Machine Learning from Data Assignment 12

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Neural Networks and Backpropagation.

(a) *Gradient*

Identity output node:

$$G_1 = \begin{pmatrix} -.0322 & -.0322 \\ -.0322 & -.0322 \\ -.0322 & -.0322 \end{pmatrix} \qquad G_2 = \begin{pmatrix} -.2162 \\ -.1373 \\ -.1373 \end{pmatrix}$$

And for the tanh() output node:

$$G_1 = \begin{pmatrix} -.0267 & -.0267 \\ -.0267 & -.0267 \\ -.0267 & -.0267 \end{pmatrix} \qquad G_2 = \begin{pmatrix} -.1791 \\ -.1137 \\ -.1137 \end{pmatrix}$$

(b) *Perturbation*

Identity output node:

$$G_1 = \begin{pmatrix} -.03224 & -.03224 \\ -.03224 & -.03224 \\ -.03224 & -.03224 \end{pmatrix} \qquad G_2 = \begin{pmatrix} -.2162 \\ -.1373 \\ -.1373 \end{pmatrix}$$

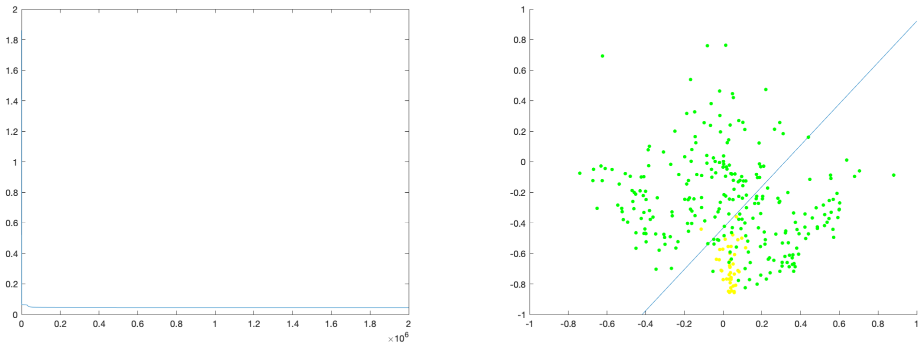
And for the tanh() output node:

$$G_1 = \begin{pmatrix} -.0267 & -.0267 \\ -.0267 & -.0267 \\ -.0267 & -.0267 \end{pmatrix} \qquad G_2 = \begin{pmatrix} -.1790 \\ -.11372 \\ -.11372 \end{pmatrix}$$

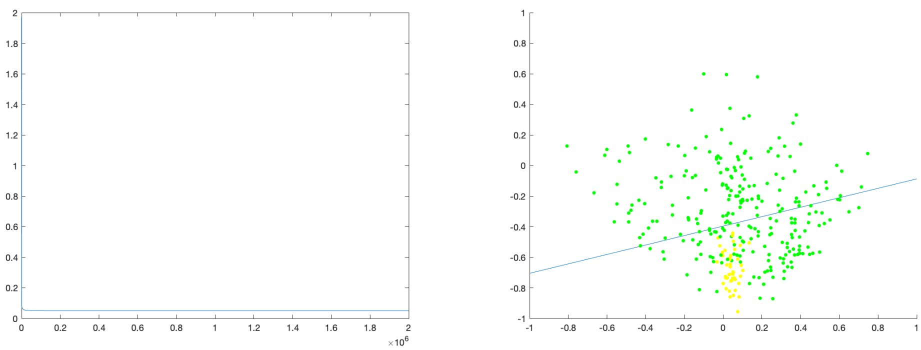
These results are very very close to the gradient results (although not clearly shone, they typically differ in the 1e-5 place of the decimal).

Neural Networks for Digits.

(a)



(b)



(c)

Support Vector Machines.

(a)

For the two data points given, since there are only two points, the optimal hyperplane is that which maximizes the distance from both points. If it is *not* the perpendicular bisector, then the distance to the boundary from one point will be small compared to the distance from the other, and can be increased while maintaining this state. Thus the only solution is the hyperplane which is equidistant for both points, i.e. the perpendicular bisector.

