Machine Learning from Data Assignment 11

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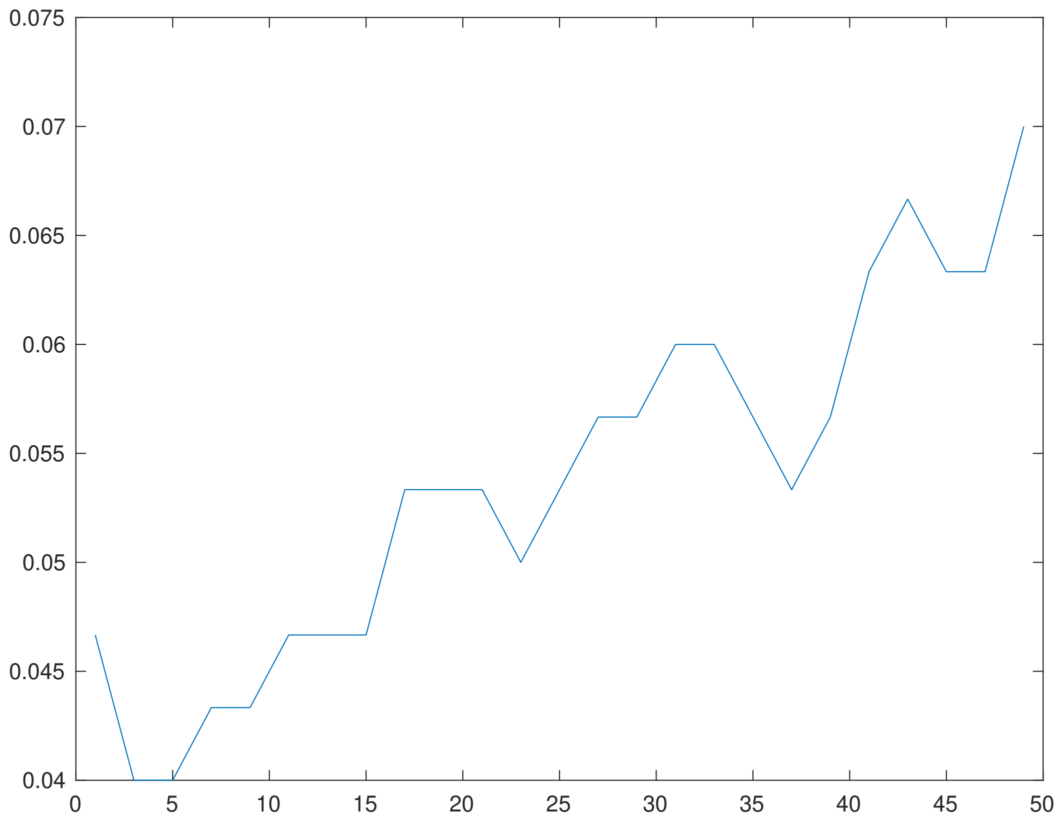
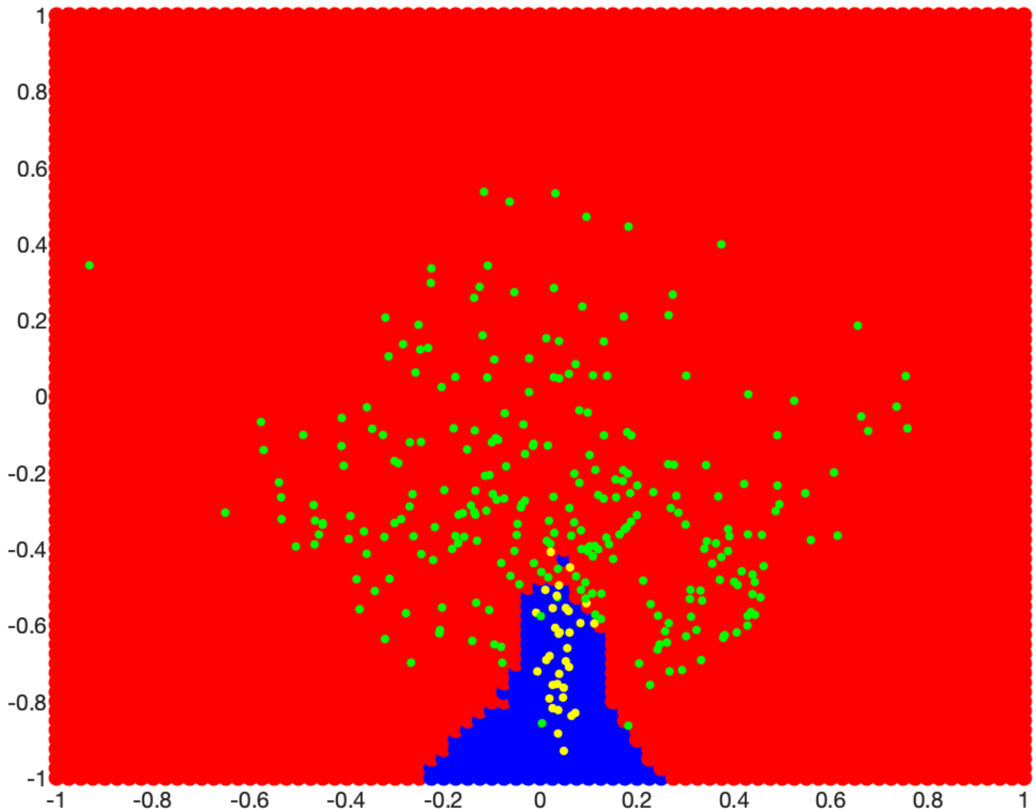
k-NN Rule.

