Middlemen, Liquidity Support, and Product Market Distortions *

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February 14, 2026

Abstract

We develop a model in which a middleman finances suppliers by advancing payments to cover their liquidity needs. We show that, paradoxically, funding these advances requires suppliers to extend trade credit to the middleman, resulting in *liquidity cross-subsidization* among suppliers. We further demonstrate that the middleman's matching efficiency and financing scope are not intrinsic complements; rather, their relationship is governed by the external cost of funds. Crucially, the model identifies a transmission channel from financial constraints to product markets: the middleman strategically raises retail prices to economize on liquidity usage. This generates *liquidity-induced double marginalization*. Thus, while financing enhances supply resilience, the resulting price distortion taxes consumers and can harm consumer welfare.

Keywords: *Middlemen*, *Liquidity Pooling*, *Trade Credit*, *Double Marginalization*

*An earlier version of this paper circulated under the title "A Model of Supply Chain Finance." We are benefited from comments, discussions and suggestions by Briana Chang, Jay Choi, Piero Gottardi, Yoske Igarashi, Bruno Jullien, Todd Keister, Peter Kondor, Noritaka Kudoh, Nan Li, Cyril Monnet, Volker Nocke, Yoshiaki Ogura, Jean Tirole, Hirofumi Uchida, Yu Zhu, and participants in Kyoto Workshop 2023 on Digitalization and Macroprudential Policies, Workshop on Platform Economics at Yonsei University in Korea, the 2023 Summer Workshop Money, Banking, Payment, and Finance in Switzerland, the 2023 Autumn meeting of the Japanese Applied Economics Association, the 2024 Spring meeting of the Japanese Economic Association, the Essex-UCL Workshop on Decentralized Financial Markets, the 2024 Asian Meeting of the Econometric Society in Ho Chi Minh City. The remaining errors are ours. Hu acknowledges the financial support from the Natural Science Foundation of China (Grant 72003041), the Shanghai Pujiang Program (Grant 21PJC011), and the Shanghai Institute of International Finance and Economics. Watanabe acknowledges the financial support from JSPS KAKENHI (Grants JP23H00054, JP22K20161, and JP23K17286) and the Murata Science Foundation. Zhang acknowledges the financial support from the Ministry of Education of China (Grants #IRT_17R24 and #2023JZDZ018).

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1 Introduction

Middlemen in product markets (e.g., retailers, merchants, trading companies) often provide liquidity to their suppliers. Historically, middlemen and liquidity provision were inextricably linked. During the colonial era, European trading companies like the Dutch East India Company offered merchant credit to local producers in Africa, Asia, and the Americas by advancing payments for goods such as spices, cotton, and tobacco.

Despite the development of modern credit markets, this function has not diminished. Rather, financing suppliers has become increasingly critical for the operation of the middleman, particularly when suppliers face severe liquidity constraints. Facilitated by Fintech providers that streamline credit enforcement, major global intermediaries such as Walmart, Alibaba, and JD.com have aggressively adopted this financing approach to support their suppliers.[1]

A deeper examination of the middleman-provided finance reveals several unique features that distinguish it from traditional bank lending:

- While a middleman provides suppliers with liquidity through accelerated payments, it also relies on participating suppliers to extend trade credit to it. For instance, the *JingBaoBei* program operated by JD.com, the largest e-commerce platform in China, is primarily funded by liquidity pools sourced from suppliers themselves via extended accounts payable. In other words, the program intended to provide liquidity is essentially funded by the very supplier base it aims to serve. Indeed, the extension of trade credit by suppliers is a common feature of middleman-provided finance.
- Access to middleman-provided finance programs is typically invitation-only. The middleman proactively selects a subset of suppliers from her broader base to participate. For example, when the UK supermarket chain Co-op launched its finance program during the pandemic in 2020, it selected fewer than 100 suppliers from a base of thousands. Similarly, Amazon Lending operated as an invitation-only program for over a decade, utilizing strict metrics to filter participants.[2]

Beyond these operational features, a fundamental difference between middlemen financiers

[1]Retailers dominate middleman finance. According to a FreightWaves analysis, retailers, along with consumer packaged goods and telecom companies, account for fully two-thirds of the middleman finance outstanding; see "Corporate Disclosures Highlight Heavy Use of Supply Chain Financing," *FreightWaves*, June 8, 2023, available at *https://www.freightwaves.com/news/corporate-disclosures-highlight-heavy-use-of-supply-chain-financing* (accessed on Jan 6, 2025). For examples of middleman finance beyond the retailers listed in the text, see "Companies Offer Supply-Chain Financing to Vendors as They Bulk Up on Inventory, Push Out Payment Terms," *The Wall Street Journal*, July 20, 2022, available at *https://www.wsj.com/articles/companies-offer-supply-chain-financing-to-vendors-as-they-bulk-up-on-inventory-push-out-payment-terms-11658316600* (accessed on Jul 17, 2023). Commonly termed supplier finance or supply chain finance (SCF), this market was valued at $1.8 trillion in 2021 globally (BCR Publishing).

[2]This selectivity reflects a careful weighing of the benefits and costs of including each supplier, consistent with best practices highlighted in the International Finance Corporation's *Supply Chain Finance Knowledge Guide*. The guide emphasizes prioritizing suppliers based on relationship depth and financial need. For details on Co-op's program, see *https://scfcommunity.org/briefing/news/2020-retail-and-apparel-winner-co-operative-group* (accessed on Jul 17, 2023). For Amazon Lending details, see *https://www.junglescout.com/blog/amazon-lending-program* (accessed on Jul 17, 2023). Amazon Lending was shut down in 2024, marking a strategic shift away from in-house financing.

and traditional banks lies in their market position: *middlemen directly face consumers and set final product prices.* This dual role implies that the middleman's upstream liquidity provision is inextricably linked to its downstream pricing strategy—a connection that remains underexplored.

In this paper, we investigate the economic rationale behind these stylized features. Specifically, we ask: Why do programs designed to relieve supplier liquidity constraints rely on supplier-provided trade credit? How does the middleman determine the financing pool? And critically, how does this arrangement affect retail prices and consumer welfare?

We develop a model in which a middleman acts as both merchant and liquidity provider. We show that supplier liquidity pooling and selective inclusion are intrinsic to this dual role. Furthermore, we demonstrate that middleman finance extends beyond purely financial frictions; it generates *liquidity-induced double marginalization*, effectively passing the cost of supply-side resilience to consumers.

Sections 2 and 3 introduce a benchmark model featuring a retail market that operates across two sequential sub-periods: early and late. Consumers have unit demand for all products. Production occurs exclusively in the early sub-period, requiring suppliers to incur a cost c in numeraire (e.g., cash for raw materials). However, suppliers lack an initial endowment, creating a temporal mismatch between cost obligations and revenue receipt. If a supplier matches with consumers early, revenue arrives in time to cover costs, allowing production to proceed. Conversely, if consumers match late, the supplier lacks the funds to finance production, precluding trade. We denote the probability of this late-period matching—effectively a liquidity shock—by λ.

This setup captures scenarios where a supplier's liquidity is endogenous to their retail technology. Inferior logistics or display technologies often prevent suppliers from securing the early payments necessary to fund production. We assume suppliers are heterogeneous in their production costs c and liquidity risk λ, both of which are publicly observable.

We introduce an intermediary who, while unable to produce, possesses a superior retail technology that reduces suppliers' liquidity shocks from λ to $m\lambda$, where $m < 1$.

Crucially, the intermediary also possesses a financial technology that allows her to pool trade credit: she retains early retail revenues and reallocates them as upfront payments to suppliers facing liquidity needs. This enforcement technology, while costly, enables the intermediary to function as a liquidity provider.

Given her retail advantage, the intermediary optimally offers retail services to all available suppliers. Her key decision is whether to extend financing (advanced payments) to specific suppliers. This decision is constrained by the aggregate liquidity pool: the intermediary must ensure that total advanced payments do not exceed her available funds, which consist of her internal capital and the retail revenue generated from early sales. Consequently, she evaluates potential candidates for financial intermediation based not only on their individual profitability but also on their net contribution to the liquidity pool.

Our central result is that the intermediary's financing program functions as a mechanism of profit-based liquidity cross-subsidization. Suppliers with high liquidity risk (high λ) and low costs (low c) are highly profitable but need liquidity; conversely, suppliers with low liquidity risk and low costs act as net contributors to the liquidity pool. The intermediary optimally reallocates trade credit from the latter to the former. This portfolio approach outperforms both universal inclusion and selection based solely on standalone profitability. The intermediary thus acts as a strategic gatekeeper, curating a mix of net liquidity providers and net liquidity users within the supplier base to maximize total profit, akin to the multiproduct assortment problem in Rhodes, Watanabe and Zhou (2021).

Second, we characterize a non-monotonic relationship between the intermediary's retail efficiency and liquidity provision. This equilibrium stems from two countervailing forces. On the supply side, improved retail efficiency (lower m) accelerates revenue realization, expanding the pool of internal funds—a *liquidity supply effect*. On the demand side, however, faster sales reduce suppliers' exposure to late-period shocks, lowering their need for credit—a *liquidity demand effect*. We show that when the external cost of capital is high, the liquidity supply effect dominates, making retail efficiency and financing *complements*. Conversely, when external costs are low, the demand effect prevails, rendering them *substitutes*. In the limit, as retail frictions vanish, the demand for liquidity disappears. This mechanism rationalizes recent market shifts, such as Amazon's 2024 decision to terminate its in-house lending program, effectively replacing liquidity provision with superior retail efficiency.

In Section 4, we extend the baseline framework to general demand to examine how upstream financial constraints transmit to downstream product market outcomes. We identify a novel distortion arising from the intermediary's dual role: *liquidity-induced double marginalization*. When the aggregate liquidity constraint binds, the intermediary optimally raises retail prices above the standard monopoly level. Crucially, this differs from classic double marginalization driven by vertical markups; here, the distortion stems from the intermediary's *shadow cost of liquidity*. By raising prices and contracting sales, the intermediary reduces the upfront capital required to fund production, thereby relaxing the liquidity constraint. In effect, she levies a "tax" on consumers to subsidize the liquidity required to fund production.

Finally, in Section 5, we analyze the welfare implications. We show that middleman finance strictly reduces consumer surplus for products with high production costs and low liquidity risk. Intuitively, for such products, the "liquidity tax" embedded in higher retail prices outweighs the modest gain from improved supply reliability. We examine how the scope of this welfare loss responds to market conditions. We find that tighter financial conditions (higher cost of capital) expand the region of consumer harm, as the intermediary raises prices more aggressively to conserve liquidity. More surprisingly, improvements in retail technology can yield a counterintuitive result: better matching diminishes the value of the financing guarantee more than it alleviates the pricing distortion, thereby expanding the region of *potential* consumer harm. We emphasize, however, that realized welfare loss depends on whether the intermediary chooses to finance these specific products in equilibrium—a decision driven by the cross-subsidization incentives of her portfolio and the cost of her financial technology.

The remainder of this section is dedicated to a literature review. All proofs are provided in the Appendix. Additional results and further extensions are available in the Online Appendix.

Related Literature

Classical theories rationalize the emergence of intermediaries as a response to various market frictions—such as search costs (e.g., Rubinstein and Wolinsky, 1987; Watanabe, 2010, 2018, 2020), information asymmetry and quality certification (e.g., Biglaiser, 1993; Biglaiser and Li, 2018; Lizzeri, 1999), or inventory risks (Li, Murry, Tian and Zhou, 2024). In particular, our paper is related to the emerging literature on hybrid platforms (e.g., Tirole and Bisceglia, 2023; Madsen and Vellodi, 2025; Gautier, Hu and Watanabe, 2023; Etro, 2023; Shopova, 2023; Kang and Muir, 2022), where existing studies typically view first-party and third-party sales as either competitive substitutes—raising self-preferencing concerns (e.g., Hagiu, Teh and Wright, 2022; Padilla, Perkins and Piccolo, 2022; Zennyo, 2022)—or strategic complements via "testing groun" or "one-stop-shop" effects (e.g., Madsen and Vellodi, 2025; Anderson and Bedre-Defolie, 2022). However, the intermediary's dual role as both a reseller and a liquidity provider remains underexplored.

More specifically, our analysis relates closely to Nocke and Thanassoulis (2014), who examine how credit constraints influence vertical relationships. In their framework, buyer firms face credit constraints regarding future investments and thus demand "profit insurance" from upstream suppliers in the form of a fixed payment (e.g., a slotting fee). In response, suppliers recoup these fixed payments by raising wholesale prices above the production cost, inducing double marginalization. We identify a distinct transmission mechanism linking upstream liquidity needs to downstream pricing. In our model, raising prices serves as a strategic choice to relax binding liquidity constraints: it reduces the associated upfront capital requirement. Unlike Nocke and Thanassoulis (2014) where the price distortion arises from a motive to share risk, here it arises from the incentive to economize on liquidity.

