A Model of Supplier Finance*

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Abstract

This article develops a theoretical model of supplier finance, where an intermediary (e.g., a large buyer firm) pools trade credit and stands ready to provide liquidity to her suppliers. We show that the optimal supplier finance involves cross-subsidization of liquidity among selected suppliers who differ in profitability and financial health. Surprisingly, we find that the number of financed suppliers can actually expand—which is welfare improving—even when the financing becomes more costly for the intermediary. Intermediation and liquidity provision can be complements or substitutes, depending on the financing cost.

Keywords: *Supplier Finance*, *Liquidity Pooling*, *Trade Credit*, *Liquidity Cross-subsidization*, *Intermediary*

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1 Introduction

Trade finance has a long history, dating back to at least the 14th century. In recent years, the rise of fintech innovation and increased financial accessibility have fueled the popularity of a new type of trade finance known as "supplier finance" (also known as "supply chain finance" or "reverse factoring"). At its core, supplier finance involves a buyer firm—often a large, creditworthy enterprise—partnering with a financial provider to offer suppliers the option to *receive early payment* for confirmed invoices, while the buyer retains *extended payment terms.* This arrangement not only optimizes working capital for buyers but also provides suppliers with critical liquidity.

The adoption of supplier finance has surged among major corporations across diverse industries. This growth has been further propelled by specialized digital platforms—including Taulia, C2FO, PrimeRevenue, and Tradeshift—that streamline and scale supplier finance operations. According to a 2019 PwC survey, 68% of companies in Europe and North America now utilize supplier finance. The global corporate supplier finance market was valued at $1.8 trillion in 2021 (BCR Publishing Ltd.), comparable in size to the global private equity market ($4.1 trillion, Preqin) and rapidly approaching the scale of the global securitization market ($2.3 trillion, SIFMA). Industry estimates from McKinsey & Company and PwC suggest double-digit growth, driven by increased adoption and technological advancements.

Despite its widespread adoption and huge success in business, supplier finance has received limited attention in the economics literature. To fill in the gap, we develop a theoretical framework of supplier finance and model the buyer enterprise, e.g. Walmart, as a retailer intermediary that connects suppliers and final consumers. Our model captures four essential features of real-world supplier finance programs: (1) A large buyer firm, acting as an intermediary, initiates the program and selects suppliers from a diverse pool. (2) The selection aims to enhance not only profitability but also the financial health of the whole buyer-supplier network. (3) Suppliers are required to extend trade credit to the buyer, allowing the latter to defer payments for a significant period. (4) Participating suppliers gain access to early payment options, providing them with much-needed liquidity.[1]

Specifically, we consider a retail market that operates across two sequential sub-periods: early and late. All production must occur in the early sub-period, requiring suppliers to pay costs c upfront using numeraire (e.g., cash payments for raw materials). However, suppliers begin with no numeraire endowment, creating a potential mismatch between when costs must be paid and when revenue arrives. If a supplier matches with consumers early, they receive revenue in time to cover production costs, enabling trade as in standard frictionless markets. However, if they

[1] To illustrate these features, consider the supplier finance program launched by Coop, a major UK supermarket chain, in 2020. Coop partnered with fintech platform PrimeRevenue to offer early payments to its suppliers. This program involved Coop selecting eligible suppliers, extending payment terms, and allowing suppliers to access early payment from PrimeRevenue at a low interest rate. Coop reported improvements in both supplier financing access and its own cash flow.

must wait until the late sub-period, they cannot finance production costs and no trade occurs. We denote the probability of this late-period matching—effectively a liquidity shock—by λ.

This setup reflects real-world scenarios where suppliers' liquidity depends critically on their retail technologies. The timing of consumer payments is determined by suppliers' technological capabilities, and limited technologies often prevent suppliers from securing early payments. For instance, a supplier lacking advanced display technologies may only be able to showcase products late in the season (probability λ), missing early-paying customers who purchase only after inspection. Similarly, those with limited inventory capabilities may only manage late-period deliveries (probability λ), failing to serve customers who pay upon delivery. Even suppliers without efficient production-to-order systems must rely on inventory-based production (probability λ), unable to leverage advance payments from customers.

In each case, better retail technologies—whether in product display, delivery systems, or production flexibility—would increase the probability of early-period transactions (to $1 - \lambda$), enabling trade that would otherwise be impossible. Suppliers vary in both their production costs c and their liquidity shock probability λ, with the pair (λ, c) capturing supplier heterogeneity. We assume these characteristics are publicly observable.

In this economy, we introduce an intermediary who cannot produce goods but possesses superior retail and enforcement technologies. The intermediary's retail advantage manifests in her enhanced ability to match with consumers, represented by a lower probability of experiencing liquidity shocks ($m\lambda$ where $m < 1$). This operational efficiency stems from her advanced retail capabilities: superior advertisement technologies that increase early product display opportunities, enhanced inventory systems that facilitate timely delivery, and improved communication channels that enable efficient production-to-order processes. The parameter $m < 1$ reflects the intermediary's technological advantage in reducing liquidity risk.

The intermediary also possesses enforcement technologies for credit arrangements with suppliers. While this finance service incurs costs, it enables two crucial functions: the pooling of trade credit (unused money from suppliers who obtain early retail profits) and the provision of early payments (allocating the pooled money to suppliers who need funds to cover production costs c).

Given the reatail technological advantages, the intermediary optimally offers retail intermediation to all available suppliers. Her key decision then becomes whether to extend the costly finance service to specific suppliers. This decision requires careful assessments: the intermediary must ensure that her total liquidity—comprising her own balance and trade credit from early-trading suppliers—sufficiently covers early payment obligations to participating suppliers. Thus, when evaluating potential finance service recipients, the intermediary considers not only their profitability but also their potential contribution to the overall liquidity pool.

Our model provides three insights. First, our model demonstrates that supplier finance enables profit-driven *liquidity cross-subsidization* across suppliers. Suppliers with higher liquidity shock probabilities (λ) and lower production costs (c) are more likely to contribute positively to profits. Conversely, suppliers with lower liquidity shock (λ) and lower costs (c) are more likely to contribute positively to the liquidity pool. The intermediary optimally uses trade credit from less profitable suppliers to support those who are more profitable but are in need of liquidity. This cross-subsidization strategy proves more profitable than selecting suppliers based solely on profits. Moreover, it explains the puzzling observation that small suppliers extend credit to large buyers (Klapper, Laeven and Rajan, 2012): rather than mere credit extraction, the intermediary actively redistributes liquidity to enable otherwise impossible production and trade, thereby playing a vital role in the supply network.

The significance of liquidity cross-subsidization hinges on the intermediary's shadow value of liquidity, which equals the external funding cost (interest rate) at the optimum. Pooled liquidity acts as a buffer against rising external financing costs. As these costs increase, supplier finance gradually shifts toward relying solely on this internal pool. The transition point to complete pool-dependency is determined by the collective liquidity capacity of participating suppliers.

Second, our model uncovers a novel relationship between the intermediary's operational efficiency and supplier finance adoption. When the intermediary becomes more efficient at matching (lower m), two competing effects emerge. On one hand, better matching reduces suppliers' liquidity risk, which decreases the value of the intermediary's financing service. On the other hand, it increases the amount of early-period revenue that can be pooled from suppliers, making credit extraction less costly. The dominant effect depends on prevailing interest rates: at low rates, reduced liquidity risk leads to less supplier finance adoption, creating a substitution effect; at high rates, enhanced liquidity pooling leads to expanded adoption, creating a complementary effect. This finding provides crucial guidance for regulators evaluating supplier finance programs, as it reveals how program resilience varies with both intermediation efficiency and funding costs.

Third, our model yields a counterintuitive welfare result: higher funding costs can actually improve overall welfare through the intermediary's strategic response. When funding costs are low, the intermediary simply selects suppliers based on their individual profitability. However, as funding costs rise, she strategically broadens participation in supplier finance to strengthen the liquidity pool. This expansion increases total trading volume and, consequently, welfare. The welfare improvement becomes more pronounced when the intermediary is operationally efficient, as efficiency both reduces the likelihood of liquidity shocks and enhances suppliers' liquidity contributions. This allows the intermediary to include more suppliers in the liquidity pool, effectively counterbalancing the higher external funding costs.

In Section 4, we demonstrate the robustness of our model through several extensions. Our findings remain valid when suppliers have equal or lower liquidity costs than the intermediary, when the retail market exhibits standard downward-sloping demand, and when suppliers face general outside options. We complement our theoretical analysis with real-world evidence supporting our model's implications and discuss its policy relevance.

In Section 5, we extend our analysis to manufacturing settings, where a manufacturer sources intermediate goods from heterogeneous suppliers to produce final goods for consumers. This setting differs fundamentally from retail intermediation: the timing of consumer purchases (early or late) affects only the final goods and is independent of intermediate goods sourcing. We analyze the manufacturer's dual sourcing strategy—combining financing contracts for liquidity-constrained suppliers with wholesale market purchases. Our analysis shows that supplier finance remains an equilibrium feature and maintains the liquidity cross-subsidization pattern found in the retail setting, where financially stable suppliers support those needing liquidity. This result holds across various production functions, from simple linear forms to more complex specifications like CES. A novel finding is that retail market structure (the proportion of early versus late consumers) affects production efficiency through the manufacturer's strategic supplier selection. Specifically, the interplay between demand timing, supplier heterogeneity, and the manufacturer's liquidity management shapes overall production efficiency. All proofs are included in the Appendix. The remainder of this section is dedicated to a literature review.

Related Literature

Our paper contributes to several strands of literature. First, we build on the trade credit literature, which emphasizes suppliers' monitoring advantages over banks (Petersen and Rajan, 1997, Burkart and Ellingsen, 2004, Cuñat, 2007, Giannetti, Burkart and Ellingsen, 2011, Garcia-Appendini and Montoriol-Garriga, 2013, Nocke and Thanassoulis, 2014, Bottazzi, Gopalakrishna and Tebaldi, 2023). We consider supplier finance as a type of financing that enables early payment to suppliers based on the trade credit they provide, ultimately leading to liquidity reallocation among suppliers.

Second, we contribute to the operations management literature on supplier finance. This literature has primarily focused on bilateral relationships between buyers and suppliers, analyzing various financial arrangements (Tunca and Zhu, 2017, Devalkar and Krishnan, 2019, Kouvelis and Xu, 2021). Our approach differs by examining supplier finance as a multilateral arrangement between an intermediary and a heterogeneous pool of suppliers, moving beyond the one-to-one framework.

Third, our work relates to the classical banking literature following Diamond and Dybvig (1983). Our model extends this literature by introducing ex-ante selection of heterogeneous depositors (suppliers), a feature absent in traditional banking models. This selection mechanism enables more targeted liquidity provision and, unlike demand deposits, avoids bank runs through limited liquidity commitments. Moreover, while financial inclusion is peripheral in banking models, it plays a central role in our framework.

Finally, we contribute to the literature on market-making intermediaries. Previous work has shown how intermediaries enhance market efficiency through various channels: maintaining market presence (Rubinstein and Wolinsky, 1987; Nosal, Wong and Wright, 2015), holding inventories (Watanabe, 2010, 2018, 2020; Li, Murry, Tian and Zhou, 2024), offering product variety (Camera, 2001; Johri and Leach, 2002; Shevchenko, 2004; Smith, 2004; Dong, 2010), and providing quality assurance (Biglaiser, 1993; Li, 1998a, 1999). Recently, using a simple two-dimensional statistic, Rhodes, Watanabe and Zhou (2021) study the product assortment problem of a multi-product intermediary. Our key innovation is to highlight intermediaries' role in liquidity provision, an aspect previously unexplored in this literature.[2]

2 The model

Consider a one-period economy with three types of agents: a mass one of consumers, a mass one of suppliers (he), and one intermediary (she). Each supplier produces a unique and indivisible good at a constant marginal cost c. Suppliers differ in $c \in [\underline{c}, \bar{c}]$, where $\bar{c} > \underline{c} > 0$, and c is publicly observable. Consumers are homogeneous and have unit demand for each good, with a common utility $u \geq \bar{c}$. The intermediary neither produces nor consumes. Instead, she can act as a middleman, buying goods from suppliers and reselling them to consumers. She also has access to a costly financing technology that allows her to delay payments to suppliers and use the funds to support suppliers needing liquidity. The details of these technologies will be specified below.

A numeraire good, which we refer to as money or liquidity, facilitates retail payments in this economy. We assume consumers have sufficient money endowments to make retail purchases. In contrast, suppliers begin with no money endowment (we will relax this assumption in Section 4). The intermediary can choose to hold money, denoted by $L \geq 0$, which she obtains from banks or money markets at a nominal interest rate $i \geq 0$. These external funding sources are not explicitly modeled.

Agents trade in a retail market where each good is sold either by its supplier or by the intermediary. If the intermediary sells a good, she purchases it from the supplier, who then exits the retail market (if suppliers are indifferent, they join the intermediary rather than selling directly in the market). Regardless of whether the supplier or the intermediary sells the good, the seller can reach all consumers.

[2]Our paper is also related to the burgeoning literature on hybrid or dual-mode platform economies, e.g., Tirole and Bisceglia (2023), Madsen and Vellodi (2023), Gautier, Hu and Watanabe (2023), Etro (2023), Etro (2024), Shopova (2023), Hagiu, Teh and Wright (2022), Anderson and Bedre-Defolie (2022), Kang and Muir (2022), Padilla, Perkins and Piccolo (2022), Zennyo (2022), Etro (2021a), and Etro (2021b). However, these papers focus on platforms that act as intermediaries between consumers and third-party sellers while also offering their own first-party products.

When a consumer and a seller (supplier or intermediary) meet, trade occurs bilaterally, and the trade surplus is split equally. The equilibrium retail price is thus given by

$$p - c = \frac{u - c}{2}. \tag{1}$$

While we use the Nash bargaining solution in the benchmark model, our conclusions are robust to alternative pricing mechanisms (see Section 4).

Liquidity shocks. Suppliers need the numeraire to cover production costs c to produce a good. In a Walrasian market, this is not an issue because suppliers can do so using their retail revenue. However, suppliers' finance matters when there exists a disparity in the timing between production and trade, and a liquidity shock prevents them from receiving revenue before production.

To be more precise, retail trade occurs in two sequential sub-periods: *early* and *late*. All production must take place in the early sub-period. Before production, an idiosyncratic shock is realized, indicating whether a given supplier's good will match with consumers in the early or late subperiod. With probability $1 - \lambda$, consumers match with the good early and thus they pay in the early subperiod. In this case, the supplier can use this immediate revenue to cover production costs, enabling production and trade even without initial money holdings. However, with probability λ, the good matches late, meaning payment occurs only in the late sub-period. Since production costs must be paid early but revenue arrives late, the supplier faces a liquidity shortage and cannot produce. Thus, the timing mismatch between required early production and potential late revenue creates liquidity risk, with late consumer arrival effectively generating a liquidity shock.

This setup captures real-world scenarios where suppliers' liquidity depends on their retail technologies. No trade occurs because of limited retail technologies possessed by suppliers to convince consumers to pay early rather than late. For instance,

- Display/advertisement: A supplier can display his good to consumers in the early subperiod with probability $1 - \lambda$ and in the late subperiod with probability λ. If consumers buy only after inspection, then it is only in the former case that the supplier can produce and trade. Better advertisement technologies increase the chance of early display.

- Delivery/inventory: A supplier can deliver his good to consumers in the early subperiod with probability $1 - \lambda$, and in the late subperiod with probability λ. If consumers pay only after delivery, then it is only in the former case that the supplier can produce and trade. Better inventory technologies increase the chance of early delivery.

- Production-to-order: A supplier has access to "production-to-order" technology with probability $1 - \lambda$ and can only "produce to inventory" with probability λ. Production-to-order allows suppliers to produce goods after receiving an order and payment from consumers.

Then it is only when this technology is accessible that the supplier can produce and trade. Better promotion or communication with consumers, facilitated by competent sales persons, increases the chance of production to order.

We assume that the probability of a liquidity shock varies among different goods and is publicly observable. Suppliers' ex-ante heterogeneity can be indexed by a pair (λ, c). Denote the two-dimensional space where (λ, c) belongs to by $\Omega \equiv [0, 1] \times [\underline{c}, \bar{c}]$ with $0 < \underline{c} < \bar{c} < u$. (λ, c) follows a continuous distribution which has a cumulative distribution function G, and a density function g that is everywhere positive in Ω.

Finally, due to a lack of enforcement technologies, there's no credit market among suppliers. Consequently, individual suppliers are unable to hedge against liquidity shocks or combine liquidity resources by themselves.

The retail technology. The intermediary can sell goods on behalf of suppliers with superior retail technologies, giving her a matching advantage over suppliers. This is in the same spirit as Rubinstein and Wolinsky (1987) where the very reason of the emergence of intermediary is due to their relative advantage in matching efficiency over suppliers. When the intermediary sells a good of λ, her advanced retail technologies reduce the probability of late matching from λ to $m\lambda$, where $m \in (0, 1)$ measures her matching advantage. Thus, the probability of early matching increases from $1 - \lambda$ to $1 - m\lambda$ ($> 1 - \lambda$). For example, the intermediary has:

- better advertisement technologies that increase the chance of early display;
- better inventory technologies that increase the chance of early delivery;
- better promotion/communication technologies with consumers that increase the chance of production to order.

