Quiz 1 Crib Sheet

Chapter 2

Sets

- order & repeated elements dont matter
- $\mathbb{Q} = \{r | r = \frac{a}{b}; a \in \mathbb{Z}, b \in \mathbb{N}\}$
- $\mathbb{Z} = \{0, \pm 1, ...\}$
- $A \subseteq B \rightarrow$ every element of A is in B
- $A \subset B \rightarrow$ every element of A is in B and at least 1 element of B is not in A
- $A = B \rightarrow A \subseteq B$ *and* $B \subseteq A$
- **Power Set:** all subsets of a set $A = \{a, b\}$; $\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
- $A \cap B \rightarrow$ elements in both A and B
- $A \cup B \rightarrow$ elements from combining A with B
- $\overline{A} \rightarrow$ all elements not in A

Combining Set Operations

1. Associative: $(A \cap B) \cap C = A \cap (B \cap C)$
   $(A \cup B) \cup C = A \cup (B \cup C)$
2. Commuatative: $A \cap B = B \cap A$
   $A \cup B = B \cup A$
3. Complements: $\overline{(\overline{A})} = A$
   $\overline{A \cap B} = \overline{A} \cup \overline{B}$
   $\overline{A \cup B} = \overline{A} \cap \overline{B}$
4. Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sequences

- order & repitition is important

Graphs

- capture relationships between objects
- set of vertices
- affiliation & conflict graphs

Axiom: self-evident statment that is asserted as true without proof

Conjecture: claim that is believed true but is not true until proven

Theorem: proven truth

The Well-Ordering Principle: Any non-empty subset of $\mathbb{N}$ has a minimum element.

Chapter 3

Truth Table:

p	q	$\neg p$	$p \vee q$	$p \wedge q$	$p \rightarrow q$
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	T	F	T
F	F	T	F	F	T

Logical Connectors Manipulation

1. Associative: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
   $(p \vee q) \vee r \equiv p \vee (q \vee r)$
2. Commutative: $p \wedge q \equiv q \wedge p$
   $p \vee q \equiv q \vee p$
3. Negations: $\neg(\neg p) \equiv p$
   $\neg(p \wedge q) \equiv \neg p \vee \neg q$
   $\neg(p \vee q) \equiv \neg p \wedge \neg q$
4. Distributive: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
   $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
5. Implication: $p \rightarrow q \equiv \neg q \rightarrow \neg p$
   $p \rightarrow q \equiv \neg p \wedge q$

Predicates

Example 1: All cars have four wheels

Predicate: $P(c) =$ "car c has four wheels"

Domain: $C = \{c | c \text{ is a car }\}$

"for all c in C, the statement $P(c)$ is true." $= \forall c \in C : P(c)$

Example 2: There exists a creature with blue eyes and blonde hair

Predicate 1: $G(a) =$ "a has blue eyes"

Predicate 2: $H(a) =$ "a has blonde hair"

Domain: $A = \{a | a \text{ is a creature}\}$

"there exists a in A for which the statement $G(a)$ and $H(a)$ are true." $= \exists a \in A : (G(a) \wedge H(a))$

- $\exists$ claims need just one case to prove it is true
- $\forall$ claims need just one contradiction to be false

$$\neg(\forall x : P(x)) \equiv \exists x : \neg P(x)$$
$$\neg(\exists x : P(x)) \equiv \forall x : \neg P(x)$$

Chapter 4: Proofs

Direct Proof: $p \rightarrow q$

Assume p is T. Argue q must be T.

Proving $\forall x \in \mathcal{D} : P(x)$ using general x:

Assume $x \in \mathcal{D}$.

Determine properties x must have for $x \in \mathcal{D}$

Show $P(x)$ is T for x, then claim must be true.

Contraposition: $p \rightarrow q$

Assume q is F. Argue p must be F.

A direct proof of the contraposition $(\neg q \rightarrow \neg p)$

Contradiction: p is T

Assume p is F.

Derive **FISHY** statement. Means p is T.

Contradiction: $p \to q$

Assume p is T & show q is F.

Assume q is F.

Derive **FISHY** statement

Either p is F or q is T. So $p \to q$ is T.

Equivalence: $p \iff q$

Prove $p \to q$.

Prove $q \to p$.

Set Proofs: *(Formal Proof)* Show:

$A \subseteq B : x \in A \to x \in B$

$A \not\subseteq B : \exists x \in A : x \notin B$

$A = B : A \subseteq B \ \& \ B \subseteq A$

Chapter 5: Induction

Induction Proof: $\forall n \geq 1 : P(n)$

Base Case: Show $P(1)$ is T.

Induction Step: Show $P(n) \to P(n+1)$ for $n \geq 1$.

Use Direct Proof or Contraposition:

Direct: Assume $P(n)$: T. Show $P(n+1)$: T.

Contra: Assume $P(n+1)$: F. Show $P(n)$: F.

$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1)$

Just plug whatever is on top $(n+1)$ into i

Chapter 6: Strong Induction

Strong Induction: To prove $P(n)\forall n \geq 1$, use induction to prove a stronger claim:

$Q(n):$ each of $P(1), P(2), ..., P(n)$ are T

Leaping Induction: Prove $\forall n \geq 1 : P(n)$

Show $P(1), ..., P(k)$ base cases are T.

Show $P(n) \to P(k)$ for $n \geq 1$

Then $\forall n \geq 1 : P(n)$