Our methodological approach parallels Rhodes, Watanabe and Zhou (2021), who analyze the optimal product assortment of a multiproduct intermediary. They show that the middleman's problem can be described as the choice of a set of points in a two-dimensional space defined by sufficient statistics. The middleman's optimal product assortment includes high-value products with low profitability, which generate a direct loss for the middleman, and low-value products with high profitability, which recoup those losses. We show that this logic extends to middleman finance: our middleman manages a portfolio of suppliers defined by profitability and liquidity contribution. Analogous to their traffic-generating products, our intermediary optimally includes net lenders—suppliers who may yield lower direct profit but provide the essential liquidity required to finance the high-yield, constrained suppliers.

Finally, unlike the liquidity pooling in Diamond and Dybvig (1983), our middleman engages in *targeted* liquidity provision, where the trade-off between profitability and liquidity contribution necessitates the strategic exclusion of certain suppliers.[3] Liquidity cross-subsidization offers a novel resolution to a long-standing puzzle in the trade credit literature: why small suppliers extend credit to apparently cash-rich buyers (e.g., Klapper, Laeven and Rajan, 2012). We argue that this flow reflects not mere credit extraction by powerful buyers, but a redistribution of liquidity that enables production and trade that would otherwise fail.

2 The Baseline Model

Consider a one-period economy populated by three types of agents: a mass of consumers, a continuum of suppliers (he), and a single intermediary (she). Each supplier produces a unique, indivisible good at constant marginal cost c. Suppliers are heterogeneous in their costs, $c \in [\underline{c}, \bar{c}]$, where $\bar{c} > \underline{c} > 0$. Consumers are homogeneous and have unit demand for each good, deriving common utility $u \geq \bar{c}$. In the following section, we extend our analysis to general demand. The intermediary produces nothing but possesses a retailing technology to buy from suppliers and resells to consumers. She also has access to a costly finance technology that allows her to fund suppliers' need for liquidity.

A numeraire good, which we refer to as "money" or "liquidity," facilitates payments. Consumers hold sufficient endowment to transact. Suppliers have no initial endowment. The intermediary holds a cash balance $L \geq 0$, funded by external capital at a nominal interest rate $i \geq 0$.

Agents trade in a retail market. If the intermediary sells a good, she purchases it from the supplier, who then exits the market. A seller (supplier or intermediary) can reach all consumers. When a consumer and a seller meet, trade occurs bilaterally, and the trade surplus is split equally, yielding an equilibrium retail price:[4]

$$p = \frac{u + c}{2}. \tag{1}$$

[3] Relatedly, Donaldson, Piacentino and Thakor (2018) link warehouse technology to credit extension. Closer to our specific setting, Li and Pegoraro (2022) model platform-provided credit.

[4] Our results are robust to alternative pricing mechanisms. See the discussion following Theorem 2 and the analysis in Section 4.

Liquidity Shocks. There are two sequential subperiods: *early* and *late*. Production requires the supplier to incur cost c in the *early* sub-period. We interpret c as production expenses—such as wages, rent, or supplier payments—that must be paid by the early sub-period. In a frictionless market, revenue realizes instantly to cover costs. However, liquidity becomes a problem when a timing disparity exists between production and trade. We assume that while production must occur early, matching with consumers is stochastic. With probability $1 - \lambda$, the match with all consumers occurs in the early sub-period, allowing the supplier to cover c with immediate revenue. With probability λ, the match occurs in the *late* sub-period. Since the supplier lacks an initial endowment, a late match creates a liquidity shortage that prevents production. We refer to this event—where a viable trade fails due to a timing mismatch—as a *liquidity shock*.

This setup captures real-world scenarios where suppliers' liquidity depends on their retail technologies. No trade occurs because of limited retail technologies possessed by suppliers to have consumers matched early rather than late. For instance,

- Display/advertisement: A supplier can display his good to consumers in the early subperiod with probability $1 - \lambda$ and in the late subperiod with probability λ. For experience goods where quality must be verified, consumers buy only after inspection; thus, it is only in the former case that the supplier can produce and trade. Better advertising technologies (e.g., live-streaming on platforms like TikTok) increase the chance of early display.

- Delivery/inventory: A supplier can deliver his good to consumers in the early subperiod with probability $1 - \lambda$, and in the late subperiod with probability λ. As is common in online marketplaces preventing fraud, consumers' payments are released to sellers only after delivery. Thus, it is only in the former case that the supplier can produce and trade. Better inventory technologies (e.g., automated warehousing or integrated logistics networks) increase the chance of early delivery.

- Production-to-order: A supplier has access to "production-to-order" technology with probability $1 - \lambda$ and can only "produce to inventory" with probability λ. Production-to-order allows suppliers to produce goods after receiving an order and payment from consumers. Then it is only when this technology is accessible that the supplier can produce and trade. Better promotion or communication with consumers, facilitated by competent sales persons, increases the chance of production to order.

We assume the probability of a liquidity shock is specific to each product/supplier and is publicly observable. Thus, each supplier is characterized by a type $(\lambda, c) \in \Omega \equiv [0, 1] \times [\underline{c}, \bar{c}]$, distributed according to a CDF $G(\cdot)$ with strictly positive density g on Ω. Finally, we assume financial autarky among suppliers: due to a lack of enforcement technologies, suppliers cannot borrow from one another to hedge against these shocks.

The Intermediary. Following Rubinstein and Wolinsky (1987), the intermediary possesses a superior matching technology. While a supplier faces a late-matching probability λ, the intermediary reduces this risk to $m\lambda$, where $m \in (0, 1)$ represents the degree of matching efficiency—lower values of m correspond to higher matching efficiency. This advantage reflects superior search, delivery, or inventory management systems, etc.

Crucially, the intermediary also possesses a *financing technology* that enables her to manage cash flows across suppliers and over time sub-periods. This technology has two functions: (1) *liquidity provision*—she advances production cost c to suppliers in the early subperiod; (2) *revenue pooling*—she delays payouts to suppliers who have already sold in the early sub-period, retaining that retail revenue to finance other suppliers. Utilizing this technology incurs a fixed cost k per supplier, where $k \in (0, \bar{k})$.[5]

The intermediary serves each supplier under one of two operating modes:

1. **Middleman Mode** (*M* Mode). The intermediary retails the good but provides no liquidity support. She pays the supplier a transfer $f_M(\lambda, c) \geq 0$ only upon receiving consumer payment. Because the supplier lacks liquidity, production occurs *only* if consumers match early (probability $1 - m\lambda$). If consumers match late, the supplier cannot produce.

2. **Finance Mode** (*F* Mode). The intermediary retails the good and provides liquidity support by advancing the production cost c to suppliers in the early sub-period. Under this mode, the intermediary retains all consumer revenue and pays the supplier a deferred transfer $f_F(\lambda, c) \geq 0$ in the late sub-period. The intermediary finances these advances using revenue pooled from early retail sales and her own numeraire holding L. This arrangement guarantees production regardless of when consumers match.

Let $q(\lambda, c) \in \{0, 1\}$ denote the selection function, where $q = 1$ ($q = 0$) indicates the supplier is invited to an *F* (*M*) mode. The intermediary's strategy is a triple of functions (q, f_F, f_M).[6] Notably, the *F* mode combines two distinct functions—retailing *and* liquidity provision—making the intermediary a hybrid entity. This dual role connects our framework to the literature on hybrid platforms that simultaneously engage in first-party and third-party activities (as discussed in the Related Literature section).

Our focus on these two modes is without loss of generality. Given the intermediary's superior matching technology ($m < 1$) and ability to extract surplus, she strictly prefers to facilitate trade whenever possible. Consequently, her strategic choice reduces to a binary decision regarding liquidity: either incur the fixed cost k to guarantee production (Finance Mode), or forgo this investment and expose the trade to liquidity risk (Middleman Mode).

[5]We define the upper bound $\bar{k} \equiv (u - \underline{c})^2$ to ensure financing is profitable for at least some types. See footnote 8.
[6]Formally, f_F is defined on $\{(\lambda, c) : q(\lambda, c) = 1\}$ and f_M is defined on $\{(\lambda, c) : q(\lambda, c) = 0\}$.

Timing. The sequence of events includes three stages.

1. The intermediary chooses a numeraire holding L and announces her strategy (q, f_F, f_M). Observing this, suppliers decide whether to participate.

2. **Early sub-period:** Consumer matching timing is realized. Suppliers who do not join the intermediary trade directly with early-matching consumers. For participating suppliers: Under *M-mode*, suppliers produce and trade *if and only if* consumers match early; the intermediary pays f_M immediately upon sale. Under *F-mode*, the intermediary advances c to suppliers and retains the revenue from early matches.

3. **Late sub-period:** Late-matching consumers trade with suppliers. The intermediary collects revenue from these late transactions, settles the deferred transfer f_F to all *F-mode* suppliers, and repays L.

It is worth noting that our results do not require the intermediary to directly observe consumer matching timing. In the *M-mode*, payments are contingent on the receipt of consumer revenue, which is verifiable. In the *F-mode*, the transfer f_F is a fixed payment independent of matching timing. Thus, the model's predictions are robust to whether the intermediary directly observes matching timing.

3 The Equilibrium

In this section, we characterize the equilibrium of the baseline model. We solve the problem using backward induction: first, taking the liquidity endowment L as given, we determine the intermediary's optimal strategy and the resulting shadow value of liquidity; second, we determine the optimal liquidity holding L, given the external funding cost i; finally, we conduct comparative statics to examine how the funding cost and matching efficiency affect the equilibrium.

Suppliers' Participation Constraints. The intermediary makes offers subject to suppliers' participation constraints. If a supplier of type (λ, c) rejects the intermediary's offer, he produces and trades only if he matches with consumers early. His expected profit is:

$$(1-\lambda)(p-c) = (1-\lambda)(u-c)/2,$$

where we have substituted p from (1). The intermediary can condition rewards f_F and f_M on type (λ, c). To induce participation, it is sufficient to offer the supplier the value of his outside option:

$$f_F(\lambda, c) = \frac{(1-\lambda)(u-c)}{2}, \tag{2}$$

and

$$f_M(\lambda, c) = \frac{(1-\lambda)(u-c)/2}{1-m\lambda} + c. \tag{3}$$

The transfer f_M differs from f_F in that it compensates the supplier for the production cost c and is scaled by the matching probability $1 - m\lambda$.

Suppliers' Profit and Liquidity Contributions. In M mode, the intermediary's expected profit from a supplier (λ, c) is:

$$\pi_M(\lambda, c) = (1 - m\lambda)\left(p - f_M(\lambda, c)\right) = (1-m)\lambda(u-c)/2, \tag{4}$$

where we have inserted p and f_M from (1) and (3). Since $m < 1$, $\pi_M(\lambda, c) > 0$. This profit stems from the intermediary's matching advantage ($1 - m\lambda$ versus $1 - \lambda$), allowing her to capture the net surplus generated by this efficiency gain.

In F mode, the intermediary guarantees production regardless of matching timing. The expected profit from a (λ, c)-type supplier is:

$$\pi_F(\lambda, c) = p - c - f_F(\lambda, c) - k = \lambda(u-c)/2 - k. \tag{5}$$

Here, the intermediary collects revenue p, advances cost c, and pays the reward f_F. The second equality follows from substituting p and f_F. $\pi_F(\lambda, c)$ increases with λ (as the supplier's outside option value decreases) and decreases with c (as the margin tightens).

We define $\Delta\pi$ as the incremental profit gain from financing a supplier (λ, c):

$$\Delta\pi(\lambda, c) \equiv \pi_F(\lambda, c) - \pi_M(\lambda, c) = m\lambda(u-c)/2 - k.$$

Beyond profitability, financing is constrained by liquidity. While F mode secures production, it imposes a liquidity requirement: the intermediary must advance the production cost c to all participating suppliers in the early sub-period. These outflows are funded by the intermediary's endowment liquidity L and the revenue p collected from successful early matches. Therefore, the net liquidity contribution of a (λ, c)-supplier in the early sub-period is:

$$\theta_F(\lambda, c) = (1 - m\lambda)p - c = (1 - m\lambda)\frac{(u+c)}{2} - c. \tag{6}$$

The Selection of Suppliers. Let Θ be the total liquidity contributed by suppliers that are financed by the intermediary ($q(\cdot) = 1$):

$$\Theta = \int_\Omega \left[q(\lambda, c)\theta_F(\lambda, c)\right] dG.$$

Then the liquidity constraint that the intermediary faces can be written as:

$$\Theta + L \geq 0. \tag{7}$$

The liquidity constraint stipulates that the intermediary's internal funding capacity—generated by the aggregate net contributions of financed suppliers—plus her endowment L, must be sufficient to cover all early production costs.[7]

The intermediary's problem of selecting suppliers into M or F modes can be formulated as

$$V^m(L) \equiv \max_{\{q(\lambda,c)\in\{0,1\}\}_{(\lambda,c)\in\Omega}} \int_\Omega [q(\lambda,c)\Delta\pi(\lambda,c)]\, dG, \text{ s.t. } (7). \tag{8}$$

The problem can be understood as follows: the intermediary earns the baseline profit $\pi_M(\lambda, c)$ from all invited suppliers, and additionally decides whether to finance them ($q(\lambda, c) = 1$) to capture the incremental gain $\Delta\pi(\lambda, c)$, subject to the aggregate liquidity constraint (7).

