Middleman Finance and Product Market Distortions *

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Abstract

We develop a model in which a middleman finances suppliers by advancing payments to cover their liquidity needs. We show that, paradoxically, funding these advances requires suppliers to extend trade credit to the middleman, resulting in *liquidity cross-subsidization* among suppliers. We further demonstrate that the middleman's matching efficiency and financing scope are not intrinsic complements; rather, their relationship is governed by the external cost of funds. Crucially, the model identifies a transmission channel from financial constraints to product markets: the middleman strategically raises retail prices to economize on liquidity usage. This generates *liquidity-induced double marginalization*. Thus, while financing enhances supply resilience, the resulting price distortion taxes consumers and can reduce total surplus.

Keywords: *Middlemen*, *Liquidity Pooling*, *Trade Credit*, *Liquidity Cross-subsidization*, *Liquidity-induced Double Marginalization*

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1 Introduction

Middlemen in product markets (e.g., retailers, merchants, trading companies) often provide liquidity to their suppliers. Historically, middlemen and liquidity provision were inextricably linked. During the colonial era, European trading companies like the Dutch East India Company offered merchant credit to local producers in Africa, Asia, and the Americas by advancing payments for goods such as spices, cotton, and tobacco.

Despite the development of modern credit markets, this function has not diminished. Rather, financing suppliers has become increasingly critical for the operation of the middleman, particularly when suppliers face severe liquidity constraints. Facilitated by Fintech providers that streamline credit enforcement, major global intermediaries such as Walmart, Alibaba, and JD.com have aggressively adopted this financing approach to support their suppliers.[1]

A deeper examination of the middleman-provided finance reveals several unique features that distinguish it from standard bank lending:

- **Reverse trade credit.** While a middleman provides suppliers with liquidity through accelerated payments, it also relies on participating suppliers to extend trade credit to it. For instance, the *JingBaoBei* program operated by JD.com, the largest e-commerce platform in China, is primarily funded by liquidity pools sourced from suppliers themselves via extended accounts payable. In other words, the program intended to provide liquidity is essentially funded by the very supplier base it aims to serve.
- **Selective supplier participation.** Access to middleman-provided finance programs is rarely universal. Instead, the middlemen select a subset of suppliers from their broader base. For example, when the UK supermarket chain Co-op launched its finance program during the pandemic in 2020, it selected fewer than 100 suppliers from a base of thousands. Similarly, Amazon Lending operated as an invitation-only program for over a decade, utilizing strict metrics to filter participants.[2]

Beyond these operational features, a fundamental difference between middlemen financiers and traditional banks lies in their market position: *middlemen directly face consumers and set final product prices.* This dual role implies that the middleman's upstream liquidity provision is inextricably linked to its downstream pricing strategy—a connection that remains underexplored.

[1] Middleman finance is dominated by retailers, which together with consumer packaged goods companies and telecom companies account for fully two-thirds of SCF outstanding," (see *https://www.freightwaves.com/news/corporate-disclosures-highlight-heavy-use-of-supply-chain-financing*, accessed on Jan 6, 2025). For middleman finance beyond the retailers listed in the text, see the Wall Street Journal report *https://www.wsj.com/articles/companies-offer-supply-chain-financing-to-vendors-as-they-bulk-up-on-inventory-push-out-payment-terms-11658316600* (accessed on Jul 17, 2023). Commonly termed supplier finance or supply chain finance (SCF), this market was valued at $1.8 trillion in 2021 globally (BCR Publishing).

[2] This selectivity reflects a careful weighing of the benefits and costs of including each supplier, consistent with best practices highlighted in the International Finance Corporation's *Supply Chain Finance Knowledge Guide*. The guide emphasizes prioritizing suppliers based on relationship depth and financial need. For details on Co-op's program, see *https://scfcommunity.org/briefing/news/2020-retail-and-apparel-winner-co-operative-group/* (accessed on Jul 17, 2023). For Amazon lending details, see *https://www.junglescout.com/blog/amazon-lending-program* (accessed on Jul 17, 2023).

In this paper, we investigate the economic rationale behind these stylized features. Specifically, we ask: Why do programs designed to relieve supplier liquidity constraints rely on supplier-provided trade credit? How does the middleman determine the financing pool? And critically, how does this arrangement affect retail prices and consumer welfare? We develop a model of a middleman who simultaneously acts as a merchant and a liquidity provider to suppliers. We show that reverse trade credit and selective inclusion are intrinsic to this dual role. Furthermore, we demonstrate that middleman finance transcends purely financial frictions; it generates *liquidity-induced double marginalization*, effectively passing the cost of supply-side resilience to consumers.

Sections 2 and 3 introduce a benchmark model featuring a retail market that operates across two sequential sub-periods: early and late. Production occurs exclusively in the early sub-period, requiring suppliers to incur a cost c in numeraire (e.g., cash for raw materials). However, suppliers lack an initial endowment, creating a temporal mismatch between cost obligations and revenue receipt. If a supplier matches with consumers early, revenue arrives in time to cover costs, allowing production to proceed. Conversely, if consumers match late, the supplier lacks the funds to finance production, precluding trade. We denote the probability of this late-period matching—effectively a liquidity shock—by λ. We initially assume retail prices are fixed to isolate the mechanics of liquidity provision from pricing distortions.

This setup captures scenarios where a supplier's liquidity is endogenous to their retail technology. Inferior logistics or display technologies often prevent suppliers from securing the early payments necessary to fund production. We assume suppliers are heterogeneous in their production costs c and liquidity risk λ, both of which are publicly observable.

We introduce an intermediary who, while unable to produce, possesses a superior retail technology that yields higher matching efficiency, as captured by a reduced probability of liquidity shocks, $m\lambda$ (where $m < 1$). This formulation follows Rubinstein and Wolinsky (1987), where intermediaries emerge from an advantage in matching efficiency.

Crucially, the intermediary also possesses a financial technology that allows her to pool trade credit: she retains early retail revenues and reallocates them as upfront payments to suppliers facing liquidity needs. This enforcement technology, while costly, enables the intermediary to function as a liquidity provider.

Given her retail advantage, the intermediary optimally offers retail services to all available suppliers. Her key decision is whether to extend financing (advanced payments) to specific suppliers. This decision is constrained by the aggregate liquidity pool: the intermediary must ensure that total advanced payments do not exceed her available funds, which consist of her internal capital and the trade credit generated from early sales. Consequently, she evaluates potential candidates for financial intermediation based not only on their individual profitability but also on their net contribution to the liquidity pool.

Our central result is that the intermediary's financing program functions as a mechanism of profit-based liquidity cross-subsidization. Suppliers with high liquidity risk (high λ) and low costs (low c) are highly profitable but need liquidity; conversely, suppliers with low liquidity risk and low costs act as net contributors to the liquidity pool. The intermediary optimally reallocates trade credit from the latter to the former. This portfolio approach outperforms both universal inclusion and selection based solely on standalone profitability. The intermediary thus acts as a strategic gatekeeper, curating a mix of net liquidity providers and net liquidity users within the supplier base to maximize total profit, akin to the multiproduct assortment problem in Rhodes, Watanabe and Zhou (2021).

Second, we characterize a non-monotonic relationship between the intermediary's matching efficiency and liquidity provision. This equilibrium stems from two countervailing forces. On the supply side, improved matching efficiency (lower m) accelerates revenue realization, expanding the pool of internal funds—a *liquidity supply effect*. On the demand side, however, faster sales reduce suppliers' exposure to late-period shocks, lowering their need for credit—a *liquidity demand effect*. We show that when the external cost of capital is high, the liquidity supply effect dominates, making matching efficiency and financing *complements*. Conversely, when external costs are low, the demand effect prevails, rendering them *substitutes*. In the limit, as matching frictions vanish, the demand for liquidity disappears. This mechanism rationalizes recent market shifts, such as Amazon's 2024 decision to terminate its in-house lending program, effectively substituting financial intermediation with superior retail efficiency.

In Section 4, we endogenize retail pricing to bridge upstream financial constraints and downstream product market outcomes. We identify a novel distortion arising from the intermediary's dual role: *liquidity-induced double marginalization*. That is, when the aggregate liquidity constraint binds, the intermediary optimally raises retail prices above the standard monopoly level. Distinct from classic double marginalization driven by vertical markups, this distortion is driven by the intermediary's *shadow cost of liquidity*. By raising prices and contracting trade volume, the intermediary reduces the upfront capital required to fund production, thereby relaxing the liquidity constraint. In effect, she imposes a "tax" on the consumer to subsidize the liquidity required to sustain production.

Regarding welfare, the financing arrangement generates a sharp trade-off. On the extensive margin, it mitigates liquidity risk, ensuring trade execution. On the intensive margin, however, it distorts prices upward via the liquidity-induced markup. We show that if the intermediary's cost of funds is high or matching is sufficiently efficient, the price distortion dominates, and financing strictly reduces total surplus. We further identify a "conflict zone" characterized by moderate liquidity risk. In this region, financing enhances total surplus but reduces consumer welfare, as the intermediary leverages market power to pass the cost of supply chain resilience onto downstream buyers.

The remainder of this section is dedicated to a literature review. All proofs are included in the Appendix.

Related Literature

First, we contribute to the literature at the intersection of industrial organization and corporate finance by exploring how liquidity constraints shape vertical contracts and product market outcomes. Our analysis relates closely to Nocke and Thanassoulis (2014), who examine how credit constraints influence vertical relationships. In their framework, financial constraints render the downstream firm effectively risk-averse regarding liquid assets, leading it to demand risk-sharing contracts that induce double marginalization as an insurance premium against demand shocks.

We identify a distinct transmission mechanism linking upstream liquidity needs to downstream pricing. In our model, raising prices serves as a strategic instrument to relax the binding liquidity constraint: it simultaneously increases per-unit revenue and contracts trade volume, thereby reducing the upfront capital requirement. Consequently, distinct from Nocke and Thanassoulis (2014) where double marginalization represents the cost of *insuring* firm assets, we show it acts as a tax on consumers to *fund* supply chain resilience.

Second, our methodological approach parallels Rhodes, Watanabe and Zhou (2021), who analyze the optimal product assortment of a multiproduct intermediary. They show that the middleman's problem can be described as the choice of a set of points in a two-dimensional space defined by sufficient statistics. The middleman's optimal product assortment includes high-value products with low profitability, which generate a direct loss for the middleman, and low-value products with high profitability, which recoup those losses. We show that this logic extends to middleman finance: our middleman manages a portfolio of suppliers defined by profitability and liquidity contribution. Analogous to their traffic-generating products, our intermediary optimally includes net lenders—suppliers who may yield lower direct profit but provide the essential liquidity required to finance the high-yield, constrained suppliers.

Third, we contribute to the literature on market-making intermediaries by analyzing liquidity provision as a distinct function. While the role of intermediaries in mitigating search and information frictions is well-established,[3] the interaction between matching technology and financial intermediation remains underexplored. We uncover a novel functional relationship between the middleman's matching and financing roles. Crucially, we show that these activities are not intrinsic complements or substitutes; rather, their relationship is endogenous to the external cost of funds.[4]

[3]Classical theories rationalize the emergence of intermediaries as a response to various market frictions, such as search costs (e.g., Rubinstein and Wolinsky (1987); Watanabe (2010, 2018, 2020)), information asymmetry and quality certification (e.g., Biglaiser (1993); Biglaiser and Li (2018); Lizzeri (1999)), or inventory risks (Li, Murry, Tian and Zhou (2024)).