The finance technology. The intermediary can also act as a financier, providing liquidity to suppliers through two functions: (1) extending credit to suppliers who need early payment, and (2) pooling retail revenue from early-paying consumers to support suppliers who need liquidity. The second functions works by delaying payments to suppliers who receive early revenue, effectively obtaining trade credit from them (detailed in the contracts below). Using this finance technology incurs a fixed cost k per financed supplier, where $k \in (0, \bar{k})$ with $\bar{k} \equiv \frac{(u-\underline{c})^2}{2(u+\underline{c})}$ (see footnote 4).

The contracts offered by the intermediary. Observing (λ, c), the intermediary offers a contract to each supplier. Given her matching advantage in the retail market, the intermediary will offer intermediation service to every supplier through one of two contracts.

A pure *middleman contract* stipulates that: (1) The intermediary sells the good on behalf of the supplier; (2) The intermediary pays the supplier a reward $f_M(\lambda, c) \geq 0$ *immediately after receiving payment from consumers*. If no payment is received, the supplier receives nothing. Under this contract, suppliers can produce only if the intermediary matches with consumers early (probability $1 - m\lambda$), as they need the revenue to cover production costs.

A *middleman-finance contract* (hereafter, the *finance contract*) stipulates that: (1) The intermediary sells the good on behalf of the supplier; (2) The intermediary pays the supplier a reward $f_F(\lambda, c) \geq 0$ *at the end of the period*; (3) The intermediary advances the production cost c to the supplier in the early subperiod.

The finance contract differs from the middleman contract in two aspects. First, in the finance contract, payments to suppliers are postponed to the end of the period. With the finance technology, this deferral allows the intermediary to leverage the delayed payments as a liquidity source to fund suppliers that are in need of liquidity. Second, the finance contract extends liquidity support of c at the time of production, which ensures the supplier can always produce and trade even if the supplier has no money in hand. Given utility is transferable and there is no asymmetric information, these two contract types are sufficient to capture the optimal contract the intermediary can offer.

Suppliers' outside values matter for the contract offer. A supplier who does not accept a intermediary's offer sells directly to consumers, in which case the supplier can produce and trade only if he is not hit by the shock (i.e. if he is matched with consumers early). Let $q(\lambda, c) \in \{0, 1\}$ be the selection function, where $q(\lambda, c) = 1$ if a (λ, c)-supplier is offered a finance contract, and $q(\lambda, c) = 0$ if he is offered a middleman contract. With this, we can summarize the set of the intermediary's offers by a triple:

$$\{q(\lambda, c), f_F(\lambda, c), f_M(\lambda, c)\}_{(\lambda, c) \in \Omega}.$$

Timing. The intermediary first determines the numeraire holding L. She then announces offers to each supplier based on (λ, c), indicating selection and contract type (middleman or finance). Selected suppliers decide whether to accept. Second, liquidity shocks are realized and trade occurs. Finance contract suppliers may request early payment of c. Middleman contract suppliers receive $f_M(\cdot)$ after consumer payment. Finally, the intermediary settles all outstanding supplier payments $f_F(\cdot)$ at the end of the period.

3 The Equilibrium

We now solve the equilibrium. Supposing the intermediary holds a real balance of L, we follow backward induction and work on the supplier selection problem first.

Suppliers' participation constraints. The intermediary makes offers subject to suppliers' participation constraints. If a supplier of (λ, c) chooses not to participate in the intermediary, then he can produce and trade only if he matches with consumers early. Thus, the expected profits are given by

$$(1-\lambda)(p-c) = (1-\lambda)(u-c)/2,$$

where we have inserted p from (1). Since the intermediary can observe (λ, c), she can make the rewards f_F and f_M dependent on (λ, c). To entice the supplier to participate, it is sufficient for the intermediary to offer him the value of his outside option so that

$$f_F(\lambda, c) = \frac{(1-\lambda)(u-c)}{2}, \tag{2}$$

and

$$f_M(\lambda, c) = \frac{(1-\lambda)(u-c)/2}{1-m\lambda} + c. \tag{3}$$

f_M differs from f_F because, in a middleman contract, production costs c are covered by the supplier, not the intermediary. The reward f_M is given to the supplier only when the intermediary successfully trades, which happens with probability $1 - m\lambda$ rather than $1 - \lambda$. With these fees, all the active suppliers are induced to accept an offered contract.

Suppliers' profit and liquidity contributions. Next, we derive the profit and liquidity that the intermediary obtains by having a supplier in a middleman or finance contract. With a middleman contract, the intermediary's expected profit from a supplier (λ, c) is

$$\pi_M(\lambda, c) = (1-m\lambda)\left(p - f_M(\lambda, c)\right) = (1-m)\lambda(u-c)/2, \tag{4}$$

which is positive since $m < 1$. The second equality follows from (1) and (3). The source of the profit is the middleman's matching advantage: the supplier does not receive a liquidity shock (and can trade successfully by himself) with probability $1 - \lambda$, whereas the middleman can do so with probability $1 - m\lambda$; the difference is given by $1 - \lambda - (1 - m\lambda) = (1 - m)\lambda$. Note that without incurring the finance technology cost k, the intermediary cannot enforce any credit deals that allow her to fund suppliers. Instead, she only provides intermediation services.

If financed by the intermediary, participating suppliers can produce even if matches occur in the late subperiod. Thus, with a finance contract, the intermediary's expected profit from a supplier (λ, c) is

$$\pi_F(\lambda, c) = p - c - f_F(\lambda, c) - k = \lambda(u-c)/2 - k, \tag{5}$$

where the intermediary receives payment p from consumers, covers the supplier's production costs c in the form of advanced payment, and rewards the supplier by f_F at the end. The second equality follows from (1) and (2). The expected profit is higher with a higher λ (as the supplier is less likely to trade if he chooses to operate independently) and a lower c (as the good has a higher profit margin).

When providing the finance service, liquidity is actually an issue because the intermediary needs to cover the production cost c for all the participating suppliers at the time of production. The source of this funding is the revenue p from early matches which occur with probability $1 - m\lambda$ for a supplier (or good) of type λ. Hence, the net expected amount of money that a supplier (λ, c) contributes to the intermediary at the time of production is

$$\theta_F(\lambda, c) = (1 - m\lambda)p - c = (1 - m\lambda)(u + c)/2 - c. \tag{6}$$

The intermediary's supplier selection problem. Let Θ be the total liquidity contributed by all the suppliers that are financed by the intermediary:

$$\Theta = \int_{\Omega} \Big[q(\lambda, c)\theta_F(\lambda, c) \Big] dG.$$

Then the liquidity constraint that the intermediary faces can be written as:

$$\Theta + L \geq 0. \tag{7}$$

The liquidity constraint states that the total liquidity contribution of financed suppliers, plus the available liquidity $L \geq 0$ that is held by the intermediary herself should be non-negative.[3]

Using (4) and (5), and defining $\Delta\pi$ as the incremental change in profits when a supplier (λ, c) is financed compared to not being financed:

$$\Delta\pi(\lambda, c) \equiv \pi_F(\lambda, c) - \pi_M(\lambda, c) = m\lambda(u - c)/2 - k,$$

the intermediary's problem of selecting suppliers into either middleman or finance contracts can be formulated as

$$V^m(L) \equiv \max_{\{q(\lambda,c)\}_{(\lambda,c)\in\Omega}} \int_{\Omega} [q(\lambda, c)\Delta\pi(\lambda, c)]\, dG, \text{ s.t. (7)}. \tag{8}$$

The problem can be understood as the intermediary obtaining $\pi_M(\lambda, c)$ for all the active suppliers and additionally deciding whether to finance suppliers to earn $\Delta\pi(\lambda, c)$ subject to liquidity constraint (7).

The intermediary's problem defined above is an optimization of functionals, and the optimal solution can be derived by using the following Lagrange method (see e.g., Rhodes, Watanabe and Zhou 2021). Let $\mu \geq 0$ be the multiplier associated with the liquidity constraint (7). We can

[3] We have a continuum of suppliers, each facing independent liquidity shocks. This allows us to leverage the Law of Large Numbers. As such, we interpret the requirement for the liquidity constraint to hold "almost surely" in a probabilistic sense. For a continuum, a literal "worst case" of every supplier defaulting simultaneously is measure-theoretically irrelevant.

construct the Lagrangian:

$$\mathcal{L} = \int_{\Omega} \Big[q(\lambda, c)\Big(\Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c)\Big)\Big] dG(\lambda, c).$$

Note that $\Delta\pi(\lambda, c)$ and $\theta_F(\lambda, c)$ can be positive or negative depending on the parameters. In particular, given the cost k of using the finance technology, it is not profitable to fund all suppliers, i.e., there exist suppliers with negative $\Delta\pi(\cdot)$.

Using this Lagrangian, the solution to the intermediary's problem can be obtained as an optimal selection policy that depends not only on (λ, c) but also on μ. With a slight abuse of notation, we shall refer to this optimal policy to finance a supplier as $q(\lambda, c, \mu)$, which is given by:

$$q(\lambda, c, \mu) = \begin{cases} 1 & \text{if } \Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0; \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Condition (9) indicates that $q(\lambda, c, \mu) = 1$ consists of three possible scenarios:

$$\Delta\pi(\lambda, c) \geq 0,\ \theta_F(\lambda, c) \geq 0, \tag{10a}$$

$$\Delta\pi(\lambda, c) > 0,\ \theta_F(\lambda, c) < 0,\ -\Delta\pi/\theta_F \geq \mu, \tag{10b}$$

$$\Delta\pi(\lambda, c) < 0,\ \theta_F(\lambda, c) > 0,\ -\Delta\pi/\theta_F \leq \mu. \tag{10c}$$

In scenario (10a), the intermediary selects suppliers with positive increments in profits $\Delta\pi$ and positive liquidity contribution θ_F to finance. In scenario (10b), the intermediary selects suppliers with positive increments in profits $\Delta\pi$ and negative liquidity contribution θ_F to finance, provided the gross return of liquidity, measured by $-\Delta\pi/\theta_F$, is higher than the shadow value of liquidity μ. In the last scenario (10c), the intermediary selects suppliers with negative $\Delta\pi$ and positive θ_F to finance, as these suppliers contribute to the aggregate liquidity of the middleman. The cost of getting one unit of liquidity from these suppliers is $-\Delta\pi/\theta_F$, and the intermediary should absorb liquidity from these suppliers if $-\Delta\pi/\theta_F \leq \mu$.

To illustrate the three scenarios in a figure, we insert $\Delta\pi(\cdot)$ and $\theta_F(\cdot)$ and obtain three boundaries that lie in Ω:

$$\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\theta_F}(\lambda) \equiv \frac{1 - m\lambda}{1 + m\lambda}u, \tag{11a}$$

$$\Delta\pi(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\Delta\pi}(\lambda) \equiv u - \frac{2k}{m\lambda}, \tag{11b}$$

$$\Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq b(\lambda, \mu) \equiv \frac{m\lambda u - 2k + \mu(1 - m\lambda)u}{m\lambda + \mu(1 + m\lambda)}. \tag{11c}$$

Note that the right-hand side of (11c) is a "weighted average" of the right-hand sides of the first two. We plot these three boundaries in Figure 1, annotated by $\theta_F(\lambda, c) = 0$, $\Delta\pi(\lambda, c) = 0$, and $\Delta\pi + \mu\theta_F = 0$, respectively. The intersection is denoted by (λ_0, c_0). Any suppliers below $\theta_F(\lambda, c) = 0$ contribute to the liquidity pool, and any suppliers below $\Delta\pi(\lambda, c) = 0$ contribute to

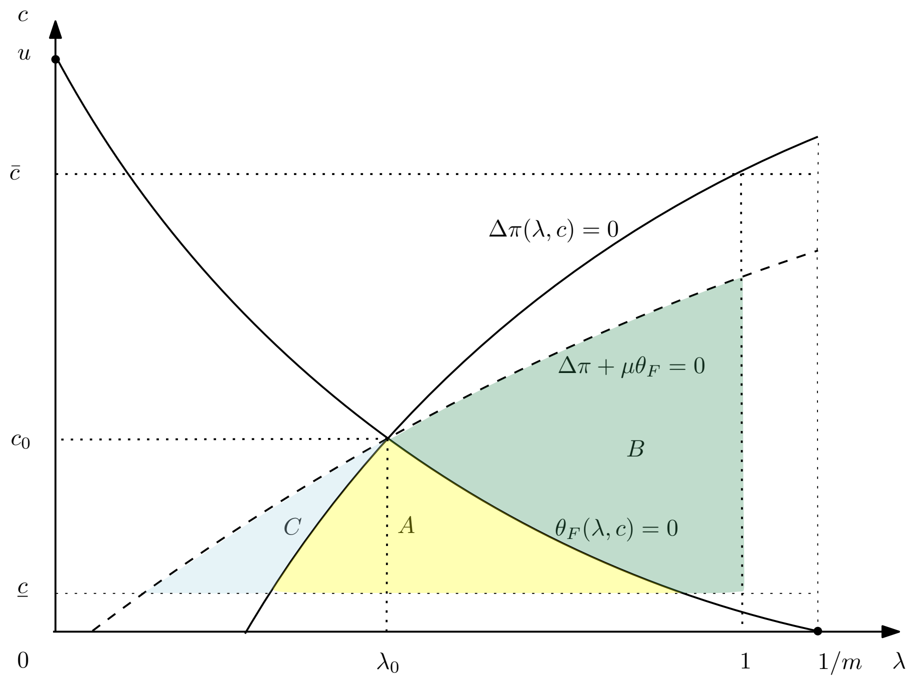


Figure 1: Middleman's selection of suppliers

the intermediary's profits.[4]

The overlapping region A represents suppliers in scenario (10a), which are financed by the intermediary because they contribute to both profits $\Delta\pi$ and liquidity θ_F. Suppliers in region B, corresponding to scenario (10b), have net liquidity needs, $\theta_F < 0$, while contributing to profits $\Delta\pi > 0$. Suppliers in region C, corresponding to scenario (10c), when included in a finance contract, give the intermediary lower profits $\Delta\pi < 0$, but contribute to the liquidity pool. Suppliers outside A, B or C are not financed.

Overall, the intermediary adopts what we call a *profit-based liquidity cross-subsidization* strategy. This involves using the positive net liquidity contributions from suppliers in regions A and C to address the liquidity needs of suppliers in region B. In particular, when the intermediary uses liquidity contributions from region C, it incurs a cost in the form of reduced (or negative) profits to these suppliers. However, when providing liquidity support to suppliers in region B, the intermediary expects a return in the form of positive profits from these suppliers.

In the standard liquidity pooling (a la Diamond and Dybvig 1983), agents are homogeneous and so, translated into our context, only those who make a positive profit contribution would be selected. With heterogeneous agents, however, we show this is suboptimal; whether a supplier will be included into the intermediary's liquidity risk-sharing program hinges not just on its liquidity contribution but also on its profit contribution.

It remains to determine μ, the shadow value of liquidity for the intermediary. If (7) is binding,

[4]Note that $k < \bar{k}$ ensures that $c_0 > \underline{c}$. Also, Figure 1 is drawn with $\lambda_0 < 1$ and $\bar{c} > c_0$. The complete analysis, including cases where $\lambda_0 \geq 1$ or $\bar{c}(i) < c_0$, is provided in the proof.

then μ is determined by

$$L = -\Theta(\mu) \equiv -\int_{\Omega} q(\lambda, c, \mu)\theta_F(\lambda, c)dG. \tag{12}$$

If (7) is slack, $\mu = 0$. In this case, the intermediary selects suppliers irrespective of liquidity concerns, i.e., the optimal selection policy is to select suppliers solely based on $\Delta\pi$.

Lemma 1 *If $\Theta(0) + L < 0$, then there exists a unique $\mu > 0$ that satisfies (12); and otherwise, $\mu = 0$.*

The liquidity value μ can be zero if the participating suppliers provide a sufficiently large amount of liquidity, $-\Theta(0) \leq L$. Note that this can be either with a positive liquidity pool $\Theta(0) \geq 0$ or a negative liquidity pool $\Theta(0) < 0$. Otherwise, the intermediary's endowment has a positive liquidity value, $\mu > 0$.

For the finance contracts to be active, the set of suppliers with $\Delta\pi > 0$ need to be non-empty. A necessary and sufficient condition is $\Delta\pi(1, \underline{c}) > 0$, or equivalently, $\frac{k}{m} < \frac{u-\underline{c}}{2}$. This, along with $c_0 > \underline{c}$ (implied by $k < \bar{k}$), ensures that region A in Figure 1 always exists. Selecting suppliers in region A (those with both a positive $\Delta\pi$ and a positive θ_F) to finance is feasible with regard to the liquidity constraint and results in positive profits.

Lemma 2 *There exist suppliers that are financed by the middleman if and only if $k/m < (u - \underline{c})/2$.*

The intuition of this lemma is as follows. Whether the supplier finance is activated or not hinges on two parameters. First, the finance service requires a per-supplier cost k. When k is large, the incremental profit $\Delta\pi$ decreases, making it less attractive to finance suppliers. Second, when m is small, indicating that the intermediary has a high matching advantage, consumers are likely to be matched in the early subperiod. This reduces the benefit of financing suppliers. In the extreme case, if $m \to 0$, the liquidity issue is eliminated altogether, making the finance contracts unnecessary. We summarize the results so far in the following theorem.