(a) *Use cross validation with the training set to select the optimal k. Plot E_{CV} vs k.*



From this we select $k = 3$ as the optimal k.

(b) *Plot the decision boundary for that k. Give the in sample and cross validation errors.*



The cross validation error is $E_{cv} = .04 = 4\%$.

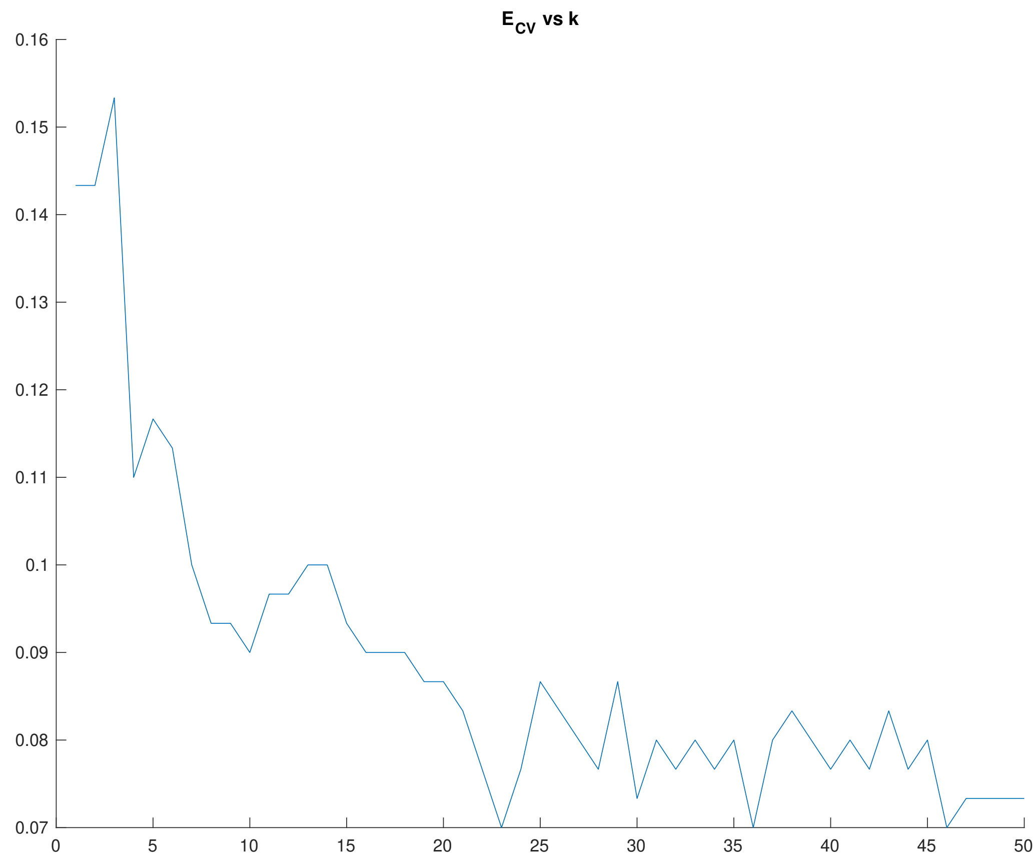
The in-sample error is $E_{in} = .06 = 6\%$.

(c) *Give the test error.*

The test error here is $E_{test} = .0609 = 6.09\%$.