Since the objective and constraint are linear integrals, problem (8) reduces to a pointwise maximization of the integrand for each supplier type (λ, c). The optimal solution can be derived using the Lagrange method (see e.g., Rhodes, Watanabe and Zhou 2021). Let $\mu \geq 0$ be the multiplier associated with constraint (7). We can construct the Lagrangian:

$$\mathcal{L} = \int_\Omega \left[q(\lambda,c)\Big(\Delta\pi(\lambda,c) + \mu\theta_F(\lambda,c)\Big)\right] dG(\lambda,c).$$

Note that $\Delta\pi(\lambda, c)$ and $\theta_F(\lambda, c)$ can be positive or negative across supplier types. In particular, given the positive cost k, financing is not universally profitable; specifically, $\Delta\pi(\cdot)$ is strictly negative for some supplier types.

The optimal selection policy derived from this Lagrangian depends on both the supplier type (λ, c) and the shadow value μ. With a slight abuse of notation, we denote this financing rule as:

$$q(\lambda,c,\mu) = \begin{cases} 1 & \text{if } \Delta\pi(\lambda,c) + \mu\theta_F(\lambda,c) \geq 0; \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Condition (9) indicates that $q(\lambda, c, \mu) = 1$ consists of three possible scenarios:

$$\Delta\pi(\lambda,c) \geq 0,\ \theta_F(\lambda,c) \geq 0, \tag{10a}$$

$$\Delta\pi(\lambda,c) > 0,\ \theta_F(\lambda,c) < 0,\ -\Delta\pi/\theta_F \geq \mu, \tag{10b}$$

$$\Delta\pi(\lambda,c) < 0,\ \theta_F(\lambda,c) > 0,\ -\Delta\pi/\theta_F \leq \mu. \tag{10c}$$

In scenario (10a), the intermediary selects suppliers with positive increments in profits $\Delta\pi$ and positive liquidity contribution θ_F to finance. In scenario (10b), the intermediary selects suppliers

[7] Since there is a continuum of suppliers facing independent liquidity shocks, the Law of Large Numbers ensures that aggregate liquidity is deterministic. Consequently, the liquidity constraint is interpreted to hold "almost surely." In this framework, a "worst-case" scenario—where every supplier requires liquidity simultaneously—has zero measure and is therefore irrelevant to the intermediary's decision.

with positive increments in profits $\Delta\pi$ and negative liquidity contribution θ_F to finance, provided the marginal profitability of liquidity usage, measured by $-\Delta\pi/\theta_F$, is higher than the shadow value of liquidity μ. In the last scenario (10c), the intermediary selects suppliers with negative $\Delta\pi$ and positive θ_F to finance, as these suppliers contribute to the aggregate liquidity of the intermediary. The cost of getting one unit of liquidity from these suppliers is $-\Delta\pi/\theta_F$, and the intermediary should extract the liquidity from these suppliers if $-\Delta\pi/\theta_F \le \mu$.

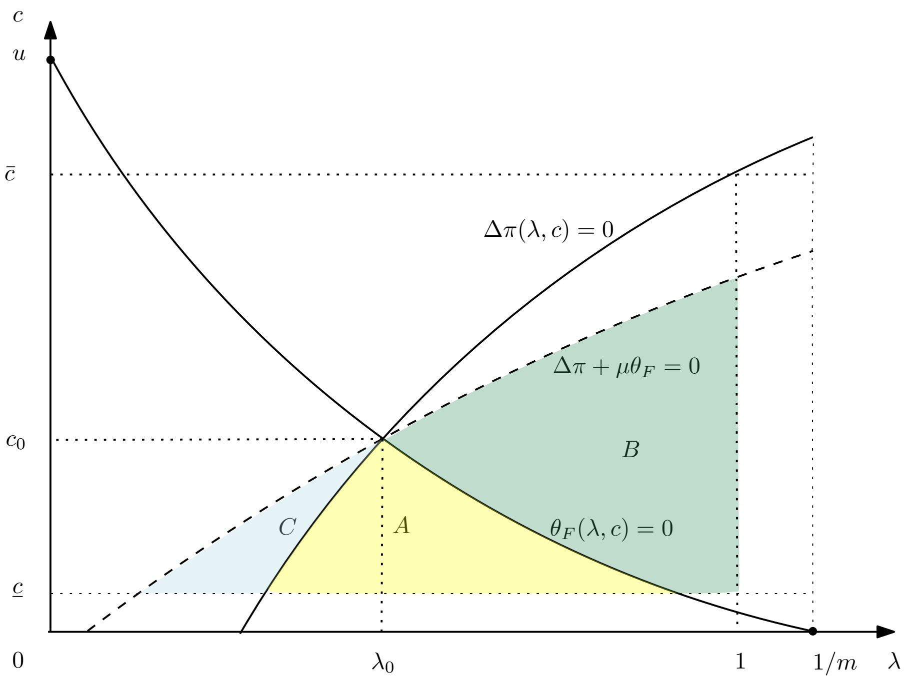


Figure 1: The intermediary's selection of suppliers

To illustrate the three scenarios in a figure, we insert $\Delta\pi(\cdot)$ and $\theta_F(\cdot)$ into (10), and obtain three boundaries in Ω:

$$\theta_F(\lambda, c) \ge 0 \Leftrightarrow c \le c_{\theta_F}(\lambda) \equiv \frac{1 - m\lambda}{1 + m\lambda} u, \tag{11a}$$

$$\Delta\pi(\lambda, c) \ge 0 \Leftrightarrow c \le c_{\Delta\pi}(\lambda) \equiv u - \frac{2k}{m\lambda}, \tag{11b}$$

$$\Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \ge 0 \Leftrightarrow c \le b(\lambda, \mu) \equiv \frac{m\lambda u - 2k + \mu(1 - m\lambda)u}{m\lambda + \mu(1 + m\lambda)}. \tag{11c}$$

Equation (11c) defines the **financing boundary** $c = b(\lambda, \mu)$, whose right-hand side is a weighted average of the right-hand sides of the first two. We plot these three boundaries in Figure 1, annotated by $\theta_F(\lambda, c) = 0$, $\Delta\pi(\lambda, c) = 0$, and $\Delta\pi + \mu\theta_F = 0$, respectively. The intersection is denoted by (λ_0, c_0). Any suppliers below $\theta_F(\lambda, c) = 0$ contribute to the liquidity pool, and any suppliers below $\Delta\pi(\lambda, c) = 0$ contribute to the intermediary's profits.[8]

The overlapping region A represents suppliers in scenario (10a), which are financed by the

[8]Note that $k < \bar{k}$ ensures that $c_0 > \underline{c}$. Also, Figure 1 is drawn with $\lambda_0 < 1$ and $\bar{c} > c_0$. The complete analysis, including cases where $\lambda_0 \ge 1$ or $\bar{c} < c_0$, is provided in the proof.

intermediary because they contribute to both profits $\Delta\pi$ and liquidity θ_F. Suppliers in region B, corresponding to scenario (10b), have net liquidity needs, $\theta_F < 0$, while contributing to profits $\Delta\pi > 0$. Suppliers in region C, corresponding to scenario (10c), when included in the F mode, give the intermediary lower profits $\Delta\pi < 0$, but contribute to the liquidity pool. Suppliers outside these regions are not financed and thus remain in the M mode.

Overall, the intermediary adopts a *profit-based liquidity cross-subsidization* strategy. This involves using the positive net liquidity contributions from suppliers in regions A and C to address the liquidity needs of suppliers in region B. In particular, when the intermediary uses liquidity contributions from region C, it incurs a cost in the form of reduced (or negative) profits to these suppliers. However, when providing liquidity support to suppliers in region B, the intermediary expects to earn positive profits from them.

In standard liquidity pooling (e.g., Diamond and Dybvig 1983), agents are typically homogeneous; in such settings, the intermediary would select only those suppliers who contribute positive profits. With heterogeneous agents, however, we show that this selection is suboptimal. Instead, the decision to finance a supplier depends on both his liquidity contribution and his incremental profit contribution.

Note that for the F mode to be active, the set of suppliers with $\Delta\pi > 0$ needs to be nonempty. The necessary and sufficient condition is $\Delta\pi(1, \underline{c}) > 0$, or equivalently, $\frac{k}{m} < \frac{u-\underline{c}}{2}$. This, along with $c_0 > \underline{c}$ (implied by $k < \bar{k}$), ensures that region A in Figure 1 always exists.

Lemma 1 *There exist suppliers that are financed by the intermediary if and only if $k/m < (u - \underline{c})/2$.*

The intuition is as follows. A large k reduces the incremental profit $\Delta\pi$, making financing less attractive. Conversely, a small m implies high matching efficiency, meaning consumers are likely to match early and thus reducing the need for liquidity support. In the limit as $m \to 0$, the intermediary's superior matching technology renders financial intervention entirely redundant.

The shadow value of liquidity, μ, is determined as follows. If the liquidity constraint (7) binds, μ satisfies:

$$L = -\Theta(\mu) \equiv -\int_{\Omega} q(\lambda, c, \mu)\theta_F(\lambda, c)dG. \tag{12}$$

Otherwise, if the constraint is slack ($L \geq -\Theta(0)$), then $\mu = 0$. In this case, the selection rule reduces to whether the incremental profit $\Delta\pi$ is positive.

Lemma 2 *If $\Theta(0) + L < 0$, then there exists a unique $\mu > 0$ that satisfies (12); and otherwise $\mu = 0$.*

The intermediary's available liquidity L governs the modes through the shadow value μ. Intuitively, μ is strictly decreasing in L over the interval $[0, -\Theta(0)]$; this reflects the fact that an additional unit of the intermediary's liquidity endowment is most valuable when her existing holdings are scarce.

Lemma 3 *$\mu(L) > 0$ is strictly decreasing in L if $\Theta(0) + L < 0$.*

We summarize the results so far in the following theorem.

Theorem 1 *For a given liquidity endowment $L \geq 0$ (and assuming $k/m < (u - \underline{c})/2$), the intermediary's unique profit-maximizing strategy involves active financing, characterized by the selection rule $q(\lambda, c, \mu)$ in (9), rewards f_F, f_M in (2)–(3), and a shadow value $\mu \geq 0$ determined by Lemma 2. The optimal policy exhibits liquidity cross-subsidization across suppliers if $\mu > 0$.*

The Intermediary's Liquidity Holding. Having characterized the selection policy for a fixed L, we now endogenize the intermediary's numeraire holding. Assume she chooses L to maximize total profits net of funding costs:

$$\max_{L \geq 0} V^m(L) - i \cdot L,$$

where $V^m(L)$ is the optimized value function from (8). Applying the Envelope Theorem, $V^{m\prime}(L) = \mu(L)$, yielding the first-order condition:

$$i \geq \mu(L), \tag{13}$$

which holds with equality if $L > 0$.

The optimal holding $L^*(i)$ depends on the scarcity of liquidity within the supplier pool. Recall $\mu(0)$ is the shadow value when the intermediary holds $L = 0$. Following Lemma 2, if the pooled liquidity from suppliers is sufficient to fund all profitable production under the F mode (that is, $\Theta(0) \geq 0$), then $\mu(0) = 0$ and the intermediary holds no liquidity.

In contrast, if the pool faces a deficit ($\Theta(0) < 0$), the choice of L depends on the interest rate i. When $i \geq \mu(0)$, external funding is too costly relative to the benefit of relaxing the liquidity constraint, so the intermediary relies solely on internal pooling and sets $L^*(i) = 0$. However, when $i < \mu(0)$, she uses external funding until its marginal benefit equals the interest rate, setting $L^*(i) > 0$ such that $\mu(L^*) = i$. This characterization is summarized in the following proposition.

Proposition 1 *The optimal liquidity holdings of the intermediary follow $L^*(i) = -\Theta(i) > 0$ if $i < \mu(0)$, and $L^*(i) = 0$ otherwise. The value of liquidity with the intermediary's optimal liquidity holdings is given by*

$$\mu^*(i) = \min\left(i, \mu(0)\right). \tag{14}$$

Together, these results establish the existence of a unique equilibrium.

Theorem 2 *A unique equilibrium exists. It is characterized by the intermediary's strategy*

$$(q(\lambda, c, \mu^*(i)), f_M(\lambda, c), f_F(\lambda, c), L^*(i), \mu^*(i)),$$

as defined in Theorem 1 and Proposition 1.

It is worth noting that our results are invariant to the specific form of the supplier's outside option. Suppose we generalize the supplier's expected payoff of direct selling to an arbitrary function $\Phi(\lambda, c)$. To ensure participation, the intermediary adjusts rewards f_F and f_M to deliver expected value Φ. In the F mode, she pays $f_F = \Phi(\lambda, c)$, yielding profit $\pi_F = (p - c) - \Phi(\lambda, c) - k$. In the M mode, where production is probabilistic, she pays $f_M = c + \frac{\Phi(\lambda,c)}{1-m\lambda}$ upon success, yielding profit $\pi_M = (1 - m\lambda)(p - c) - \Phi(\lambda, c)$.

Crucially, the incremental profit $\Delta\pi = \pi_F - \pi_M = m\lambda(p - c) - k$ remains independent of Φ. Because the cost of satisfying the outside option enters both profit functions linearly, it cancels out in the marginal comparison. Since the net liquidity contribution θ_F is also unaffected, the selection problem (8) and the resulting equilibrium properties remain identical to those characterized above.