Theorem 1 (Selection of suppliers) *Taking $L \geq 0$ as given, and assuming $k/m < (u - \underline{c})/2$, the intermediary's profit-maximizing strategy exists uniquely with the selection policies to finance suppliers $q(\lambda, c, \mu)$, satisfying (9), the reward to suppliers $f_F(\lambda, c)$ and $f_M(\lambda, c)$ satisfying (2) and (3), and the shadow value of liquidity $\mu \geq 0$ uniquely determined in Lemma 1.*

The intermediary's available liquidity L shapes the feasibility of the finance contracts via the liquidity constraint and especially μ. It is intuitive that μ is strictly decreasing in $L \in [0, -\Theta(0)]$: an additional unit of the intermediary's money holding is appreciated more when her money holdings are relatively low. If L is higher, the curve $\Delta\pi + \mu\theta_F = 0$ is closer to $\Delta\pi = 0$, and the intermediary selects suppliers primarily based on profits. If L is lower (which leads to a larger μ), $\Delta\pi + \mu\theta_F = 0$ is closer to $\theta_F = 0$. Then liquidity becomes more important when selecting suppliers, and the intermediary relies more on liquidity cross-subsidization among suppliers. It is important to note that even a supplier that has high profits may not be chosen for the finance contract if he contributes little to the liquidity pool.

Corollary 1 *$\mu(L) > 0$ is strictly decreasing in L if $\Theta(0) + L < 0$.*

The intermediary's liquidity holding. Next, we turn to the intermediary's numeraire holding, which is determined by the following profit-maximization problem:

$$\max_{L \geq 0} V^m(L) - i \cdot L,$$

where $V^m(L)$ is the intermediary's profits if she holds a numerical of measure L as is defined in (8) and the second term is the corresponding funding cost.

Applying the Envelop condition $V^{m\prime}(L) = \mu(L)$, the first order condition for L is:

$$i \geq \mu(L). \tag{13}$$

Recall that $\mu(0)$ is the shadow value of liquidity to the intermediary if her liquidity holding $L = 0$. From Lemma 1, we have $\mu(0) > 0$ if and only if $\Theta(0) < 0$, and $\mu(0) = 0$ otherwise. Then we can characterize the intermediary's optimal liquidity holdings by comparing i and $\mu(0)$.

There are two scenarios to consider. In the first scenario, $\Theta(0) < 0$, which implies $\mu(0) > 0$ (see Corollary 1). If the nominal interest rate is relatively high, namely $i \geq \mu(0)$, the optimal money holding is $L(i) = 0$, indicating that the funding source is entirely given by the pooled liquidity of suppliers. If the nominal interest rate is relatively low, namely $i < \mu(0)$, then (13) holds with equality, and the intermediary holds a positive amount of money $L(i) = -\Theta(i) > 0$. In the second scenario, $\Theta(0) > 0$, which implies $i > \mu(0) = 0$ (see Lemma 1), and the intermediary can finance all the suppliers with positive profit contributions $\Delta\pi$, without holding liquidity.

Proposition 1 (Intermediary's liquidity holdings) *The optimal liquidity holdings of the intermediary follow $L(i) = -\Theta(i) > 0$ if $i < \mu(0)$, and $L(i) = 0$ otherwise. The value of liquidity with the intermediary's optimal liquidity holdings is given by*

$$\mu^*(i) = \min(i, \mu(0)). \tag{14}$$

With the intermediary's optimal selection policy (Theorem 1) and numeraire holding rule (Lemma 1) now established, we proceed to the equilibrium.

Theorem 2 (The Equilibrium) *An equilibrium exists and is unique where the intermediary operates with $(q(\lambda, c, \mu^*(i)), f_M(\lambda, c), f_F(\lambda, c), L(i), \mu^*(i))$, as characterized by Theorem 1 and Lemma 1.*

3.1 The impact of funding cost i

We now examine how changes in the nominal interest rate affect both the intermediary's optimal liquidity holdings and her selection of suppliers for financing. Our analysis reveals that suppliers' internal liquidity pool serves as an effective buffer against external funding cost shocks, yielding important implications for policy makers.

Following Proposition 1, when $i < \mu(0)$, the shadow value of liquidity $\mu^*(i) = i$ is strictly increasing in the funding cost i. This means that as the cost of external funds increases, the value of each unit of liquidity held by the intermediary also increases. Consequently, the intermediary's liquidity holdings, $L(i) = -\Theta(\cdot)$, are positive and strictly decreasing in i. In other words, the intermediary holds less liquidity as the cost of that liquidity rises. This effect is depicted in Figure 1 by the selection curves' $\Delta\pi + \mu\theta_F = 0$ rotating clockwise around the point (λ_0, c_0).

The threshold $\mu(0)$ represents the inherent liquidity richness of the supplier pool. A small $\mu(0)$ indicates abundant liquidity among suppliers, making the program less sensitive to changes in external funding costs. Conversely, a large $\mu(0)$ suggests a less liquid supplier pool, making the program more vulnerable to increases in i. When $i > \mu(0)$, the intermediary no longer holds the numeraire and relies entirely on cross-subsidizing liquidity among suppliers.

Regulators and practitioners worry that a sudden increase in funding costs (e.g., due to a financial crisis, a credit crunch, or driven by monetary policy) could disrupt supplier finance programs, potentially leading to widespread supplier failures and substantial aggregate output losses. This vulnerability to fluctuations in market funding costs becomes a ”sleeping risk” associated with the growing use of supplier finance.[5]

Our result demonstrates that this concern, while valid, may be overstated in certain circumstances. Supplier finance programs, as modeled here, draw liquidity from *both* the market (external liquidity with cost i) and the liquidity pool among suppliers (internal liquidity with cost $\mu(0)$). As i rises, the intermediary optimally shifts its reliance toward the internal liquidity pool, effectively reducing its dependence on external financing. When external funding costs i exceed the threshold $\mu(0)$, the intermediary ceases to hold numeraire and relies entirely on the internal liquidity pool, which acts as a direct buffer, absorbing the impact of changes in i.

Our model, therefore, suggests that regulators and investors should pay attention to the *liquidity characteristics* of supplier pools. In this context, policies that improve the transparency of supplier finance programs are highly valued. For example, the Financial Accounting Standards Board (FASB), starting in 2023, is requiring corporations to disclose the terms and size of their supplier finance programs in their financial statements. These disclosures help investors understand the internal liquidity capacity of the supplier pool, which is a crucial factor in evaluating a program's resilience to increases in external funding costs and thus its overall risk.

[5] For more on this, see the Wall Street Journal report titled “Supply-Chain Finance Is New Risk in Crisis”: `hchttps://www.wsj.com/articles/supply-chain-finance-is-new-risk-in-crisis-11585992601`. Accessed on Jul 17, 2023.

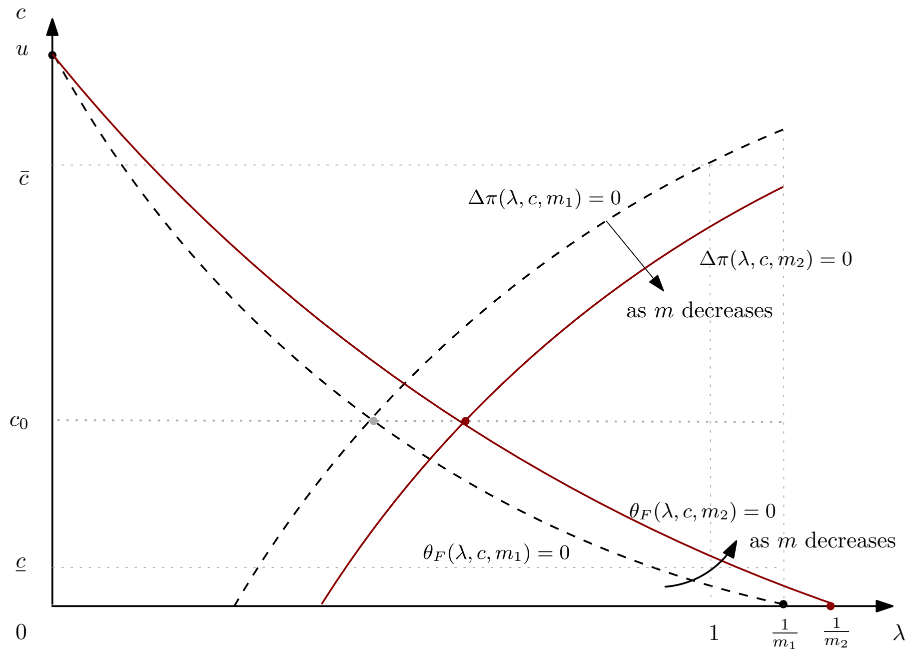


Figure 2: Effects of a decrease in m on $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$

3.2 Comparative statics of matching efficiency m

We now turn to how matching efficiency m affects the operation of the intermediary. We start by showing how the curves $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ change as m decreases which captures an improvement in matching efficiency. Figure 2 illustrates that as m decreases from m_1 to m_2, the incremental profits $\Delta\pi(\lambda, c)$ decrease, causing the curve $\Delta\pi(\lambda, c) = 0$ to shift downwards. At the same time, since the middleman is more likely to match with consumers early, the liquidity contribution of suppliers improves. As a result, $\theta_F(\lambda, c) = 0$ curve rotates upwards.

Notably, $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ intersect at $(c_0, \lambda_0) = (k + u - \sqrt{k^2 + 4uk}, \frac{k+\sqrt{k^2+4ku}}{2mu})$. Since c_0 does not depend on m, as m decreases, the two curves intersect along the horizontal line of $c = c_0$, and the intersection point moves to the right. The intersection point lies within the set Ω as long as $\lambda_0 \le 1$, or equivalently,

$$m \ge \tilde{m} \equiv \frac{k + \sqrt{k^2 + 4uk}}{2u}. \tag{15}$$

When $m > \tilde{m}$, all the three regions of (10) are non-empty, as illustrated in Figure 1, that is, there are suppliers in Ω with positive $\Delta\pi(\cdot)$ and negative $\theta_F(\cdot)$. Thus, liquidity cross-subsidization is plausible in equilibrium.

In contrast, when $m \le \tilde{m}$, the $\Delta\pi(\cdot) = 0$ curve lies entirely below the $\theta_F(\cdot) = 0$ curve. Figure 3 illustrates the case of $m = \tilde{m}$ where $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ intersect at $\lambda = 1$. In this case, all suppliers who bring a positive $\Delta\pi$ (the shaded region) also give the intermediary a positive liquidity contribution, leading to $\mu = 0$. Indeed, a necessary condition for a binding liquidity

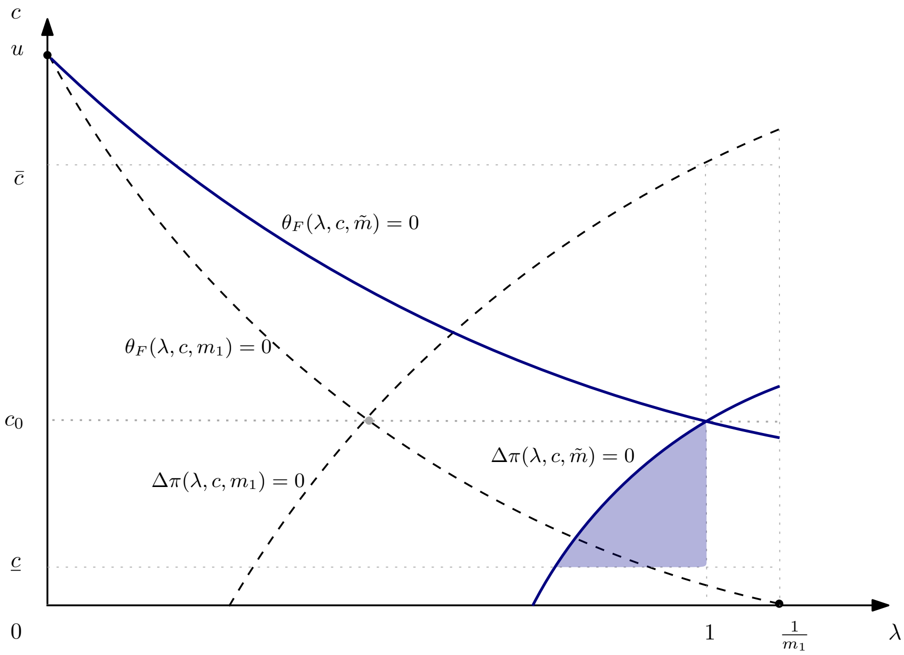


Figure 3: The curves of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$ when $m = \tilde{m}$ and $m = m_1 > \tilde{m}$

constraint for the intermediary is $m > \tilde{m}$.

Matching efficiency and finance inclusion. Improved matching efficiency influences the trade-off between the finance mode and the middleman mode. We are interested in how this trade-off shapes the optimal set of financed suppliers. We focus on the scenario where the intermediary's liquidity holding is interior ($L > 0$), implying an effective funding cost of $\mu^*(i) = i$.[6]

To analyze the marginal suppliers, Figure 4 illustrates four distinct supplier groups, categorized by their profit and liquidity contributions. Suppliers in $S1$ and $S2$ generate positive profits but require liquidity ($\Delta\pi(\cdot) > 0$, $\theta_F(\cdot) < 0$), while suppliers in $S3$ and $S4$ provide liquidity but generate negative profits ($\Delta\pi(\cdot) < 0$, $\theta_F(\cdot) > 0$).

The following lemma then introduces a threshold rate i_0 that determines the slope of the selection curve. When $i < i_0$, the selection curve is upward-sloping (see Figure 1), and the marginally financed suppliers—those suppliers who lie exactly on the selection curve $\Delta\pi + \mu\theta_F = 0$ and are thus indifferent between being financed or not—are in regions $S1$ and $S3$. Conversely, when $i > i_0$, the selection curve is downward-sloping, and these marginally financed suppliers are in regions $S2$ and $S4$.

Lemma 3 *Let* $i_0 = \left(k + \sqrt{k^2 + 4uk}\right) / (2u)$. *Given* $\mu^*(i) = i$, *if* $\max\{i, \mu(0)\} < i_0$, *then* $b'_\lambda(\lambda, i) > 0$; *if* $\min\{i, \mu(0)\} > i_0$, *then* $b'_\lambda(\lambda, i) < 0$.

[6] This requires $\mu(0) > 0$ and thus $m > \tilde{m}$. Lemma 2 establishes that the finance mode is active if and only if $m > \frac{k}{(u-\underline{c})/2}$. When $m \in \left(\frac{k}{(u-\underline{c})/2}, \tilde{m}\right)$, all suppliers selected for finance contracts contribute positive liquidity, and no cross-subsidization occurs. Since the liquidity constraint is not binding, the selection rule for financing suppliers is simply $\Delta\pi(\lambda, c) \geq 0$. As m decreases, the set of financed suppliers contracts.

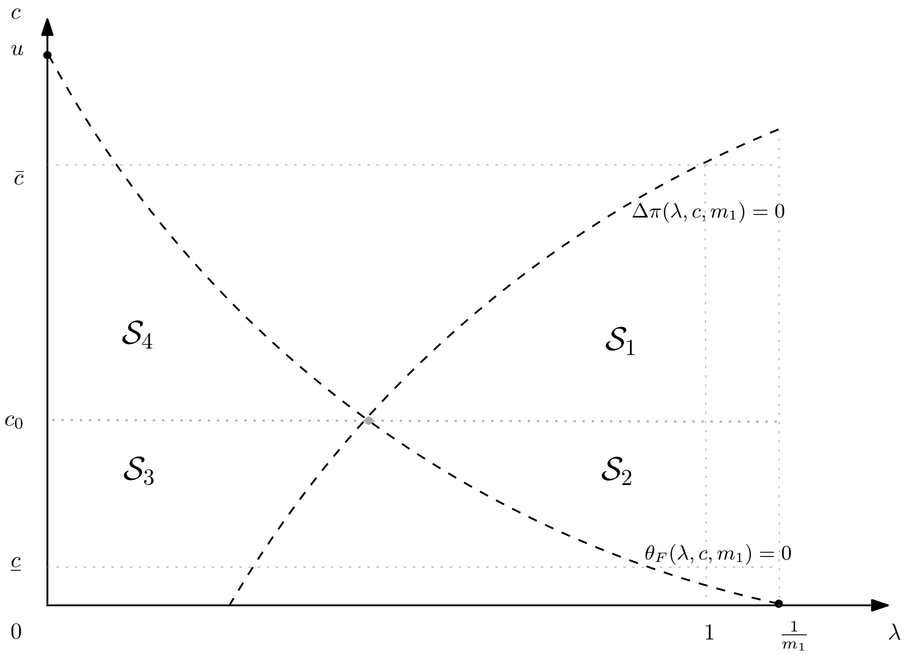


Figure 4: Four Sets of Suppliers by Profit and Liquidity Contributions

To analyze how improved matching efficiency affects the set of financed suppliers, we examine the profit-liquidity ratio $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$ of suppliers. This ratio measures the profit gained (or cost incurred) per unit of liquidity for each supplier. Since μ^* is fixed, the change in this ratio as m decreases determines whether more or fewer suppliers will be financed.

