

```
random access
dynamically allocated
- increases size exponentially, so
  push_back become  $O(1)$ 
push_back()
pop_back()
front() and back() retrn
  values of them
iterators random access
erase(itr)  $O(n)$ 
- returns iterator pointing to new
  loc. of element following
insert(itr, val)  $O(n)$ 
- returns itr pointing to first of
  newly inserted elements
begin() - iterator @ top
end() - iterator @ end
rbegin() - reverse itr
rend() - reverse itr
```

```
doubly linked
begin, end, rbegin, rend all
    work
front() and back() return
    values of them
iterators bidirectional
erase(itr) O(1)
- returns itr pointing to element
  following last element erased
insert(itr, val) O(1)
- returns itr pointing to first of
  new inserted elements
push_back(), push_front()
pop_back(), pop_front()
has all the same
    iterator functions
```

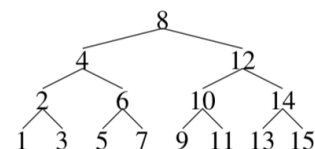
```

Binary Search trees made of pairs of
const key_val and variable val
std::map<key_type, val_type> map;
unique keys.
begin() returns first element in
the sorted order O(log n)
end() returns past-the-end. can
decrement. O(log n)
empty() returns true if size = 0
find(key) finds the key sought in
the map. returns iterator to
loc or end iter O(log n)
insert(std::make_pair(things))
if already in, returns pair w/
iter to existing pair & false
if not in, returns iter to new
pair in map & true
O(log n)
map[key] = val O(log n)
if key in map, changes val
if key not in map, inserts it
with that new value

```

- Just like maps but with only key
- same  $O(\log n)$  efficiency
- classic trees

Sets, but tree like. yeah trees.  
With sets, i.e. **binary search trees**, start with root and have scheme for adding nodes.  
to know:  
take balanced tree



```
BREADTH FIRST goes
8 4 12 2 6 10 14 1 3 5 7 9 11 13 15
IN ORDER TRAVERSAL goes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
POST-ORDER TRAVERSAL goes
1 3 2 5 7 6 4 9 11 10 13 15 14 12 8
PRE-ORDER TRAVERSAL goes
8 4 2 1 3 6 5 7 12 10 9 11 14 13 15
***all "order" trvals are depth
```

```

PRE ORDER
F B A D C E G I H
IN ORDER
A B C D E F G H I
POST ORDER
A C E D B H I G F
BREADTH FIRST
F B G A D I C E H

```

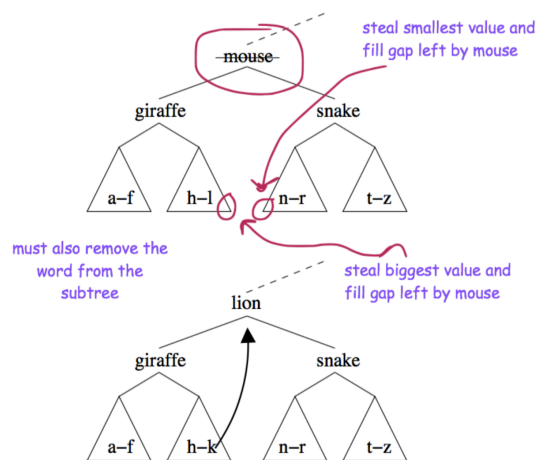
```

binary search tree with following:
1. each node either red or black
2. NULL child pointers are black
3. Both children of every red pointer
   are black (parent of red also
   black)
4. all paths from particular node to
   NULL child pointer contain same
   number of black nodes
erase:

```

```

graph TD
    16 --> 5
    16 --> 20
    5 --> 1
    5 --> 10
    1 --> 2
    2 --> 1
    2 --> 4
    10 --> 1
    10 --> 1
    style 10 stroke:#f00,stroke-width:2px
    style 1 stroke:#f00,stroke-width:2px
    style 1 stroke:#f00,stroke-width:2px
  
```



```

generally,
    ++ => in order successor
    -- => in order predecessor
i.e., binary tree ++:
tree_iterator<T> & operator++() {
    if (ptr_>right != NULL) {
        // find the leftmost child of the
        right node
        ptr_ = ptr_>right;
        while (ptr_>left != NULL) {
            ptr_ = ptr_>left;
        }
    } else {
        // go upwards along right
        branches... stop after the first
        left
        while (ptr_>parent != NULL &&
ptr_>parent->right == ptr_) {
            ptr_ = ptr_>parent;
        }
        ptr_ = ptr_>parent;
    }
    return *this;
}

binary tree - - first part for
decrementing from the end
tree_iterator<T> & operator--() {
    if (ptr_ == NULL) {
        ptr_ = set->root_;
        while (ptr_>right != NULL) {
            ptr_ = ptr_>right;
        }
        return *this;
    }
    if (ptr_>left != NULL) {
        ptr_ = ptr_>left;
        while (ptr_>right != NULL)
{ ptr_ = ptr_>right; }
    } else {
        while (ptr_>parent != NULL &&
ptr_>parent->left == ptr_) { ptr_ =
ptr_>parent; }
        ptr_ = ptr_>parent;
    }
    return *this;
}