[4]Our analysis is related to the emerging literature on hybrid platforms. See, e.g., Tirole and Bisceglia (2023), Madsen

Finally, our framework complements the banking literature. Unlike the liquidity pooling in Diamond and Dybvig (1983), our middleman engages in *targeted* liquidity provision, where the trade-off between profitability and liquidity contribution necessitates the strategic exclusion of certain suppliers.[5] Liquidity cross-subsidization offers a novel resolution to a long-standing puzzle in the trade credit literature: why small suppliers extend credit to apparently cash-rich buyers (e.g., Klapper, Laeven and Rajan, 2012). We argue that this flow reflects not mere credit extraction by powerful buyers, but a redistribution of liquidity that enables production and trade that would otherwise fail. This perspective connects our work to the burgeoning literature on supply chain finance (e.g., Kouvelis and Xu, 2021).

2 The Baseline Model

Consider a one-period economy populated by three types of agents: a mass of consumers, a continuum of suppliers (he), and a single intermediary (she). Each supplier produces a unique, indivisible good at constant marginal cost c. Suppliers are heterogeneous in their costs, $c \in [\underline{c}, \bar{c}]$, where $\bar{c} > \underline{c} > 0$. Consumers are homogeneous and have unit demand for each good, deriving common utility $u \geq \bar{c}$. The intermediary produces nothing but possesses a retailing technology to buy from suppliers and resell to consumers. She also has access to a costly finance technology that allows her to fund suppliers' need for liquidity.

A numeraire good, which we refer to as "money" or "liquidity," facilitates payments. Consumers hold sufficient endowment to transact. Suppliers have no initial endowment. The intermediary holds a cash balance $L \geq 0$, funded by external capital at a nominal interest rate $i \geq 0$.

Agents trade in a retail market. If the intermediary sells a good, she purchases it from the supplier, who then exits the market. A seller (supplier or intermediary) can reach all consumers. When a consumer and a seller meet, trade occurs bilaterally, and the trade surplus is split equally, yielding an equilibrium retail price:[6]

$$p = \frac{u + c}{2}. \tag{1}$$

Liquidity Shocks. There are two sequential subperiods: *early* and *late*. Production requires the supplier to incur cost c in the *early* sub-period. In a frictionless market, revenue realizes instantly to cover costs. However, liquidity becomes a problem when a timing disparity exists between

and Vellodi (2023), Gautier, Hu and Watanabe (2023), Etro (2023), Shopova (2023), Kang and Muir (2022). Existing studies typically view first-party and third-party sales as either competitive substitutes—raising self-preferencing concerns (e.g., Hagiu, Teh and Wright, 2022; Padilla, Perkins and Piccolo, 2022; Zennyo, 2022)—or strategic complements via "testing ground" or "one-stop-shop" effects (e.g., Madsen and Vellodi, 2023; Anderson and Bedre-Defolie, 2022).

[5] Relatedly, Donaldson, Piacentino and Thakor (2018) link warehouse technology to credit extension. Closer to our specific setting, Li and Pegoraro (2022) model platform-provided credit.

[6] Our results are robust to alternative pricing mechanisms. See the discussion following Theorem 2. We endogenize the retail price in Section 4.

production and trade. We assume that while production must occur early, consumers' arrival is stochastic. With probability $1-\lambda$, all consumers arrive early, allowing the supplier to cover c with immediate revenue. With probability λ, all consumers arrive in the *late* sub-period. Since the supplier lacks an initial endowment, a late match creates a liquidity shortage that prevents production. We refer to this event—where a viable trade fails due to timing mismatch—as a *liquidity shock*.

This setup captures real-world scenarios where suppliers' liquidity depends on their retail technologies. No trade occurs because of limited retail technologies possessed by suppliers to have consumers matched early rather than late. For instance,

- Display/advertisement: A supplier can display his good to consumers in the early subperiod with probability $1-\lambda$ and in the late subperiod with probability λ. If consumers buy only after inspection, then it is only in the former case that the supplier can produce and trade. Better advertising technologies increase the chance of early display.

- Delivery/inventory: A supplier can deliver his good to consumers in the early subperiod with probability $1-\lambda$, and in the late subperiod with probability λ. If consumers pay only after delivery, then it is only in the former case that the supplier can produce and trade. Better inventory technologies increase the chance of early delivery.

- Production-to-order: A supplier has access to "production-to-order" technology with probability $1-\lambda$ and can only "produce to inventory" with probability λ. Production-to-order allows suppliers to produce goods after receiving an order and payment from consumers. Then it is only when this technology is accessible that the supplier can produce and trade. Better promotion or communication with consumers, facilitated by competent sales persons, increases the chance of production to order.

We assume the probability of a liquidity shock is specific to each supplier-good pair and is publicly observable. Thus, each supplier is characterized by a type $(\lambda, c) \in \Omega \equiv [0,1] \times [\underline{c}, \bar{c}]$, distributed according to a CDF $G(\cdot)$ with strictly positive density g on Ω. Finally, we assume financial autarky among suppliers: due to a lack of enforcement technologies, suppliers cannot borrow from one another to hedge against these shocks.

The Intermediary. Following Rubinstein and Wolinsky (1987), the intermediary possesses a superior matching technology. While a supplier faces a late-matching probability λ, the intermediary reduces this risk to $m\lambda$, where $m \in (0,1)$. This advantage reflects superior search, delivery, or inventory management systems.

Crucially, the intermediary also possesses an *financing technology* that allows her to decouple payments to suppliers from receipts from consumers. This enables two distinct functions: (1)

liquidity provision, where she advances c to suppliers before sales occur; and (2) *revenue pooling*, where she delays payouts to suppliers who have already sold goods, using the retained revenue to fund the suppliers. Utilizing this technology incurs a fixed cost k per supplier, where $k \in (0, \bar{k})$.[7]

The intermediary offers each supplier one of two contract types:

1. **Middleman Contract** (M Contract). The intermediary retails the good but provides no liquidity. She pays the supplier a transfer $f_M(\lambda, c) \geq 0$ strictly contingent on the receipt of consumer payment. Consequently, production occurs *only* if consumers match in the early sub-period (probability $1 - m\lambda$). If the match occurs late, the supplier is liquidity-constrained, and trade fails.

2. **Finance Contract** (F Contract). The intermediary retails the good and provides liquidity to cover suppliers' costs. She advances the production cost c in the early sub-period, ensuring production regardless of the matching time. In exchange, she retains all consumer payment and pays the supplier a deferred reward $f_F(\lambda, c) \geq 0$ at the end of the late sub-period.

Let $q(\lambda, c) \in \{0, 1\}$ denote the selection function, where $q = 1$ ($q = 0$) indicates the supplier is offered an F (M) contract. The intermediary's strategy is the triple $\{q, f_F, f_M\}$.[8]

Timing. The sequence of events includes three stages.

1. Contracting: The intermediary chooses a numeraire holding L and offers a contract $\{q, f_F, f_M\}$ to each supplier based on their type (λ, c). Suppliers accept or reject.

2. Early sub-period: Consumer arrival times are realized. Under the *M-Contract*, trade executes only if consumers arrive early; the intermediary pays f_M immediately. Under the *F-Contract*, the intermediary advances c to all suppliers. She pools all revenue received from early consumer matches.

3. Late sub-period: Remaining matches are consummated. The intermediary settles obligations by paying the deferred reward f_F to F-contract suppliers and repays L.

3 The Equilibrium

In this section, we characterize the equilibrium of the baseline model. We solve the problem using backward induction: first, taking the liquidity endowment L as given, we determine the

[7] We define the upper bound $\bar{k} \equiv (u - \underline{c})^2$ to ensure financing is profitable for at least some types. See footnote 10.

[8] Let $q_F =$ if $q = 0$. We restrict our attention to these two contracts without loss of generality. Since the intermediary has a strict retailing advantage ($m < 1$) and can extract the supplier's surplus, she always retails. The remaining decision is binary: either incur cost k to eliminate the liquidity friction (F-Contract) or save k and accept the risk of trade failure (M-Contract).

intermediary's optimal strategy and the resulting shadow value of liquidity; second, we endogenize L by accounting for the opportunity cost of capital i. Finally, we conduct comparative statics to examine how the interest rate i and matching inefficiency m affect the equilibrium.

Suppliers' Participation Constraints. The intermediary makes offers subject to suppliers' participation constraints. If a supplier of type (λ, c) rejects the intermediary's offer, he produces and trades only if he matches with consumers early. His expected profit is:

$$(1-\lambda)(p-c) = (1-\lambda)(u-c)/2,$$

where we have substituted p from (1). Since (λ, c) is observable, the intermediary can condition rewards f_F and f_M on type. To induce participation, it is sufficient to offer the supplier the value of his outside option:

$$f_F(\lambda, c) = \frac{(1-\lambda)(u-c)}{2}, \tag{2}$$

and

$$f_M(\lambda, c) = \frac{(1-\lambda)(u-c)/2}{1-m\lambda} + c. \tag{3}$$

Note that f_M differs from f_F because, under the M contract, the supplier incurs the production cost c himself. Furthermore, f_M is paid only upon successful early matching, which occurs with probability $1 - m\lambda$. These transfers ensure all suppliers accept the offered contracts.

Suppliers' Profit and Liquidity Contributions. Under an M contract, the intermediary's expected profit from a supplier (λ, c) is:

$$\pi_M(\lambda, c) = (1-m\lambda)\left(p - f_M(\lambda, c)\right) = (1-m)\lambda(u-c)/2, \tag{4}$$

where we have inserted p and f_M from (1) and (3). Since $m < 1$, $\pi_M(\lambda, c) > 0$. This profit derives entirely from the intermediary's superior matching technology: while the supplier trades with probability $1-\lambda$, the intermediary increases this probability to $1 - m\lambda$, capturing the net surplus generated by this efficiency gain.

Under an F contract, the intermediary guarantees production regardless of matching timing. The expected profit from a (λ, c)-type supplier is:

$$\pi_F(\lambda, c) = p - c - f_F(\lambda, c) - k = \lambda(u-c)/2 - k. \tag{5}$$

Here, the intermediary collects revenue p, advances cost c, and pays the reward f_F. The second equality follows from substituting p and f_F. Expected profit increases with λ (as the supplier's outside option value decreases) and decreases with c (as the margin tightens).