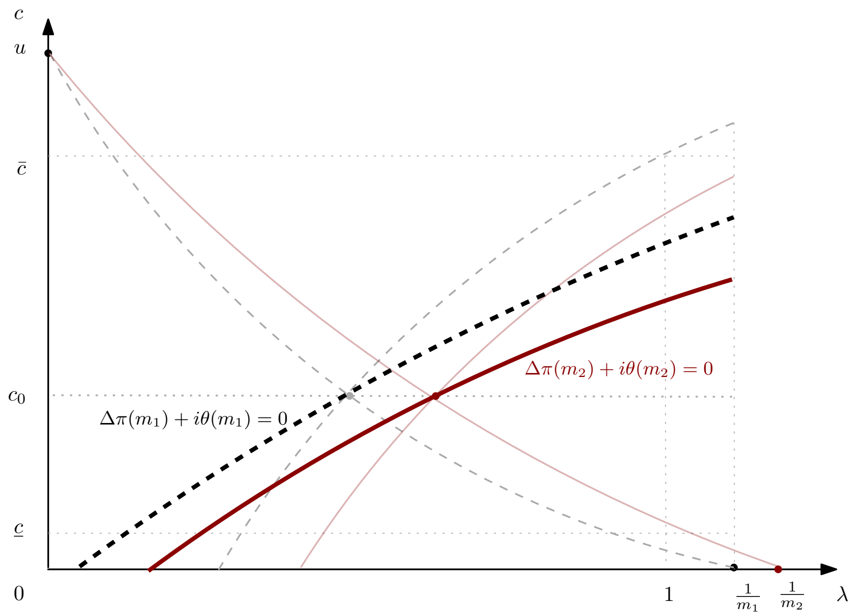
For suppliers in regions $S1$ and $S2$ who require liquidity ($\theta_F < 0$), this ratio is:

$$\frac{\Delta\pi(\cdot)}{-\theta_F(\cdot)} = \frac{m\lambda(u-c) - 2k}{m\lambda(u+c) + c - u}.$$

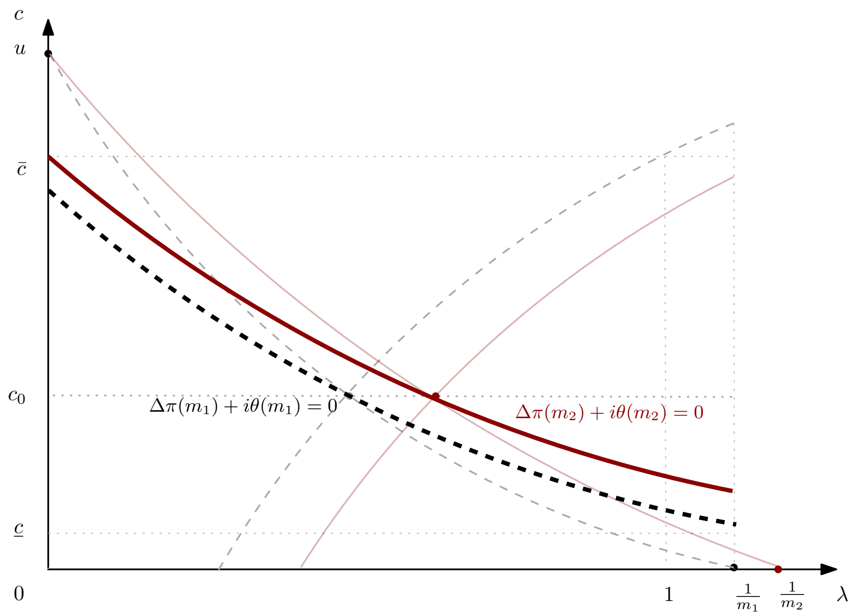
As m increases, both the profit and required liquidity increase, but at different rates. The profit increases at rate $\lambda(u-c)$, while the liquidity requirement increases at rate $\lambda(u+c)$. Since $\lambda(u+c) > \lambda(u-c)$, the liquidity requirement generally grows faster than profit. However, the supplier's cost parameter c plays a crucial role through its effect on the denominator. When c is high, the larger liquidity requirement dampens the relative increase in required liquidity as m increases. This causes the profit effect to dominate, making the profit-liquidity ratio increase with m. When c is low, the liquidity requirement effect dominates, causing the ratio to decrease with m. When $c = c_0$, the two effects strike a balance. A parallel analysis applies to suppliers in regions $S3$ and $S4$. Lemma 4 summarizes how the profit-liquidity ratio varies with m:

Lemma 4 *Given $m > \tilde{m}$, as matching efficiency improves (i.e., m decreases), the profit-liquidity ratio $-\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)}$ decreases for suppliers in $S1$ and $S4$ and increases for suppliers in $S2$ and $S3$.*

We now connect this result to the impact of matching efficiency on the scope of supplier finance. If $i < i_0$, the selection curve is upward-sloping, and marginal suppliers are in $S1$ and



(a) $\mu^*(i) = i < i_0$, and the selection curve is upward-sloping



(b) $\mu^*(i) = i > i_0$, and the selection curve is downward-sloping

Figure 5: Change in Supplier Finance Selection Curve as Matching Efficiency Increases (m Decreases)

$S3$ (see Figure 5(a)). Improved matching efficiency (lower m) reduces the return on financing suppliers in $S1$ and increases the cost of acquiring liquidity from $S3$. This leads to a contraction of supplier finance, suggesting that retail matching efficiency and supplier finance act as substitutes.

Conversely, if $i > i_0$, the selection curve is downward-sloping, and marginal suppliers are in $S2$ and $S4$ (see Figure 5(b)). As m decreases, the return on financing suppliers in $S2$ increases, and the cost of acquiring liquidity from $S4$ decreases. This leads to an expansion of supplier finance, indicating a complementary relationship between retail matching efficiency and supplier finance.

Proposition 2 *Suppose the effective funding cost satisfies $\mu^*(i) = i$. If $i < i_0$, a decrease in matching frictions (i.e., improved matching efficiency, represented by a lower m) reduces the optimal scope of supplier finance, suggesting a substitution relationship between supplier finance and retail matching efficiency. Conversely, if $i > i_0$, a decrease in matching frictions expands the optimal scope of supplier finance, indicating a complementary relationship with retail matching efficiency.*

3.3 Welfare

Next, we examine social welfare in our economy. Since a profit-maximizing intermediary does not internalize consumer surplus, the equilibrium outcome differs from the social optimum even with supplier finance. Surprisingly, we find that increasing the funding cost can improve social welfare. We provide conditions under which welfare exhibits a non-monotonic relationship with the nominal interest rate.

Social optimum. Consider a planner who takes i as given, selects suppliers from Ω and decides whether to finance them.[7] Let $s(\lambda, c)$ be a binary function which equals one if a supplier of (λ, c) is financed, and let $L \geq 0$ be the liquidity chosen by the planner. The total welfare, excluding funding costs, is given by

$$\mathcal{W} = \max_{s(\cdot)\in\{0,1\}} \int_\Omega \left\{ s(\lambda, c)(u - c - k) + \Big(1 - s(\lambda, c)\Big)(1 - m\lambda)(u - c) \right\} dG,$$
$$\text{s.t.} \int_\Omega s(\lambda, c)\theta_F(\lambda, c) dG + L \geq 0.$$

The total surplus for the goods is $u - c - k$ if the supplier is financed and is $(1 - m\lambda)(u - c)$ if not. Following the procedure in the previous sections, let $\mu^s(L)$ be the multiplier associated with the liquidity constraint. The planner's optimal selection rule into the finance mode can be written as

$$s(\lambda, c, \mu^s) = \begin{cases} 1 & \text{if } \Delta v(\lambda, c) + \mu^s \theta(\lambda, c) \geq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

[7] Generally, the planner assigns suppliers to direct selling, middleman contracts, or middleman-finance contracts. Since $m < 1$ makes the intermediary technology more efficient than individual suppliers at matching goods with demand, all suppliers are allocated to either middleman or middleman-finance contracts. Additionally, if the planner could choose the funding cost, setting $i = 0$ would dominate by removing the liquidity constraint.

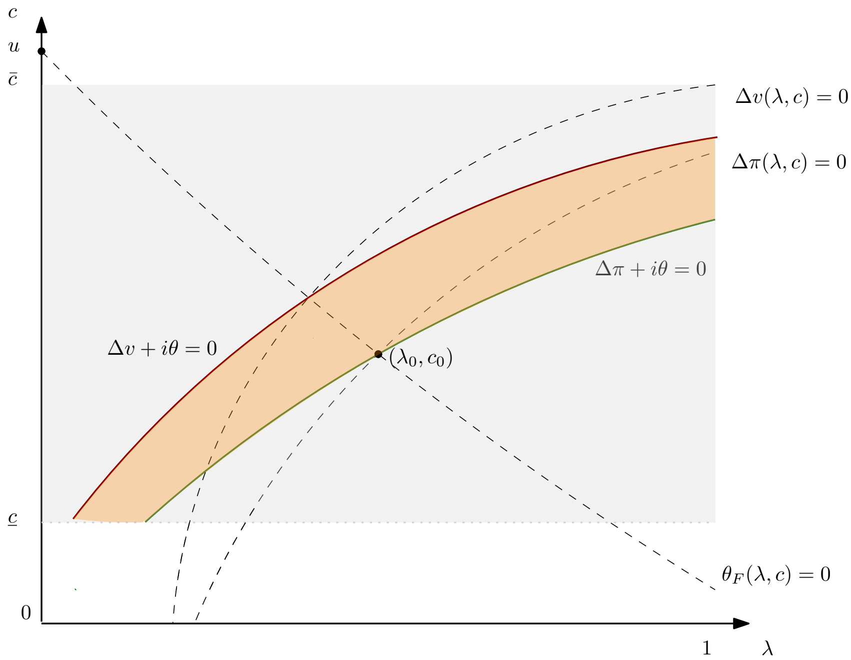


Figure 6: Compare the planner and the intermediary's selection rules

where $\Delta v(\lambda, c) \equiv m\lambda(u - c) - k > \Delta\pi(\lambda, c)$ is the gain of surplus, and crucially, it is larger than the gain of intermediary profits since the latter ignore the surplus of the consumer side. Furthermore, $\mu^s = 0$ if $\Theta(0) + L \geq 0$, and otherwise μ^s is pinned down by $\Theta(\mu^s) + L = 0$. Then the planner's liquidity holding is determined by comparing $\mu^s(0)$ and i where $L > 0$ only if $i < \mu(0)$.

Like the intermediary's optimal choice, the planner's solution also features liquidity cross-subsidization but is based on surplus gains. Through the rest of the analysis, we assume $i < \min\{\mu^s(0), \mu(0)\}$ so that the planner has the same liquidity value as the intermediary. Then, the planner chooses a positive amount of liquidity holding $L > 0$ and the liquidity value $\mu^s = i$. According to (16), the planner selects suppliers with positive surplus gain and liquidity ($\Delta v > 0, \theta > 0$), as well as those with negative surplus gain but positive liquidity ($\Delta v < 0, \theta > 0$ with $-\Delta v/\theta \leq i$), and uses the pooled liquidity and her own endowment L to fund suppliers with a liquidity need ($\Delta v > 0, \theta < 0$) if $-\Delta v/\theta \geq i$.

Since the social surplus gain exceeds the profit gain ($\Delta v(\lambda, c) > \Delta\pi(\lambda, c)$), the planner's financing decision encompasses a broader set of suppliers than the intermediary's. Figure 6 illustrates this difference: the orange region represents suppliers who are financed by the planner but not by the intermediary. These suppliers fall into two categories: those who contribute positively to the liquidity pool and those who require liquidity support. Although the intermediary's profit-maximizing selection never achieves the social optimum, the supplier finance program still enhances welfare by facilitating liquidity flows from early-revenue suppliers to those facing liquidity constraints.

Funding cost and welfare. The liquidity cross-subsidization mechanism has important implications for social welfare. We demonstrate that increasing the funding cost can, counterintuitively, improve welfare through enhanced liquidity cross-subsidization.

To establish this result, we focus on cases where $\mu(0) > 0$ for $i \in [0, \varepsilon)$, with ε being a small positive number. When this condition does not hold, the outcome becomes trivial: the intermediary's financing decision remains unchanged with interest rate, only financing suppliers with $\Delta\pi > 0$. Given $\mu(0) > 0$, a marginal increase in i from zero implies $\mu(i) = i$ (see equation (14)). Figure 7 illustrates this scenario, where the grey region represents the set of all suppliers (Ω), with different subsets labeled by capital letters.

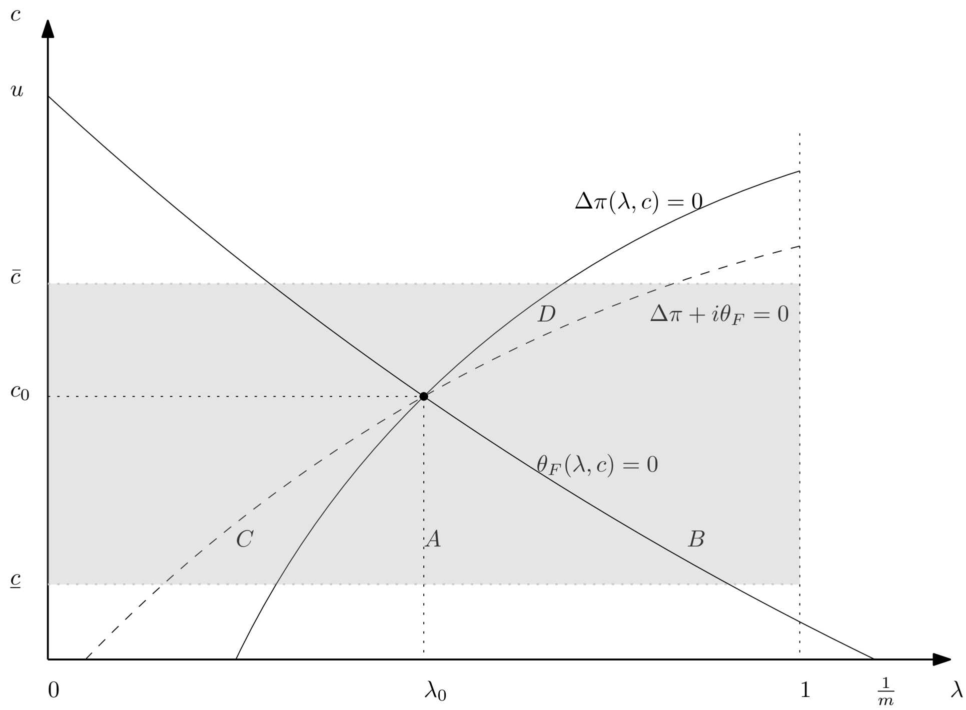


Figure 7: Friedman rule and welfare

When the nominal interest rate is zero, the intermediary finances all suppliers with positive profitability, covering regions A, B, and D in the figure. As the interest rate rises, the higher funding cost induces the intermediary to engage in liquidity cross-subsidization. This process involves excluding suppliers with positive profitability but negative liquidity (region D), while including suppliers with negative profitability but positive liquidity (region C). Consequently, the intermediary's profits decrease. Suppliers' profits do not change since they are indifferent between being funded and not funded by the intermediary. However, there is a potential for an increase in consumer surplus if the total trading volume increases. Ultimately, whether social welfare is improved or not depends on the dominance of the consumer surplus effect.

To analyze the impact on trading volume, let $\lambda^{\pi}(c)$ represent the combinations of (λ, c) for which $\Delta\pi(\lambda, c) = 0$, and let $\lambda^{\mu}(c)$ represent the combinations of (λ, c) such that $\Delta\pi(\lambda, c) +$

$\mu\theta(\lambda, c) = 0$. Excluding suppliers in region D leads to a decreased trading volume given by:

$$m \int_{c_0}^{\bar{c}} \int_{\lambda^{\pi}(c)}^{\lambda^{\mu}(c)} \lambda g(\lambda, c) d\lambda dc,$$

while including suppliers in region C leads to an increased trading volume given by:

$$m \int_{\underline{c}}^{c_0} \int_{\lambda^{\mu}(c)}^{\lambda^{\pi}(c)} \lambda g(\lambda, c) d\lambda dc.$$

This is because, for instance, each of the newly added suppliers, measured by $\int_{c_0}^{\bar{c}} \int_{\lambda^{\pi}(c)}^{\lambda^{\mu}(c)} g(\lambda, c) d\lambda dc$, will become available to consumers even when he is hit by a liquidity shock, which occurs with probability $m\lambda$.

Comparing the above two volumes, we can see that when c_0 is large enough, the former volume can be made arbitrarily small, while when c_0 is small enough, the latter volume can be made arbitrarily small. Thus, for a sufficiently large c_0, the consumer surplus effect dominates, and i deviating from the Friedman rule $i = 0$ improves welfare. In other words, c_0 crucial determines the number of suppliers to exclude (those who contribute to negative liquidity) and the number of suppliers to include (those who contribute to positive liquidity). c_0 is determined as the intersection of $\theta_F(\lambda, c) = 0$ with $\Delta\pi(\lambda, c) = 0$. As k decreases, c_0 increases accordingly: for $k \to 0$, $c_0 \to u > \bar{c}$, and for $k \to \bar{k}$, $c_0 \to \underline{c}$. Therefore, with k sufficiently small, increased trading volume outweighs decreased trading volume.

