m} \Big[\frac{\kappa}{x^2} + \frac{\kappa}{x} - 1\Big] dx.$$

In this case, $H(m,\kappa)$ has the following properties:

- $H(m,\kappa)$ is strictly decreasing in $m$ for $m \in [\tilde{m}, 1]$ since $\frac{\partial H(m,\kappa)}{\partial m} = \frac{\kappa + \kappa m - m^2}{m^2} < 0$ for $m > \tilde{m}$.
- $H(\tilde{m}(\kappa), \kappa) \geq 0$ since $\frac{\kappa}{x^2} + \frac{\kappa}{x} - 1 \geq 0$ for $x \leq \tilde{m}$. $H(\tilde{m}(\kappa), \kappa) = 0$ only if $\kappa = \bar{\kappa}$.
- $H(1,\kappa) < 0$. To see this, using (20)

$$H(1,\kappa) = \Big[-\lambda + \kappa\Big(\log(\lambda) - 1/\lambda\Big)\Big]_{\underline{\lambda}}^{1} = (\underline{\varepsilon} - 1)(\kappa - \frac{1}{\underline{\varepsilon}}) - \kappa\log(\kappa\underline{\varepsilon}) \equiv h(\kappa).$$

  Note that $h'(\kappa) = \underline{\varepsilon} - 2 - \log(\underline{\varepsilon}\kappa) > 0$ since $\underline{\varepsilon} > 2$ and $\kappa\underline{\varepsilon} = \underline{\lambda} < 1$. Then $h(\kappa) < h(\bar{\kappa}) < h(1/\underline{\varepsilon}) = 0$ (since $\bar{\kappa} < \frac{1}{\underline{\varepsilon}}$). Thus, $H(1,\kappa) < 0$.

These properties together imply that there must exist $m^*(\kappa) \in [\tilde{m}(\kappa), 1)$ such that $H(m,\kappa) > 0$ iff $m < m^*(\kappa)$.

Figure 13 illustrates the parameter space of $(\kappa, m)$ where $m = \tilde{m}(\kappa)$ (upward concave curve) and $m = \bar{\varepsilon}\kappa$ (upward straight line) are plotted. As argued above, we shall focus on $m > \tilde{m}(\kappa)$ since only in that case it is plausible that $\mu(0,0) > 0$. Whether $m$ larger or smaller than $\bar{\varepsilon}\kappa$ (the

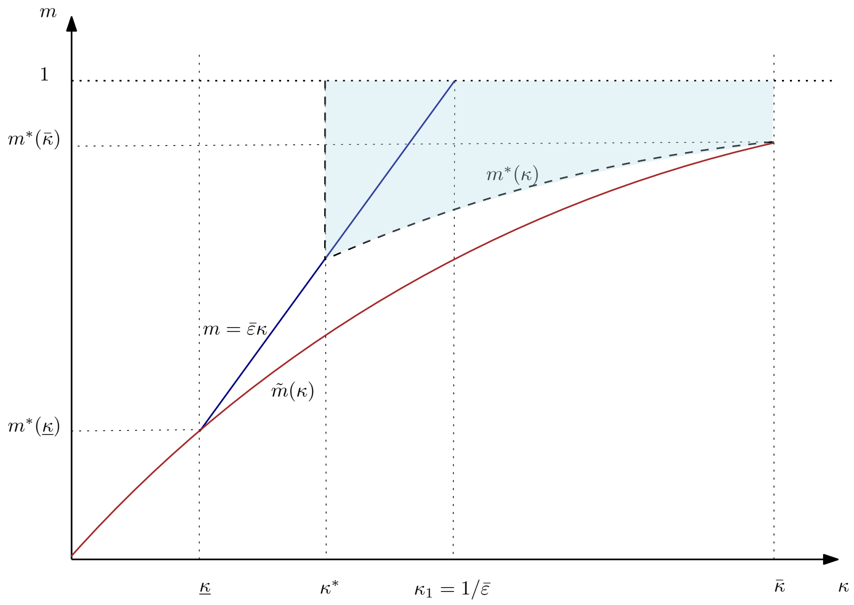



Figure 13: The parameter space that determines the sign of $H(m,\kappa)$ (in shaded region $H(\cdot) < 0$.)

upward straight line) determines the upper bound of the integral in $H(m,\kappa)$.

In this figure, we have applied the following properties of $m = \tilde{m}(\kappa)$: (1) $\tilde{m}(0) = 0$, and $\tilde{m}(\bar{\kappa}) < 1$; (2) It is increasing and concave on $\kappa \in [0,\bar{\kappa}]$: $\tilde{m}'(\kappa) = \frac{1}{2}\left(1 + \frac{\kappa+2}{\sqrt{\kappa^2+4\kappa}}\right) > 0$, $\tilde{m}''(\kappa) = -\frac{2}{(4\kappa+\kappa^2)^{3/2}} < 0$. We also applied the following properties of $m = \bar{\varepsilon}\kappa$: $\bar{\varepsilon}\bar{\kappa} > 1$, and $\bar{\varepsilon}\underline{\kappa} = \tilde{m}(\underline{\kappa})$.

