Machine Learning from Data Assignment 2

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Exercise 1.8

If $\mu = 0.9$, what is the probability that a sample of 10 marbles will have $\nu \leq 0.1$?

There can be either 0 or 1 red marbles in the 10 marble sample. So we can calculate the probability of having 0 or any 1 of the marbles be red.

All not red: $0.1^{10} = 10^{-10}$

One red: $10 \cdot (0.1)^9 \cdot .9 = 9 \times 10^{-9}$

So the overall probability of this event is

$$9.1 \times 10^{-9}$$

Exercise 1.9

$$\mathbb{P}[|\nu - \mu| > \epsilon] \leq 2e^{-2\epsilon^2 N}$$
$$\mathbb{P}[|0.1 - 0.9| > \epsilon] \leq 2e^{-20\epsilon^2}$$

Choosing ϵ very close to the actual difference, 0.8, we obtain

$$\mathbb{P}[|0.1 - 0.9| > 0.79999] \leq 5.52 \times 10^{-6}$$

This is much larger than the probability obtained in the previous exercise, but it is an upper bound, whereas in the previous exercise we computed the actual probability.

Exercise 1.10

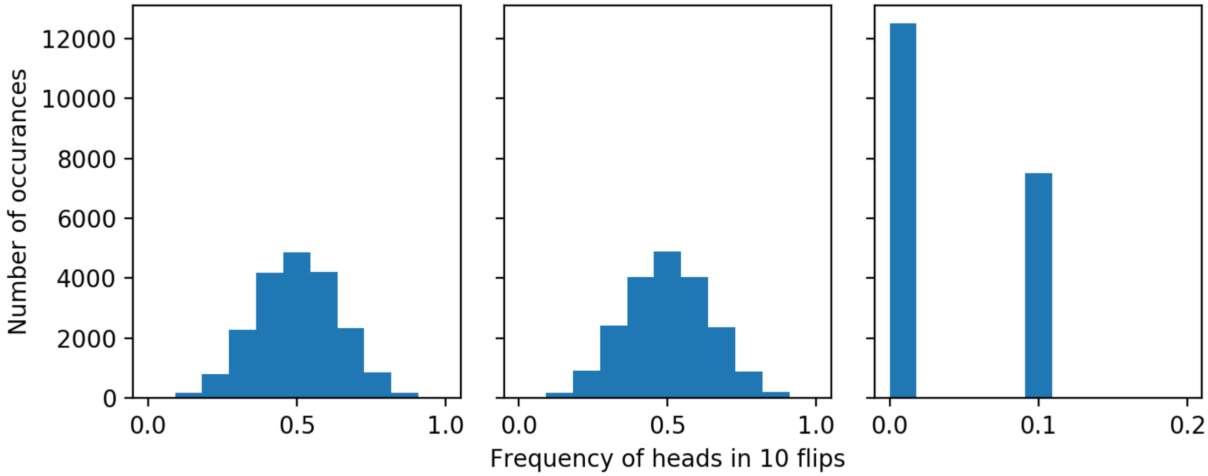
Experiment: Run a computer simualtion for flipping 1000 fair coins. Flip each coin independently 10 times. Focus on 3 coins as follows: c_1 is the first coin flipped; c_{rand} is a coin you choos at random; c_{min} is the coin that had the minimum frequency of heads (picking the earlier one in case of a tie). Let $\nu_1, \nu_{rand}, \nu_{min}$ be the fraction of heads you obtain for the respective three coins.