```

```
use to define a type to make it
easier to use
Syntax:
typedef [clunky def] {new def};
e.g.
typedef tree_iterator<T> iter;
Place these in the public section of
the class you're making for ease of
use
```

```

TreeNode<T>*
copy_tree(TreeNode<T>* old_root,
TreeNode<T>* the_parent) {
    if (old_root == NULL)
        return NULL;
    TreeNode<T> *answer = new
TreeNode<T>();
    answer->value = old_root->value;
    answer->left =
copy_tree(old_root->left, answer);
    answer->right =
copy_tree(old_root->right, answer);
    answer->parent = the_parent;
    return answer;
}

void destroy_tree(TreeNode<T>* p) {
    if (!p) return;
    destroy_tree(p->right);
    destroy_tree(p->left);
    delete p;
}

iterator find(const T& key_value,
TreeNode<T>* p) {
    if (!p) return end();
    if (p->value > key_value)
        return find(key_value, p-
>left);
    else if (p->value < key_value)
        return find(key_value, p-
>right);
    else
        return iterator(p, this);
}

std::pair<iterator, bool>
insert(const T& key_value,
TreeNode<T>* p, TreeNode<T>*
the_parent) {
    if (!p) {
        p = new TreeNode<T>(key_value);
        p->parent = the_parent;
        this->size++;
        return
std::pair<iterator, bool>(iterator(p, t
his), true);
    }
    else if (key_value < p->value)
        return insert(key_value, p-
>left, p);
    else if (key_value > p->value)
        return insert(key_value, p-
>right, p);
    else
        return
std::pair<iterator, bool>(iterator(p, t
his), false);
}

int erase(T const& key_value,
TreeNode<T>* &p) {
    if (!p) return 0;

    // look left & right
    if (p->value < key_value)
        return erase(key_value, p-
>right);
    else if (p->value > key_value)
        return erase(key_value, p-
>left);

    // Found the node. Let's delete
it
    assert (p->value == key_value);
    if (!p->left && !p->right) { //
leaf
        delete p;
        p=NULL;
        this->size--;

```

```

    } else if (!p->left) { // no left
child
        TreeNode<T>* q = p;
        p=p->right;
        assert (p->parent == q);
        p->parent = q->parent;
        delete q;
        this->size--;
    } else if (!p->right) { // no
right child
        TreeNode<T>* q = p;
        p=p->left;
        assert (p->parent == q);
        p->parent = q->parent;
        delete q;
        this->size--;
    } else { // Find rightmost node
in left subtree
        TreeNode<T>* q = p->left;
        while (q->right) q = q->right;
        p->value = q->value;
        // recursively remove the value
from the left subtree
        int check = erase(q->value, p-
>left);
        assert (check == 1);
    }
    return 1;
}

void print_in_order(std::ostream&
ostr, const TreeNode<T>* p) const {
    if (p) {
        print_in_order(ostr, p->left);
        ostr << p->value << "\n";
        print_in_order(ostr, p->right);
    }
}

void
print_as_sideways_tree(std::ostream&
ostr, const TreeNode<T>* p, int
depth) const {
    if (p) {
        print_as_sideways_tree(ostr, p-
>right, depth+1);
        for (int i=0; i<depth; ++i)
            ostr << " ";
        ostr << p->value << "\n";
        print_as_sideways_tree(ostr, p-
>left, depth+1);
    }
}

from lec 19
// height calculation (longest path)
// avg runtime: O(n) -> have to go
through all nodes
// avg memory usage: O(log n) ->
stack grows & shrinks w/ recursive
function
template <class T>
int height( TreeNode<T>* root ) {
    if( root == NULL ) {
        return 0;
    }
    return 1 + std::max( height(root-
>left), height(root->right) );
}

// finding shortest path to a node
template <class T>
int shortestpath( TreeNode<T>* root )
{
    if( root == NULL ) {
        return 0;
    }
    return 1 +
std::min( shortestpath(root->left),
shortestpath(root->right) );
}
template <class T>

```

```

int shortestpath_bf( TreeNode<T>*
root ) {
    int which_level;
    if( p != NULL ) {
        std::list<TreeNode<T>*>
current_level;
        current_level.push_back(p);
        while( current_level.size() !=
0 ) {
            std::list<TreeNode<T>*>
next_level;

            for( std::list<TreeNode<T>*>::iterato
r itr = current_level.height()
            )
        }
    }
}

generate ideas
• play with examples! develop a
strategy for solving the problem? try
any strategy on several
examples.possible to map this
strategy into algorithm?
• solving simpler version of the
problem first and learn from the
exercise or generalize.
• Does problem look like another
problem you know how to solve?
• If given partial solution, could
extend to complete sol'n?
• can split prob 1/2 and solve halves
(recursively) separately?
• Does sorting the data help?
• Can split prob in different cases,
handle cases separately?
• Can discover something fund. about
prob that makes it easier to solve or
makes you able to solve it more
efficiently?
• have idea you think works, evaluate
it: indeed works? other ways to
approach might be better / faster? if
not work, why not?

mapping to code
• How to represent the data? most
efficient? what is easiest?
• use classes to organize data? What
data stored and manipulated as unit?
What info needs stored for each
object? What ops (past simple
accessors) helpful?
• How divide prob in 2 units of logic
that become functions? Can reuse any
code u have already? any logic u
write be re-usable?
• going to use recursion or
iteration? What info need to handle
during loops or recursive call, how
is it "carried along"?
• How effective is sol'n? Is ur sol'n
gen'l? How's performance? (order
not'n of no. operations)? think -
better ideas/approaches?
• Make notes about logic of code as
you write. These become invariants -
what should be true at begin and end
of each iteration / recursive call.

details
• everything initialized correctly,
e.g. bool flag vars, accumulation
vars, max/min vars?
• Is logic of conditionals correct?
Check several times and test examples
by hand.
• have bounds on loops correct?
Should you end at n, n-1 or n-2?
• Tidy "notes" to formalize
invariants.

```