A similar analysis applies to λ. If λ_0 is sufficiently large, according to the cross-subsidization strategy, as i marginally increases from $i = 0$, the intermediary excludes fewer suppliers and includes more suppliers in finance contracts. Just like before, m determines λ_0. With m sufficiently small e.g., close to $\tilde{m}$, λ_0 is larger and closer to $\lambda = 1$. As a result, the increased trading volume outweighs the decreased trading volume.

The following proposition provides sufficient conditions for critical values of k and m such that the decreased trading volume is smaller than the increased trading volume if k or m is lower than the critical value. Social welfare increases because there is a significant improvement in consumer surplus that outweighs the decrease in the intermediary's profits. Hence, deviating from the Friedman rule is welfare-improving

Proposition 3 *Let $\kappa \equiv \frac{k}{u} \in (0, \frac{\bar{k}}{u})$ and define $\tilde{m}(\kappa) = \frac{1}{2}\left(\kappa + \sqrt{\kappa^2 + 4\kappa}\right)$. Suppose that (λ, c) follows a uniform distribution and $\mu(0,0) > 0$. There exists a critical value $\kappa^* \in (0, \frac{\bar{k}}{u}]$ and $m^*(\kappa) \in (\tilde{m}(\kappa), 1]$, such that under $m < m^*(\kappa)$ or $\kappa < \kappa^*$ that marginally increases i from $i = 0$ improves welfare.*

The proposition also establishes sufficient conditions for a non-monotonic impact of i on welfare. Figure 8 illustrate how welfare changes with i using a numerical example with $u = 1, k = 0.1, m = 1, \underline{c} = 0.1, \bar{c} = 0.6$, and a uniform distribution of (λ, c). Under these values, $\mu(0) = 0.26$. The figure shows that (1) the aggregate profits ($\Delta\Pi$, red curve) exhibit a monotonic decrease in i

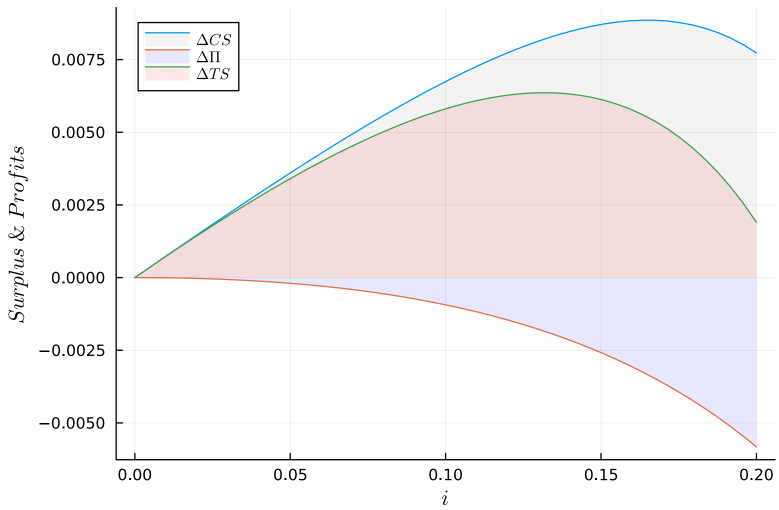


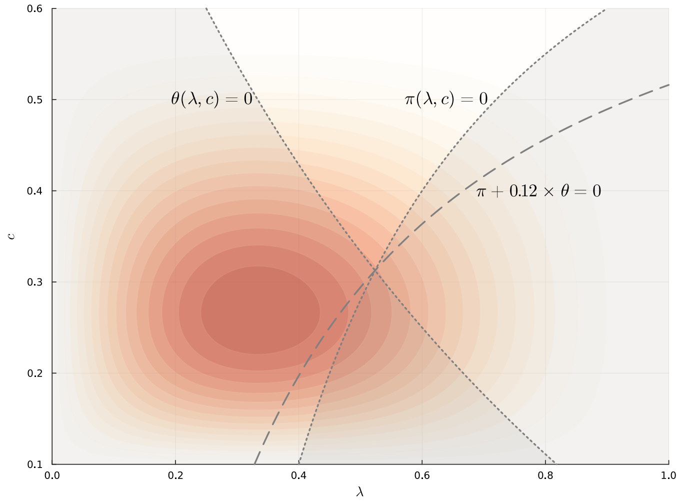
Figure 8: Welfare is non-monotonic in i under uniform distribution of (λ, c)

due to the exclusion of suppliers with positive π and the inclusion of suppliers with negative π; and (2) the total consumer surplus (ΔCS blue curve) follows an inverted U-shape because total trading volume first increases and then decreases. The effect of consumer surplus dominates. Consequently, the total surplus (ΔTS green curve) first increases and then decreases at relatively higher levels of i.

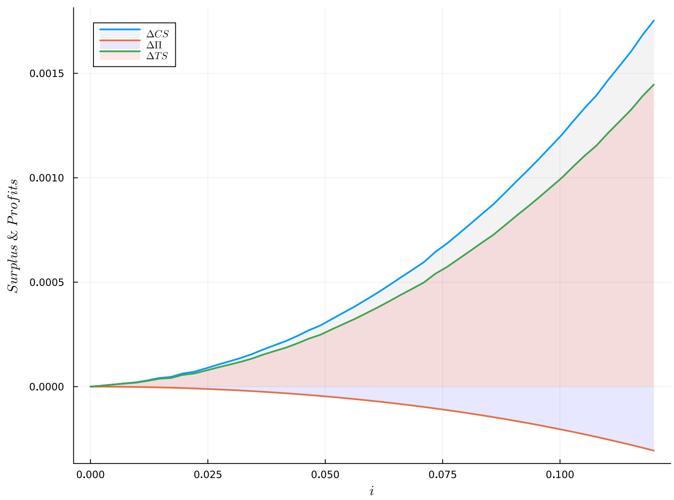
Of course, the suboptimality of the Friedman rule can occur with non-uniform distributions. Figure 9 provides a numerical exercise with $u = 1, k = 0.18, m = 1, \underline{c} = 0.1, \bar{c} = 0.6$, and both λ, and c follow a $Beta(2, 3)$ distribution. Under these values, $\mu(0) = 0.137$. Panel (a) shows the implied densities (contour graph in red) and a particular selection rule of $\pi + 0.12 \times \theta = 0$. Panel (b) shows that, as i increases, the blue curve representing total consumer surplus (ΔCS) increases monotonically, which outweighs the decrease in total profits ($\Delta\Pi$, red curve), resulting in a monotonically increasing total surplus (ΔTS, green curve) for the shown range of nominal rate.

4 Discussions and extensions

Our benchmark model provides a framework for understanding key features of real-world supplier finance programs. This section proceeds in as follows. First, we connect the model's core mechanisms to three prominent characteristics observed in supplier finance practice. Second, we extend the benchmark model in several ways, relaxing simplifying assumptions made for tractability and exploring the robustness of our findings.



(a) Density and selection rule ($\mu = 0.12$)



(b) Welfare change

Figure 9: Welfare increases in i if $(\lambda, c) \sim Beta(2, 3)$

4.1 Linking theory to practice in supplier finance

We examine three key features prevalent in supplier finance practices: the selective participation of suppliers, the cross-subsidization of liquidity across suppliers, and the role of the intermediary's matching efficiency in shaping the demand for supplier finance. These characteristics, observed across diverse real-world programs, anchor our theoretical model.

Selective supplier participation. Real-world supplier finance programs typically involve a selective process. For example, when Coop launched its program in 2020, it selected fewer than 100 suppliers from its network of thousands. Similarly, Amazon Lending, offered to third-party merchants, is an invitation-only program with customized credit terms.[8] This selectivity reflects a careful weighing of the benefits and costs of including each supplier, echoing the best practices emphasized in the *Supply Chain Finance Knowledge Guide* published by the International Finance Corporation. The guide highlights the importance of prioritizing suppliers based on their relationship with the buyer company and their financial needs. Our model captures these considerations: suppliers with stronger relationships are more crucial to the buyer's value creation (profit contributions), while the intermediary's liquidity constraint reflects the consideration of suppliers' financial needs. The model's selection mechanism thus provides an economic rationale for the observed selectivity in supplier finance programs.

Liquidity cross-subsidization. A key feature of many supplier finance programs is liquidity cross-subsidization, where larger, more financially stable suppliers effectively provide liquidity that is used to finance smaller, more liquidity-constrained suppliers. The JingBaoBei program operated by JD.com, the largest e-commerce platform in China, illustrates this. JingBaoBei allows suppliers to request advance payment based on their receivables from JD.com, providing funding to over 200,000 vendors with a total amount of more than 730 billion RMB. Crucially, the program is primarily funded by pooled liquidity, much of which originates from the suppliers themselves, particularly through trade credit. JD.com's reliance on supplier trade credit is evident in their financial reporting. For instance, on 2021, the increase in accounts payable constituted over 77% of JD's net cash inflow from operating activities.[9] While JD.com has introduced asset-backed securities to further finance JingBaoBei, supplier trade credit remains a substantial funding source. Large suppliers like Lenovo, Philips, and Bosch, who rarely request advance payments, effectively subsidize liquidity to smaller suppliers who need it more. Our model captures this feature through the heterogeneity in supplier liquidity needs, with some suppliers acting as net providers of liquidity while others are net users.

Matching Efficiency, liquidity, and supplier finance growth. Our model highlights the interplay between the intermediary's matching efficiency and suppliers' liquidity needs in driving the growth of supplier finance. This is evident in recent market trends. For example, disruptions during the pandemic led to increased inventories and extended payment terms, prompting retailers to increasingly utilize supplier financing to support their suppliers' cash flow. These disruptions can be modeled as a decrease in m, reflecting reduced matching efficiency. Our model

[8] For details of Coop's supplier finance program, see `https://scfcommunity.org/briefing/news/2020-retail-and-apparel-winner-co-operative-group/`. Accessed on Jul 17, 2023.For details about Amazon Lending, see `https://www.junglescout.com/blog/amazon-lending-program`. Accessed on Jul 17, 2023.

[9] From 2015 to 2018, JD's accounts payable turnover days have gone up from 41.9 to 58.1 days. This means, for instance, in 2018, it took more than on average 58 days for JD to pay off its suppliers. On the other hand, JD's accounts receivable turnover is quite short, with payments being received from customers within five days of a sale. Combining these numbers with a 30-day inventory turnover, JD can efficiently use supplier trade credit for about 23 ($= 58 - 5 - 30$) days before having to pay it off. Notably, this strategy has proven successful for JD, as its cash position has consistently improved alongside its total revenue.

predicts that, when funding cost i is relatively low (to be precise, $i < i_0$), a decrease in m expands the scope of supplier finance, consistent with observed market behavior.[10]

The pandemic is not the sole driver of growing supplier liquidity demand. For example, inventory turnover days for sellers on Taobao, China's leading e-commerce platform, increased significantly from 6 days in 2017 to over 20 days in 2020, indicating a reduced likelihood of suppliers matching with consumers. This trend, captured by a decrease in m, predates the pandemic and may be driven by increased competition on the platform. Consistent with our model's predictions, the supplier finance program by Ant Finance (Alibaba's financial subsidiary) expanded substantially, with credit balances rising from 647.5 billion RMB in 2017 to 2,153.6 billion RMB in 2020.[11] Our model thus provides a theoretical framework for understanding the growing demand for supplier finance in a world of evolving retail technologies and market dynamics.

4.2 Suppliers' access to money market

We first show that supplier finance can still exist even when suppliers can borrow from the money market. Suppose suppliers can access the money market or bank credit and hold liquidity for their needs. Assume $\bar{c} > c_0 > \underline{c}$ and $\lambda_0 < 1$. Let $i^s > 0$ be the interest rate suppliers face, which may be higher or lower than the intermediary's rate i. Let $z^s = z^s(c)$ be the real balance held by a supplier with cost c. A supplier (λ, c) with $z^s(c)$ has a retail market value of:

$$z^s + \left((1-\lambda) + \lambda \min\left\{ \frac{z^s}{c}, 1 \right\} \right) \frac{u-c}{2}.$$

Here, $\lambda \min\left\{ \frac{z^s}{c}, 1 \right\}$ shows that, with a liquidity shock, the supplier can use money holdings to produce and sell to $\min\{z^s/c, 1\}$ consumers. The supplier's money-holding problem is:

$$\max_{z^s} \left\{ \left[z^s + \left((1-\lambda) + \lambda \min\left\{ \frac{z^s}{c}, 1 \right\} \right) \frac{u-c}{2} \right] - (1+i^s) z^s \right\}.$$

Suppliers never hold $z^s > c$, as it's inefficient. The first-order condition shows that suppliers with (λ, c) satisfying $\frac{\lambda(u-c)}{2} > i^s c$ hold money. This simplifies to:

$$c < c^s(\lambda, i^s) \equiv \frac{\lambda}{\lambda + 2i^s} u. \tag{17}$$

Thus, suppliers with $c < c^s(\lambda, i^s)$ hold $z^s(c) = c$, while those with $c \geq c^s(\lambda, i^s)$ hold $z^s(c) = 0$.

Next, we consider the intermediary's problem. She can only offer finance contracts to suppli-

[10] For example, Constellation Brands Inc., a New York-based producer of Corona beer and Svedka vodka—launched a supplier finance program in 2022 in response to significant inventory growth and extended days of payables outstanding. Similarly, VF Corp., the parent company of popular brands such as Vans, North Face, and Supreme, initiated a supplier finance program in 2022 under similar circumstances. For further information, see the Wall Street Journal report at *https://www.wsj.com/articles/companies-offer-supply-chain-financing-to-vendors-as-they-bulk-up-on-inventory-push-out-payment-terms-11658316600?*, accessed on Jul 17, 2023.

[11] The inventory turnover days are obtained from *https://www.gurufocus.com/*. The credit balances of the supplier finance program are obtained from the *Ant Group Co., Ltd. Initial Public Offering and Listing on the STAR Market Prospectus*.

ers who don't hold money. The feasible set of suppliers is:

$$\tilde{\Omega}(i^s) = \{(\lambda, c) \in \Omega | c \geq c^s(\lambda, i^s)\},$$

which is nonempty. Her supplier selection problem is:

$$\max_{\{q(\cdot)\}_{(\lambda,c)\in\tilde{\Omega}(i^s)}} \int_{\tilde{\Omega}(i^s)} q(\lambda, c)\Delta\pi(\lambda, c) dG,$$

subject to the liquidity constraint:

$$\int_{\tilde{\Omega}(i^s)} q(\lambda, c)\theta_F(\lambda, c) dG + L \geq 0,$$

where i^s and L are given.

In earlier sections, we showed finance contracts are profitable when $\lambda_0 < 1$ because region A in Figure 1 is nonempty (see Lemma 2 for details). But when suppliers can access the money market, finance contracts may not always activate under the same conditions.

Proposition 4 *Suppose $\lambda_0 < 1$, $\underline{c} > 0$, $i < \frac{k\bar{\lambda}}{mu\bar{\lambda}-2k}$, and suppliers face money market rate i^s. There exist thresholds $i < \underline{i}^s < \bar{i}^s \equiv \frac{(u-\underline{c})\bar{\lambda}}{2\underline{c}}$ such that:*

- *If $i^s \leq \underline{i}^s$, suppliers with $c \leq c^s(\lambda, i^s)$ hold money for liquidity, and supplier finance stays inactive.*
- *If $i^s \geq \bar{i}^s$, no suppliers hold money, and supplier finance activates for some suppliers.*
- *If $i^s \in (\underline{i}^s, \bar{i}^s)$, suppliers with $c \leq c^s(\lambda, i^s)$ hold money, while supplier finance activates for others suppliers.*

Figure 10 shows the third case. Suppliers with costs below $c^s(\lambda, i^s)$ (region E, dark blue) hold money and skip the intermediary's finance. Those with costs above $c^s(\lambda, i^s)$ don't hold money. Among them, suppliers in regions A, B, and C join the finance contract. This can happen even if $i^s < i$, meaning suppliers borrow cheaper than the intermediary. Supplier finance still works because the intermediary uses liquidity more efficiently, leveraging the law of large numbers. This requires $c^s(\lambda, i^s)$ to cross the selection curve $\Delta\pi + \mu\theta_F = 0$ below c_0, ensuring region A exists, as shown in Figure 10.