In the Online Appendix, we relax the assumption of financial autarky by allowing suppliers to access external credit markets. We demonstrate that while external options may alter the composition of the pool, the core mechanism of liquidity cross-subsidization remains robust: the intermediary continues to finance a strategic mix of net liquidity contributors and users to maximize portfolio value.

Funding Costs and Liquidity Scarcity. As established in Proposition 1, when $i < \mu(0)$, the shadow value of liquidity equates to the market rate ($\mu^* = i$). As i increases, the intermediary reduces her external position $L^*(i)$, causing the selection boundary (defined by $\Delta\pi + \mu\theta_F = 0$) in Figure 1 to rotate clockwise around (λ_0, c_0). This rotation reflects the rising relative price of liquidity: as external funds become more costly, the intermediary optimally tightens the selection criterion, substituting away from liquidity-consuming suppliers toward those making positive net contributions to the pool.

The threshold $\mu(0)$ captures the *intrinsic liquidity scarcity* of the supplier base. A low $\mu(0)$ implies relative abundance, making the selection policy less sensitive to external rate shocks. Crucially, once i exceeds $\mu(0)$, the intermediary ceases external borrowing altogether. In this regime ($i \geq \mu(0)$), the set of financed suppliers becomes independent of further interest rate hikes, as the program relies exclusively on internal cross-subsidization.

Regulators and practitioners often worry that sudden spikes in funding costs could paralyze middleman finance programs, leading to contagion and output losses. Our results suggest that such concerns may be overstated. The internal trade credit pool functions as a strategic buffer: as external funding becomes prohibitively expensive, the intermediary extracts more liquidity from within the pool, insulating the core of the financing program from further interest rate volatility.[9]

[9]See, for example, "Supply-Chain Finance Is New Risk in Crisis," *The Wall Street Journal*, April 4, 2020, available at *https://www.wsj.com/articles/supply-chain-finance-is-new-risk-in-crisis-11585992601* (accessed on Jul 17, 2023).

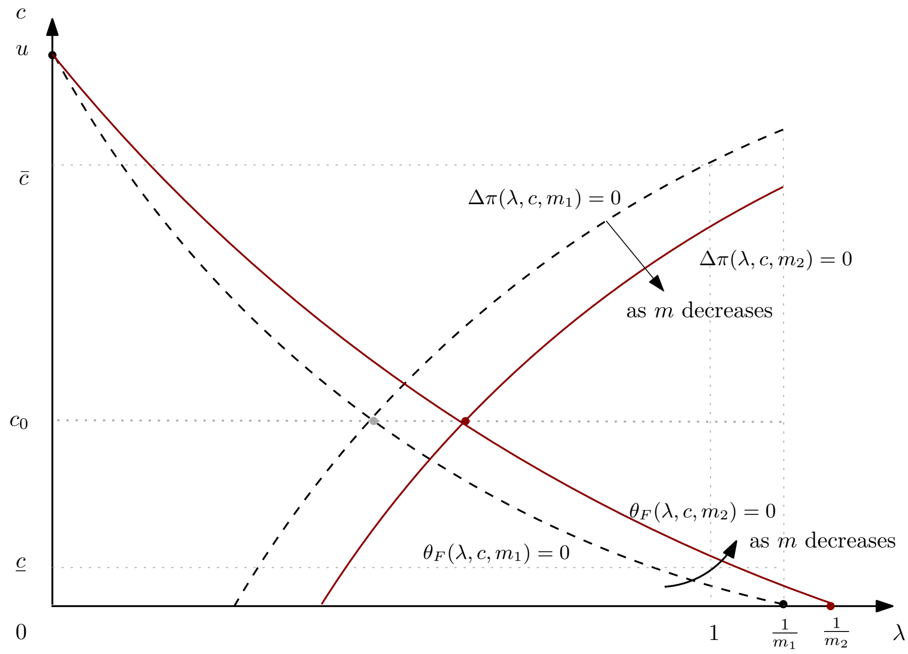


Figure 2: Effects of a decrease in m on the boundaries of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$

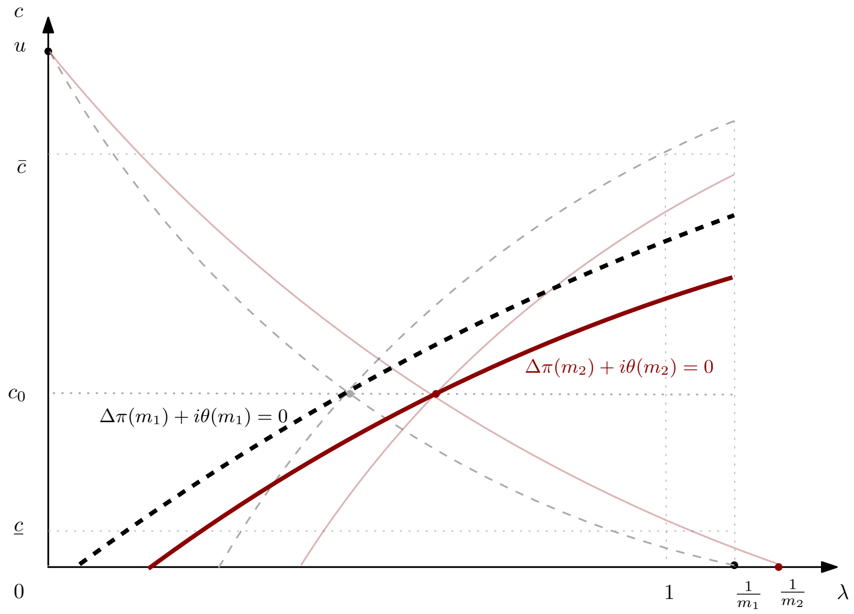
Retail Efficiency and Liquidity Provision. An improvement in matching efficiency (a lower m) exerts two countervailing forces on the equilibrium scope of the F mode, as illustrated by the shifts in Figure 2 as m decreases from m_1 to m_2. First, as the intermediary becomes more efficient in early matching, the benefit of guaranteeing production via financing diminishes. This *profit effect* shifts the incremental profit boundary $\Delta\pi(\cdot) = 0$ downward, meaning fewer suppliers generate positive incremental profits from being financed. Second, higher matching efficiency accelerates revenue realization, increasing liquidity inflows. This *liquidity effect* rotates the zero-contribution boundary $\theta_F(\cdot) = 0$ upward, thereby expanding the set of suppliers who contribute liquidity once included in the F mode.[10]

We focus on the regime with $L > 0$ such that $\mu^*(i) = i$. As shown in Figure 3, how the financing boundary responds to matching improvements depends qualitatively on i relative to the threshold i_0.

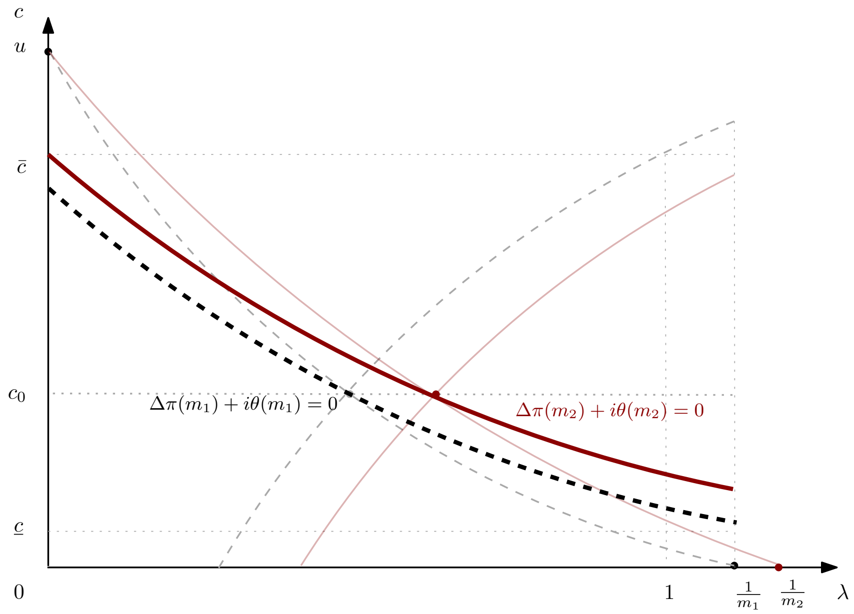
When $i < i_0$ (Figure 3(a)), selection is primarily profit-driven. As retail matching becomes more efficient (lower m), suppliers' baseline profitability improves, reducing their relative need for liquidity support. Consequently, the intermediary narrows the scope of the F mode, treating retail efficiency and financing as substitutes.

Conversely, when $i > i_0$ (Figure 3(b)), selection is constrained by liquidity. Improved matching efficiency accelerates revenue realization, thereby expanding the internal liquidity pool. The

[10]The boundaries $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ intersect at point $(c_0, \lambda_0) = (k + u - \sqrt{k^2 + 4uk}, \frac{k+\sqrt{k^2+4ku}}{2mu})$. Since c_0 is independent of m, a decrease in m causes the intersection to move to the right along the line $c = c_0$. This intersection remains within the space Ω as long as $m \geq \tilde{m} \equiv (k + \sqrt{k^2 + 4uk})/(2u)$. When $m > \tilde{m}$, all three regions defined in Figure 1 are nonempty, ensuring that liquidity cross-subsidization is feasible.



(a) $\mu^*(i) = i < i_0$: Profit Dominance (Contraction)



(b) $\mu^*(i) = i > i_0$: Liquidity Dominance (Expansion)

Figure 3: The effect of improved matching efficiency (lower m) on the scope of the F mode

intermediary leverages this increased slack to finance a broader set of suppliers. In this regime, retail efficiency and financing act as complements.

This dichotomy challenges the view that technological advancements in retail matching uniformly complement financing. Instead, the interaction depends on funding costs: improved matching technology crowds out financing when external funds are cheap (acting as a substitute), but facilitates it when they are expensive (acting as a complement).

Proposition 2 *Let* $i_0 \equiv \left(k + \sqrt{k^2 + 4uk}\right) / (2u)$. *Let* $c = b(\lambda, i)$ *denote the financing boundary derived in (11c). Given* $\mu^*(i) = i$, *the slope of the financing boundary with respect to risk* λ *satisfies:*

- *If* $i < i_0$, *the boundary is upward-sloping* ($b'_\lambda > 0$).
- *If* $i > i_0$, *the boundary is downward-sloping* ($b'_\lambda < 0$).

4 General Demand

In this section, we extend the baseline framework to general demand to examine how upstream liquidity constraints affect downstream market outcomes. We maintain the assumptions of the baseline model, with the only change being that each product (λ, c) faces downward-sloping demand $D(p)$ (twice continuously differentiable). The intermediary then chooses prices to optimally trade off profit maximization against her liquidity needs.

In the direct-selling channel, a supplier produces only if consumers match early (probability $1 - \lambda$). In this state, he sets the monopoly price $p^*(c) \equiv \arg\max_{p \geq c}(p - c)D(p)$. We assume the profit function is strictly quasi-concave, yielding a unique $p^*(c)$. The resulting maximum profit is $\pi^*(c)$.

In M mode, the intermediary retails the good but provides no liquidity support. Production remains contingent on early consumer arrival, which occurs with probability $1 - m\lambda$. Accordingly, the intermediary sets the monopoly price $p^*(c)$ and pays the supplier his outside option, $(1 - \lambda)\pi^*(c)$. The intermediary's expected profit is:

$$\pi_M(\lambda, c) = (1 - m)\lambda\pi^*(c). \tag{15}$$

In F mode, the intermediary guarantees production by advancing the cost $c \cdot D(p)$ in the early subperiod. Her expected profit at a chosen price p is:

$$\pi_F(\lambda, c, p) = (p - c)D(p) - (1 - \lambda)\pi^*(c) - k. \tag{16}$$

Crucially, the choice of p now determines the net liquidity contribution $\theta_F(\lambda, c, p)$, which is the expected revenue received early minus the upfront production cost:

$$\theta_F(\lambda, c, p) = (1 - m\lambda)pD(p) - cD(p). \tag{17}$$

Equation (17) highlights that the price that maximizes profit (π_F) differs from the one that maximizes the net liquidity contribution (θ_F). This misalignment becomes crucial when the intermediary needs to balance her desire for profit against the need to maintain an adequate liquidity pool. Consequently, when the aggregate liquidity constraint binds, the intermediary deviates from the standard monopoly price to increase the net liquidity contribution from the product market.

Liquidity-Driven Pricing. The intermediary determines which products to finance and the corresponding retail prices, while setting $p^*(c)$ for those not financed. This decision is subject to the aggregate liquidity constraint, which requires that the net liquidity generated by the portfolio plus the intermediary's endowment, L, remain non-negative. Formally:

$$\max_{\{q(\cdot),p(\cdot)\}_{(\lambda,c)\in\Omega}} \int_\Omega q(\lambda,c)\Delta\pi(\lambda,c,p(\lambda,c))dG(\lambda,c),$$

$$\text{s.t.} \int_\Omega q(\lambda,c)\theta_F(\lambda,c,p(\lambda,c))dG(\lambda,c) + L \geq 0. \tag{18}$$

As established in Section 3, when the intermediary holds positive internal liquidity ($L > 0$), the shadow value of the constraint is pinned down by the external cost of funds, $\mu = i$. We treat μ as a parameter henceforth.

