Machine Learning from Data Assignment 7

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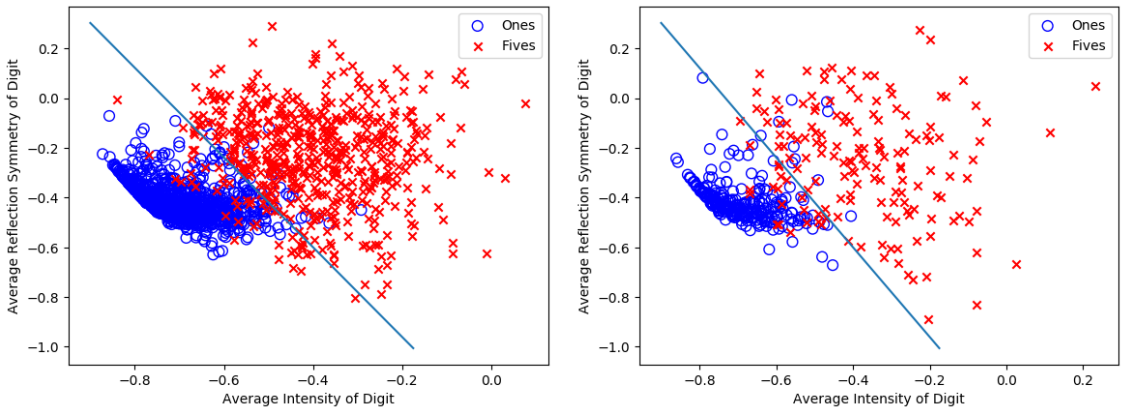
October 22, 2018

Classifying Handwritten Digits

(a) *Give separate plots of the training and test data, together with the separators.*

On the **left** is the training data, and on the **right** is the test data:



(b) *Compute E_{in} on your training data and E_{test}, the test error on the test data.*

$E_{in} = 0.05061 = 5.061\%$

$E_{test} = 0.07311 = 7.311\%$

(c) *Obtain a bound on the true out-of-sample error. You should get two bounds, one based on E_{in} and one based on E_{test}. Use a tolerance $\delta = 0.05$. Which is the better bound?*

Using E_{in}, we have

$$\begin{aligned} E_{out} &\leq E_{in} + \sqrt{\frac{8}{N} \ln \frac{4(2N)^{d_{VC}} + 4}{\delta}} \\ &\leq 0.05061 + \sqrt{\frac{8}{1561} \ln \frac{4(2 \cdot 1561)^3 + 4}{0.05}} \\ &\leq .05061 + .38232 = .43293 \\ &\leq 43.3\% \end{aligned}$$

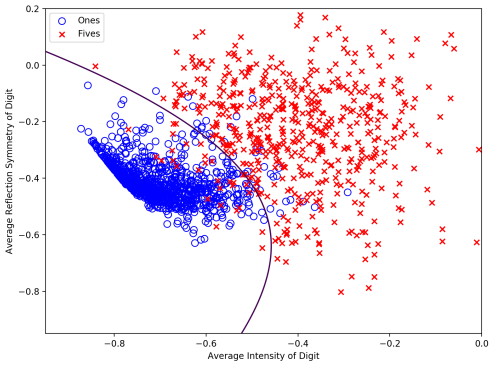
and using E_{test}, we have

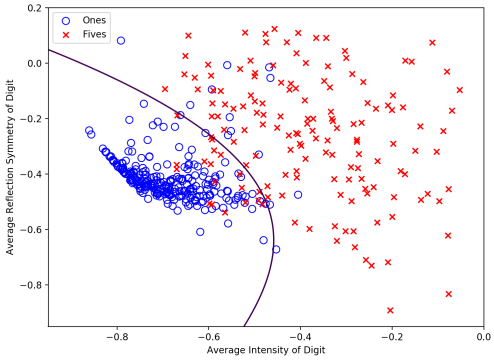
$$\begin{aligned} E_{out} &\leq E_{test} + \sqrt{\frac{1}{2N} \ln \frac{2M}{\delta}} \\ &\leq .07311 + \sqrt{\frac{1}{2 \cdot 424} \ln \frac{2}{.05}} = .07311 + 0.06596 \\ &\leq 13.91\% \end{aligned}$$

Obviously, the bound obtained from E_{test} is the better of the two.

(d) *Now repeat using a 3rd order polynomial transform.*

On the **left** is the training data, and on the **right** is the test data:





$$E_{in} = 0.04228 = 4.228\% \qquad E_{test} = 0.07547 = 7.547\%$$

The error bars are as follows.

$$\begin{aligned} E_{out} &\leq E_{in} + \sqrt{\frac{8}{N} \ln \frac{4(2N)^{d_{VC}} + 4}{\delta}} \\ &\leq 0.04228 + \sqrt{\frac{8}{1561} \ln \frac{4(2 \cdot 1561)^{10} + 4}{0.05}} \\ &\leq .04228 + .65941 = .70169 \\ &\leq 70.17\% \end{aligned}$$

and using E_{test}, we have

$$\begin{aligned} E_{out} &\leq E_{test} + \sqrt{\frac{1}{2N} \ln \frac{2M}{\delta}} \\ &\leq .07547 + \sqrt{\frac{1}{2 \cdot 424} \ln \frac{2}{.05}} = .07547 + 0.06596 \\ &\leq 14.14\% \end{aligned}$$

(e) *As your final deliverable to a customer, would you use the linear model with or without the 3rd order polynomial transform? Explain.*

I'd deliver the linear model without a 3rd order transform to the customer. It has a very slightly lower bound for E_{out}, and is simpler, so it's less at risk for overfitting, unlike the 3rd order transform, which certainly is at risk. For the unsatisfied customer, though, the 3rd order transform result could be delivered if they are incredulous about the simplicity of the linear result, then quickly snatched away again, like a cruel aunt teasing her toddler nephew with a piece of candy.