The equation of this hyperplane is $x_1 = 0$.

(b)

i.

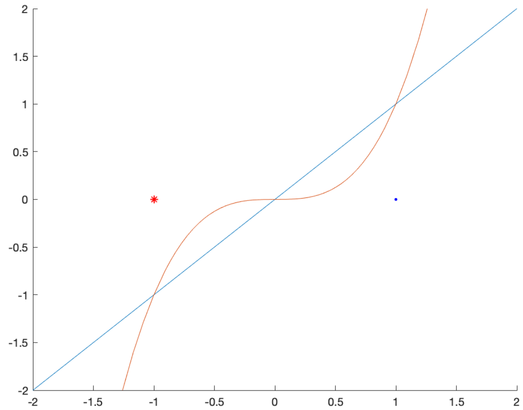
$$\mathbf{z}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \mathbf{z}_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

ii.

The optimal hyperplane in this case is the same, since the points are the same, but this is in Z-space:

$$z_1 = 0$$

(c)



(d)

$$
\begin{aligned}
K(\mathbf{x}, \mathbf{y}) &= \mathbf{z}(\mathbf{x}) \cdot \mathbf{z}(\mathbf{y}) \\
&= \begin{pmatrix} x_1^3 - x_2 \\ x_1 x_2 \end{pmatrix} \cdot \begin{pmatrix} y_1^3 - y_2 \\ y_1 y_2 \end{pmatrix} \\
&= (x_1^3 - x_2)(y_1^3 - y_2) + x_1 x_2 y_1 y_2
\end{aligned}
$$

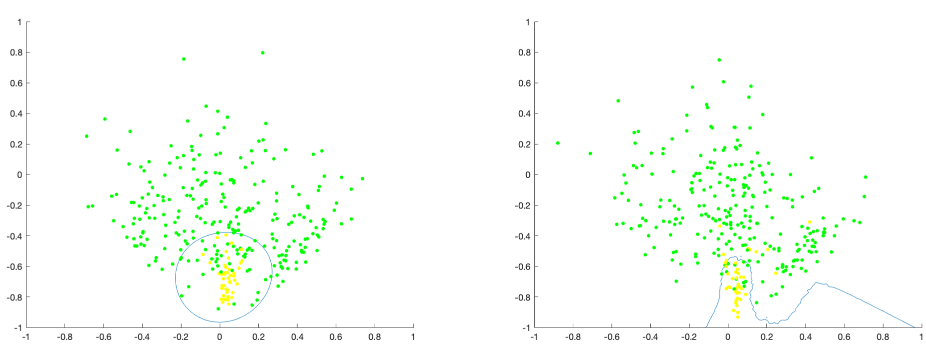
(e)

$$x_1^3 - x_2 = 0$$

SVM with Digits.

(a)

We have C = 1 on the left and C = 5000 on the right:

