Foundations of Computer Science HW 8

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22.14(a)-(c) Answer True or False

(a) *A bijection must be an injection.*

(b) *There is a bijection from* $\mathbb{Q}$ *to* $\mathbb{R}$

(c) *There is a bijection from* $\mathbb{Q}$ *to* $\mathbb{R}$

22.18(a)-(c) Prove or Disprove

(a) $\mathbb{Z}^2$ *is the set of pairs* $\{(z_1, z_2) \mid z_1, z_2 \in \mathbb{Z}\}$. $\mathbb{Z}^2$ *is countable.*

(b) $\mathbb{Q}$ *is the set of rational numbers,* $\mathbb{Q} = \{r \mid r = \frac{a}{b}, a \in \mathbb{Z}, \text{ and } b \in \mathbb{N}\}$. $\mathbb{Q}$ *is countable.*

(c) F *is the set of all functions from* $\mathbb{N}$ *to* $\mathbb{N}, F = \{f \mid f : \mathbb{N} \to \mathbb{N}\}$. F *is countable.*

24.5(d)-(f) Give DFA's for the following

(d) *The strings which begin with 10 and end with 01.*

(e) $\mathscr{L} = \{01^{\bullet n} \mid n \geq 0\}$

(f) *The language with all strings whose length is divisible by 3.*

25.4(a)-(c) Give a DFA and a CFG for the following

(a) $\mathscr{L} = \{01^{\bullet n} \mid n \geq 0\}$

(b) $\mathscr{L} = \{0^{\bullet n}1^{\bullet n} \mid 0 \leq n \leq 5\}$

(c) $\mathscr{L} = \{$*strings which end in a 1*$\}$

25.15(a)-(c) Consider the language $\mathscr{L} = \{\epsilon, 1, 11, 111, \dots\} = \{1\}*$

(a) *Show that the CFG* $S \to \epsilon \mid 1 \mid 1S$ *generates* $\mathscr{L}$. *Give a derivation of 111.*

(b) *Show that the CFG* $S \to \epsilon \mid 1 \mid SS$ *generates* $\mathscr{L}$. *Give two different derivations of 111.*

(c) *A leftmost (rightmost) derivation replaces the leftmost (rightmost) variable at every step. For the grammar in (b), give left and rightmost derivations of 111.*

25.16(a)-(c) Given the CFG $S \to A1B; A \to \epsilon \mid 0A; B \to \epsilon \mid 0B \mid 1B$, Give leftmost and rightmost derivations, and parse trees, of

(a) *00101*

(b) *1001*

(c) *00011*

26.4(a)-(b) Give high-level pseudocode for Turing Machines that solve these problems. In some cases you are asked for a decider. In others, you are asked for a transducer.

(a) *Regular languages:* $\mathscr{L}_1 = \{*01*\}$ *and* $\mathscr{L}_2 = \{*01\}$

(b) *Not CFL:* $\mathscr{L} = \{0^{\bullet n}\#1^{\bullet n}\#0^{\bullet n}\}$ *where # is a punctuation symbol.*