We define $\Delta\pi$ as the incremental profit gain from financing a supplier (λ, c):

$$\Delta\pi(\lambda, c) \equiv \pi_F(\lambda, c) - \pi_M(\lambda, c) = m\lambda(u - c)/2 - k.$$

While the F contract secures production, it imposes a liquidity requirement: the intermediary must advance the production cost c to all participating suppliers in the early sub-period. These outflows are funded by the intermediary's endowment liquidity L and the revenue p collected from successful early matches. Therefore, the net liquidity contribution of a (λ, c)-supplier by the end of the early sub-period is:

$$\theta_F(\lambda, c) = (1 - m\lambda)p - c = (1 - m\lambda)\frac{(u + c)}{2} - c. \tag{6}$$

The Selection of Suppliers. Let Θ be the total liquidity contributed by suppliers that are financed by the intermediary ($q(\cdot) = 1$):

$$\Theta = \int_\Omega \Big[q(\lambda, c)\theta_F(\lambda, c)\Big]dG.$$

Then the liquidity constraint that the intermediary faces can be written as:

$$\Theta + L \geq 0. \tag{7}$$

The liquidity constraint stipulates that the intermediary's internal funding capacity—generated by the aggregate net contributions of financed suppliers—plus her endowment L, must be sufficient to cover all early production costs.[9]

The intermediary's problem of selecting suppliers into M or F contracts can be formulated as

$$V^m(L) \equiv \max_{\{q(\lambda,c)\}_{(\lambda,c)\in\Omega}} \int_\Omega [q(\lambda, c)\Delta\pi(\lambda, c)]\, dG, \ \text{ s.t. } (7). \tag{8}$$

The problem can be understood as the intermediary obtaining $\pi_M(\lambda, c)$ for all the invited suppliers and additionally deciding whether to finance suppliers to earn $\Delta\pi(\lambda, c)$ subject to liquidity constraint (7).

Since the objective and constraint are linear integrals, problem (8) reduces to a pointwise maximization of the integrand for each supplier type (λ, c). The optimal solution can be derived using the Lagrange method (see e.g., Rhodes, Watanabe and Zhou 2021). Let $\mu \geq 0$ be the multiplier associated with constraint (7). We can construct the Lagrangian:

$$\mathcal{L} = \int_\Omega \Big[q(\lambda, c)\Big(\Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c)\Big)\Big]dG(\lambda, c).$$

Note that $\Delta\pi(\lambda, c)$ and $\theta_F(\lambda, c)$ can be positive or negative across supplier types. In particular,

[9] Since there is a continuum of suppliers facing independent liquidity shocks, the Law of Large Numbers ensures that aggregate liquidity is deterministic. Consequently, the liquidity constraint is interpreted to hold "almost surely." In this framework, a "worst-case" scenario—where every supplier requires liquidity simultaneously—has zero measure and is therefore irrelevant to the intermediary's decision.

given the cost k of using finance technology, it is not profitable to fund all suppliers, that is, there exist suppliers with negative $\Delta\pi(\cdot)$.

Using this Lagrangian, the solution to the intermediary's problem can be obtained as an optimal selection policy that depends not only on (λ, c) but also on μ. With a slight abuse of notation, we shall refer to this optimal policy to finance a supplier as $q(\lambda, c, \mu)$, which is given by:

$$q(\lambda, c, \mu) = \begin{cases} 1 & \text{if } \Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0; \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Condition (9) indicates that $q(\lambda, c, \mu) = 1$ consists of three possible scenarios:

$$\Delta\pi(\lambda, c) \geq 0,\ \theta_F(\lambda, c) \geq 0, \tag{10a}$$

$$\Delta\pi(\lambda, c) > 0,\ \theta_F(\lambda, c) < 0,\ -\Delta\pi/\theta_F \geq \mu, \tag{10b}$$

$$\Delta\pi(\lambda, c) < 0,\ \theta_F(\lambda, c) > 0,\ -\Delta\pi/\theta_F \leq \mu. \tag{10c}$$

In scenario (10a), the intermediary selects suppliers with positive increments in profits $\Delta\pi$ and positive liquidity contribution θ_F to finance. In scenario (10b), the intermediary selects suppliers with positive increments in profits $\Delta\pi$ and negative liquidity contribution θ_F to finance, provided the marginal profitability of liquidity usage, measured by $-\Delta\pi/\theta_F$, is higher than the shadow value of liquidity μ. In the last scenario (10c), the intermediary selects suppliers with negative $\Delta\pi$ and positive θ_F to finance, as these suppliers contribute to the aggregate liquidity of the intermediary. The cost of getting one unit of liquidity from these suppliers is $-\Delta\pi/\theta_F$, and the intermediary should extract the liquidity from these suppliers if $-\Delta\pi/\theta_F \leq \mu$.

To illustrate the three scenarios in a figure, we insert $\Delta\pi(\cdot)$ and $\theta_F(\cdot)$ into (10), and obtain three boundaries in Ω:

$$\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\theta_F}(\lambda) \equiv \frac{1 - m\lambda}{1 + m\lambda}u, \tag{11a}$$

$$\Delta\pi(\lambda, c) \geq 0 \Leftrightarrow c \leq c_{\Delta\pi}(\lambda) \equiv u - \frac{2k}{m\lambda}, \tag{11b}$$

$$\Delta\pi(\lambda, c) + \mu\theta_F(\lambda, c) \geq 0 \Leftrightarrow c \leq b(\lambda, \mu) \equiv \frac{m\lambda u - 2k + \mu(1 - m\lambda)u}{m\lambda + \mu(1 + m\lambda)}. \tag{11c}$$

The right-hand side of (11c) is a "weighted average" of the right-hand sides of the first two. We plot these three boundaries in Figure 1, annotated by $\theta_F(\lambda, c) = 0$, $\Delta\pi(\lambda, c) = 0$, and $\Delta\pi + \mu\theta_F = 0$, respectively. The intersection is denoted by (λ_0, c_0). Any suppliers below $\theta_F(\lambda, c) = 0$ contribute to the liquidity pool, and any suppliers below $\Delta\pi(\lambda, c) = 0$ contribute to the intermediary's profits.[10]

The overlapping region A represents suppliers in scenario (10a), which are financed by the intermediary because they contribute to both profits $\Delta\pi$ and liquidity θ_F. Suppliers in region B,

[10] Note that $k < \bar{k}$ ensures that $c_0 > \underline{c}$. Also, Figure 1 is drawn with $\lambda_0 < 1$ and $\bar{c} > c_0$. The complete analysis, including cases where $\lambda_0 \geq 1$ or $\bar{c} < c_0$, is provided in the proof.

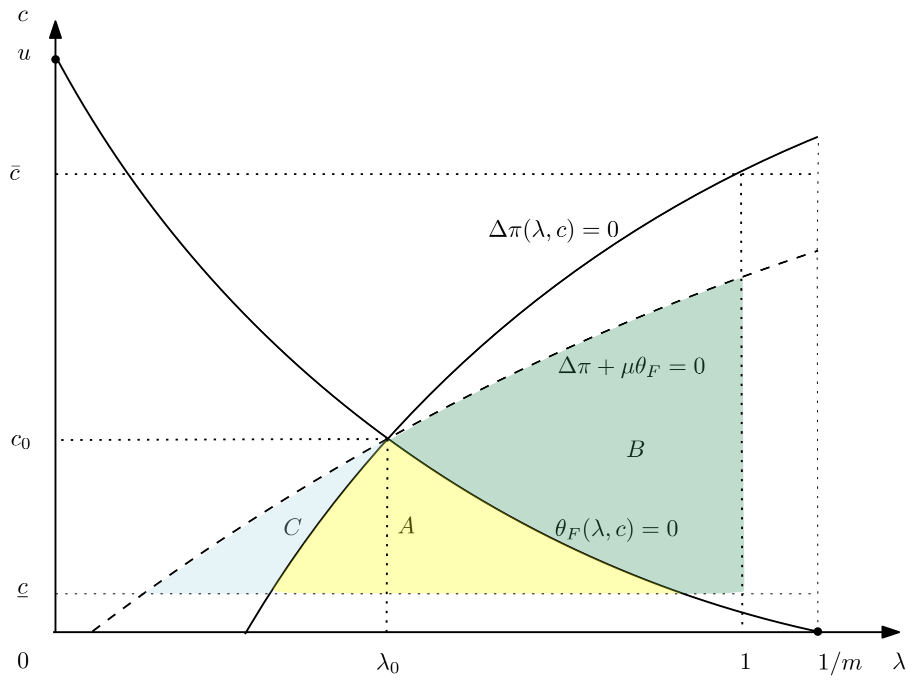


Figure 1: The intermediary's selection of suppliers

corresponding to scenario (10b), have net liquidity needs, $\theta_F < 0$, while contributing to profits $\Delta\pi > 0$. Suppliers in region *C*, corresponding to scenario (10c), when included in a finance contract, give the intermediary lower profits $\Delta\pi < 0$, but contribute to the liquidity pool. Suppliers outside *A*, *B* or *C* are not financed.

Overall, the intermediary adopts a *profit-based liquidity cross-subsidization* strategy. This involves using the positive net liquidity contributions from suppliers in regions *A* and *C* to address the liquidity needs of suppliers in region *B*. In particular, when the intermediary uses liquidity contributions from region *C*, it incurs a cost in the form of reduced (or negative) profits to these suppliers. However, when providing liquidity support to suppliers in region *B*, the intermediary expects to earn positive profits from them.

In the standard liquidity pooling (a la Diamond and Dybvig 1983), agents are homogeneous and so, translated into our context, only those who make a positive profit contribution would be selected. However, with heterogeneous agents, we show that this is suboptimal; whether a supplier will be included in the intermediary's funding program depends not only on his liquidity contribution but also on his profit contribution.

Note that for finance contracts to be active, the set of suppliers with $\Delta\pi > 0$ needs to be nonempty. The necessary and sufficient condition is $\Delta\pi(1, \underline{c}) > 0$, or equivalently, $\frac{k}{m} < \frac{u-\underline{c}}{2}$. This, along with $c_0 > \underline{c}$ (implied by $k < \bar{k}$), ensures that region *A* in Figure 1 always exists.

Lemma 1 *There exist suppliers that are financed by the intermediary if and only if* $k/m < (u - \underline{c})/2$.

The intuition is as follows. When k is large, the incremental profit $\Delta\pi$ decreases, making it less attractive to finance suppliers. When m is small, the intermediary has a high matching advantage, and consumers are likely to match in the early subperiod. This reduces the benefit of financing suppliers. In the extreme case, if $m \to 0$, the intermediary's superior matching technology eliminates the need for financial intervention, rendering the F contract redundant.

It remains to determine μ, the shadow value of liquidity. If (7) is binding, μ is determined by

$$L = -\Theta(\mu) \equiv -\int_{\Omega} q(\lambda, c, \mu)\theta_F(\lambda, c) dG. \tag{12}$$

If (7) is slack, i.e., $-\Theta(0) \leq L$, then $\mu = 0$. Consequently, the intermediary selects suppliers solely based on whether $\Delta\pi \geq 0$ or not.

Lemma 2 *If* $\Theta(0) + L < 0$, *then there exists a unique* $\mu > 0$ *that satisfies (12); and otherwise* $\mu = 0$.

The intermediary's available liquidity L governs the contracts through the shadow value μ. Intuitively, μ is strictly decreasing in L over the interval $[0, -\Theta(0)]$; this reflects the fact that an additional unit of the intermediary's liquidity endowment is most valuable when her existing holdings are scarce.

Lemma 3 $\mu(L) > 0$ *is strictly decreasing in* L *if* $\Theta(0) + L < 0$.

We summarize the results so far in the following theorem.