First consider the case of $m > \bar{\varepsilon}\kappa$, then $H(m,\kappa) > 0$ iff $\kappa < \kappa^*$. Define $\kappa_1 = \frac{1}{\bar{\varepsilon}}$ the value of $\kappa$ where $m = \bar{\varepsilon}\kappa$ intersects with $m = 1$. $\kappa^*$ must lie between $\underline{\kappa}$ and $\kappa_1$. $\kappa^* > \underline{\kappa}$ because $H(1,\underline{\kappa}) > 0$ (when $\kappa = \underline{\kappa}$, we have $c_0 = \bar{c}$ and a marginal increase in $i$ only selected more suppliers into the finance contract without dropping suppliers out of it.) $\kappa^* < \kappa_1$ because $H(1,\kappa_1) < 0$.

Second, consider the case where $m < \bar{\varepsilon}\kappa$. In this scenario, $H(m,\kappa) > 0$ if and only if $m < m^*(\kappa)$. As shown in the figure, $m^*(\kappa)$ connects $(\bar{\kappa}, \tilde{m}(\bar{\kappa}))$ at one end and $(\kappa^*, \bar{\varepsilon}\kappa^*)$ at the other end. We have $m^*(\bar{\kappa}) = \tilde{m}(\bar{\kappa})$ because when $\kappa = \bar{\kappa}$ and $m = \tilde{m}(\bar{\kappa})$, $\Delta\mathcal{W}(\mu)$ is constantly zero and independent of $\mu$. By definition of $\kappa^*$, $H(m^*(\kappa^*),\kappa^*) = 0$. ■

## A.7 Proof of Proposition 4

Let $\Pi(i^s, i) \equiv \int_{\tilde{\Omega}(i^s)} q(\lambda,c)\Delta\pi(\lambda,c)dG$ be the maximized profits of the intermediary from activating the finance service taking nominal interest rate $i < i_1$ as given. Let $c_{\Delta\pi}(\lambda) = u - \frac{2k}{m\lambda}$ denote the curve of $(\lambda, c)$ such that $\Delta\pi(\lambda,c) = 0$. It can be shown that $c^s(\lambda, i^s)$ and $c_{\Delta\pi}(\lambda)$ cross each other at most once.

If $c^s(1,i^s) > c_{\Delta\pi}(1)$, or equivalently, $i^s < \frac{k}{mu-2k}$, then $c^s(\lambda,i^s) > c_{\Delta\pi}(\lambda)$ for all $\lambda \in [0,1]$, meaning that all suppliers with positive profits $\Delta\pi(\lambda,c)$ are excluded from $\tilde{\Omega}(i^s)$. Thus, we must have $\Pi(i^s,i) = 0$. On the other hand, if $i^s \geq \bar{i}^s \equiv \frac{u-\underline{c}}{2\underline{c}}$, then $\tilde{\Omega}(i^s) = \Omega$, resulting in $\Pi(i^s,i) > 0$. Note that $\lambda_0 < 1$ implies $c_{\Delta\pi}(1) > \underline{c}$, which is equivalent to $\bar{i}^s > \frac{k}{mu-2k}$.

Finally, $\Pi(\cdot)$ is weakly increasing in $i^s$, because as $i^s$ increases, the set of feasible suppliers $\tilde{\Omega}(i^s)$ becomes larger. Therefore, $\underline{i}^s \in [\frac{k}{mu-2k}, \bar{i}^s)$ must exist. Combined with the suppliers' money-holding decision rule (see condition (17) in the main text), this proves the claims in the proposition. ■

## A.8 Proof of the existence of solutions in Section 5.2

We apply Brouwer's fixed-point Theorem to show that a solution exists. First, take $\mu \in [0, \bar{\mu}]$, where $\bar{\mu} > 0$ is an upper bound ensuring $\mu\Theta = 0$. Since $Q$ is bounded and each supplier's profit contribution is finite, such a $\bar{\mu} < \infty$ exists. Define a mapping $T : [0, \bar{\mu}] \to [0, \bar{\mu}]$: (1) For each $\mu$, set $q(\lambda, c) = 1$ if $(1 + \mu\alpha)\frac{\delta Q}{\delta q} > \lambda c(1 + \mu) + k - \mu(1 - \lambda)w(c)$ (finance), else $q = 0$ (wholesale). (2) Compute $\Theta = \alpha Q(q) - \int [q\lambda c + (1 - q)(1 - \lambda)w(c)]dG$. (3) Set $T(\mu) = 0$ if $\Theta + L \geq 0$; otherwise, set $T(\mu) = \min\{\mu + \beta|\Theta + L|, \bar{\mu}\}$, where $\beta > 0$ is small. $T$ maps $[0, \bar{\mu}]$ to itself and is continuous: $Q$ is continuous in $q$ under the $L^1(\Omega, G)$ topology, and $G$'s atomless nature ensures $\Theta$ adjusts gradually with $\mu$. By Brouwer's theorem, there exists $\mu^* = T(\mu^*)$. Then, if $\mu^* = 0$, $\Theta + L \geq 0$; if $\mu^* > 0$, $\Theta + L = 0$. The corresponding $q^*$ satisfies the decision rule and the liquidity constraint $\Theta + L \geq 0$. ■