The Lagrangian can be written as:

$$\mathcal{L} = \int_\Omega q(\lambda,c)\tilde{S}(\lambda,c,p(\lambda,c))dG(\lambda,c) + \mu L,$$

where $\tilde{S}(\lambda,c,p) \equiv \Delta\pi(\lambda,c,p) + \mu\theta_F(\lambda,c,p)$ represents the net value of financing supplier (λ,c) at price p. Substituting the expressions for π_F and θ_F yields:

$$\tilde{S}(\lambda,c,p) = [1+\mu(1-m\lambda)]\,(p-\tilde{c})D(p) - k - (1-m\lambda)\pi^*(c), \tag{19}$$

where $\tilde{c} \equiv \gamma c$ is the *effective marginal cost* with the liquidity wedge

$$\gamma \equiv \frac{1+\mu}{1+\mu(1-m\lambda)} \geq 1.$$

γ is increasing in the liquidity cost of the intermediary μ and the liquidity risk of the specific supplier λ, and it strictly exceeds 1 whenever both of them are strictly positive ($\mu\lambda > 0$).[11]

Equation (19) implies that for a financed supplier, the pricing decision is equivalent to maximizing standard monopoly profit with an effective marginal cost $\tilde{c}$. This leads to our main characterization of retail pricing:

Proposition 3 *Under general demand, the intermediary sets retail prices $p^*(\tilde{c})$ for financed products, where $\tilde{c} = \gamma c$ is the effective marginal cost. Whenever $\mu > 0$ and $\lambda > 0$, $\gamma > 1$, implying a strict upward*

[11] For comparison, standard bank finance at rate μ implies a marginal cost of $(1+\mu)c$. In contrast, middleman finance yields a strictly lower effective cost: $\gamma < 1+\mu$. This gain depends on the likelihood of early repayment: the gap $(1+\mu)-\gamma = \frac{\mu(1+\mu)(1-m\lambda)}{1+\mu(1-m\lambda)}$ is strictly increasing in the probability of early sales, $1-m\lambda$. Intuitively, the faster the intermediary can "recycle" revenue back into the liquidity pool, the greater her advantage over a fixed-term bank loan.

price distortion:

$$p^*(\tilde{c}) > p^*(c).$$

The wedge $p^*(\tilde{c}) - p^*(c)$ represents a *liquidity-induced double marginalization*. This distortion differs from classical double marginalization, which arises from pricing externalities between vertical firms with market power. Here, the upstream supplier is passive, and there is no markup due to market power. Instead, the distortion is driven entirely by the liquidty friction: financing production requires upfront capital c, while revenue returns early only with probability $1 - m\lambda$. Because the intermediary must settle costs in the early subperiod using scarce funds (shadow cost μ), she perceives production as disproportionately costly ($\tilde{c} > c$). To conserve liquidity, she optimally raises retail prices to contract sales volume, effectively "taxing" consumers to subsidize the liquidity pool.[12]

Strategic Value of Pricing. Substituting the optimal price back into (19), the optimized value becomes

$$S(\lambda, c) = [1 + \mu(1 - m\lambda)]\pi^*(\tilde{c}) - k - (1 - m\lambda)\pi^*(c). \tag{20}$$

The intermediary finances a supplier if and only if $S(\lambda, c) \geq 0$.

To isolate the novel economic force introduced by liquidity-driven pricing, we decompose $S(\lambda, c)$ into a baseline component and a strategic pricing gain:

$$S(\lambda, c) = S_{base}(\lambda, c) + \mathcal{G}(\lambda, c).$$

The first term $S_{base}(\lambda, c)$ is the value of financing the supplier if price is fixed to the standard monopoly price $p^*(c)$, ignoring the shadow cost of liquidity. It encapsulates the baseline trade-off between profit and liquidity analyzed in Section 3:

$$S_{base}(\lambda, c) = \underbrace{m\lambda\pi^*(c) - k}_{\equiv \Delta\pi_{base}} + \mu \underbrace{[(1 - m\lambda)p^*(c) - c]\, D(p^*(c))}_{\equiv \theta_{F,base}}.$$

The profit contribution $\Delta\pi_{base}$ increases with λ and decreases with c, while the liquidity contribution $\theta_{F,base}$ decreases in both λ and c. Thus, just as in the baseline model, profit-based liquidity cross-subsidization allows the intermediary to finance those suppliers with $S_{base}(\lambda, c) > 0$.

The second term, $\mathcal{G}(\lambda, c)$, captures the additional value generated by re-optimizing the retail price to $p^*(\tilde{c})$. By deviating from $p^*(c)$, the intermediary sacrifices some profit ($\Delta\pi(p^*(\tilde{c})) < \Delta\pi_{base}$) but extracts more liquidity ($\theta_F(p^*(\tilde{c})) > \theta_{F,base}$). Optimality ensures the net gain $[\Delta\pi(\tilde{p}) - \Delta\pi_{base}] + \mu[\theta_F(\tilde{p}) - \theta_{F,base}]$ is strictly positive. To see this, note that the net gain can be rewritten

[12] To quantify the price distortion, let $\tau \equiv \tilde{c} - c$ be the cost wedge. The *average liquidity pass-through rate* is $\bar{\rho} \equiv \frac{1}{\tau}\int_c^{\tilde{c}} \rho(z)dz$, where $\rho(z) \equiv p^{*\prime}(z)$ is the local pass-through. The magnitude depends on demand curvature: if demand is log-concave (e.g., linear), $\rho(z) \leq 1$; if log-convex (e.g., constant elasticity), $\rho(z) \geq 1$.

as

$$\mathcal{G}(\lambda, c) = (1 + \mu(1 - m\lambda)) \left[\pi^*(\tilde{c}) - \pi(p^*(c), \tilde{c})\right],$$

with $\pi^*(\tilde{c}) = \max_p \pi(p, \tilde{c})$. Since $p^*(c)$ is suboptimal under the effective cost $\tilde{c}$, $\mathcal{G}$ is strictly positive. $\mathcal{G}(\lambda, c)$ reflects the intermediary's ability to use her market power as a tool for liquidity management. The following lemma characterizes its properties.

Lemma 4 *The pricing gain $\mathcal{G}(\lambda, c)$ is strictly positive whenever $\mu > 0$ and $\lambda > 0$. It is strictly increasing in liquidity risk λ, the shadow cost of funds μ, and retail inefficiency m.*

The intuition is as follows. As liquidity risk rises (higher λ), the shadow cost of funds increases (higher μ), or matching efficiency declines (higher m), the liquidity wedge γ widens, and the intermediary uses her pricing power more aggressively to extract liquidity from the product market. Consequently, the value of adjusting increasing retail price captured by $\mathcal{G}$ grows alongside these frictions.[13]

Proposition 4 *Compared to the fixed-price benchmark, the set of financed suppliers is strictly larger, and the value of financing suppliers increases more for higher-risk products: $\partial S(\lambda, c)/\partial\lambda > \partial S_{base}(\lambda, c)/\partial\lambda$.*

The second part of the proposition follows from the property that $\mathcal{G}(\cdot)$ is increasing in liquidity risk λ. Intuitively, pricing flexibility acts as a strategic buffer, allowing the intermediary to expand the set of financed products to include high-risk suppliers that would otherwise be excluded.

Figure 4 illustrates supplier selection with linear demand $D(p) = u - p$ (see Online Appendix for derivation). The purple region depicts the baseline financing set when the retail price is fixed at the standard monopoly level $p^*(c) = (u + c)/2$. The yellow region shows the expansion enabled by strategic price increases: as established in Proposition 4, the intermediary optimally raises prices to $p^*(\tilde{c}) = (u + \gamma c)/2$ to generate the additional liquidity needed to finance higher-cost suppliers. As shown in the figure, this expansion becomes more pronounced as liquidity risk λ increases.

5 Welfare Implications

How does middleman finance affect retail market welfare? We focus our attention on consumer surplus, defined as $CS(p) \equiv \int_p^{\bar{p}} D(z)dz$, where $\bar{p}$ is the choke price satisfying $D(\bar{p}) = 0$ (or ∞ if no such price exists). We prioritize consumer surplus for two reasons. First, antitrust and regulatory authorities typically weigh consumer welfare more heavily than industry profits. Second, and more importantly, by raising retail prices to address liquidity frictions, the middleman imposes

[13] The monotonicity of $\mathcal{G}$ with respect to c depends on demand curvature: $\mathcal{G}$ increases in c if demand is log-concave (e.g., linear) and decreases if demand is log-convex (e.g., constant elasticity), provided $\tilde{c} - c$ is not excessively large.

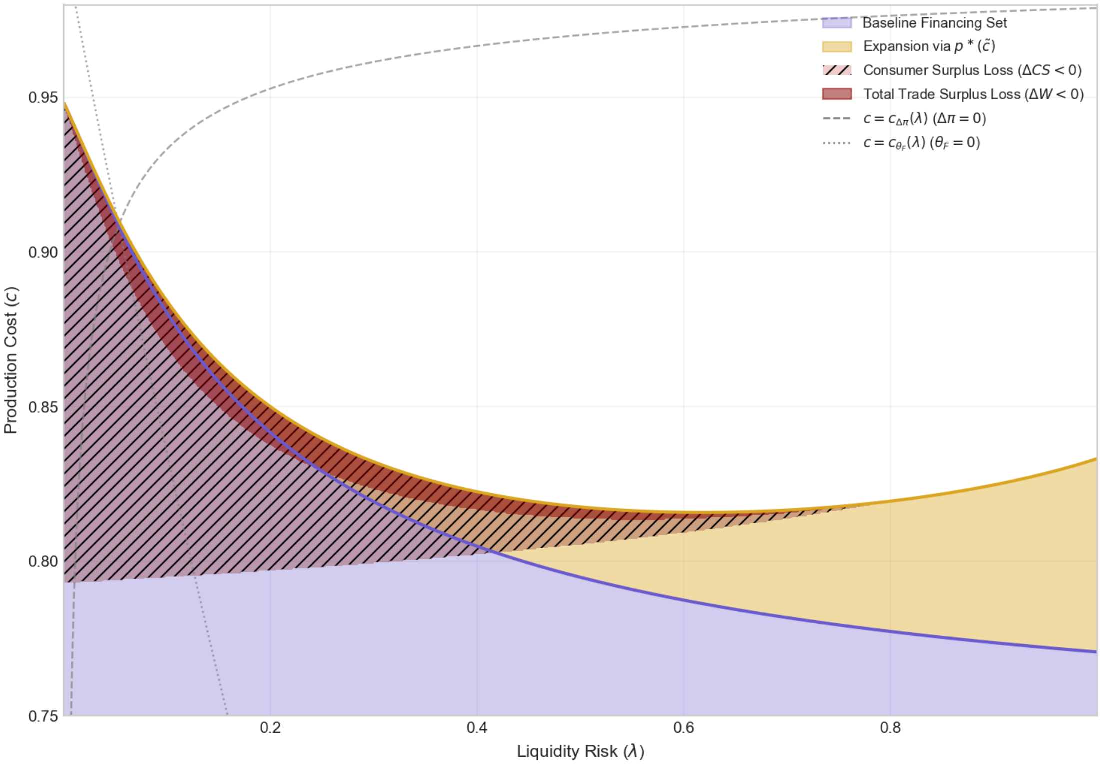


Figure 4: **Financing Set under Linear Demand and Welfare Implications**

Note: This figure illustrates equilibrium outcomes with linear demand $D(p) = u - p$ and parameter values $u = 1$, $m = 0.9$, $k = 10^{-4}$, and $\mu = 0.15$. The **purple region** represents the baseline financing set where suppliers are financed under the standard monopoly price $p^*(c)$. The **yellow region** illustrates the expansion of the financing set enabled by adjusting the retail price upward to $p^*(\tilde{c})$. The **hatched area** identifies the set of suppliers where consumer surplus decreases upon being financed ($\Delta CS < 0$), despite the elimination of liquidity risk. The **dark red area** identifies the set of suppliers where the total welfare effect is negative ($\Delta W < 0$), indicating that the price distortion dominates the insurance gain.

a "liquidity tax" that transfers value from consumers to herself—even when financing improves supply reliability. We conclude the section with a brief discussion of total welfare.

We define the consumer surplus change as the difference between surplus in the F mode (guaranteed trade at price $p^*(\tilde{c})$) and the baseline (risky trade at price $p^*(c)$):

$$\Delta CS(\lambda, c) \equiv CS(p^*(\tilde{c})) - (1 - m\lambda)CS(p^*(c)).$$

We say that the F mode harms consumers for product (λ, c) if $\Delta CS(\lambda, c) < 0$. Our goal is to characterize the set of products where this loss occurs.