Theorem 1 (Selection into Finance Contracts) *For a given liquidity endowment* $L \geq 0$ *and assuming* $k/m < (u - \underline{c})/2$, *the intermediary's profit-maximizing strategy exists and is unique. It is characterized by the selection rule* $q(\lambda, c, \mu)$ *defined in (9), the supplier rewards* $f_F(\lambda, c)$ *and* $f_M(\lambda, c)$ *as specified in (2) and (3), and the shadow value of liquidity* $\mu \geq 0$ *uniquely determined by Lemma 2.*

The Intermediary's Liquidity Holding. Having characterized the selection policy for a given L, we now endogenize the intermediary's numeraire holding. The intermediary chooses L to maximize total profits net of funding costs:

$$\max_{L \geq 0} V^m(L) - i \cdot L,$$

where $V^m(L)$ is the optimized value function from (8). Applying the Envelope Theorem, $V^{m\prime}(L) = \mu(L)$, yielding the first-order condition:

$$i \geq \mu(L), \tag{13}$$

which holds with equality if $L > 0$.

The optimal holding $L^*(i)$ depends on the scarcity of liquidity within the supplier pool. Recall $\mu(0)$ is the shadow value when the intermediary holds $L = 0$. Following Lemma 2, if the pooled liquidity from suppliers is sufficient to fund all profitable F contracts ($\Theta(0) \geq 0$), then $\mu(0) = 0$ and the intermediary holds no liquidity ($L = 0$).

If, however, the pool faces a deficit ($\Theta(0) < 0$), the choice of L depends on the interest rate i. With $i \geq \mu(0)$, external funding is more expensive than the marginal benefit of relaxing the liquidity constraint. The intermediary relies solely on internal pooling and sets $L^*(i) = 0$. With $i < \mu(0)$, the intermediary uses external funding until its marginal benefit equals the interest rate, setting $L^*(i) > 0$ such that $\mu(L^*) = i$. This characterization is summarized in the following proposition.

Proposition 1 (Intermediary's Liquidity Holdings) *The optimal liquidity holdings of the intermediary follow $L^*(i) = -\Theta(i) > 0$ if $i < \mu(0)$, and $L^*(i) = 0$ otherwise. The value of liquidity with the intermediary's optimal liquidity holdings is given by*

$$\mu^*(i) = \min\left(i, \mu(0)\right). \tag{14}$$

Combining the selection rule with $L^*(i)$, we establish the existence of a unique equilibrium.

Theorem 2 (Equilibrium) *A unique equilibrium exists. It is characterized by the intermediary's strategy $(q(\lambda, c, \mu^*(i)), f_M(\lambda, c), f_F(\lambda, c), L^*(i), \mu^*(i))$, as defined in Theorem 1 and Proposition 1.*

It is worth noting that our results are invariant to the specific form of the supplier's outside option. Suppose we generalize the supplier's expected payoff of direct selling to an arbitrary function $\Phi(\lambda, c)$. To ensure participation, the intermediary adjusts rewards f_F and f_M to deliver expected value Φ. Under the F contract, she pays $f_F = \Phi(\lambda, c)$, yielding profit $\pi_F = (p - c) - \Phi(\lambda, c) - k$. Under the M contract, where production is probabilistic, she pays $f_M = c + \frac{\Phi(\lambda,c)}{1-m\lambda}$ upon success, yielding profit $\pi_M = (1 - m\lambda)(p - c) - \Phi(\lambda, c)$.

Crucially, the incremental profit from financing, $\Delta\pi = \pi_F - \pi_M = m\lambda(p - c) - k$, is independent of Φ. Because the cost of satisfying the outside option enters both profit functions linearly, it cancels out in the marginal comparison. Since the supplier's net liquidity contribution θ_F is also unaffected by the fixed transfer Φ, the intermediary's selection problem (8) and all subsequent equilibrium properties remain unchanged.

3.1 Comparative Statics

Funding Cost i. The nominal interest rate i determines the opportunity cost of external funds. As established in Proposition 1, when $i < \mu(0)$, the shadow value of liquidity equates to the market rate ($\mu^* = i$). As i increases, the intermediary reduces her external position $L^*(i)$, causing the selection boundary in Figure 1 to rotate clockwise around (λ_0, c_0). This rotation reflects the rising relative price of liquidity: as funds become costlier, the intermediary optimally tightens the selection criterion, substituting away from liquidity-consuming suppliers toward those making positive net contributions to the pool.

The threshold $\mu(0)$ captures the *intrinsic liquidity scarcity* of the supplier base. A low $\mu(0)$ implies relative abundance, rendering the selection policy less sensitive to external rate shocks. Conversely, a high $\mu(0)$ indicates structural scarcity. Crucially, once i exceeds $\mu(0)$, the intermediary ceases external borrowing. Beyond this threshold, the set of financed suppliers becomes independent of further rate increases, as the program relies exclusively on internal cross-subsidization.

This mechanism suggests that concerns regarding the vulnerability of middleman finance to "credit crunches" (sudden spikes in i) may be overstated. The internal trade credit pool functions as a strategic buffer; as external funding becomes prohibitively expensive, the intermediary optimally substitutes external debt with internal liquidity extraction, insulating the core of the financing program from further volatility.[11]

Matching Efficiency m. An improvement in matching efficiency (a lower m) exerts two countervailing forces on the equilibrium scope of F contracts. First, as the intermediary becomes more efficient in early matching, the benefit of guaranteeing production via the F contract diminishes. This *profit effect* shifts the incremental profit boundary $\Delta\pi(\cdot) = 0$ downward. Second, higher matching efficiency accelerates revenue realization, increasing liquidity inflows. This *liquidity effect* rotates the zero-contribution boundary $\theta_F(\cdot) = 0$ upward. Figure 2 illustrates these shifts as m decreases from m_1 to m_2.[12]

We focus on the regime where the liquidity constraint binds ($L > 0$), such that the shadow cost of funds is determined by the external rate, $\mu^*(i) = i$. The net impact of m on the scope of financing depends on the interest rate i relative to a threshold i_0. Let $c = b(\lambda, i)$ denote the financing boundary derived in (11c). As shown in Figure 3, the geometry of this boundary—and its response to matching improvements—pivots around i_0.

Lemma 4 *Let* $i_0 \equiv \left(k + \sqrt{k^2 + 4uk}\right) / (2u)$. *Given* $\mu^*(i) = i$, *the slope of the financing boundary with respect to risk* λ *satisfies:*

- *If* $i < i_0$, *the boundary is upward-sloping* ($b'_\lambda > 0$).
- *If* $i > i_0$, *the boundary is downward-sloping* ($b'_\lambda < 0$).

[11] Regulators and practitioners often worry that sudden increases in funding costs—driven by monetary policy or financial crises—could paralyze middleman finance programs, leading to contagion and output losses. See, for example, "Supply-Chain Finance Is New Risk in Crisis," *The Wall Street Journal*, April 4, 2020. `https://www.wsj.com/articles/supply-chain-finance-is-new-risk-in-crisis-11585992601`

[12] The boundaries $\Delta\pi(\cdot) = 0$ and $\theta_F(\cdot) = 0$ intersect at a pivot point $(c_0, \lambda_0) = (k + u - \sqrt{k^2 + 4uk}, \frac{k+\sqrt{k^2+4ku}}{2mu})$. Since c_0 is independent of m, a decrease in m causes the intersection to move to the right along the line $c = c_0$. This intersection remains within the type space Ω as long as $m \geq \tilde{m} \equiv (k + \sqrt{k^2 + 4uk})/(2u)$. When $m > \tilde{m}$, all three regions defined in Figure 1 are nonempty, ensuring that liquidity cross-subsidization is feasible.

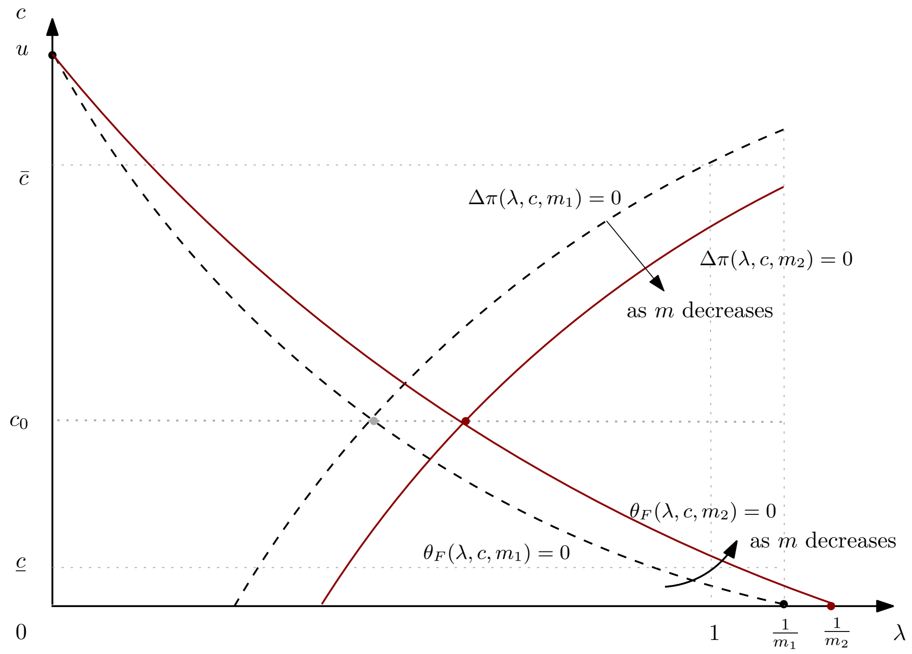


Figure 2: Effects of a decrease in m on the Profit Boundary $\Delta\pi = 0$ and Liquidity Boundary $\theta_F = 0$

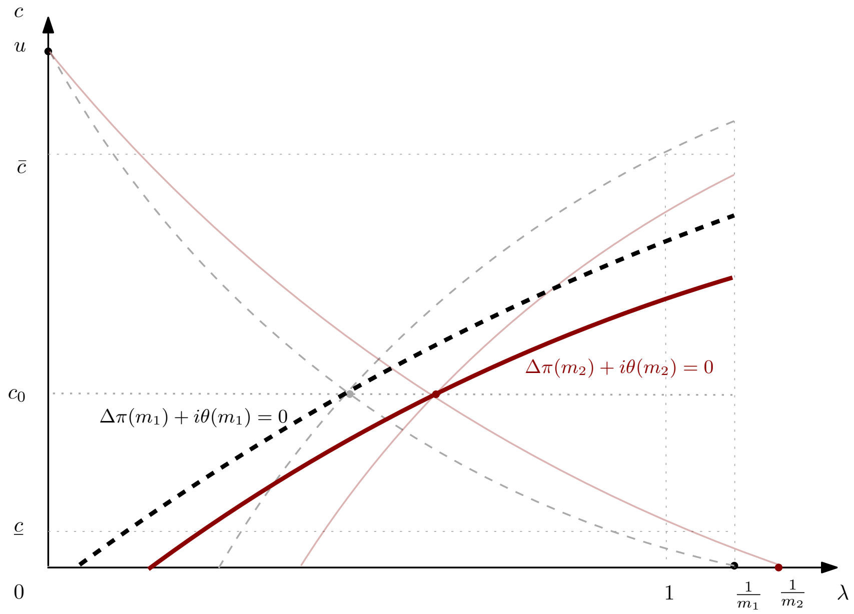
When funding costs are low ($i < i_0$, Figure 3(a)), the intermediary's selection for F contract is primarily profit-driven. As matching efficiency improves, the retail technology becomes sufficiently effective that the gain from financing suppliers diminishes. Consequently, the profit effect dominates, and improved efficiency leads to a *contraction* of the *F*-contract pool. Here, retail efficiency substitutes for financial intermediation.