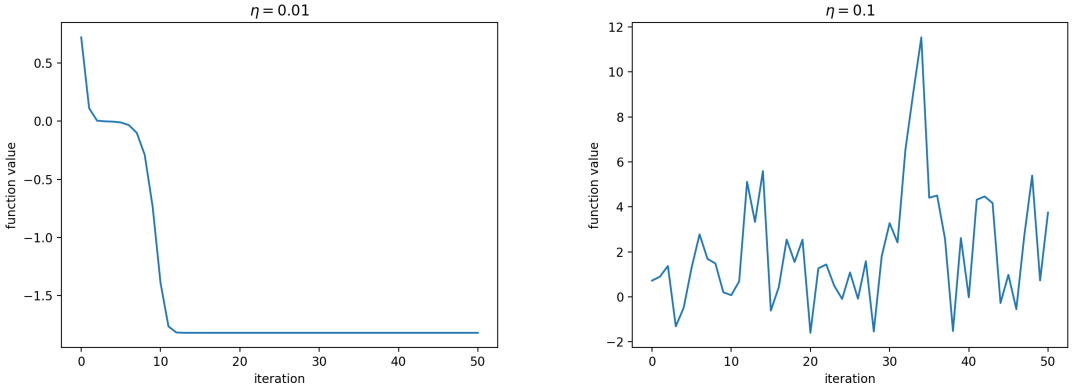
Gradient Descent on "Simple" Function

Consider

$$f(x, y) = x^2 + 2y^2 + 2\sin(2\pi x)\sin(2\pi y)$$

(a) *Implement gradient descent to minimize using the given starting values and learning rate. Number of iterations = 50. Give a plot of how the function value drops with the number of iterations performed. Repeat for* $\eta = 0.1$

The two plots are



Clearly, the larger learning rate caused the descent algorithm to overshoot, and it couldn't come back from it, perhaps entering a region with another local minimum or something like that.

(b) *Do the same for the given pairs of points and the better* η.

(x_0, y_0)	(x_f, y_f)	$f(x_f, y_f)$
$(0.1, 0.1)$	$(0.243, -0.238)$	-1.82
$(1.0, 1.0)$	$(1.218, 0.713)$	0.593
$(-0.5, -0.5)$	$(-0.731, -0.238)$	-1.33
$(-1.0, -1.0)$	$(-1.218, -0.713)$	0.593

Problem 3.16

Concerned with using logistic regression to get hard classification using risk matrix. The final hypothesis is

$$g(\mathbf{x}) = \mathbb{P}[y = +1 \mid \mathbf{x}],$$

which is the estimate of the probability that $y = +1$. *The cost matrix is given by*

		True	*Classification*
		+1 (correct person)	*-1 (intruder)*
you	*+1*	*0*	c_a
say	*-1*	c_r	*0*

For a fingerprint $\mathbf{x}$, $g(\mathbf{x})$ *needs to be computed and used to decide acceptance or rejection. Accept if* $g(\mathbf{x}) \geq \kappa$, *where* κ *is the threshold.*

(a) *Define cost(accept) as your expected cost if you accept the person. Also define cost(reject). Show*

$$cost(accept) = (1 - g(\mathbf{x}))c_a$$
$$cost(reject) = g(\mathbf{x})c_r$$

The cost of accepting 1 person if they ought be accepted is of course 0, and c_a if it's an intruder. Since $g(\mathbf{x})$ is defined as the chance of y being $+1$ (acceptance) given the fingerprint. The cost(accept), or expectation of cost, is then given by the probability of each event given *you* accept multiplied by the cost:

$$cost(accept) = g(\mathbf{x}) \cdot 0 + (1 - g(\mathbf{x}))c_a = (1 - g(\mathbf{x}))c_a.$$

The expected cost of rejection is calculated the same way:

$$cost(reject) = g(\mathbf{x})c_r + (1 - g(\mathbf{x})) \cdot 0 = g(\mathbf{x})c_r.$$

(b) *Use part (a) to derive a condition on $g(\mathbf{x})$ for accepting the person and hence show that*

$$\kappa = \frac{c_a}{c_a + c_r}.$$

We want cost(accept) - cost(reject) ≤ 0. So we can write

$$\begin{aligned}
(1 - g(\mathbf{x}))c_a - g(\mathbf{x})c_r &\leq 0 \\
c_a - (c_a + c_r)g(\mathbf{x}) &\leq 0 \\
c_a &\leq (c_a + c_r)g(\mathbf{x}) \\
\frac{c_a}{c_a + c_r} &\leq g(\mathbf{x})
\end{aligned}$$

so we have a threshold for $g(\mathbf{x})$, given by $\frac{c_a}{c_a+c_r}$, meaning that for κ we have

$$\kappa = \frac{c_a}{c_a + c_r}$$

(c) *Use cost matrices for the supermarket and CIA applications in example 1.1 to compute the threshold κ for each of these two cases. Give some intuition for the threshold you get.*

i. Supermarket.
$\kappa = \frac{1}{1+10} = .090909...$

ii. CIA
$\kappa = \frac{1000}{1000+1} = 0.999001$

These are not very surprising values. In the CIA, you'd essentially never want to have a false positive—admitting an intruder could be detrimental, so the threshold for acceptance should be very high. A false rejection is merely an inconvenience in this case.

Conversely, the threshold for acceptance for the supermarket is very low. It's not worth being so secure to the supermarket, because it's not worth the cost of a false rejection, which could actually end up costing the business of a customer who was actually getting rewards previously. A false accept just gives a small one time discount to someone who shouldn't have gotten it—and perhaps they'd come back because of this anyway!