Region of Consumer Harm We first establish conditions under which this loss region is nonempty. At $\lambda = 0$, there is no liquidity risk and hence no wedge: $\tilde{c} = c$ and $\Delta CS(0, c) = 0$. At the other extreme, as liquidity-driven failure becomes arbitrarily severe (i.e., as $m\lambda \to 1$ so that the baseline trade probability $1 - m\lambda \to 0$), the second term in $\Delta CS(\lambda, c)$ vanishes, while the F mode still delivers positive surplus, implying $\Delta CS(\lambda, c) > 0$ for sufficiently high risk.[14] Therefore, a consumer-loss region exists whenever consumer surplus initially *declines* as financing is introduced for low-risk products.

Differentiating ΔCS at $\lambda = 0$, we find that financing the supplie reduces consumer surplus for sufficiently small positive λ if $\partial \Delta CS(\lambda, c)/\partial\lambda|_{\lambda=0} < 0$, or

$$\frac{\mu c}{1+\mu} \cdot \frac{\partial p^*(c)}{\partial c} \cdot D(p^*(c)) > CS(p^*(c)). \tag{21}$$

Intuitively, this condition compares the marginal cost and benefit of financing a supplier with negligible risk ($\lambda \to 0$). The left-hand side captures the welfare loss from the liquidity-driven price increase. The right-hand side represents the welfare gain from eliminating the risk of production failure. Condition (21) establishes that for low-risk products, the consumer surplus destroyed by the price distortion exceeds the expected surplus gained from guaranteed supply. Since financing benefits consumers at sufficiently high risk levels, this condition implies the existence of a specific "consumer surplus loss region" at the lower end of the risk spectrum.

Lemma 5 *For each cost c satisfying condition (21), there exists a threshold $\lambda^*(c) \in (0, 1/m)$ such that financing the supplier strictly reduces consumer surplus for all $\lambda < \min\{\lambda^*(c), 1\}$.*

The threshold $\lambda^*(c)$ partitions the product space: financing reduces consumer surplus for low-risk products ($\lambda < \lambda^*(c)$) while enhancing it for sufficiently high-risk products ($\lambda > \lambda^*(c)$). We treat $\lambda^*(c)$ as unique throughout. This is guaranteed if $\Delta CS(\cdot, c)$ is quasi-convex in λ, a condition satisfied by standard demand specifications.[15]

[14]Formally, probabilities require $m\lambda \leq 1$, i.e., $\lambda \leq 1/m$. If the empirical support of types is bounded above by 1, then our welfare statements are understood on $[0, 1]$ and the loss region is truncated accordingly (hence the $\min\{\cdot, 1\}$ notation below).

[15]Without quasi-convexity, the marginal impact of risk, $-\frac{d}{d\lambda}CS(p^*(\tilde{c}(\lambda)))$, would need to intersect the benchmark $m \cdot$

We next show that the consumer-loss threshold $\lambda^*(c)$ is increasing in cost c. The slope of this threshold depends on the semi-elasticity of consumer surplus to cost, defined as:

$$\Psi(c) \equiv -\frac{d \ln CS(p^*(c))}{dc} = \frac{D(p^*(c))\,\rho(c)}{CS(p^*(c))},$$

where $\rho(c) \equiv p^{*\prime}(c)$ is the pass-through rate. $\Psi(c)$ captures the percentage rate at which surplus erodes as costs rise.

Proposition 5 *Suppose condition (21) holds and the surplus semi-elasticity $\Psi(c)$ is non-decreasing (or does not decrease too rapidly). Then there exists a unique, strictly increasing boundary $\lambda^*(c)$ such that financing the supplier reduces consumer surplus if and only if $\lambda < \lambda^*(c)$.*

Since consumer harm occurs for risks *below* this threshold ($\lambda < \lambda^*(c)$), a higher cost c yields a higher cutoff $\lambda^*(c)$, expanding the range of consumer harm. This implies that consumer welfare loss is concentrated in the "high-cost, low-risk" segment of the market. Intuitively, high production costs exacerbate the liquidity burden, necessitating larger price distortions than erode surplus. Conversely, low risk limits the insurance value of guaranteed supply, making it insufficient to offset the higher prices.

Figure 4 illustrates this welfare partition for linear demand $D(p) = u - p$, where the hatched region ($\Delta CS < 0$) exhibits the upward slope predicted by Proposition 5. The condition holds because $\Psi(c) = \frac{2}{u-c}$ is increasing in cost.

While the existence of a welfare loss region is robust, its geometry can vary with demand functions. The upward-sloping boundary arises when the surplus semi-elasticity $\Psi(c)$ is sufficiently sensitive to costs. For isoelastic demand $D(p) = p^{-\varepsilon}$, by contrast, $\Psi(c) = \frac{\varepsilon-1}{c}$ is decreasing. In this case, the boundary becomes vertical: consumer harm depends primarily on liquidity risk λ, independent of production costs.

Having characterized the shape of the welfare loss region, we now examine how changes in the environment—specifically, the shadow cost of liquidity μ and retail efficiency m—affect the extent of consumer harm.

Corollary 1 *The consumer welfare loss region expands ($\lambda^*(c)$ increases) when (i) credit becomes more costly (higher μ), or (ii) the intermediary's retail efficiency improves (lower m). Consequently, tighter financial conditions and technological progress in retailing both increase the scope of consumer harm.*

The intuition for the comparative statics of μ is straightforward. When credit becomes more expensive (higher μ), the intermediary raises prices more aggressively to extract liquidity from the product market. This magnifies the "liquidity tax" on consumers, expanding the region

$CS(p^*(c))$ at multiple points. Such oscillatory behavior is unlikely under standard demand specifications. Condition (21) holds under standard demand specifications, though with differing structural implications. For linear demand $D(p) = u - p$, the condition holds when production costs are sufficiently high ($c > \frac{1+\mu}{1+3\mu}u$). For isoelastic demand $D(p) = p^{-\varepsilon}$, the condition is cost-independent and holds when $\frac{\mu}{1+\mu} > \frac{1}{\varepsilon-1}$.

where consumers are harmed. In other words, when funding is scarce, more products fall into the category where the price distortion outweighs the supply reliability gain.

The effect of retail efficiency (m) is subtler but can be understood by considering the *effective liquidity risk*, $x \equiv m\lambda$. From the consumer's perspective, the trade-off between the price distortion and the supply assurance depends directly on this effective risk x, rather than on m and λ separately. Let $x^*(c)$ denote the threshold of effective risk below which financing harms consumers (i.e., $\Delta CS < 0$ for $x < x^*(c)$). In terms of the raw supply risk λ, this threshold corresponds to $\lambda^*(c) = x^*(c)/m$. As the intermediary becomes more efficient (lower m), the scaling factor $1/m$ increases, which lifts the boundary $\lambda^*(c)$ and expands the range of suppliers falling into the consumer-loss region. Thus, greater retail efficiency paradoxically widens the scope of consumer welfare loss.

Realized Consumer Harm. Identifying the region of potential consumer harm is not sufficient to establish realized welfare loss; one must also determine whether these products in the consumer-loss region are actually financed in equilibrium. Let

$$\mathcal{F} \equiv \{(\lambda, c) : S(\lambda, c) \geq 0\}$$

denote the equilibrium financing set, and

$$\mathcal{H} \equiv \{(\lambda, c) : \Delta CS(\lambda, c) < 0\}$$

denote the consumer-loss set. Realized consumer harm occurs only in the intersection $\mathcal{F} \cap \mathcal{H}$; absent this overlap, middleman finance is unambiguously beneficial to consumers.

The geometry of these sets is driven by distinct economic forces. The loss region $\mathcal{H}$ is determined by the trade-off between price distortions and supply assurance, parameterized by demand curvature, matching efficiency m, and the shadow cost μ. The financing set $\mathcal{F}$, however, is constrained by the intermediary's profitability, which depends critically on the fixed cost k. In the linear demand case (Figure 4), we observe that this intersection is non-empty when k is sufficiently small, though this condition would be different for other demand functions.

While we do not seek to provide a general characterization of this intersection, we note a structural conflict between the intermediary's portfolio choice and consumer welfare. To sustain the liquidity pool, the intermediary must finance "liquidity providers"—low-risk suppliers who generate reliable early revenue. Yet, precisely because their default risk is low, financing these suppliers offers minimal insurance value to consumers, who nonetheless bear the cost of higher prices.

Meanwhile, financing low-risk suppliers serves a broader purpose within the cross-subsidization mechanism. By extracting liquidity from these suppliers, the intermediary generates the slack required to support high-risk "liquidity users"—suppliers who would otherwise fail with high probability. Including these risky products creates significant consumer value. Consequently, the aggregate consumer welfare impact depends on the net balance: whether the surplus destruction in low-risk "provider" markets is outweighed by the surplus creation in high-risk ones.

Total Welfare. Finally, we evaluate the total welfare impact of middleman finance. We define the change in total welfare $\Delta W(\lambda, c)$ as the sum of the change in consumer surplus and the change in the intermediary's producer surplus. Crucially, the intermediary's surplus contribution $S(\lambda, c)$ (defined in (20)) accounts for both the direct profit change $\Delta\pi$ and the value of the liquidity contribution $\mu\theta_F$. Since the shadow value μ corresponds to the opportunity cost of external capital (i.e., $\mu^* = i$ in equilibrium), including this term implies treating the cost of funds as a real resource cost. Thus, we have:

$$\Delta W(\lambda, c) \equiv \Delta CS(\lambda, c) + S(\lambda, c).$$

Since the intermediary only finances a supplier when $S(\lambda, c) \geq 0$, it follows immediately that any reduction in total welfare must be driven by a decline in consumer surplus.

Corollary 2 *A reduction in total surplus (*$\Delta W(\lambda, c) < 0$*) can occur only if consumer surplus decreases (*$\Delta CS(\lambda, c) < 0$*).*

This corollary implies that the region of total welfare loss is a subset of the consumer loss region. As illustrated in Figure 4, total welfare declines only in the portion of the hatched area adjacent to the selection boundary. In this area, although financing is profitable, the intermediary's gain $S(\lambda, c)$ is marginal—barely exceeding zero. Consequently, it is insufficient to offset the consumer harm driven by the liquidity-driven price increase, resulting in a net destruction of social value.

6 Conclusion

This paper analyzes the dual role of middlemen as merchants and liquidity providers. We show that middleman finance is not merely a transfer of funds but a system of liquidity cross-subsidization among suppliers. The middleman pools trade credit from cash-rich suppliers and reallocates it to finance high-profit but liquidity-constrained suppliers who would otherwise fail to produce. This pooling arrangement explains two key empirical patterns: why financial inclusion is necessarily selective, and why even cash-rich firms effectively borrow from their suppliers to fund the system.

Our analysis highlights two key implications for retail intermediaries who increasingly integrate financial services into their core operations.

First, we uncover a non-monotonic relationship between retail efficiency and supplier financing. When external funding is expensive, better retail technology complements financial intermediation: faster sales accelerate revenue realization, expanding the liquidity pool available for financing. Conversely, when external funding is cheap, better retail technology reduces the need for financing: as sales velocity increases, suppliers face lower liquidity risk, diminishing their demand for liquidity support. In the limit of high retail efficiency, retailers may abandon supplier financing altogether.

Second, we identify a new channel through which liquidity constraints affect consumers: liquidity-induced double marginalization. To fund supplier needs, the intermediary raises retail prices above standard monopoly levels. This "liquidity tax" generates a stark welfare trade-off. On one hand, financing ensures that constrained suppliers can produce, creating value through guaranteed trade. On the other hand, higher prices destroy consumer surplus, even for products that would have traded anyway. Consequently, financing suppliers may reduce consumer welfare for high-cost, low-risk products—precisely where the supply reliability gain is small but the price distortion is large. Antitrust authorities must weigh these competing effects when evaluating e-commerce intermediaries' pricing practices.

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A Appendix

A.1 Proof of Lemma 1

In text. ■

A.2 Proof of Lemma 2

The intersection point of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$ is $(\lambda_0, c_0) = \left(\frac{k+\sqrt{k^2+4ku}}{2mu},\ u + k - \sqrt{k^2+4ku}\right)$. We can derive that

$$\frac{\partial b(\lambda, \mu)}{\partial \mu} = \frac{2\left(km\lambda + k - u(m\lambda)^2\right)}{(m\lambda\mu + m\lambda + \mu)^2},$$

which is positive if $\lambda < \lambda_0$ and negative if $\lambda > \lambda_0$. That is, as μ increases, $c = b(\lambda, \mu)$ rotates around (λ_0, c_0) clockwise, which implies that more suppliers with positive θ_F are selected (i.e. $q(\lambda, c, \mu)$ is increasing in μ for (λ, c) such that $\theta_F(\lambda, c) > 0$) and fewer suppliers with negative θ_F are selected (i.e. $q(\lambda, c, \mu)$ is decreasing in μ for (λ, c) such that $\theta_F(\lambda, c) < 0$).

Taking $\bar{c}$ as given. If $c_0 \in [\underline{c}, \bar{c}]$, since $g(\cdot)$ is everywhere positive in Ω, it holds that $\Theta(\mu, i) = \int_\Omega q(\lambda, c, \mu)\theta_F(\lambda, c)dG$ is strictly increasing in μ.