Conversely, when funding costs are high ($i > i_0$, Figure 3(b)), the intermediary is liquidity-constrained. Improved matching efficiency accelerates revenue collection and relaxes the intermediary's liquidity constraint. In this regime, the liquidity effect dominates: the intermediary leverages the efficiency gains to *expand* *F*-contract offerings. Thus, matching efficiency and financing become complements.

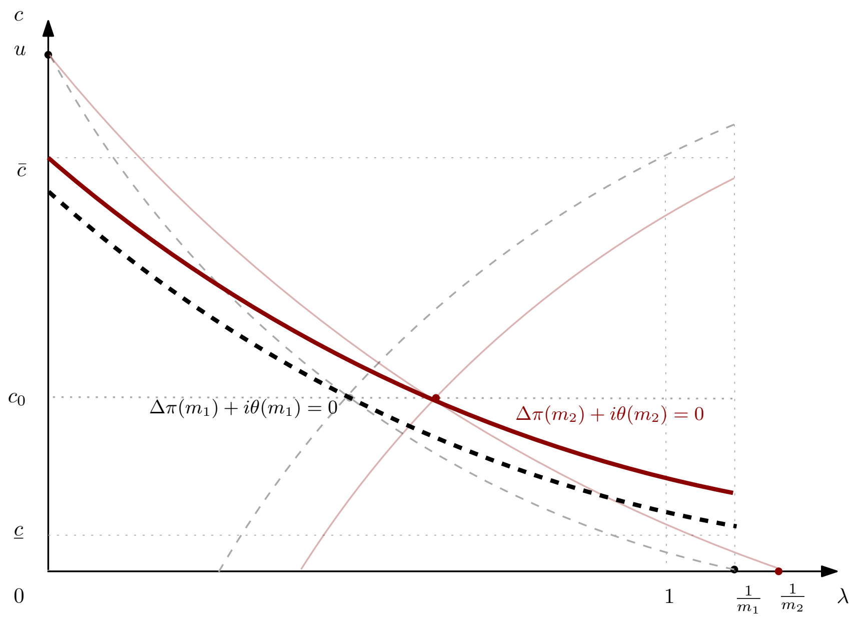
This duality suggests that technological advancements in retail matching do not uniformly supplement financial intermediation. Rather, the relationship is endogenous to the external credit costs: better matching technology crowds out financing when capital is cheap, but facilitates it when capital is expensive.

4 Endogenous Pricing

Our previous analysis fixed the retail price to isolate the financing mechanism. We now relax this assumption to examine how upstream liquidity constraints transmit to downstream product market pricing. Demand for each product is represented by a downward-sloping, twice continu-



(a) $\mu^*(i) = i < i_0$: Profit Dominance (Contraction)



(b) $\mu^*(i) = i > i_0$: Liquidity Dominance (Expansion)

Figure 3: The effect of improved matching efficiency (lower m) on the scope of supplier finance

ously differentiable function $D(p)$. The product cost is c per unit. The intermediary now chooses retail prices to jointly manage profitability and liquidity.

In the direct-selling channel, a supplier produces only if consumers arrive early (probability $1-\lambda$). In this state, he sets the monopoly price $p^*(c) \equiv \arg\max_{p \geq c}(p-c)D(p)$. We assume the profit function is strictly quasi-concave, yielding a unique $p^*(c)$. The resulting maximum profit is $\pi^*(c)$.

Under an M contract, the intermediary retails the good but provides no liquidity. Production remains contingent on early consumer arrival, which occurs with probability $1-m\lambda$. Since the intermediary cannot alleviate the liquidity friction, she simply sets the monopoly price $p^*(c)$ and pays the supplier his outside option, $(1-\lambda)\pi^*(c)$. Her expected profit is:

$$\pi_M(\lambda, c) = (1-m)\lambda\pi^*(c). \tag{15}$$

Under an F contract, the intermediary guarantees production by advancing the cost $c \cdot D(p)$ in the early subperiod. Her expected profit at a chosen price p is:

$$\pi_F(\lambda, c, p) = (p-c)D(p) - (1-\lambda)\pi^*(c) - k. \tag{16}$$

Crucially, the choice of p now determines the *liquidity requirement*. The net liquidity contribution $\theta_F(\lambda, c, p)$ is the expected revenue received early minus the upfront production cost:

$$\theta_F(\lambda, c, p) = (1-m\lambda)pD(p) - cD(p) = \left[(1-m\lambda)p - c\right] D(p). \tag{17}$$

Equation (17) reveals the fundamental tension: maximizing profit (π_F) generally requires a different price than maximizing liquidity contribution (θ_F). When the aggregate liquidity constraint binds, the intermediary must deviate from the standard monopoly price to obtain liquidity from the product market.

The intermediary's problem is to jointly determine which suppliers to finance ($q(\lambda, c) = 1$) and the optimal retail price $p(\lambda, c)$ for each financed product. This decision is subject to the aggregate liquidity constraint, which requires that the net liquidity generated by the portfolio plus the intermediary's endowment, L, remain non-negative. Formally:

$$\begin{aligned} &\max_{\{q(\cdot),p(\cdot)\}_{(\lambda,c)\in\Omega}} \int_\Omega q(\lambda, c)\Delta\pi(\lambda, c, p(\lambda, c))dG(\lambda, c), \\ &\qquad\qquad \text{s.t.} \int_\Omega q(\lambda, c)\theta_F(\lambda, c, p(\lambda, c))dG(\lambda, c) + L \geq 0. \end{aligned} \tag{18}$$

As established in Section 3, when the intermediary holds positive internal liquidity ($L > 0$), the shadow value of the constraint is pinned down by the external cost of funds, $\mu = i$. We treat μ as a parameter henceforth.

The Lagrangian can be written as:

$$\mathcal{L} = \int_{\Omega} q(\lambda, c)\tilde{S}(\lambda, c, p(\lambda, c))dG(\lambda, c) + \mu L,$$

where $\tilde{S}$ represents the value of financing supplier (λ, c). Substituting π_F and θ_F yields:

$$\tilde{S}(\lambda, c, p) = [1 + \mu(1 - m\lambda)]\,(p - \tilde{c})D(p) - k - (1 - m\lambda)\pi^*(c), \tag{19}$$

where the *effective marginal cost* is defined as $\tilde{c} \equiv \gamma c$, with the liquidity wedge

$$\gamma \equiv \frac{1 + \mu}{1 + \mu(1 - m\lambda)} \geq 1.$$

γc captures the effective cost of liquidity. γ strictly exceeds 1 whenever the constraint binds ($\mu > 0$) and liquidity risk exists ($\lambda > 0$). γ is increasing in μ and λ.

Equation (19) shows that for a financed supplier, the intermediary chooses p to maximize a profit function with effective cost $\tilde{c}$. This yields the optimal price for a financed good of type (λ, c):

$$p(\lambda, c) = p^*(\tilde{c}).$$

Since $\tilde{c} > c$, we have $p^*(\tilde{c}) > p^*(c)$. The wedge $p^*(\tilde{c}) - p^*(c)$ is the *liquidity-induced double marginalization*. Unlike classic double marginalization driven by vertical markups, this distortion arises because the intermediary internalizes the opportunity cost of the cash tied up in production. By raising the price, she lowers the quantity $D(p)$, thereby reducing the upfront capital requirement.[13]

Substituting the optimal price back into (19), the value of financing becomes:

$$S(\lambda, c) = [1 + \mu(1 - m\lambda)]\pi^*(\tilde{c}) - k - (1 - m\lambda)\pi^*(c). \tag{20}$$

The intermediary finances a supplier if and only if $S(\lambda, c) \geq 0$.

To isolate the novel economic force introduced by liquidity-driven pricing, we express the virtual value $S(\lambda, c)$ as the sum of a baseline component and a strategic pricing gain:

$$S(\lambda, c) = S_{base}(\lambda, c) + \mathcal{G}(\lambda, c).$$

The first term $S_{base}(\lambda, c)$ is the value of financing the supplier if price is fixed to the standard monopoly price $p^*(c)$, ignoring the shadow cost of liquidity. It encapsulates the baseline trade-off between profit and liquidity found in Section 3:

$$S_{base}(\lambda, c, \mu) = \underbrace{m\lambda\pi^*(c) - k}_{\equiv \Delta\pi_{base}} + \mu \underbrace{[(1 - m\lambda)p^*(c) - c]\, D(p^*(c))}_{\equiv \theta_{F,base}}.$$

[13] To quantify this distortion, let $\tau \equiv \tilde{c} - c$ be the cost wedge. The *average liquidity pass-through rate* is $\bar{\rho} \equiv \frac{1}{\tau}\int_c^{\tilde{c}} \rho(z)dz$, where $\rho(z) \equiv p^{*\prime}(z)$ is the local pass-through. The magnitude depends on demand curvature: if demand is log-concave (e.g., linear), $\rho(z) \leq 1$ (under-shifting); if log-convex (e.g., constant elasticity), $\rho(z) \geq 1$ (over-shifting).

The profit contribution $\Delta\pi_{base}$ increases with λ and decreases with c, while the liquidity contribution θ_{base} decreases in both λ and c. Thus, just as in the baseline model, profit-based liquidity cross-subsidization allows the intermediary to finance those suppliers with $S_{base}(\lambda, c) > 0$.

The term $\mathcal{G}(\lambda, c)$ captures the value added by re-optimizing the price to account for the shadow cost of liquidity. Since $p^*(c)$ uniquely maximizes standalone profit, setting $p^*(\tilde{c}) \neq p^*(c)$ entails a profit sacrifice:

$$\Delta\pi(p^*(\tilde{c})) < \Delta\pi_{base}. \tag{21}$$

However, since $\tilde{p}$ is optimized to account for the shadow cost μ, it generates a higher liquidity:

$$\theta_F(p^*(\tilde{c})) > \theta_{F,base}. \tag{22}$$

The intermediary engages in price-adjusted liquidity cross-subsidization because the value of the increased liquidity outweighs the profit sacrifice. The net valuation gain, $[\Delta\pi(\tilde{p}) - \Delta\pi_{base}] + \mu[\theta_F(\tilde{p}) - \theta_{F,base}]$, which is the second term $\mathcal{G}$ can be rewritten as

$$\mathcal{G}(\lambda, c) = (1 + \mu(1 - m\lambda))\left[\pi^*(\tilde{c}) - \pi(p^*(c), \tilde{c})\right],$$

where $\pi^*(\tilde{c}) = \max_p \pi(p, \tilde{c})$. Since the optimized profit strictly exceeds the suboptimal profit, $\mathcal{G}(\cdot) > 0$.

The following lemma characterizes how this gain varies with supplier characteristics.

Lemma 5 *If $\gamma > 1$, the pricing gain $\mathcal{G}(\lambda, c)$ is strictly positive and increasing in λ. Its monotonicity with respect to c depends on demand curvature: $\mathcal{G}$ increases in c if demand is log-concave (e.g., linear) and decreases if demand is log-convex (e.g., constant elasticity), provided $\tilde{c} - c$ is not excessively large.*

The pricing gain $\mathcal{G}(\cdot)$ acts as a strategic buffer. By distorting prices, the intermediary generates additional liquidity per unit sold, effectively relaxing the intermediary's liquidity constraint. This buffer leads to our next result.