4.3 General consumer demand

In the benchmark model, we assumed unit demand for simplicity. Here, we extend our results to a general setting where each supplier is a monopolist facing a downward-sloping demand function $Q(p)$. A seller (supplier or intermediary) sets the price to maximize profit:

$$\max_{p\in\mathbb{R}_+} (p - c)Q(p),$$

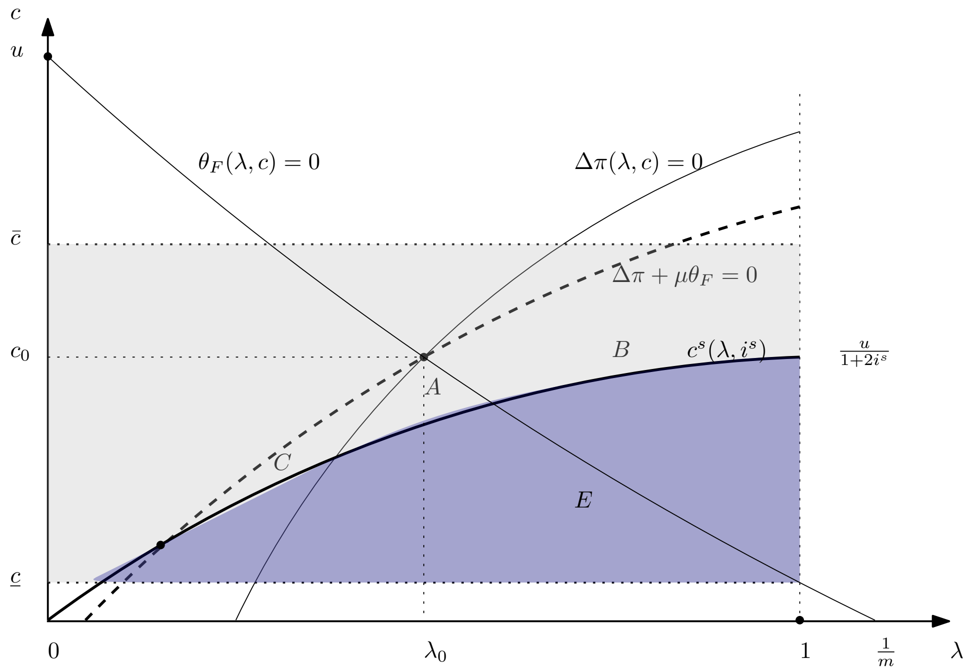


Figure 10: Suppliers' money holdings and intermediary finance coexist

where c is the constant production cost, as before. This problem is the same whether consumers arrive early or late. Assume $Q(p)$ yields a single-peaked profit function with a unique optimal price $p^m(c)$ and maximized profit $\tilde{\pi}(c)$. Under standard conditions, $\tilde{\pi}(c)$ is continuously differentiable, strictly decreasing, and convex. As in the benchmark, each good has a probability $1-\lambda$ of early consumer arrivals and λ of late arrivals.

When a supplier (λ, c) joins a middleman contract, the profit contribution is $\pi_M(\lambda, c) = (1-m)\lambda\tilde{\pi}(c)$. For a finance contract, it's $\pi_F(\lambda, c) = \tilde{\pi}(c) - k$. The difference is:

$$\Delta\pi(\lambda, c) = \pi_F(\lambda, c) - \pi_M(\lambda, c) = \tilde{\pi}(c) - k - (1-m)\lambda\tilde{\pi}(c) = m\lambda\tilde{\pi}(c) - k,$$

which rises with λ and falls with c. The liquidity contribution from a finance contract is:

$$\theta_F(\lambda, c) = (1 - m\lambda)(p^m(c) - c) - c,$$

which decreases in both λ and c. Under these conditions, the existence of the selection curve, and the existence of supplier finance (provided k is sufficiently large and m is not too small, as in the benchmark model), can be established.

As a parametric example, suppose $Q(p) = u - p$. Then $\tilde{\pi}(c) = (u-c)^2/4$, so $\Delta\pi(\lambda, c) = m\lambda(u-c)^2/4 - k$ and $\theta_F(\lambda, c) = (1-m\lambda)(u-c)/2 - c = (1-m\lambda)(u+c)/2 - c$. Note that the functional form of θ_F is exactly the same to that in the benchmark model. The relevant boundary conditions now become:

$$\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\theta_F}(\lambda) \equiv \frac{1 - m\lambda}{1 + m\lambda} u,$$
$$\Delta\pi(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\Delta\pi}(\lambda) \equiv u - \frac{2\sqrt{k}}{\sqrt{m\lambda}}.$$

4.4 Generalizing supplier outside options

In the benchmark model, suppliers' outside option value is $(1 - \lambda)(u - c)/2$, reflecting their ability to sell directly to consumers when matched early. We now examine the robustness of the intermediary's problem to a general outside option denoted by $w(\lambda, c)$, which may or may not depend on λ and c, subject to a condition ensuring all suppliers participate with the intermediary, at least under the middleman contract. Specifically, we assume $w(\lambda, c) \leq (1 - m\lambda)\frac{u-c}{2}$ for all $(\lambda, c) \in \Omega$. This condition guarantees that the intermediary's middleman contract is attractive relative to suppliers' outside options across Ω.

Under the *finance contract*, the intermediary pays c upfront, ensures production with certainty, and sets $f_F = w(\lambda, c)$ to match the outside option, yielding profit $\pi_F = \frac{u-c}{2} - w(\lambda, c) - k$, with liquidity contribution unchanged at $\theta_F = (1 - m\lambda)\frac{u+c}{2} - c$.

For the *middleman contract*, production and the cost c occur only if the match is early (probability $1 - m\lambda$. The supplier receives f_M when trade occurs, with expected profit $(1 - m\lambda)(f_M - c)$. Setting this equal to $w(\lambda, c)$, we have $f_M = c + \frac{w(\lambda,c)}{1-m\lambda}$, and the intermediary's profit becomes $\pi_M = (1 - m\lambda)\frac{u-c}{2} - w(\lambda, c)$.

The profit difference is $\Delta\pi = \pi_F - \pi_M = m\lambda\frac{u-c}{2} - k$, which matches the benchmark exactly. Since $\Delta\pi$ and θ_F remain identical to the benchmark model, the intermediary's selection problem (8) and all subsequent results, including Theorems 1 and 2, hold unchanged. This invariance demonstrates that the intermediary's optimal strategy depends solely on her matching advantage (m) and financing cost (k), not on the specific form of $w(\lambda, c)$.

5 Manufacturing supplier finance

We now shift our focus to model supplier finance for manufacturers, who procure intermediate goods from suppliers to produce final goods for consumers. This pooling of diverse intermediate goods into a unified output alters the operation of supplier finance, necessitating a tailored framework. Nonetheless, we demonstrate that supplier finance persists in equilibrium and retains liquidity cross-subsidization—where financially stable suppliers support those with liquidity needs—as a central feature, consistent with the benchmark model.

Consider a manufacturer (M) who produces final goods using intermediate goods sourced from suppliers and sells these goods to consumers in a retail market across two subperiods: early and late. A fraction α of consumers purchase in the early subperiod, with the remaining $1-\alpha$ purchasing in the late subperiod. The retail price is normalized to 1 per unit. Consumers buy all final goods produced by M at this fixed price, with demand split between early (fraction α) and late (fraction $1-\alpha$) subperiods, and the total quantity demanded equals the manufacturer's output, ensuring the market clears perfectly at the normalized price.

There is a measure one of suppliers indexed by the pair (λ, c). Each supplier can produce one unit of an intermediate good with cost $c \in [\underline{c}, \bar{c}]$. With probability λ, a supplier cannot initiate production due to insufficient funds to purchase required inputs from input producers (who are passive in the model). The joint distribution of (λ, c) is described by an atomless C.D.F. $G(\lambda, c)$ defined on $\Omega = [0,1] \times [\underline{c}, \bar{c}]$.

The manufacturer sources intermediate goods through two channels. The first channel is the supplier financing contracts: M offers financing contracts to a selected set of suppliers, providing them with working capital of c if they are hit by the liquidity shocks (observable to M). In return, these suppliers commit to delivery of one unit of the intermediate good in the early subperiod and receive a reward $f(\lambda, c)$ in the late subperiod. The second channel is a competitive wholesale market that operates in the early subperiod following the realization of liquidity shocks. Only suppliers with adequate working capital (those not hit by liquidity shocks) can participate in this market since production must occur before market entry. We do not restrict the pricing mechanism in the wholesale market. The only constraint being that the wholesale price denoted by $w(c)$ lies in $[c, 1]$ and is non-decreasing in c.

The timing is as follows: (1) At the beginning of the period, M announces the set of suppliers selected for financing contracts and the set that is targeted for wholesale market purchases. This announcement includes M's commitment to provide liquidity support to financed suppliers. (2) In the early subperiod, a fraction α of consumers (early consumers) arrives. Their payments generate immediate revenue that M uses for supplier financing and wholesale market transactions. (3) Upon realization of liquidity shocks, M provides working capital of c to financed suppliers who experience shocks. Simultaneously, unshocked non-financed suppliers enter the wholesale market, where M immediately pays $w(c)$ for their output. Using intermediate goods from both financing and wholesale channels, M produces final goods. A portion α of these goods serves the early consumers' demand, while the remainder is stored for late consumers. (4) In the late subperiod, the remaining measure $(1-\alpha)$ of late consumers arrive and purchase from M. Finally, M settles the pre-committed payments $f(\lambda, c)$ with financed suppliers.

5.1 Linear production function

To establish connections with our benchmark model, we first consider a linear production function with homogeneous intermediate goods. Since the wholesale price satisfies $w(c) < 1$, the manufacturer finds it profitable to source from all suppliers in Ω. We maintain the assumption that the manufacturer has sufficient liquidity to do so.[12] Under these conditions, the manufacturer's decision for each supplier is binary: either provide financing ($q(\cdot) = 1$) or purchase through the wholesale market ($q(\cdot) = 0$). The total effective input I is the sum of the intermediate goods:

$$I = \int_{\Omega} \Big(q(\lambda, c) + (1 - q(\lambda, c))(1 - \lambda) \Big) dG,$$

and the production function is $Q(I) = I$.

Each supplier can sell in the wholesale market at $w(c)$. Thus, the expected outside value is $(1 - \lambda)(w(c) - c)$. To ensure supplier participation in the financing contract, the manufacturer must give reward $f(\lambda, c)$ at least the outside option value. With the suppliers' participation constraint, it follows that a financed supplier of (λ, c) contributes to M a profit of $\pi_F(\lambda, c) = 1 - c - (1 - \lambda)(w(c) - c) - k$. We assume the per-supplier operting cost $k \in (0, 1 - \underline{c})$. The financed supplier also contributes liquidity of $\theta_F(\lambda, c) = \alpha - \lambda c$. Here, α captures the expected early-period revenue, and λc captures the expected funding request. For suppliers from whom M purchases in the wholesale market, the contributions to profit and liquidity of M are scaled by the probability of market participation $(1 - \lambda)$. The profit contribution is $\pi_W(\lambda, c) = (1 - \lambda)(1 - w(c))$, and the liquidity contribution is $\theta_W(\lambda, c) = (1 - \lambda)(\alpha - w(c))$.

The manufacturer's problem is to choose $q(\lambda, c)$ to maximize:

$$\int_{\Omega} \Big(q(\lambda, c)\pi_F(\lambda, c) + (1 - q(\lambda, c))\pi_W(\lambda, c) \Big) dG$$

subject to the liquidity constraint:

$$\int_{\Omega} \Big(q(\lambda, c)\theta_F(\lambda, c) + (1 - q(\lambda, c))\theta_W(\lambda, c) \Big) dG + L \geq 0.$$

With μ denoting the multiplier on the liquidity constraint and defining $\Delta\pi \equiv \pi_F - \pi_W = \lambda(1 - c) - k$ and $\Delta\theta \equiv \theta_F - \theta_W = \lambda(\alpha - c) + (1 - \lambda)w(c)$, the optimal financing decision follows that $q(\cdot) = 1$ if and only if $\Delta\pi + \mu\Delta\theta \geq 0$, or

$$\Big(\lambda(1 - c) - k \Big) + \mu \Big(\lambda(\alpha - c) + (1 - \lambda)w(c) \Big) \geq 0$$

Let $c_{\Delta\pi}(\lambda) = 1 - k/\lambda$ be the cost threshold where $\Delta\pi(\lambda, c) = 0$, which is concave and strictly increasing in Ω. Similarly, let $c_{\Delta\theta}(\lambda)$ be the threshold of c such that $\Delta\theta(\lambda, c) = 0$. While $c_{\Delta\theta}(\lambda)$ depends on the unspecified wholesale price function $w(c)$, it is bounded by:

$$\frac{\alpha\lambda}{2\lambda - 1} \equiv c^L_{\Delta\theta}(\lambda) \leq c_{\Delta\theta}(\lambda) \leq c^H_{\Delta\theta}(\lambda) \equiv \frac{1}{\lambda} - (1 - \alpha)$$

where the upper and lower bounds, $c^H_{\Delta\theta}(\lambda)$ and $c^L_{\Delta\theta}(\lambda)$, correspond to $w(c) = 1$ and $w(c) = c$, respectively. For $\lambda \in [\frac{1}{2-\alpha}, 1]$, both bounds are non-negative, and they intersect at the interval

[12]Formally, we assume $\int_{\Omega}(1 - \lambda)(w(c) - c)dG + L \geq 0$.

endpoints of $\lambda = \frac{1}{2-\alpha}$ and $\lambda = 1$. In Figure 11, we plot the three bounds $c = c_{\Delta\theta}(\lambda)$ (solid blue), $c = c^L_{\Delta\theta}(\lambda)$, and $c = c^H_{\Delta\theta}(\lambda)$ (dash blue). $c_{\Delta\theta}(\lambda)$ is bounded by $c^L_{\Delta\theta}(\lambda)$ and $c^H_{\Delta\theta}(\lambda)$, and they conincide at the endpoints.

The curves $c_{\Delta\pi}(\lambda)$ and $c_{\Delta\theta}(\lambda)$ intersect in Ω if and only if $\alpha < 1-k$. When this condition holds, there can exist liquidity cross-subsidization among suppliers. Figure 11 illustrates this by plotting $c_{\Delta\pi}(\lambda)$ along with the bounds $c^L_{\Delta\theta}(\lambda)$ and $c^H_{\Delta\theta}(\lambda)$. And if we let M to choose the liquidity L to hold, then at optimality, $\mu^* = \max\{i, \mu(0)\}$. Conversely, when $\alpha \geq 1-k$, any supplier that contributes positively to profits ($c \leq c_{\Delta\pi}(\lambda)$) also contributes positively to liquidity, eliminating the need for cross-subsidization. In this case, the supplier finance program still exists and profitable with $\mu^* = 0$.

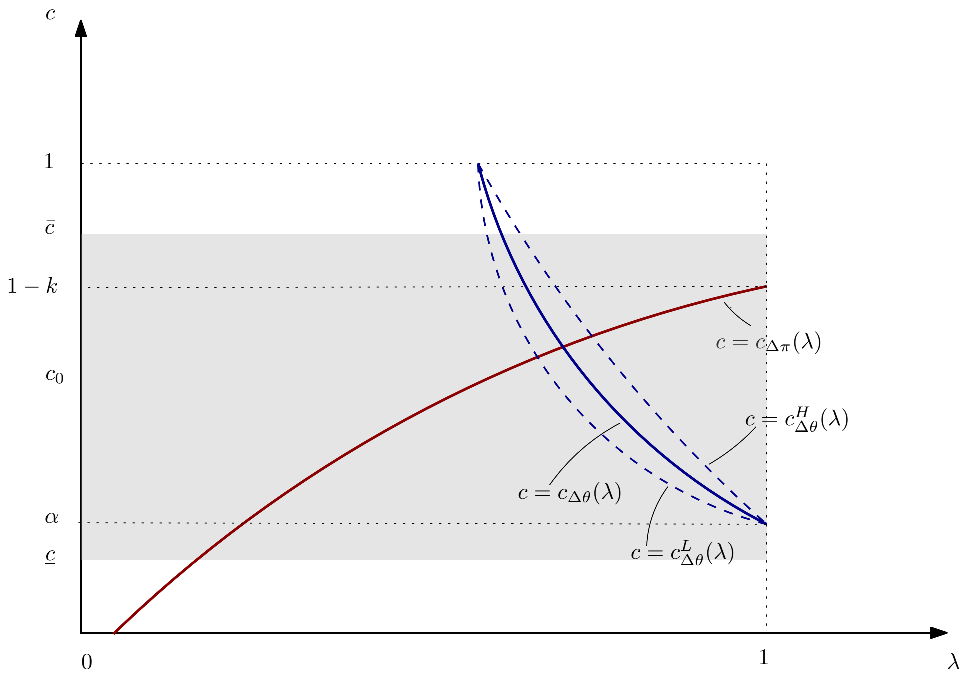


Figure 11: Manufacturer financing selection under linear Q

Discussions: Retail demand and production efficiency. In this model, production efficiency equates to the manufacturer's profit, $\int_\Omega [q(\lambda,c)\pi_F(\lambda,c) + (1-q(\lambda,c))\pi_W(\lambda,c)]dG$, since consumers are passive and suppliers earn fixed outside values. The retail market's demand timing, captured by α—the share of early consumers—alters production efficiency via the supplier financing channel, shaping which suppliers contribute to production.

For $\alpha < 1-k$, scarce early revenue tightens liquidity, rasing μ. The manufacturer then prioritizes suppliers with low liquidity needs, despite higher costs and lower profits, over efficient, low-c, high-λ suppliers whose liquidity drain prevails when μ is high. As α nears $1-k$, liquidity eases, μ drops, and more low-c suppliers are financed, boosting efficiency. Above $1-k$, with $\mu = 0$, financing targets all $c \leq c_{\Delta\theta}$, maximizing profit unconstrained. Thus, α tunes efficiency by steering the supplier mix.