RBF-Network.

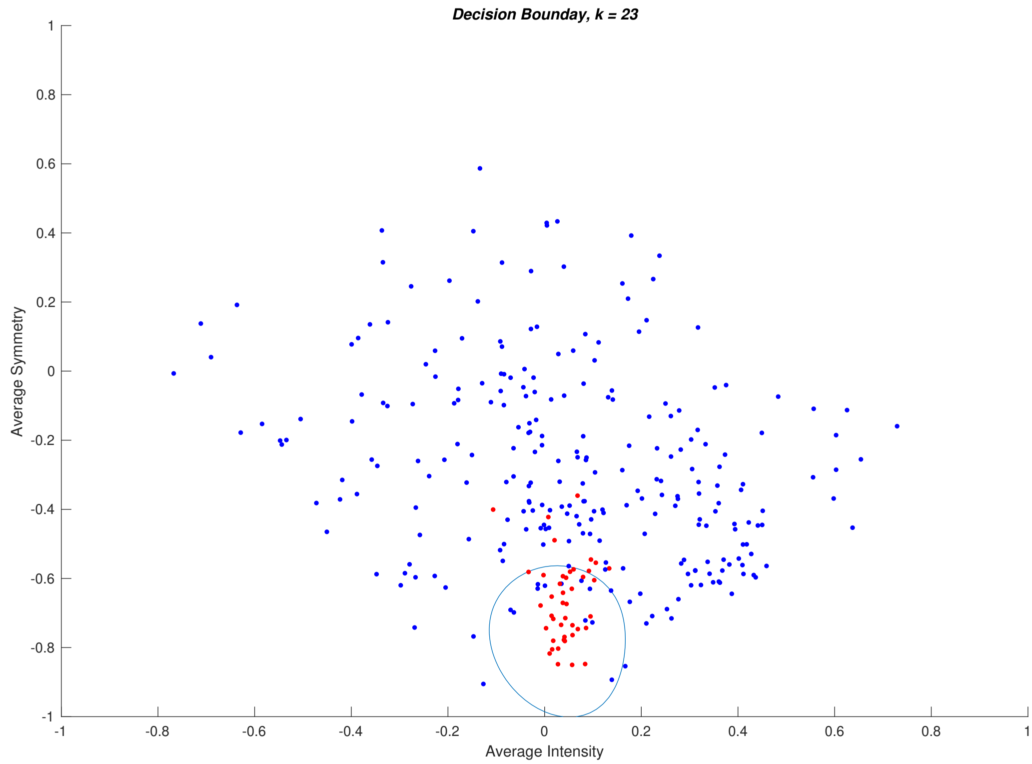
(a) *For RBF with Gaussian kernel, set scale* $r = \frac{2}{\sqrt{k}}$ *where* k *is the number of centers. Use cross validation with the training set to select optimal number of centers* k*. Plot* E_{CV} *vs* k*.*



The optimal value of k was 23; i.e. there are 23 centers used for the decision boundary.

(b) *Plot the decision boundary for that k. Give the in sample error and cross validation error.*

The following boundary was obtained with the pseudo inverse linear regression and a hint of regularization.



The cross validation error for this case was .07, or 7%.

The in-sample error E_{in} is .063, or 6.3%.

(c) *Give the test error.*

The test error is $E_{test} \approx .062 = 6.2\%$

Linear vs k-NN vs RBF-Network.

Compare the final test errors from the three attempts done to solve this problem. Make some intelligent comments.

Of the three, the 8^{th} order Legendre transform has the highest test error, and thus the highest E_{out}, at 6.7%. Next smallest, we have the RBF network, with an approximated E_{out} of 6.2%, and the 3-NN boundary, with E_{out} of approximately 6.1%.

These last two are very similar in E_{out}, which is not terribly surprising considering the similarity of the methods. The result of the 3-NN rule is in some ways superior as it's about as good as the RBF network result, but takes significantly less time to compute, thanks to the simplicity of the calculation. RBF involves much more computation, so it of course takes longer. However, only pseudo inverse regression with regularization was used for this calculation; the result could likely be improved if more work was done, for example by running the pocket algorithm. While I haven't computed it here, the improvement from doing so may be worth it.

The result of the 8^{th} order transform is the worst of the bunch, and goes to show that adding complexity and degrees of freedom to the transform does not necessarily increase the quality of the result. The risk of overfitting is also increased, though regularization helps. However, the computation is quick, which has benefits.