Proposition 2 (Expansion and Resilience) *Liquidity-driven pricing strictly expands the scope of financing and enhances resilience to liquidity shocks compared to the fixed-price benchmark $p^*(c)$. Specifically:*

1. ***Expansion:*** *Since $S(\lambda, c) > S_{base}(\lambda, c)$, the set of financed suppliers is strictly larger than in the benchmark.*
2. ***Resilience:*** *The valuation of financed suppliers is less sensitive to liquidity risk: $\frac{\partial S(\lambda,c)}{\partial\lambda} > \frac{\partial S_{base}(\lambda,c)}{\partial\lambda}$.*

Intuitively, under $p^*(c)$, an increase in λ mechanically drains the liquidity pool because early revenue becomes less likely. Under $p^*(\tilde{c})$, the intermediary counters this by raising prices. This sacrifices trade volume but increases the liquidity contribution per unit, effectively "taxing" consumers to sustain high-risk suppliers that would otherwise be excluded.

Corollary 1 *The pricing gain $\mathcal{G}(\lambda, c)$ is strictly increasing in the shadow cost of liquidity μ and the matching inefficiency m.*

As matching efficiency declines (m increases) or the shadow cost of liquidity rises (μ increases), the liquidity wedge γ widens. The intermediary responds by leveraging her retail pricing power to "buy" the necessary liquidity from the product market. This effectively transforms downstream pricing into a funding instrument for upstream suppliers.

5 Welfare Implications

How does middleman finance affect retail market efficiency? We measure efficiency by the difference in total surplus generated under the finance contract versus the pure middleman contract:

$$\Delta W(\lambda, c) \equiv \underbrace{W\Big(p^*(\tilde{c})\Big)}_{\text{F contract surplus}} - \underbrace{\Big(1 - m\lambda\Big) W\Big(p^*(c)\Big)}_{\text{M contract surplus}},$$

where $W(p) \equiv \int_p^{\bar{p}} D(z)dz + (p - c)D(p)$, with $\bar{p} < \infty$ if there exits $D(\bar{p}) = 0$, and $\bar{p} = \infty$ otherwise.

ΔW captures the fundamental trade-off between supply reliability and price distortion. Financing eliminates liquidity risk, ensuring trade occurs with probability 1 rather than $1 - m\lambda$. However, it raises the retail price from $p^*(c)$ to $p^*(\tilde{c})$. Whether middleman finance improves welfare depends on which force dominates.

To facilitate the analysis, let $x \equiv m\lambda \in [0, 1]$ denote the effective liquidity risk. We write $\Delta W(x) \equiv W(p^*(\tilde{c}(x))) - (1 - x)W(p^*(c))$. It is straightforward to verify that $\Delta W(0) = 0$ and $\Delta W(1) > 0$. Furthermore, the marginal effect near zero is negative $\Delta W'(0) < 0$ if and only if

$$\frac{\mu c}{1 + \mu} \cdot \frac{\partial p^*(c)}{\partial c} \cdot D(p^*(c)) > W(p^*(c)). \tag{23}$$

When condition (23) holds, the price increase destroys more value than the trade assurance creates near zero; that is, $\Delta W(x) < 0$ for sufficiently small x. Consequently, there exists a range of products for which adopting middlemen financing reduces surplus in the retail market.

Proposition 3 (Trade Surplus Loss Region) *Suppose (23) holds. There exists a threshold $\lambda^* \in (0, 1/m)$ such that middleman finance strictly reduces the product market surplus for all $\lambda < \min\{\lambda^*, 1\}$.*

The uniqueness of λ^* is guaranteed under standard conditions where $\Delta W(x)$ is quasi-convex; we assume this regularity henceforth.[14] The threshold λ^* partitions the market: financing re-

[14] In the absence of quasi-convexity, it would require the marginal welfare effect of financing to oscillate to have threshold non-unique —– specifically, the marginal impact of risk, $-\frac{d}{dx}W(p^*(\tilde{c}(x)))$, would need to intersect the constant benchmark surplus $W(p^*(c))$ at multiple points. Since such irregular behavior is unlikely under standard demand specifications, we hereby treat λ^* as unique.

duces trade surplus for low-risk products ($\lambda < \lambda^*$) while enhancing it for high-risk products ($\lambda > \lambda^*$).

Corollary 2 *Suppose $\lambda^* \in (0, 1)$. Then λ^* is increasing in the shadow cost of liquidity μ and decreasing in the matching inefficiency m.*

The shadow cost μ affects welfare exclusively through the finance contract surplus $W(p^*(\tilde{c}))$. As μ rises, the effective cost $\tilde{c}$ increases, depressing surplus and pushing the threshold λ^* upward. Intuitively, a higher μ exacerbates the price distortion; consequently, the finance contract requires a higher level of underlying risk λ to generate enough guaranteed trade value to offset the efficiency loss.

Regarding matching efficiency, since m acts only to scale physical risk λ into effective risk x, a decrease in m (higher efficiency) raises λ^*.[15] Economically, when the intermediary is highly efficient at matching, the actual risk she "covers" is small. This reduces the gross benefit of financing, making it more likely that the net effect on surplus is negative.

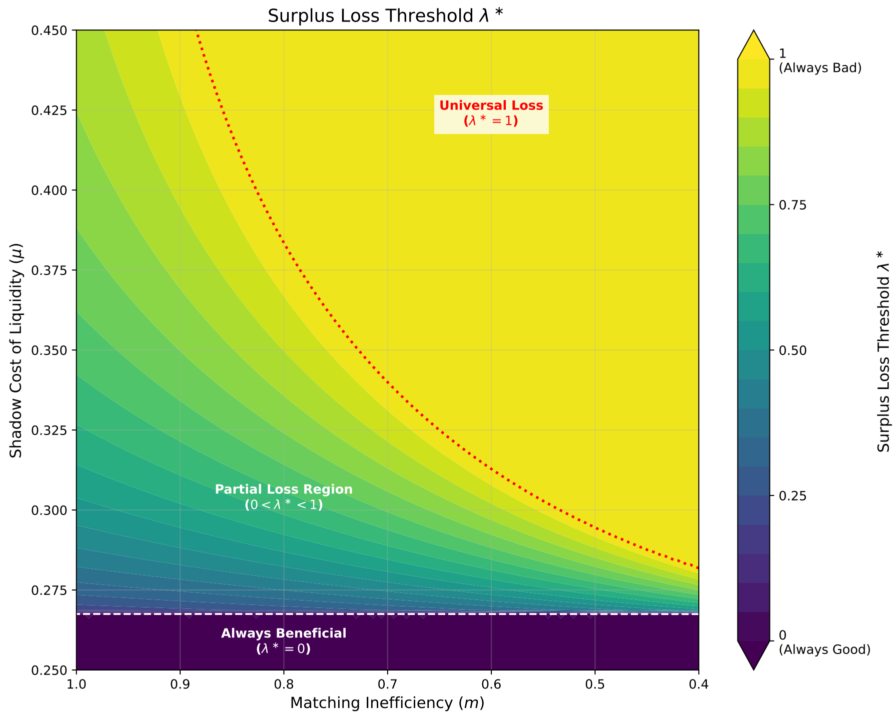


Figure 4: Welfare Loss Threshold λ^* with $D(p) = p^{-\varepsilon}$

Note: The figure illustrates the risk threshold λ^* as a function of matching inefficiency m and the shadow cost of liquidity μ, assuming an isoelastic demand with $\varepsilon = 10$. The value λ^* represents the boundary below which financing reduces product market surplus (i.e., $\Delta W(\lambda) < 0$ for all $\lambda < \lambda^*$). The plot identifies three distinct regions: Always Beneficial ($\lambda^* = 0$); Partial Loss ($0 < \lambda^* < 1$); Universal Loss ($\lambda^* = 1$).

Impose $D(p) = p^{-\varepsilon}$, Figure 4 plots the threshold λ^* as a function of matching inefficiency m

[15] Note that $\Delta W(x) = W(p^*(\tilde{c}(x))) - (1-x)W(p^*(c)) = 0$ defines a critical effective risk x independent of m.

and the shadow cost of liquidity μ. With this demand function, (23) can be written as

$$\mu > \bar{\mu} \equiv \frac{2\varepsilon - 1}{\varepsilon(\varepsilon - 1)}.$$

For $\mu < \bar{\mu}$, the price distortion is negligible, and financing improves surplus for all λ. Above this threshold, a loss region emerges. When cost of liquidity μ is sufficiently high, and (or) the m is low (corresponding to the upper-right corner), a universal surplus loss occurs: middleman finance strictly reduces trade surplus for all supplier types $\lambda \in [0, 1]$.[16]

Consumer Surplus and the Conflict Zone. Policymakers often prioritize consumer welfare over industry profits. While middleman finance can restore total surplus by preventing supply breaks, this efficiency gain comes at the cost of higher retail prices. This creates a distributional conflict: the intermediary effectively transfers the cost of liquidity provision to consumers.

Formally, we define the net consumer surplus gain as:

$$\Delta CS(\lambda, c) \equiv CS(p^*(\tilde{c})) - (1 - m\lambda) CS(p^*(c)),$$

where $CS(p) \equiv \int_p^{\bar{p}} D(z) dz$. Further define the mean consumer surplus by $M(p) \equiv CS(p)/D(p)$. The following proposition states that, consumers are likely to suffer even when total trade surplus is improved.

Proposition 4 *Fix c and suppose $\Delta W(\lambda, c) < 0$ for $\lambda < \lambda^*$. Further suppose that the mean consumer surplus $M(p) \equiv CS(p)/D(p)$ satisfies the following markup elasticity condition:*

$$\frac{p - c}{M(p)} \frac{dM(p)}{dp} < 1 \quad \text{for } p > p^*(c). \tag{24}$$

Then there exists $\lambda^*_{CS} > \lambda^*$ such that $\Delta CS(\lambda, c) < 0 < \Delta W(\lambda, c)$ for $\lambda \in (\lambda^*, \lambda^*_{CS})$.

Condition (24) is a mild requirement satisfied by a broad class of demand functions, including log-concave demand (e.g., linear) and log-convex demand (e.g., constant elasticity demand). The inequality $\lambda^*_{CS} > \lambda^*$ highlights a critical conflict zone for regulation. For goods with moderate liquidity risk $\lambda \in (\lambda^*, \lambda^*_{CS})$, intermediary finance improves total surplus but harms consumers. In this region, the intermediary leverages market power to tax consumers for upstream supply stability—a trade-off that a consumer-centric regulator would reject.

[16] Using the isoelastic demand specification ($D(p) = p^{-\varepsilon}$) as an example, the universal surplus loss occurs when $\Delta W < 0$ for $\lambda = 1$, which is equivalent to

$$\gamma(m)^{-\varepsilon} \left(\frac{\rho^2 \gamma(m) - 1}{\rho^2 - 1} \right) \leq 1 - m.$$

The set of (m, μ) is the upper-right corner in Figure 4 where $\lambda^* = 1$.