This demand-side mechanism reveals how retail market liquidity patterns affect production efficiency. The share of early consumers (α) shapes the manufacturer's liquidity position, which determines its supplier selection strategy. When early revenue is high, manufacturers can engage more cost-efficient suppliers despite their higher liquidity risks, thereby achieving greater production efficiency. Our framework's explicit modeling of supplier heterogeneity is crucial for understanding this novel efficiency channel.[13]

5.2 General production functions

Our analysis extends to general prduction functions. We define $Q(x(\lambda, c))$ as a function dependent on the input strategy $x(\lambda, c) = q(\lambda, c) + (1 - q(\lambda, c))(1 - \lambda)$, where $q : \Omega \to \{0, 1\}$ is the financing choice, $\Omega = [0, 1] \times [\underline{c}, \bar{c}]$ is compact with $|\Omega| = 1$, and G is a continuous, atomless probability measure. Let X be the space of measurable q, and $Q : X \to \mathbb{R}_+$ be bounded and continuous in the $L^1(\Omega, G)$ topology.[14] We assume Q is Fréchet differentiable, so the marginal effect of changing $x(\lambda, c)$, denoted $\frac{\delta Q}{\delta x(\lambda, c)}$, exists for almost every (λ, c) in Ω.[15] We additionally assume that sourcing intermediate goods from all suppliers through the wholesale market is both profitable and liquidity-feasible for the manufacturer. This simplifies our analysis, as we only need to determine whether each supplier should be sourced through the wholesale market or the financing channel.[16]

The manufacturer maximizes:

$$Q(x) - \int_\Omega [q(c + (1 - \lambda)(w(c) - c) + k) + (1 - q)(1 - \lambda)w(c)]dG,$$

subject to:

$$\Theta(x) + L = \alpha Q(x) - \int_\Omega [q\lambda c + (1 - q)(1 - \lambda)w(c)]dG + L \geq 0.$$

The Lagrangian is:

$$\mathcal{L} = (1 + \mu\alpha)Q(x) - \int_\Omega q[\lambda c(1 + \mu) + k - \mu(1 - \lambda)w(c)]dG - \int_\Omega (1 - q)(1 - \lambda)w(c)(1 + \mu)dG + \mu L.$$

[13]For example, fast-fashion retailers like Zara leverage high early-season demand (high α) to generate immediate revenue, enabling them to finance agile but liquidity-constrained suppliers in developing economies. These suppliers often combine low production costs with higher liquidity risk, a trade-off that becomes optimal when strong early sales provide sufficient liquidity.

[14]In the $L^1(\Omega, G)$ topology, the distance between two strategies q and q' is $\|q - q'\|_1 = \int_\Omega |q(\lambda, c) - q'(\lambda, c)|dG(\lambda, c)$, which measures the "size" of the supplier set where financing choices differ, weighted by G. Continuity ensures that if this distance is small, $Q(q)$ and $Q(q')$ are close in value.

[15]The Fréchet derivative captures the rate of change in Q. For a small perturbation $h(\lambda, c)$ in x, we have $Q(x + \epsilon h) = Q(x) + \epsilon \int_\Omega \frac{\delta Q}{\delta x(\lambda, c)} h(\lambda, c)dG + o(\epsilon)$, where $o(\epsilon)$ is a remainder that vanishes faster than ϵ. Since q is binary, $\frac{\delta Q}{\delta q} = \frac{\delta Q}{\delta x} \cdot \lambda$, reflecting the switch from $1 - \lambda$ to 1.

[16]This assumption holds if the marginal product of the intermediate good (evaluated when all suppliers contribute $x(\lambda, c) = 1 - \lambda$) exceeds the highest wholesale cost $w(\bar{c})$ for almost every supplier (λ, c). Formally, $\left.\frac{\delta Q}{\delta x(\lambda, c)}\right|_{x(\lambda, c) = 1 - \lambda \text{ for all } (\lambda, c)} > w(\bar{c})$.

Then, $q(\lambda, c) = 1$ if:

$$\lambda \left(\frac{\delta Q}{\delta x} - c \right) - k + \mu \left[\lambda \left(\alpha \frac{\delta Q}{\delta x} - c \right) + (1 - \lambda) w(c) \right] \geq 0,$$

with $\mu \geq 0$, $\Theta \geq 0$, $\mu\Theta = 0$.

Proposition 5 *Under the assumptions on the production function Q stated above, there exists $q(\cdot) \in X$ that solves the profit maximization problem subject to the liquidity constraint.*

Liquidity cross-subsidization across suppliers occurs. For instance, if $w(c) = c$, a financed supplier contributes positively to profit when $c < c_{\Delta\pi} \equiv \frac{\delta Q}{\delta x} - k/\lambda$ and to liquidity when $c < c_{\Delta\theta} \equiv \frac{\delta Q}{\delta x} \alpha\lambda/(2\lambda - 1)$. Following a similar analysis as in the linear production function, here, liquidity cross-subsidization occurs if $\alpha < 1 - k/\frac{\delta Q}{\delta x}$, akin to the linear benchmark.

Our general production function framework nests two important special cases: homogeneous intermediate goods with a nonlinear production technology and heterogeneous intermediate goods aggregated through a constant elasticity of substitution (CES) production function.

Example 3 (Nonlinear production function with homogeneous intermediate goods) *Let $q(\lambda, c) = 1$ when a supplier is financed, and let $I = \int_\Omega [q + (1 - q)(1 - \lambda)] dG$ represent the measure of intermediate goods supplied to M. The final output, $Q(I)$, is a smooth, concave function of I, with $Q'(|\Omega|) > w(\bar{c})$ ensuring a sufficiently large marginal product even when all suppliers are included. The manufacturer solves:*

$$\max_{q(\cdot) \in \{0,1\}} Q(I) - \int_\Omega \left[q(\lambda, c) \left(c + (1 - \lambda)(w(c) - c) + k \right) + (1 - q(\lambda, c))(1 - \lambda) w(c) \right] dG,$$

subject to $\Theta(I, q) + L \geq 0$, where the pooled liquidity is:

$$\Theta(I, q) = \alpha Q(I) - \int_\Omega [q(\lambda, c)\lambda c + (1 - q(\lambda, c))(1 - \lambda) w(c)] dG.$$

By forming the Lagrangian and applying pointwise optimization, we obtain that the optimal financing decision follows $q(\lambda, c) = 1$ if and only if:

$$\lambda \Big(Q'(I) - c \Big) - k + \mu \Big[\lambda \Big(\alpha Q'(I) - c \Big) + (1 - \lambda) w(c) \Big] \geq 0.$$

Suppose $w(c) = c$, a financed supplier contributes positively to profit when $c < c_{\Delta\pi} \equiv Q'(I) - k/\lambda$ and to liquidity when $c < c_{\Delta\theta} \equiv Q'(I)\alpha\lambda/(2\lambda - 1)$. Thus, liquidity cross-subsidization occurs if $\alpha < 1 - k/Q'(I)$.

Example 4 (CES production function with heterogeneous intermediate goods) *The manufacturer aggregates effective inputs $x(\lambda, c)$ from a continuum of suppliers via*

$$Q = \left(\int_\Omega x(\lambda, c)^\rho dG \right)^{\frac{1}{\rho}}, \quad \rho < 1,$$

where $x(\lambda, c) = q(\lambda, c) + (1 - q(\lambda, c))(1 - \lambda)$. Suppose it it profitable to include all suppliers.[17] *The manufacturer solves the problem of choosing $q(\cdot) \in \{0, 1\}$ to maximize profits:*

$$\Pi = Q - \int_{\Omega} \Big[q(\lambda, c) \Big(c + (1 - \lambda)(w(c) - c) + k \Big) + \Big(1 - q(\lambda, c) \Big)(1 - \lambda) w(c) \Big] dG,$$

subject to $\Theta + L \geq 0$, where the pooled liquidity is: $\Theta = \alpha Q - \int_{\Omega} [q(\lambda, c)\lambda c + (1 - q(\lambda, c))(1 - \lambda) w(c)] dG$. Forming the Lagrangian:

$$\mathcal{L} = Q - \int_{\Omega} q(\lambda, c) \Big(\lambda c + k \Big) dG + \mu \Big[\alpha Q - \int_{\Omega} \Big[q(\lambda, c) \Big(\lambda c - (1 - \lambda) w(c) \Big) \Big] dG.$$

Noting $x(\lambda, c) = 1$ if $q = 1$, or $1 - \lambda$ if $q = 0$, the change of Q when $q(\lambda, c)$ switches from 0 to 1 is $\lambda Q^{1-\rho} g(\lambda, c)$, where $Q^{1-\rho} g(\lambda, c) = \partial Q / \partial x(\lambda, c)$ at $x = 1$. Thus, the marginal impact of switching from $q = 0$ to $q = 1$ on profits is:

$$\lambda \Big(Q^{1-\rho} - c \Big) - k + \mu \Big[\lambda \Big(\alpha Q^{1-\rho} - c \Big) + (1 - \lambda) w(c) \Big].$$

And $q(\lambda, c) = 1$ if the impact above is non-negative. Liquidity cross-subsidization mirrors the general case, occurring when $\alpha < 1 - k / Q^{1-\rho}$ for some suppliers.

6 Conclusion

This paper develops a tractable model of supplier finance, capturing its key features: supplier selection, liquidity pooling, and advance payments to liquidity-constrained suppliers. Our analysis highlights the crucial role of liquidity cross-subsidization for the viability and welfare implications of supplier finance programs. We demonstrate how the intermediary's funding costs influence the trade-off between liquidity provision and profitability, showing that surprisingly, higher funding costs can even enhance social welfare. Furthermore, we examine the interplay between supplier finance and suppliers' direct access to money markets, illustrating how the interest rate premium faced by suppliers shapes the equilibrium allocation of liquidity. Our results are robust to a wide range extensions and the model also applies to the supplier finance in the manufacturing industry.

Intermediaries operating supplier finance programs often possess an informational advantage over traditional financial institutions, such as commercial banks, due to their established industry presence and long-term relationships with suppliers. Our model, based on the assumption of perfect information, provides a tractable framework for understanding supplier finance. Future research could fruitfully extend this model to incorporate information asymmetry, offer-

[17] A sufficient condition for the manufacturer to profitably source all suppliers in $\Omega = [0, 1] \times [\underline{c}, \bar{c}]$ via the wholesale market is $Q_0^{1-\rho} > w(\bar{c})$, where $Q_0 = \left(\int_{\Omega} (1 - \lambda)^{\rho} g(\lambda, c) d\lambda dc \right)^{\frac{1}{\rho}}$ is the output when all $q(\lambda, c) = 0$. The marginal product per unit of $x(\lambda, c) = 1 - \lambda$ is $Q^{1-\rho}(1 - \lambda)^{\rho - 1}$, exceeding the marginal cost $w(c)$ if $Q^{1-\rho}(1 - \lambda)^{\rho - 1} > w(c)$. Since $Q \geq Q_0$, $w(c) \leq w(\bar{c})$, and $(1 - \lambda)^{\rho - 1} \geq 1$ for $\rho < 1$, $Q_0^{1-\rho} > w(\bar{c})$ ensures this holds for all (λ, c). Furthermore, for wholesale market sourcing to be liquidity feasible, we require $\alpha Q_0 - \int_{\Omega} (1 - \lambda) w(c) dG + L \geq 0$.

ing a more nuanced perspective on the dynamics of these programs.

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A Appendix

A.1 Proof of Lemma 1

The intersection point of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$ is $(\lambda_0, c_0) = \left(\frac{k+\sqrt{k^2+4ku}}{2mu},\ u + k - \sqrt{k^2 + 4ku}\right)$. We can derive that

$$\frac{\partial b(\lambda, \mu)}{\partial \mu} = \frac{2\left(km\lambda + k - u(m\lambda)^2\right)}{(m\lambda\mu + m\lambda + \mu)^2},$$

which is positive if $\lambda < \lambda_0$ and negative if $\lambda > \lambda_0$. That is, as μ increases, $c = b(\lambda, \mu)$ rotates around (λ_0, c_0) clockwise, which implies that more suppliers with positive θ_F are selected (i.e. $q(\lambda, c, \mu)$ is increasing in μ for (λ, c) such that $\theta_F(\lambda, c) > 0$) and fewer suppliers with negative θ_F are selected (i.e. $q(\lambda, c, \mu)$ is decreasing in μ for (λ, c) such that $\theta_F(\lambda, c) < 0$).

Taking $\bar{c}$ as given. If $c_0 \in [\underline{c}, \bar{c}(i)]$, since $g(\cdot)$ is everywhere positive in Ω, it holds that $\Theta(\mu, i) = \int_\Omega q(\lambda, c, \mu)\theta_F(\lambda, c)dG$ is strictly increasing in μ.

If $c_0 > \bar{c}$, and suppose $\lambda_0 < 1$, then there exist unique threshold values, denoted by $\underline{\mu} > 0$ and $\bar{\mu} \in (\underline{\mu}, \infty)$, such that the curve of $c = b(\lambda, \mu)$ lies entirely above $c = \bar{c}$ for $\mu \in (\underline{\mu}, \bar{\mu})$, see Figure 12. For $\mu \in (\underline{\mu}, \bar{\mu})$, $\Theta = \int_\Omega \theta_F(\lambda, c)dG$ is independent of μ, which means that μ does not influence the selection of suppliers. For $\mu \in (0, \underline{\mu}) \cup (\bar{\mu}, \infty)$, by the same logic as shown above, $\Theta(\mu)$ is strictly increasing in μ. Suppse $\lambda_0 \geq 1$, then $\Theta(\mu)$ is strictly increasing in μ for $\mu \in (0, \underline{\mu})$ and keeps constant for $\mu > \underline{\mu}$.

Common to all cases is that when μ approaches infinity, only suppliers with positive θ_F are selected, thus $\Theta(\infty) > 0$.

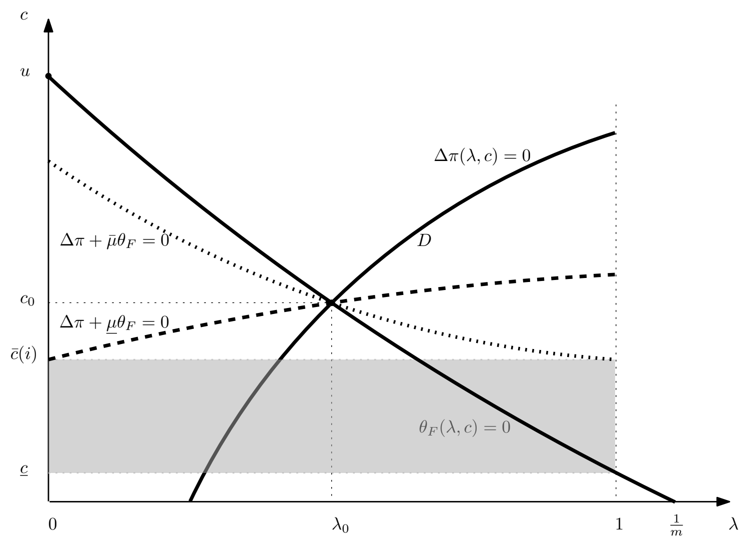

Figure 12: For $\mu \in (\underline{\mu}, \bar{\mu})$, $c = b(\lambda, \mu)$ lies above $\bar{c}$.

Now we show that μ is generically unique. Since $\Theta(\mu)$ is monotonically increasing in μ, if $\Theta(0) + L \geq 0$, then $\mu = 0$. If $\Theta(0) + L < 0$, then the liquidity constraint is binding at some $\mu \in (0, \infty)$, which is uniquely pinned down by $\Theta(\mu) + L = 0$. Note that when $L = -\int_\Omega \theta_F(\lambda, c)dG \geq 0$ and $c_0 > \bar{c}(i)$, any $\mu \in [\underline{\mu}, \bar{\mu}]$ satisfies $\Theta(\mu) + L = 0$. In this case, we define the solution μ as $\underline{\mu}$.

