

$$O = \{n \mid n = 2k + 1; k \in \mathbb{N}\}$$

$$\mathbb{R}$$

1 sequences

List of objects: order and repetition matter.

$$tap \neq tpa$$

2 graphs

Defined by sets of vertices and edges.

e.g., Friendships: what matters is
 who the people are (set of V objects)
 who is friends with whom (set of E edges)

2.1 Types of Graphs:

affiliation graphs

e.g., students and their courses

conflict graphs

e.g., courses with students in common conflict

3 proofs.

In the speed dating ritual, no one meets more than 12 people.

deductive proof

In any round a person meets at most 3 new people.

There are 4 rounds, so one can only meet $3 * 4 = 12$ people total.

3.1 when is a number a square

even squares come from even numbers and even numbers have even squares.

proof.

- **n even** $\rightarrow n = 2k \rightarrow n^2 = 2(2k^2) \rightarrow \mathbf{n^2 \text{ even}}$
- **n odd** $\rightarrow n = 2k + 1 \rightarrow n^2 = 4(k^2 + k) + 1 \rightarrow \mathbf{n^2 \text{ odd}}$

3.2 graph

thm. Any 6-node graph has a 3 person clique or a 3 person war.

proof. in two cases.

3.3 proof by contradiction.

Ex. $\sqrt{2}$ is irrational.

$$\sqrt{2} = \left\{ \frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \dots \right\}$$

where a_1, a_2, \dots are all integers and b_1, b_2, \dots are all natural numbers.

Well ordering principle \rightarrow there is a minimum b_i . Call it b_* .

$\sqrt{2} = \frac{a_*}{b_*}$, and a_* and b_* have no common factor.

But this also implies that $a_*^2 = 2b_*^2$, so a_* is even.

And if a_* is even, then $a_* = 2k \rightarrow b_*^2 = 4k^2$, so b_* is even.

This means, a_* and b_* have a common factor! But we know this to be false.

Thus $\sqrt{2}$ is irrational.

4 definitions

axioms: a self-evident statement that is asserted as true without proof.

conjectures: a claim that is believed true but is not true until proven so.

theorems: a proven truth. you can take it to the bank.