If $c_0 > \bar{c}$, and suppose $\lambda_0 < 1$, then there exist unique threshold values, denoted by $\underline{\mu} > 0$ and $\bar{\mu} \in (\underline{\mu}, \infty)$, such that the curve of $c = b(\lambda, \mu)$ lies entirely above $c = \bar{c}$ for $\mu \in (\underline{\mu}, \bar{\mu})$, see Figure 5. For $\mu \in (\underline{\mu}, \bar{\mu})$, $\Theta = \int_\Omega \theta_F(\lambda, c)dG$ is independent of μ, which means that μ does not influence the selection of suppliers. For $\mu \in (0, \underline{\mu}) \cup (\bar{\mu}, \infty)$, by the same logic as shown above, $\Theta(\mu)$ is strictly increasing in μ. Suppse $\lambda_0 \geq 1$, then $\Theta(\mu)$ is strictly increasing in μ for $\mu \in (0, \underline{\mu})$ and keeps constant for $\mu > \underline{\mu}$.

Common to all cases is that when μ approaches infinity, only suppliers with positive θ_F are selected, thus $\Theta(\infty) > 0$.

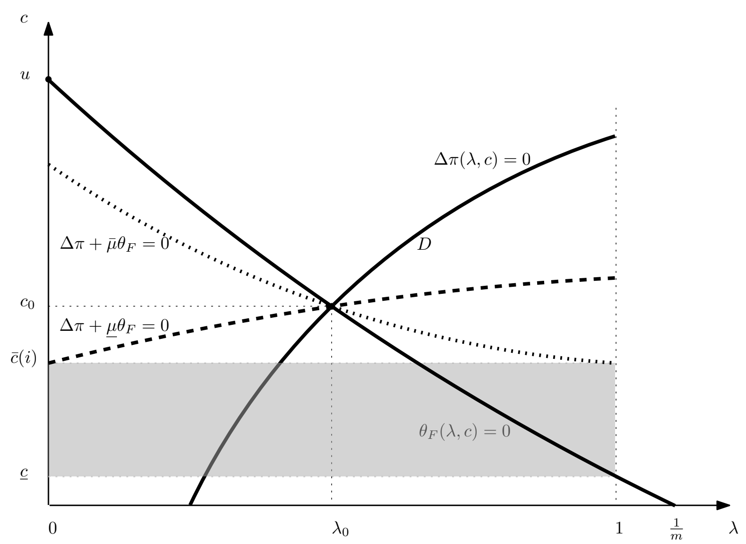

Figure 5: For $\mu \in (\underline{\mu}, \bar{\mu})$, $c = b(\lambda, \mu)$ lies above $\bar{c}$.

Now we show that μ is generically unique. Since $\Theta(\mu)$ is monotonically increasing in μ, if $\Theta(0) + L \geq 0$, then $\mu = 0$. If $\Theta(0) + L < 0$, then the liquidity constraint is binding at some $\mu \in (0, \infty)$, which is uniquely pinned down by $\Theta(\mu) + L = 0$. Note that when $L = -\int_{\Omega} \theta_F(\lambda, c) dG \geq 0$ and $c_0 > \bar{c}$, any $\mu \in [\underline{\mu}, \bar{\mu}]$ satisfies $\Theta(\mu) + L = 0$. In this case, we define the solution μ as $\underline{\mu}$. ■

A.3 Proof of Lemma 3

Given that $\Theta(0) + L < 0$, $\mu(L)$ is determined by (12). Since $\Theta(\mu)$ is strictly increasing in μ outside the interval $[\underline{\mu}, \bar{\mu}]$ provided the range exists and also note that we have selected $\mu = \underline{\mu}$ if $\Theta(\underline{\mu}) = \Theta(\bar{\mu}, i) = L$, the statement follows. ■

A.4 Proof of Theorem 1

In text. ■

A.5 Proof of Proposition 1

By the Euler equation (13), there are two cases. First, if $i \geq \mu(0)$, then $L = 0$. This case is valid either if $\mu(0) = 0$ (then $i > \mu = 0$ follows), or if $\mu(0) > 0$. Second, $i = \mu(L) > 0$ and $L > 0$, which requires that $\Theta(0) < 0$ and $i \leq \mu(0, i)$. ■

A.6 Proof of Theorem 2

In text. ■

A.7 Proof of Proposition 2

Given that $b'_\lambda(\lambda, i) = \frac{2m(k+ik-i^2u)}{(i+m\lambda+im\lambda)^2}$, it is straightforward to verify that $b'_\lambda(\cdot) > 0$ if $i < i_0 \equiv \frac{k+\sqrt{k^2+4uk}}{2u}$, and $b'_\lambda(\cdot) < 0$ if $i > i_0$.■

A.8 Proof of Proposition 3

In text. ■

A.9 Proof of Lemma 4

We first examine the monotonicity with respect to λ. Let $\Gamma(\lambda) \equiv 1 + \mu(1 - m\lambda)$. The pricing gain can be expressed as the difference in the objective function evaluated at the optimal and benchmark prices:

$$\mathcal{G}(\lambda, c) = \Gamma(\lambda) \left[\pi(p^*(\tilde{c}), \tilde{c}) - \pi(p^*(c), \tilde{c}) \right].$$

Recall that the effective cost is $\tilde{c} = \frac{1+\mu}{\Gamma(\lambda)}c$, implying that $\Gamma(\lambda)\tilde{c} = (1+\mu)c$ is constant in λ. Using this property and expanding the profit terms, we obtain:

$$\mathcal{G}(\lambda, c) = \Gamma(\lambda)\left[R(p^*(\tilde{c})) - R(p^*(c))\right] - (1+\mu)c\left[D(p^*(\tilde{c})) - D(p^*(c))\right].$$

Differentiating with respect to λ yields:

$$\frac{\partial \mathcal{G}}{\partial \lambda} = \Gamma'(\lambda)[R(p^*(\tilde{c})) - R(p^*(c))] + \underbrace{\frac{\partial \mathcal{G}}{\partial p}\bigg|_{p=p^*(\tilde{c})}\frac{\partial p^*(\tilde{c})}{\partial \lambda}}_{=0 \text{ (Envelope Thm)}}.$$

The second term vanishes by the Envelope Theorem because $p^*(\tilde{c})$ maximizes the objective function. Thus, the derivative depends only on the direct effect of λ on Γ. We have $\Gamma'(\lambda) = -\mu m < 0$. Since $\tilde{c} > c$, optimal prices are higher ($p^*(\tilde{c}) > p^*(c)$). On the elastic portion of the demand curve, this implies lower revenue ($R(p^*(\tilde{c})) < R(p^*(c))$). Therefore:

$$\frac{\partial \mathcal{G}}{\partial \lambda} = -\mu m\left[R(p^*(\tilde{c})) - R(p^*(c))\right] > 0.$$

Consider next the shadow cost μ. Applying the Envelope Theorem with respect to μ: $\frac{\partial \mathcal{G}}{\partial \mu} = \theta_F(p^*(\tilde{c})) - \theta_F(p^*(c))$. Since $p^*(\tilde{c}) > p^*(c)$, and the price is increased specifically to generate more liquidity, we have $\theta_F(p^*(\tilde{c})) > \theta_F(p^*(c))$, thus $\partial\mathcal{G}/\partial\mu > 0$.

Finally, note that m and λ always enter the pricing gain as a product $m\lambda$ (the effective liquidity risk). Since $\mathcal{G}$ is strictly increasing in λ, it follows by the chain rule that $\frac{\partial \mathcal{G}}{\partial m} = \frac{\lambda}{m}\frac{\partial \mathcal{G}}{\partial \lambda} > 0$. ■

A.10 Proof of Proposition 4

In text. ■

A.11 Proof of Lemma 5

In text. ■

A.12 Proof of Proposition 5

The threshold $\lambda^*(c)$ is defined by the indifference condition $\Delta CS(\lambda^*(c), c) = 0$. By implicit differentiation,

$$\frac{d\lambda^*}{dc} = -\frac{\partial \Delta CS(\lambda, c)/\partial c}{\partial \Delta CS(\lambda, c)/\partial \lambda}.$$

At the crossing point, $\Delta CS(\lambda, c)$ transitions from negative to positive as λ increases, so $\partial\Delta CS/\partial\lambda > 0$. Hence the sign of the slope is governed by the numerator:

$$\operatorname{sgn}\left(\frac{d\lambda^*}{dc}\right) = -\operatorname{sgn}\left(\frac{\partial \Delta CS}{\partial c}\right).$$

Let $\rho(c) \equiv p^{*\prime}(c)$ denote the pass-through rate. Since $CS'(p) = -D(p)$, the sensitivity of consumer surplus to production cost satisfies

$$\frac{d\,CS(p^*(c))}{dc} \;=\; -D\big(p^*(c)\big)\,\rho(c).$$

Using this, the marginal change in the surplus gap is

$$\frac{\partial \Delta CS}{\partial c} \;=\; -\gamma\,D\big(p^*(\tilde{c})\big)\,\rho(\tilde{c}) \;+\; (1-m\lambda)\,D\big(p^*(c)\big)\,\rho(c).$$

At the boundary $\Delta CS = 0$, we have $(1 - m\lambda) = CS(p^*(\tilde{c}))/CS(p^*(c))$. Substituting this ratio yields:

$$\left.\frac{\partial \Delta CS}{\partial c}\right|_{\Delta CS=0} \;=\; \gamma\,D\big(p^*(\tilde{c})\big)\,\rho(\tilde{c})\left[\frac{CS(p^*(\tilde{c}))}{D(p^*(\tilde{c}))\,\rho(\tilde{c})}\Big/\frac{CS(p^*(c))}{D(p^*(c))\,\rho(c)}\cdot\frac{1}{\gamma} \;-\; 1\right].$$

Define the surplus semi-elasticity as

$$\Psi(c) \;\equiv\; \frac{D(p^*(c))\,\rho(c)}{CS(p^*(c))}.$$

The term in the square brackets is equal to $\frac{1/\Psi(\tilde{c})}{1/\Psi(c)}\frac{1}{\gamma} - 1$. Thus, the condition $\partial\Delta CS/\partial c < 0$ (and hence $d\lambda^*/dc > 0$) is equivalent to:

$$\frac{\Psi(\tilde{c})}{\Psi(c)} \;>\; \frac{1}{\gamma}. \tag{22}$$

Since $\gamma > 1$ (so $1/\gamma < 1$) and $\tilde{c} > c$, this condition holds comfortably whenever $\Psi(c)$ is non-decreasing. Under condition (22), $\lambda^*(c)$ is strictly increasing in c. ■

A.13 Proof of Corollary 1

The consumer surplus change is given by $\Delta CS(\lambda, c) = CS(p^*(\tilde{c})) - (1 - m\lambda)CS(p^*(c))$. Note that the effective marginal cost $\tilde{c}$ depends on m and λ only through the product $x \equiv m\lambda$: $\tilde{c}(x) = \gamma(x)c = \frac{1+\mu}{1+\mu(1-x)}c$. Consequently, the entire expression for ΔCS can be strictly written as a function of the effective liquidity risk x and cost c:

$$\phi(x, c) \equiv CS(p^*(\tilde{c}(x))) - (1 - x)CS(p^*(c)).$$

Let $x^*(c)$ be the unique root satisfying $\phi(x^*(c), c) = 0$. The consumer loss region is defined by $x < x^*(c)$. Substituting $x = m\lambda$, this condition becomes $m\lambda < x^*(c)$, or equivalently in the (λ, c) space:

$$\lambda < \lambda^*(c) \equiv \frac{x^*(c)}{m}.$$

Since $x^*(c)$ is determined solely by demand properties and μ (independent of m), it follows directly that

$$\frac{\partial \lambda^*(c)}{\partial m} = -\frac{x^*(c)}{m^2} < 0.$$

Therefore, an improvement in retail efficiency (a decrease in m) increases the risk threshold $\lambda^*(c)$, strictly expanding the set of products where consumers are harmed.