6 Conclusion

In this paper, we analyze the dual role of intermediaries as merchants and liquidity providers. We show that middleman finance is not merely a vertical transfer of funds but a mechanism of liquidity cross-subsidization among the supplier base. By pooling trade credit from liquidity-abundant suppliers, the middleman relaxes the aggregate liquidity constraint, enabling her to finance high-yield but constrained suppliers that would otherwise fail to produce. This portfolio approach rationalizes why financial inclusion is necessarily selective and why even cash-rich buyer firms effectively borrow from their suppliers to fund the system.

Our analysis yields two broad implications for retail intermediaries, who increasingly integrate financial services into their core operations. First, we uncover an endogenous relationship between matching efficiency and financial intermediation. We show that these functions can act as either complements or substitutes depending on the cost of funds. In the limit, superior retailing technologies can render financing obsolete, a mechanism that rationalizes recent industry shifts, such as Amazon's decision to terminate its in-house lending program.

Second, we identify a novel transmission channel from upstream liquidity constraints to downstream product markets: *liquidity-induced double marginalization*. When liquidity is scarce, the intermediary strategically raises retail prices to generate higher revenue and reduce upfront capital needs, effectively taxing consumers to fund supply continuity. This generates a sharp welfare trade-off: while intermediary finance enhances total surplus by ensuring production on the extensive margin, it erodes consumer surplus on the intensive margin. Consequently, antitrust scrutiny of platform pricing must carefully weigh the costs of higher markups against the value of guaranteed trade execution.

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A Appendix

A.1 Proof of Lemma 2

The intersection point of $\Delta\pi(\lambda, c) = 0$ and $\theta_F(\lambda, c) = 0$ is $(\lambda_0, c_0) = \left(\frac{k+\sqrt{k^2+4ku}}{2mu},\ u + k - \sqrt{k^2 + 4ku}\right)$. We can derive that

$$\frac{\partial b(\lambda, \mu)}{\partial \mu} = \frac{2\left(km\lambda + k - u(m\lambda)^2\right)}{(m\lambda\mu + m\lambda + \mu)^2},$$

which is positive if $\lambda < \lambda_0$ and negative if $\lambda > \lambda_0$. That is, as μ increases, $c = b(\lambda, \mu)$ rotates around (λ_0, c_0) clockwise, which implies that more suppliers with positive θ_F are selected (i.e. $q(\lambda, c, \mu)$ is increasing in μ for (λ, c) such that $\theta_F(\lambda, c) > 0$) and fewer suppliers with negative θ_F are selected (i.e. $q(\lambda, c, \mu)$ is decreasing in μ for (λ, c) such that $\theta_F(\lambda, c) < 0$).

Taking $\bar{c}$ as given. If $c_0 \in [\underline{c}, \bar{c}]$, since $g(\cdot)$ is everywhere positive in Ω, it holds that $\Theta(\mu, i) = \int_\Omega q(\lambda, c, \mu)\theta_F(\lambda, c)dG$ is strictly increasing in μ.

If $c_0 > \bar{c}$, and suppose $\lambda_0 < 1$, then there exist unique threshold values, denoted by $\underline{\mu} > 0$ and $\bar{\mu} \in (\underline{\mu}, \infty)$, such that the curve of $c = b(\lambda, \mu)$ lies entirely above $c = \bar{c}$ for $\mu \in (\underline{\mu}, \bar{\mu})$, see Figure 5. For $\mu \in (\underline{\mu}, \bar{\mu})$, $\Theta = \int_\Omega \theta_F(\lambda, c)dG$ is independent of μ, which means that μ does not influence the selection of suppliers. For $\mu \in (0, \underline{\mu}) \cup (\bar{\mu}, \infty)$, by the same logic as shown above, $\Theta(\mu)$ is strictly increasing in μ. Suppse $\lambda_0 \geq 1$, then $\Theta(\mu)$ is strictly increasing in μ for $\mu \in (0, \underline{\mu})$ and keeps constant for $\mu > \underline{\mu}$.

Common to all cases is that when μ approaches infinity, only suppliers with positive θ_F are selected, thus $\Theta(\infty) > 0$.

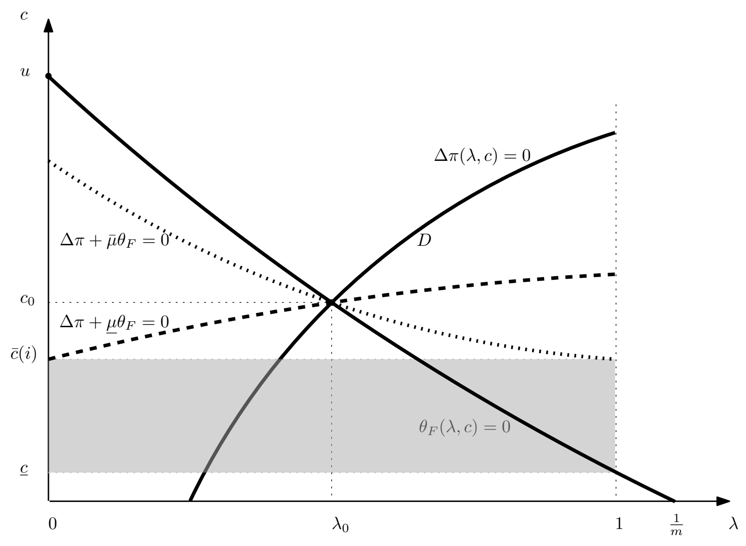

Figure 5: For $\mu \in (\underline{\mu}, \bar{\mu})$, $c = b(\lambda, \mu)$ lies above $\bar{c}$.

Now we show that μ is generically unique. Since $\Theta(\mu)$ is monotonically increasing in μ, if $\Theta(0) + L \geq 0$, then $\mu = 0$. If $\Theta(0) + L < 0$, then the liquidity constraint is binding at some $\mu \in (0, \infty)$, which is uniquely pinned down by $\Theta(\mu) + L = 0$. Note that when $L = -\int_\Omega \theta_F(\lambda, c)dG \geq 0$ and $c_0 > \bar{c}$, any $\mu \in [\underline{\mu}, \bar{\mu}]$ satisfies $\Theta(\mu) + L = 0$. In this case, we define the solution μ as $\underline{\mu}$. ■

A.2 Proof of Lemma 3

Given that $\Theta(0) + L < 0$, $\mu(L)$ is determined by (12). Since $\Theta(\mu)$ is strictly increasing in μ outside the interval $[\underline{\mu}, \bar{\mu}]$ provided the range exists and also note that we have selected $\mu = \underline{\mu}$ if $\Theta(\underline{\mu}) = \Theta(\bar{\mu}, i) = L$, the statement follows. ■

A.3 Proof of Proposition 1

By the Euler equation (13), there are two cases. First, if $i \geq \mu(0)$, then $L = 0$. This case is valid either if $\mu(0) = 0$ (then $i > \mu = 0$ follows), or if $\mu(0) > 0$. Second, $i = \mu(L) > 0$ and $L > 0$, which requires that $\Theta(0) < 0$ and $i \leq \mu(0, i)$. ■

A.4 Proof of Lemma 4

Given that $b'_\lambda(\lambda, i) = \frac{2m(k+ik-i^2u)}{(i+m\lambda+im\lambda)^2}$, it is straightforward to verify that $b'_\lambda(\cdot) > 0$ if $i < i_0 \equiv \frac{k+\sqrt{k^2+4uk}}{2u}$, and $b'_\lambda(\cdot) < 0$ if $i > i_0$.■

A.5 Proof of Lemma 5

We first examine the monotonicity with respect to λ. Let $\Gamma(\lambda) \equiv 1 + \mu(1 - m\lambda)$. The pricing gain can be written as: $\mathcal{G}(\lambda, c) = \Gamma(\lambda)(\pi^*(\tilde{c}) - \pi(p^*(c), \tilde{c}))$. Recall that the effective cost satisfies $\tilde{c}(\lambda) = \frac{1+\mu}{\Gamma(\lambda)}c$, implying that the product $\Gamma(\lambda)\tilde{c}(\lambda) = (1 + \mu)c$ is independent of λ. Using this property and expanding the profit function $\pi(p, c) = R(p) - cD(p)$, we can simplify the gain to:

$$\mathcal{G}(\lambda, c) = \Gamma(\lambda)R(p^*(\tilde{c})) - \Gamma(\lambda)R(p^*(c)) - (1 + \mu)c\left[D(p^*(\tilde{c})) - D(p^*(c))\right].$$

Differentiating with respect to λ yields:

$$\frac{\partial \mathcal{G}}{\partial \lambda} = \Gamma'(\lambda)\left[R(p^*(\tilde{c})) - R(p^*(c))\right] + \underbrace{\Gamma(\lambda)\frac{\partial \pi^*(\tilde{c})}{\partial p}\frac{\partial p^*}{\partial \lambda}}_{=0 \text{ (Envelope Thm)}},$$

where $\Gamma'(\lambda) = -\mu m$. Since the monopolist operates on the elastic portion of the demand curve, revenue $R(p)$ is decreasing in price for $p \geq p^*(c)$. Because the liquidity wedge implies $\tilde{c} > c$, we have $p^*(\tilde{c}) > p^*(c)$, and consequently $R(p^*(\tilde{c})) < R(p^*(c))$. Thus:

$$\frac{\partial \mathcal{G}}{\partial \lambda} = -\mu m\left[R(p^*(\tilde{c})) - R(p^*(c))\right] > 0.$$

Next, we consider the effect of cost c. Let $\Gamma \equiv 1 + \mu(1 - m\lambda)$. Differentiating $\mathcal{G}$ with respect to c yields

$$\frac{1}{\Gamma}\frac{\partial \mathcal{G}}{\partial c} = \gamma[D(p^*(c)) - D(p^*(\tilde{c}))] - (\tilde{c} - c)|D'(p^*(c))|\rho(c),$$

where $\rho(c) \equiv (p^*)'(c) > 0$ is the pass-through rate. The sign of $\frac{\partial \mathcal{G}}{\partial c}$ is positive if and only if:

$$\gamma > \rho(c) \frac{|D'(p^*(c))|}{\frac{D(p^*(c))-D(p^*(\tilde{c}))}{\tilde{c}-c}}. \tag{25}$$

We analyze condition (25) based on demand curvature. With a log-concave demand (e.g., linear), the pass-through rate is low ($\rho < 1$, under-shifting). Furthermore, the demand curve steepens as prices rise, making the local slope $|D'(p^*(c))|$ smaller than the average slope over the interval. Thus, the right-hand side of (25) is strictly less than 1. Since $\gamma > 1$, the condition holds, implying $\frac{\partial \mathcal{G}}{\partial c} > 0$.