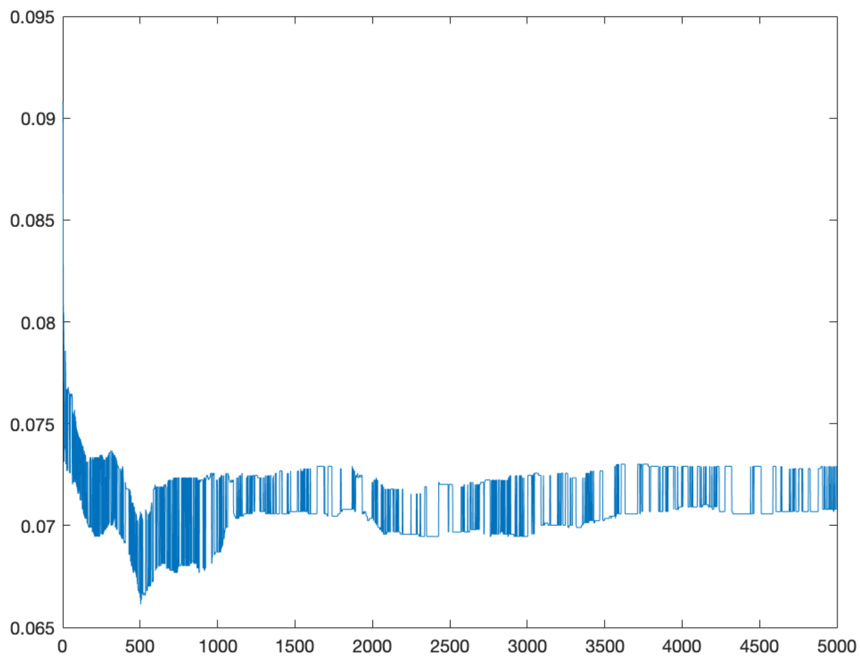


Ein is 14.33% for C = 1, and 7% for C = 5000.

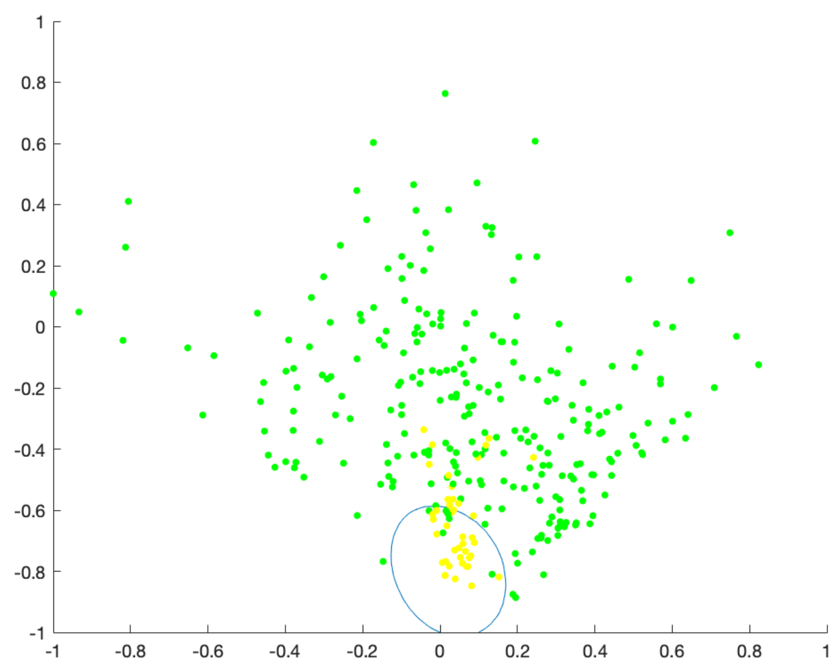
(b)

The C = 1 boundary has very little "regularization" in this case, so it is underfit. On the other hand, C = 5000 results in overfitting, which is also undesirable and increases complexity of the boundary. However, the Ein is much smaller, since it aids in fitting the data available better.

(c)



This best error value for $1 \leq C \leq 5000$ is at $C = 493$. The resulting decision boundary is



It has a test error of 6.4%.

Compare Methods.

Overall, none of the methods used ended up being that different from each other by more than a few tenths of a percentage point. The approximate test error of the 3-Nearest Neighbor boundary was the smallest of all five, at 6.1%. Next was RBF at 6.2, SVM at 6.4, Neural Network at 6.5, and the 8th order Legendre transform perceptron with 6.7% error.

One key takeaway here is no matter how powerful the learning algorithm is, it will be handicapped by somewhat poorly selected features that do not provide adequate separability in the data. This can be fixed with better defined features, or perhaps preprocessing the data so the features are better separated. There are also preprocessing methods to select new features automatically, and while this takes away possibly valuable information about what is going into those features, it could help build a better classifier.

The neural network took, by far, the longest to run, at about 30 minutes for this small training set, and given the lack of benefit in error, it makes little sense to use it versus the more efficient RBF method. In the long run, however, when more features and more digits need to be classified (other than just 1 and not-1), the neural network could prove more useful.

What's most important to recognize is that learning is a balancing act. Adding complexity to the algorithms and to the data, while tempting, may actually leave you better off when it comes to actually using the trained model. Overfitting is a significant problem. Likewise, underfitting can be detrimental, so it's important to get the technique and the data in the right shape to most effectively learn.