$\blacksquare$

A.2 Proof of Corollary 1

Given that $\Theta(0) + L < 0$, $\mu(L)$ is determined by (12). Since $\Theta(\mu)$ is strictly increasing in μ outside the interval $[\underline{\mu}, \bar{\mu}]$ provided the range exists and also note that we have selected $\mu = \underline{\mu}$ if $\Theta(\underline{\mu}) = \Theta(\bar{\mu}, i) = L$, the statement follows. $\blacksquare$

A.3 Proof of Proposition 1

By the Euler equation (13), there are two cases. First, if $i \geq \mu(0)$, then $L = 0$. This case is valid either if $\mu(0) = 0$ (then $i > \mu = 0$ follows), or if $\mu(0) > 0$. Second, $i = \mu(L) > 0$ and $L > 0$, which requires that $\Theta(0) < 0$ and $i \leq \mu(0, i)$. $\blacksquare$

A.4 Proof of Lemma 3

Given that $b'_\lambda(\lambda, i) = \frac{2m(k+ik-i^2u)}{(i+m\lambda+im\lambda)^2}$, it is straightforward to verify that $b'_\lambda(\cdot) > 0$ if $i < i_0 \equiv \frac{k+\sqrt{k^2+4uk}}{2u}$, and $b'_\lambda(\cdot) < 0$ if $i > i_0$. The relationship between $c = b(\lambda, i)$ and $c = \bar{c}(i)$ can be obtained by comparing $b(1/m, i) = \frac{u-2k}{1+2i}$ with $\bar{c}(i) = \frac{1-i}{1+i}u$. If $i < i_0$, then $b(1/m, i) < \bar{c}(i)$. If $i > i_0$, then $b(1/m, i) > \bar{c}(i)$.$\blacksquare$

A.5 Proof of Lemma 4

Let $Ratio(m) \equiv -\frac{\Delta\pi(\cdot)}{\theta_F(\cdot)} = -\frac{m\lambda(u-c)/2-k}{(1-m\lambda)(u+c)/2-c}$. Then $Ratio'(m)$ has the same sign as $c^2 - 2(k+u)c + u(u-2k)$. It follows $Ratio'(m) < 0$ if $c < c_0$, and $Ratio'(m) > 0$ if $c > c_0$.$\blacksquare$

A.6 Proof of Proposition 3

Note that $k < \bar{k}$ implies $c_0 > \underline{c}$. Let $\kappa \equiv \frac{k}{u}$ and $\bar{\kappa} \equiv \frac{\bar{k}}{u} \equiv \frac{(u-\underline{c})^2}{2u(u+\underline{c})}$. For all $i \leq i_1$, the multiplier μ at $L = 0$ does not depend on i: $\mu(0, i) = \mu(0, 0)$. We suppose the latter is positive throughout the proof, thus $\mu(i) = i$. A necessary condition of $\mu(0, 0) > 0$ is $m > \tilde{m}(\kappa)$. We focus on the parameter space where $\kappa \in (0, \bar{\kappa})$, and $m \in (\tilde{m}(\kappa), 1]$ for each value of κ.

There are two cases for $\kappa < \bar{\kappa}$. One case is that $\kappa \leq \underline{\kappa} \equiv \frac{(u-\bar{c})^2}{2u(u+\bar{c})}(< \bar{\kappa})$, then $c_0 \geq \bar{c}$. A marginal increase of i from $i = 0$ will select more suppliers into the finance contract and drop no suppliers from it. Thus, social welfare must be improved.

The other case is that $\kappa \in (\underline{\kappa}, \bar{\kappa})$, namely, $c_0 \in (\underline{c}, \bar{c})$. Then the middleman's selection rule for the finance contract is $q(\lambda, c, \mu) = 1$ iff $c \in [\underline{c}, b(\lambda, \mu)]$ whenever $b(\lambda, \mu) \geq \underline{c}$. Here, $b(\lambda, \mu) = \frac{m\lambda u - 2k + \mu(1-m\lambda)u}{m\lambda + \mu(1+m\lambda)}$. The welfare gain by having active middleman finance is

$$\Delta\mathcal{W}(\mu) = \int_{\lambda_l(\mu)}^{\lambda_h(\mu)} \int_{\underline{c}}^{b(\lambda,\mu)} \Big(m\lambda(u-c) - k\Big) g(\lambda, c) dc d\lambda + \int_{\lambda_h(\mu)}^{1} \int_{\underline{c}}^{\bar{c}} \Big(m\lambda(u-c) - k\Big) g(\lambda, c) dc d\lambda,$$

where we have imposed that $b(\lambda,\mu)$ is upward sloping with respect to λ, which is always the case when μ is in the neighborhood of $\mu=0$. Here, $\lambda_h(\mu)=\min\{1,\frac{1}{m}\frac{2k-\mu(u-\bar{c})}{(u-\bar{c})-\mu(u+\bar{c})}\}$, and $\lambda_l(\mu)=\max\{0,\frac{1}{m}\frac{2k-\mu(u-\underline{c})}{u-\underline{c}-\mu(u+\underline{c})}\}$.

Observe that $\frac{\partial\Delta\mathcal{W}(\mu)}{\partial\mu}=\int_{\lambda_l(\mu)}^{\lambda_h(\mu)}\Big(m\lambda(u-b(\lambda,\mu))-k\Big)g(\lambda,b(\mu))b'_\mu(\lambda,\mu)d\lambda$. Since (λ,c) follows a uniform distribution, g is a constant. Let $\propto$ represent "proportional to". Inserting $b(\lambda,0)=u-\frac{2k}{m\lambda}$ and $b'(0)=2u\left(\frac{\kappa}{m^2\lambda^2}+\frac{\kappa}{m\lambda}-1\right)$, we have

$$\frac{\partial\Delta\mathcal{W}(\mu)}{\partial\mu}|_{\mu=0}\propto\int_{\lambda_l}^{\lambda_h}\Big[\frac{\kappa}{m^2\lambda^2}+\frac{\kappa}{m\lambda}-1\Big]d\lambda,\tag{18}$$

where $\lambda_h\equiv\lambda_h(0)=\min\{1,\frac{1}{m}\frac{u\kappa}{(u-\bar{c})/2}\}$, and $\lambda_l\equiv\lambda_l(0)=\frac{1}{m}\frac{u\kappa}{(u-\underline{c})/2}$. Note that $\lambda_l<1$. This is because with $m>\tilde{m}$, $c_\pi(1)>c_0>\underline{c}$ where the second inequality is given by $\kappa<\bar{\kappa}$. And $c_\pi(1)>\underline{c}$ is equivalent to $\lambda_l<1$. Define $\bar{\varepsilon}=\frac{u}{(u-\bar{c})/2}$ and $\underline{\varepsilon}=\frac{u}{(u-\underline{c})/2}$. It holds that $\bar{\varepsilon}>\underline{\varepsilon}>2$. Furthermore, it is straightforward to check that $\underline{\kappa}<\frac{1}{\bar{\varepsilon}}<\bar{\kappa}<\frac{1}{\underline{\varepsilon}}$. With this, $\lambda_h=\min\{1,\frac{1}{m}\kappa\bar{\varepsilon}\}$, $\lambda_l=\frac{1}{m}\kappa\underline{\varepsilon}$.

Substitute for λ by $x\equiv m\lambda$. Then the upper and lower bounds become $x_h=\min\{m,\kappa\bar{\varepsilon}\}$, and $x_l=\kappa\underline{\varepsilon}$, and we have

$$\frac{\partial\Delta\mathcal{W}(\mu)}{\partial\mu}|_{\mu=0}\propto\int_{\kappa\underline{\varepsilon}}^{\min\{m,\kappa\bar{\varepsilon}\}}\Big[\frac{\kappa}{x^2}+\frac{\kappa}{x}-1\Big]dx\equiv H(m,\kappa).\tag{19}$$

In the following, we look for the parameter space for $H(m,\kappa)>0$.

Suppose $\bar{\varepsilon}\kappa<m$. Then, by (19) $H(m,\kappa)$ does not depend on m directly:

$$H(m,\kappa)=-\kappa(\bar{\varepsilon}-\underline{\varepsilon})+\frac{\bar{\varepsilon}-\underline{\varepsilon}}{\bar{\varepsilon}\underline{\varepsilon}}+\kappa\Big(\log(\bar{\varepsilon})-\log(\underline{\varepsilon})\Big),$$

which is positive iff $\kappa<\frac{1}{\bar{\varepsilon}\underline{\varepsilon}}\frac{1}{1-\frac{\log(\bar{\varepsilon})-\log(\underline{\varepsilon})}{\bar{\varepsilon}-\underline{\varepsilon}}}\equiv\kappa^*$.

Suppose $\bar{\varepsilon}\kappa\geq m$, then

$$H(m,\kappa)=\int_{\kappa\underline{\varepsilon}}^{m}\Big[\frac{\kappa}{x^2}+\frac{\kappa}{x}-1\Big]dx.$$

In this case, $H(m,\kappa)$ has the following properties:

- $H(m,\kappa)$ is strictly decreasing in m for $m\in[\tilde{m},1]$ since $\frac{\partial H(m,\kappa)}{\partial m}=\frac{\kappa+\kappa m-m^2}{m^2}<0$ for $m>\tilde{m}$.
- $H(\tilde{m}(\kappa),\kappa)\geq 0$ since $\frac{\kappa}{x^2}+\frac{\kappa}{x}-1\geq 0$ for $x\leq\tilde{m}$. $H(\tilde{m}(\kappa),\kappa)=0$ only if $\kappa=\bar{\kappa}$.
- $H(1,\kappa)<0$. To see this, using (19)

$$H(1,\kappa)=\Big[-\lambda+\kappa\Big(\log(\lambda)-1/\lambda\Big)\Big]_{\underline{\lambda}}^{1}=(\underline{\varepsilon}-1)(\kappa-\frac{1}{\underline{\varepsilon}})-\kappa\log(\kappa\underline{\varepsilon})\equiv h(\kappa).$$

  Note that $h'(\kappa)=\underline{\varepsilon}-2-\log(\underline{\varepsilon}\kappa)>0$ since $\underline{\varepsilon}>2$ and $\kappa\underline{\varepsilon}=\underline{\lambda}<1$. Then $h(\kappa)<h(\bar{\kappa})<h(1/\underline{\varepsilon})=0$ (since $\bar{\kappa}<\frac{1}{\underline{\varepsilon}}$). Thus, $H(1,\kappa)<0$.

These properties together imply that there must exist $m^*(\kappa)\in[\tilde{m}(\kappa),1)$ such that $H(m,\kappa)>0$ iff $m<m^*(\kappa)$.

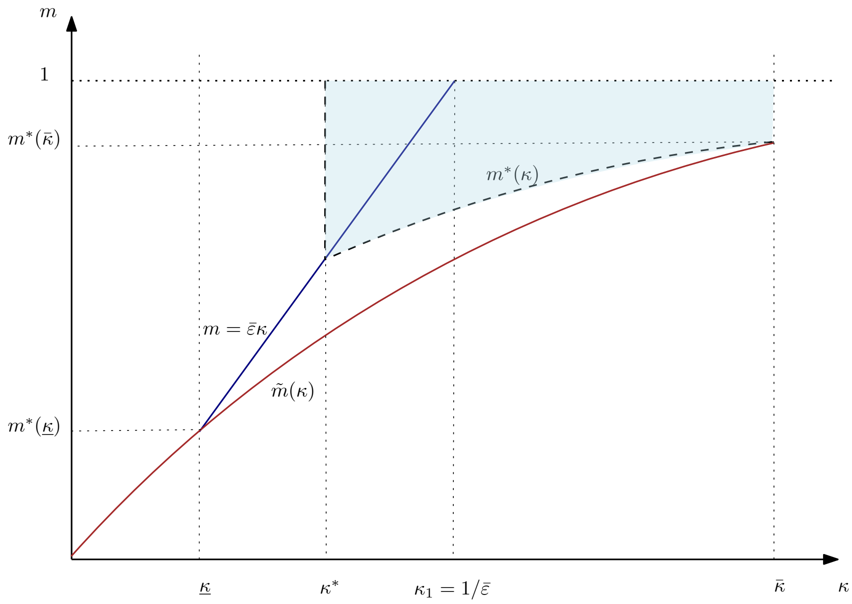

Figure 13: The parameter space that determines the sign of $H(m,\kappa)$ (in shaded region $H(\cdot) < 0$.)

Figure 13 illustrates the parameter space of (κ, m) where $m = \tilde{m}(\kappa)$ (upward concave curve) and $m = \bar{\varepsilon}\kappa$ (upward straight line) are plotted. As argued above, we shall focus on $m > \tilde{m}(\kappa)$ since only in that case it is plausible that $\mu(0,0) > 0$. Whether m larger or smaller than $\bar{\varepsilon}\kappa$ (the upward straight line) determines the upper bound of the integral in $H(m,\kappa)$.

In this figure, we have applied the following properties of $m = \tilde{m}(\kappa)$: (1) $\tilde{m}(0) = 0$, and $\tilde{m}(\bar{\kappa}) < 1$; (2) It is increasing and concave on $\kappa \in [0,\bar{\kappa}]$: $\tilde{m}'(\kappa) = \frac{1}{2}\left(1 + \frac{\kappa+2}{\sqrt{\kappa^2+4\kappa}}\right) > 0$, $\tilde{m}''(\kappa) = -\frac{2}{(4\kappa+\kappa^2)^{3/2}} < 0$. We also applied the following properties of $m = \bar{\varepsilon}\kappa$: $\bar{\varepsilon}\bar{\kappa} > 1$, and $\bar{\varepsilon}\underline{\kappa} = \tilde{m}(\underline{\kappa})$.

First consider the case of $m > \bar{\varepsilon}\kappa$, then $H(m,\kappa) > 0$ iff $\kappa < \kappa^*$. Define $\kappa_1 = \frac{1}{\bar{\varepsilon}}$ the value of κ where $m = \bar{\varepsilon}\kappa$ intersects with $m = 1$. κ^* must lie between $\underline{\kappa}$ and κ_1. $\kappa^* > \underline{\kappa}$ because $H(1,\underline{\kappa}) > 0$ (when $\kappa = \underline{\kappa}$, we have $c_0 = \bar{c}$ and a marginal increase in i only selected more suppliers into the finance contract without dropping suppliers out of it.) $\kappa^* < \kappa_1$ because $H(1,\kappa_1) < 0$.

Second, consider the case where $m < \bar{\varepsilon}\kappa$. In this scenario, $H(m,\kappa) > 0$ if and only if $m < m^*(\kappa)$. As shown in the figure, $m^*(\kappa)$ connects $(\bar{\kappa}, \tilde{m}(\bar{\kappa}))$ at one end and $(\kappa^*, \bar{\varepsilon}\kappa^*)$ at the other end. We have $m^*(\bar{\kappa}) = \tilde{m}(\bar{\kappa})$ because when $\kappa = \bar{\kappa}$ and $m = \tilde{m}(\bar{\kappa})$, $\Delta\mathcal{W}(\mu)$ is constantly zero and independent of μ. By definition of κ^*, $H(m^*(\kappa^*), \kappa^*) = 0$. ■

A.7 Proof of Proposition 4

Let $\Pi(i^s, i) \equiv \int_{\tilde{\Omega}(i^s)} q(\lambda, c)\Delta\pi(\lambda, c)dG$ be the maximized profits of the middleman from activating the finance service taking nominal interest rate $i < i_1$ as given. Let $c_{\Delta\pi}(\lambda) = u - \frac{2k}{m\lambda}$ denote the curve of (λ, c) such that $\Delta\pi(\lambda, c) = 0$. It can be shown that $c^s(\lambda, i^s)$ and $c_{\Delta\pi}(\lambda)$ cross each other at most once.

If $c^s(1, i^s) > c_{\Delta\pi}(1)$, or equivalently, $i^s < \frac{k}{mu-2k}$, then $c^s(\lambda, i^s) > c_{\Delta\pi}(\lambda)$ for all $\lambda \in [0,1]$, meaning that all suppliers with positive profits $\Delta\pi(\lambda, c)$ are excluded from $\tilde{\Omega}(i^s)$. Thus, we must have $\Pi(i^s, i) = 0$. On the other hand, if $i^s \geq \bar{i}^s \equiv \frac{u-\underline{c}}{2\underline{c}}$, then $\tilde{\Omega}(i^s) = \Omega$, resulting in $\Pi(i^s, i) > 0$. Note that $\lambda_0 < 1$ implies $c_{\Delta\pi}(1) > \underline{c}$, which is equivalent to $\bar{i}^s > \frac{k}{mu-2k}$.

Finally, $\Pi(\cdot)$ is weakly increasing in i^s, because as i^s increases, the set of feasible suppliers $\tilde{\Omega}(i^s)$ becomes larger. Therefore, $\underline{i}^s \in [\frac{k}{mu-2k}, \bar{i}^s)$ must exist. Combined with the suppliers' money-holding decision rule (see condition (17) in the main text), this proves the claims in the proposition. ■

A.8 Proof of Proposition 5

We apply Brouwer's Fixed-Point Theorem to show a solution exists. First, take $\mu \in [0, \bar{\mu}]$, where $\bar{\mu} > 0$ is an upper bound ensuring $\mu\Theta = 0$. Since Q is bounded and each supplier's profit contribution is finite, such a $\bar{\mu} < \infty$ exists—beyond it, high financing costs push q to 0, satisfying liquidity. Define a mapping $T : [0, \bar{\mu}] \to [0, \bar{\mu}]$: (1) For each μ, set $q(\lambda, c) = 1$ if $(1 + \mu\alpha)\frac{\delta Q}{\delta q} > \lambda c(1 + \mu) + k - \mu(1 - \lambda)w(c)$ (finance), else $q = 0$ (wholesale). (2) Compute $\Theta = \alpha Q(q) - \int [q\lambda c + (1 - q)(1 - \lambda)w(c)]dG$. (3) Set $T(\mu) = 0$ if $\Theta + L \geq 0$; otherwise, set $T(\mu) = \min\{\mu + \beta|\Theta + L|, \bar{\mu}\}$, where $\beta > 0$ is small. T maps $[0, \bar{\mu}]$ to itself and is continuous: Q is continuous in q under the $L^1(\Omega, G)$ topology, and G's atomless nature ensures Θ adjusts gradually with μ. By Brouwer's theorem, there exists $\mu^* = T(\mu^*)$. Then: if $\mu^* = 0$, $\Theta + L \geq 0$; if $\mu^* > 0$, $\Theta + L = 0$. The corresponding q^* satisfies the decision rule and constraint $\Theta + L \geq 0$. ■