With a log-convex demand, e.g., constant elasticity, the pass-through rate is high ($\rho > 1$, over-shifting). Additionally, convexity implies the local slope $|D'(p^*(c))|$ is steeper than the average slope, making the slope ratio greater than 1. Consequently, the right-hand side exceeds 1. Since the liquidity wedge γ is typically small (close to 1), condition (25) fails, implying $\frac{\partial \mathcal{G}}{\partial c} < 0$. ■

A.6 Proof of Corollary 1

Consider first the shadow cost μ. Applying the Envelope Theorem with respect to μ: $\frac{\partial \mathcal{G}}{\partial \mu} = \theta_F(p^*(\tilde{c})) - \theta_F(p^*(c))$. Since $p^*(\tilde{c}) > p^*(c)$, and the price is increased specifically to generate more liquidity, we have $\theta_F(p^*(\tilde{c})) > \theta_F(p^*(c))$, thus $\partial \mathcal{G}/\partial \mu > 0$. Second, note that m and λ always enter the pricing gain as a product $m\lambda$ (the effective liquidity risk). Since $\mathcal{G}$ is strictly increasing in λ (Lemma 5), it follows by the chain rule that $\frac{\partial \mathcal{G}}{\partial m} = \frac{\lambda}{m} \frac{\partial \mathcal{G}}{\partial \lambda} > 0$. ■

A.7 Proof of Proposition 4

Define the *relative surplus* for Total Surplus (R_W), Consumer Surplus (R_{CS}), and Profit (R_π) as the value under the distorted price relative to the benchmark:

$$R_Y(x) \equiv \frac{Y(p^*(\tilde{c}(x)))}{Y(p^*(c))} \quad \text{for } Y \in \{W, CS, \pi\}.$$

Financing improves welfare measure Y if and only if $R_Y(x) > 1 - x$, where $x \equiv m\lambda$. Consequently, the thresholds λ^* and λ^*_{CS} are determined by the intersection of $R_W(x)$ and $R_{CS}(x)$ with the line $1 - x$.

Note that R_W is a convex combination of consumer surplus and profit retention:

$$R_W(x) = \alpha R_{CS}(x) + (1-\alpha) R_\pi(x), \quad \text{where } \alpha \equiv \frac{CS(p^*(c))}{W(p^*(c))} \in (0, 1).$$

Therefore, to prove $\lambda^*_{CS} > \lambda^*$, it suffices to show that $R_{CS}(x) < R_\pi(x)$ for all $x > 0$, which implies $R_{CS}(x) < R_W(x)$.

At the benchmark price ($x = 0$), the Envelope Theorem implies $\pi'(p^*) = 0$ while $CS'(p^*) < 0$. Thus, consumer surplus initially decays faster than profit. To ensure this ranking persists for $x > 0$ (i.e., $p > p^*(c)$), we require the magnitude of the percentage decay of CS to exceed that of

π:

$$\left|\frac{\partial \ln CS(p)}{\partial p}\right| > \left|\frac{\partial \ln \pi(p)}{\partial p}\right| \iff \frac{D(p)}{CS(p)} > h(p) - \frac{1}{p-c}.$$

Using the definition $M(p) \equiv CS(p)/D(p)$, we have the identity $h(p) = \frac{1}{M(p)} + \frac{M'(p)}{M(p)}$. Substituting this into the inequality above yields:

$$\frac{1}{M(p)} > \left(\frac{1}{M(p)} + \frac{M'(p)}{M(p)}\right) - \frac{1}{p-c} \iff \frac{1}{p-c} > \frac{M'(p)}{M(p)}.$$

Multiplying by the markup $p - c$ establishes that the required decay ranking is equivalent to the assumed condition (24). Since (24) holds, $R_{CS}(x)$ lies strictly below $R_\pi(x)$ and thus strictly below $R_W(x)$ for all $x > 0$. Consequently, the intersection of $R_{CS}(x)$ with the breakeven line $1 - x$ must occur at a strictly higher level of risk than that of $R_W(x)$, implying $\lambda^*_{CS} > \lambda^*$. ■

B Appendix B: Suppliers' access to alternative funding sources

We now consider the robustness of the baseline model when suppliers have access to external credit sources at a rate i^s. A supplier (λ, c) can choose to hold a real balance z^s to ensure production during a liquidity shock.

Assume $\bar{c} > c_0 > \underline{c}$ and $\lambda_0 < 1$. A supplier (λ, c) with $z^s(c)$ has a retail market value of:

$$z^s + \left((1-\lambda) + \lambda \min\left\{\frac{z^s}{c}, 1\right\}\right) \frac{u-c}{2}.$$

Here, $\lambda \min\left\{\frac{z^s}{c}, 1\right\}$ shows that, with a liquidity shock, the supplier can use his own liquidity holding to produce and sell to $\min\{z^s/c, 1\}$ consumers. The supplier's liquidity holding problem is:

$$\max_{z^s} \left\{ \left[z^s + \left((1-\lambda) + \lambda \min\left\{\frac{z^s}{c}, 1\right\}\right) \frac{u-c}{2} \right] - (1+i^s)z^s \right\}.$$

Suppliers never hold $z^s > c$, as it is inefficient. The first-order condition shows that suppliers with (λ, c) satisfying $\frac{\lambda(u-c)}{2} > i^s c$ hold money. This simplifies to:

$$c < c^s(\lambda, i^s) \equiv \frac{\lambda}{\lambda + 2i^s} u. \tag{26}$$

Thus, suppliers with $c < c^s(\lambda, i^s)$ hold $z^s(c) = c$, while those with $c \geq c^s(\lambda, i^s)$ hold $z^s(c) = 0$.

Next, we consider the intermediary's problem. She can only offer finance contracts to suppliers who holds no numeraire The feasible set of suppliers is:

$$\tilde{\Omega}(i^s) = \{(\lambda, c) \in \Omega | c \geq c^s(\lambda, i^s)\},$$

which is nonempty. Her supplier selection problem is:

$$\max_{\{q(\cdot)\}_{(\lambda,c)\in\tilde{\Omega}(i^s)}} \int_{\tilde{\Omega}(i^s)} q(\lambda, c)\Delta\pi(\lambda, c) dG,$$

subject to the liquidity constraint:

$$\int_{\tilde{\Omega}(i^s)} q(\lambda, c)\theta_F(\lambda, c) dG + L \geq 0,$$

where i^s and L are given.

In earlier sections, we showed that finance contracts are profitable when $\lambda_0 < 1$ because region A in Figure 1 is nonempty (see Lemma 1 for details). But when suppliers can access the money market, finance contracts may not always be activated under the same conditions.

Proposition 5 *Suppose $\lambda_0 < 1$, $\underline{c} > 0$, $i < \frac{k\bar{\lambda}}{mu\bar{\lambda} - 2k}$, and suppliers face money market rate i^s. There exist thresholds $i < \underline{i}^s < \bar{i}^s \equiv \frac{(u-\underline{c})\bar{\lambda}}{2\underline{c}}$ such that:*

- *If $i^s \leq \underline{i}^s$, suppliers with $c \leq c^s(\lambda, i^s)$ hold money for liquidity, and supplier finance stays inactive.*
- *If $i^s \geq \bar{i}^s$, no supplier holds money, and supplier finance is activated for some suppliers.*

- *If $i^s \in (\underline{i}^s, \bar{i}^s)$, suppliers with $c \leq c^s(\lambda, i^s)$ have money, while supplier finance activates for other suppliers.*

Proof. Let $\Pi(i^s, i) \equiv \int_{\tilde{\Omega}(i^s)} q(\lambda, c)\Delta\pi(\lambda, c)dG$ be the maximized profits of the intermediary from activating the finance service taking nominal interest rate $i < i_1$ as given. Let $c_{\Delta\pi}(\lambda) = u - \frac{2k}{m\lambda}$ denote the curve of (λ, c) such that $\Delta\pi(\lambda, c) = 0$. It can be shown that $c^s(\lambda, i^s)$ and $c_{\Delta\pi}(\lambda)$ cross each other at most once.

If $c^s(1, i^s) > c_{\Delta\pi}(1)$, or equivalently, $i^s < \frac{k}{mu-2k}$, then $c^s(\lambda, i^s) > c_{\Delta\pi}(\lambda)$ for all $\lambda \in [0, 1]$, meaning that all suppliers with positive profits $\Delta\pi(\lambda, c)$ are excluded from $\tilde{\Omega}(i^s)$. Thus, we must have $\Pi(i^s, i) = 0$. On the other hand, if $i^s \geq \bar{i}^s \equiv \frac{u-\underline{c}}{2\underline{c}}$, then $\tilde{\Omega}(i^s) = \Omega$, resulting in $\Pi(i^s, i) > 0$. Note that $\lambda_0 < 1$ implies $c_{\Delta\pi}(1) > \underline{c}$, which is equivalent to $\bar{i}^s > \frac{k}{mu-2k}$.

Finally, $\Pi(\cdot)$ is weakly increasing in i^s, because as i^s increases, the set of feasible suppliers $\tilde{\Omega}(i^s)$ becomes larger. Therefore, $\underline{i}^s \in [\frac{k}{mu-2k}, \bar{i}^s)$ must exist. Combined with the suppliers' money-holding decision rule (see condition (26) in the main text), this proves the claims in the proposition. ■

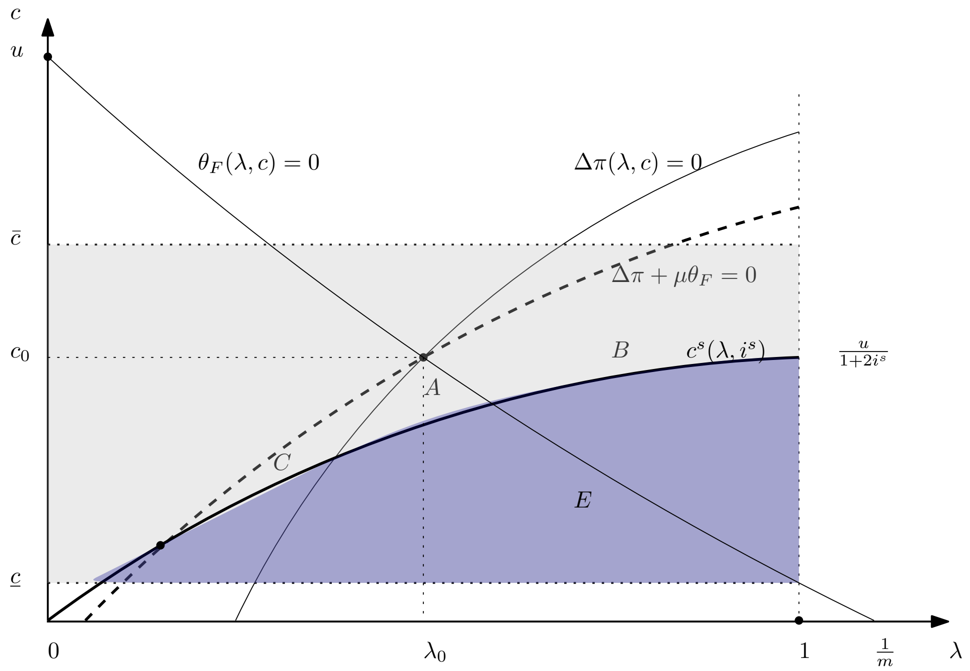

Figure 6: Suppliers' money holdings and intermediary finance coexist

Figure 6 shows the third case. Suppliers with costs below $c^s(\lambda, i^s)$ (region E, dark blue) hold money and skip the intermediary's finance. Those with costs above $c^s(\lambda, i^s)$ do not hold money. Among them, suppliers in regions A, B, and C join the finance contract. This can happen even if $i^s < i$, meaning suppliers borrow cheaper than the intermediary. Supplier finance still works because the intermediary uses liquidity more efficiently, leveraging the law of large numbers. This requires $c^s(\lambda, i^s)$ to cross the selection curve $\Delta\pi + \mu\theta_F = 0$ below c_0, ensuring region A exists, as shown in Figure 6.