Regarding the funding cost μ, notice that $\frac{\partial \gamma}{\partial \mu} > 0$. Since $\tilde{c}$ is increasing in γ, a higher μ raises the effective cost $\tilde{c}$, which in turn increases the retail price $p^*(\tilde{c})$. Because consumer surplus $CS(p)$ is decreasing in price, $CS(p^*(\tilde{c}))$ declines while the benchmark surplus $(1-x)CS(p^*(c))$ remains unchanged. Thus, for any given x, the value of $\phi(x, c)$ decreases. Since $\phi(x, c)$ is increasing in x at the crossing point $x^*(c)$, a downward shift in the function implies that the root $x^*(c)$ must increase. A higher $x^*(c)$ implies a higher $\lambda^*(c)$, expanding the loss region. ■

A.14 Proof of Corollary 2

In text. ■

B Online Appendix

B.1 Parametric Demand Analysis

The Linear Demand Case This appendix provides the analytical derivation for the linear demand specification $D(p) = u - p, u > c$ utilized in the main text. The standard monopoly price is $p^*(c) = \frac{u+c}{2}$, resulting in monopoly profit $\pi^*(c) = \frac{(u-c)^2}{4}$. Suppose first that the price is fixed at the benchmark $p^*(c)$. The value of financing a supplier with characteristics (λ, c) is given by:

$$S_{base}(\lambda, c, \mu) = \Delta\pi(\lambda, c) + \mu\theta(\lambda, c).$$

The profit contribution is $\Delta\pi(\lambda, c) = m\lambda\frac{(u-c)^2}{4} - k$. Setting $\Delta\pi \geq 0$ yields the profit-based selection boundary:

$$c \leq c_{\Delta\pi}(\lambda) \equiv u - \frac{2\sqrt{k}}{\sqrt{m\lambda}}.$$

Next, evaluating the liquidity contribution at the benchmark price yields:

$$\theta_F(\lambda, c) = \left[(1 - m\lambda)\frac{u+c}{2} - c\right]\frac{u-c}{2}.$$

Since demand is positive, the sign of θ_F is determined by the bracketed term. Solving $(1 - m\lambda)(u + c) \geq 2c$ yields the liquidity-based selection boundary:

$$c \leq c_\theta(\lambda) \equiv \frac{1 - m\lambda}{1 + m\lambda}u.$$

The financing set in the baseline model is defined by $S_{base}(\lambda, c) \geq 0$, where:

$$S_{base}(\lambda, c) = \left[m\lambda\frac{(u-c)^2}{4} - k\right] + \mu\left[\frac{(u-c) - m\lambda(u+c)}{2}\right]\frac{u-c}{2}.$$

Now consider the case where the price is optimally adjusted to account for liquidity costs. The intermediary sets $p^*(\tilde{c}) = \frac{u+\tilde{c}}{2}$, where $\tilde{c} = \gamma c$. The corresponding profit is $\pi^*(\tilde{c}) = \frac{(u-\tilde{c})^2}{4} = \frac{(u-\gamma c)^2}{4}$. The value of financing a supplier becomes:

$$S(\lambda, c) = (1 + \mu(1 - m\lambda))\frac{(u-\gamma c)^2}{4} - (1 - m\lambda)\frac{(u-c)^2}{4} - k.$$

Comparing the two value functions, it is straightforward to verify that $S(\lambda, c) > S_{base}(\lambda, c)$ for all $(\lambda, c) \in \Omega$, confirming the expansion result.

The change in consumer surplus is $\Delta CS = CS(p^*(\tilde{c})) - (1 - x)CS(p^*(c))$ where $CS(p) = \frac{(u-p)^2}{2}$. The condition for consumer surplus to decrease ($\Delta CS'(0) < 0$) simplifies to:

$$\mu > \frac{u-c}{3c-u}, \text{ for } 3c > u.$$

The Isoelastic Demand Case Consider a constant elasticity demand function $D(p) = p^{-\varepsilon}$ with $\varepsilon > 1$. The standard monopoly profit is $\pi^*(z) = \Lambda z^{1-\varepsilon}$, where $\Lambda \equiv \varepsilon^{-\varepsilon}(\varepsilon - 1)^{\varepsilon-1}$. If the intermediary is constrained to the fixed price $p^*(c)$, the value of financing a supplier with characteristics (λ, c) is:

$$S_{base}(\lambda, c) = \Lambda c^{1-\varepsilon}\left[\mu + m\lambda(1 - \mu\varepsilon)\right] - k.$$

The selection boundary $c^*_{base}(\lambda)$ is determined by $S_{base} = 0$. The slope of this boundary is determined by the sign of the term $(1 - \mu\varepsilon)$. Crucially, if the funding environment is tight or demand is highly elastic such that $\mu\varepsilon > 1$, the boundary slopes downward.

Under optimal pricing, the intermediary optimizes against the effective cost $\tilde{c} = \gamma c$. Substituting this into the valuation function yields:

$$S(\lambda, c) = \Lambda c^{1-\varepsilon}\Omega(\lambda, \mu) - k,$$

where the effective return term is $\Omega(\lambda, \mu) = (1 + \mu)^{1-\varepsilon}(1 + \mu(1 - m\lambda))^{\varepsilon} - (1 - m\lambda)$. The sensitivity of the new boundary to risk is governed by the partial derivative with respect to λ:

$$\frac{\partial S}{\partial \lambda} \propto m\left[1 - \frac{\varepsilon\mu}{\gamma^{\varepsilon-1}}\right].$$

In the baseline, the financing set expands with risk (upward-sloping boundary) only if $1 > \varepsilon\mu$. In the optimal pricing case, the condition relaxes to $1 > \varepsilon\mu/\gamma^{\varepsilon-1}$. Since $\gamma > 1$ and $\varepsilon > 1$, the denominator $\gamma^{\varepsilon-1}$ is strictly greater than 1, effectively scaling down the shadow cost of liquidity. Consequently, even in a high-distortion regime where $\varepsilon\mu > 1$, the endogenous boundary can remain upward-sloping. By passing the liquidity cost to consumers through higher prices, the intermediary recovers the viability of the high-risk segment.

B.2 Suppliers' Access to Alternative Funding Sources

In the baseline model (with unit demand), we assumed that suppliers rely exclusively on the intermediary for liquidity support. In this appendix, we relax this assumption and allow suppliers to access external credit markets directly. This extension serves two purposes. First, it demonstrates that our core mechanism—liquidity cross-subsidization—remains robust even when suppliers have outside financing options. Second, it highlights the efficiency advantage of the intermediary's centralized pooling: because the intermediary can "recycle" revenue across different products (as discussed in Footnote 11), she can often provide liquidity more cheaply than a standalone bank loan, even when facing the same nominal interest rate.

We explicitly model this by assuming suppliers have access to external credit sources at a rate i^s. A supplier (λ, c) can choose to hold a real balance z^s to ensure production.

Assume $\bar{c} > c_0 > \underline{c}$ and $\lambda_0 < 1$. A supplier (λ, c) with $z^s(c)$ has a retail market value of:

$$z^s + \left((1 - \lambda) + \lambda \min\left\{\frac{z^s}{c}, 1\right\}\right)\frac{u - c}{2}.$$

Here, $\lambda \min\left\{\frac{z^s}{c}, 1\right\}$ shows that, with a liquidity shock, the supplier can use his own liquidity holding to produce and sell to $\min\{z^s/c, 1\}$ consumers. The supplier's liquidity holding problem is:

$$\max_{z^s} \left\{ \left[z^s + \left((1-\lambda) + \lambda \min\left\{ \frac{z^s}{c}, 1 \right\} \right) \frac{u-c}{2} \right] - (1+i^s) z^s \right\}.$$

Suppliers never hold $z^s > c$, as it is inefficient. The first-order condition shows that suppliers with (λ, c) satisfying $\frac{\lambda(u-c)}{2} > i^s c$ hold money. This simplifies to:

$$c < c^s(\lambda, i^s) \equiv \frac{\lambda}{\lambda + 2i^s} u. \tag{23}$$

Thus, suppliers with $c < c^s(\lambda, i^s)$ hold $z^s(c) = c$, while those with $c \geq c^s(\lambda, i^s)$ hold $z^s(c) = 0$.

Next, we consider the intermediary's problem. She can only include into the F mode suppliers who holds no numeraire. The feasible set of suppliers is:

$$\tilde{\Omega}(i^s) = \{(\lambda, c) \in \Omega | c \geq c^s(\lambda, i^s)\},$$

which is nonempty. Her supplier selection problem is:

$$\max_{\{q(\cdot)\}_{(\lambda,c)\in\tilde{\Omega}(i^s)}} \int_{\tilde{\Omega}(i^s)} q(\lambda, c) \Delta\pi(\lambda, c) dG,$$

subject to the liquidity constraint:

$$\int_{\tilde{\Omega}(i^s)} q(\lambda, c) \theta_F(\lambda, c) dG + L \geq 0,$$

where i^s and L are given.

In earlier sections, we showed that F mode is profitable when $\lambda_0 < 1$ because region A in Figure 1 is nonempty (see Lemma 1 for details). But when suppliers can access the liquidity by external credit markets, the F mode may not always be activated under the same conditions.

Proposition 6 *Suppose $\lambda_0 < 1$, $\underline{c} > 0$, $i < \frac{k\bar{\lambda}}{mu\bar{\lambda} - 2k}$, and suppliers face outside credit market rate i^s. There exist thresholds $i < \underline{i}^s < \bar{i}^s \equiv \frac{(u-\underline{c})\bar{\lambda}}{2\underline{c}}$ such that:*

- *If $i^s \leq \underline{i}^s$, suppliers with $c \leq c^s(\lambda, i^s)$ hold liquidity, and F mode stays inactive.*
- *If $i^s \geq \bar{i}^s$, no supplier holds liquidity, and F mode is activated for some suppliers.*
- *If $i^s \in (\underline{i}^s, \bar{i}^s)$, suppliers with $c \leq c^s(\lambda, i^s)$ hold liquidity, while F mode activates for other suppliers.*

Proof. Let $\Pi(i^s, i) \equiv \int_{\tilde{\Omega}(i^s)} q(\lambda, c) \Delta\pi(\lambda, c) dG$ be the maximized profits of the intermediary from activating the finance service taking nominal interest rate $i < i_1$ as given. Let $c_{\Delta\pi}(\lambda) = u - \frac{2k}{m\lambda}$ denote the curve of (λ, c) such that $\Delta\pi(\lambda, c) = 0$. It can be shown that $c^s(\lambda, i^s)$ and $c_{\Delta\pi}(\lambda)$ cross each other at most once.

If $c^s(1, i^s) > c_{\Delta\pi}(1)$, or equivalently, $i^s < \frac{k}{mu-2k}$, then $c^s(\lambda, i^s) > c_{\Delta\pi}(\lambda)$ for all $\lambda \in [0, 1]$, meaning that all suppliers with positive profits $\Delta\pi(\lambda, c)$ are excluded from $\tilde{\Omega}(i^s)$. Thus, we must have $\Pi(i^s, i) = 0$. On the other hand, if $i^s \geq \bar{i}^s \equiv \frac{u-\underline{c}}{2\underline{c}}$, then $\tilde{\Omega}(i^s) = \Omega$, resulting in $\Pi(i^s, i) > 0$. Note that $\lambda_0 < 1$ implies $c_{\Delta\pi}(1) > \underline{c}$, which is equivalent to $\bar{i}^s > \frac{k}{mu-2k}$.

Finally, $\Pi(\cdot)$ is weakly increasing in i^s, because as i^s increases, the set of feasible suppliers $\tilde{\Omega}(i^s)$ becomes larger. Therefore, $\underline{i}^s \in [\frac{k}{mu-2k}, \bar{i}^s)$ must exist. Combined with the suppliers' liquidity-holding decision rule (23), this proves the claims in the proposition. ■

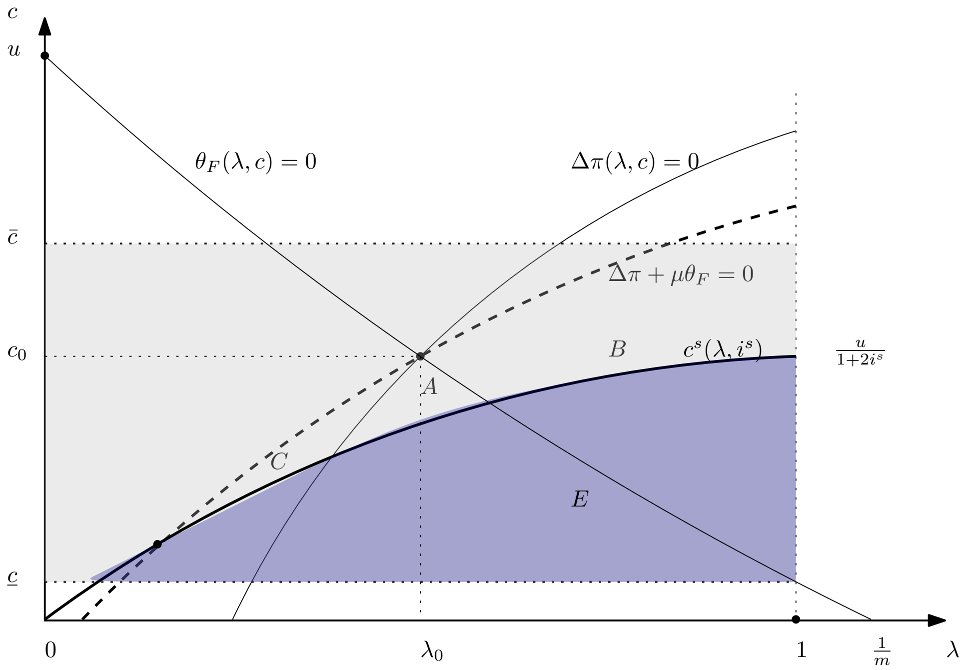

Figure 6: Suppliers' liquidity holdings and intermediary finance coexist

Figure 6 shows the third case. Suppliers with costs below $c^s(\lambda, i^s)$ (region E, dark blue) hold liquidity by themselves and skip the intermediary's finance. Those with costs above $c^s(\lambda, i^s)$ do not hold liquidity. Among them, suppliers in regions A, B, and C join the F mode. This can happen even if $i^s < i$, meaning suppliers borrow cheaper than the intermediary. Middleman finance still works because the intermediary uses liquidity more efficiently, leveraging the law of large numbers. This requires $c^s(\lambda, i^s)$ to cross the selection curve $\Delta\pi + \mu\theta_F = 0$ below c_0, ensuring region A exists, as shown in Figure 6.