(a) *What is μ for the three coins selected?*

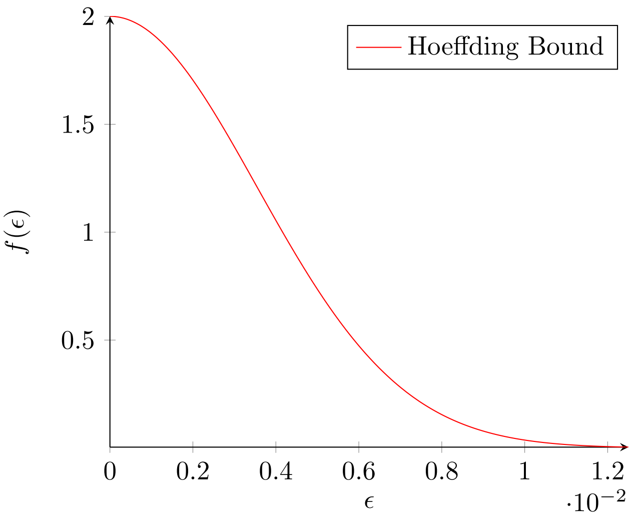
Since the coins are all fair, $\mu = 0.5$

(b) *Repeat this entire experiment a large number of times to get several instances ofthe ν's and plot the histograms of the distributions. Notice that which coins end up being c_{rand} and cmin may differ from one run to another.*

For this experiment, N = 20,000.



(c) *Using (b), plot estimates for $\mathbb{P}[|\nu - \mu| > \epsilon]$ as a function of ϵ, together with the Hoeffding bound.*



(d) *Which coins obey the Hoeffding bound, and which do not?*

c_1 and c_{rand} both obey the Hoeffding bound, while c_{min} does not. This is because c_{rand} is chosen at random each time and is independent of other choices, so as N increases, the mean or expected value of the sample tends to the actual mean. The situation is similar for c_1; although choosing c_1 each time is a determination made beforehand, the actual outcome is random, and this randomness and independence also applies to the distribution for c_1, so we see the same thing. c_{min} does not obey the Hoeffding bound because, due to the random nature of coin-flipping, in a batch of 1000 examples of 10 flips, the minimum frequency of, which just has to occur one time, is likely to be much lower than the mean.

(e) *Relate part (d) to the multiple bins in Fig. 1.10.*

The distributions for c_1 and c_{rand} are obtained in entirely different ways, but give rise to almost the exact same distribution. In both cases, the situation is analagou to reaching randomly into a bin to obtain a sample. The c_{min} distribution is obtained in a different way, and provides a completely different sample set which is not distributed like the other two because the sample taken from the "bin" is not obtained randomly, but deliberately, taking the minimum.

Exercise 1.11

(a) *Can S produce a hypothesis that is guaranteed to perform better than random on any point outside D?*

No, no such guarantee can be made. It can only perform better than random with some probability, not with certainty.

(b) *Assume for the rest of the exercise that all the examples in D have $y_n = +1$. Is it possible that the hypothesis which C produces turns out to be better than the hypothesis that S produces?*

Yes, it is possible. In this case, S would produce h_1 as it matches all of the data. However, it's possible that outside D, the rest of $\mathbb{R}$ maps to -1. This would mean C is accurate for the entirety of $\mathbb{R}$, save for the in D, which is literally infinitely better than S.

(c) *If $p = 0.9$, what is the probability that S will produce a better hypothesis than C?*

We need

$$\mathbb{P}[\mathbb{P}[g_S \approx f] > \mathbb{P}[g_C \approx f]]$$

where g_S is the resulting hypothesis from S and g_C the result from C.

Since $\mathbb{P}[f(x) = +1] = 0.9$, we can say $\mathbb{P}[g_s \approx f] = 0.9$. The analagous probability for g_C is 0.5, for both the +1 and -1 cases. We can now calculate the probability

$$\mathbb{P}[g_C \approx f] = 0.5(0.9) + 0.5(0.1) = 0.5$$

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We know that $\mathbb{P}[g_S \approx f] = 0.9$, and $0.9 > 0.5$, so

$$\mathbb{P}[\mathbb{P}[g_S \approx f] > \mathbb{P}[g_C \approx f]] = 1$$

(d) *Is there any value for p for which it is more likely than no that C will produce a better hypothesis than S?*

Yes—for $p < 0.5$ this will be the case.

Exercise 1.12

The answer is (c). If learning succeeds, and a hypothesis g is produced, then we can be reasonably sure, to a high probability, that $g \approx f$. But there is the chance the learning will fail, in which case the only thing to do is declare the failure.