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1. 首先觀察資料集有沒有 na 值，計算 na 比例 & 補植

```

PassengerId    0
Survived        0
Pclass         0
Name           0
Sex            0
Age           177
SibSp          0
Parch          0
Ticket         0
Fare           0
Cabin         687
Embarked       2
dtype: int64
*****

```

Age 遺失比例： $\frac{177}{891} = 19.865\%$

先從 Name 得到稱謂 (Mr., Mrs., Miss., ...):

```

# 取出稱謂 Mr., Mrs., Miss.: '空格' + 字母 + '.'
df['Title'] = df['Name'].str.extract(' ([A-Za-z]+)\.', expand = False)

```

再補上各稱謂範圍裡年齡的中位數：

```

# 計算個別 Title 的年齡中位數，補植
df['Age'].fillna(df.groupby('Title')['Age'].transform('median'),
inplace = True)

```

Cabin 遺失比例： $\frac{687}{891} = 77.104\%$

雖然遺失比例高達77%應該直接丟棄，但還是有23%的真實資料，且其實可以從 Pclass (票種) 資訊推測出船艙區域 (越高級的票會住在越高級的區域)。

因為只需大略知道船艙區域，所以取 Cabin 的第一個字母：

```

# 取出船艙的第一個字母，表示船艙所在的區域
df['Cabin'] = df['Cabin'].str[:1]

```

再根據各票種的船艙區域中位數補上 na 值：

```

# 計算個別 Pclass 的船艙區域中位數，補植
df['Cabin'].fillna(df.groupby('Pclass')['Cabin'].transform('median').as
type(int), inplace = True)

```

Embarked 遺失比例： $\frac{2}{891} = 0.224\%$

Embarked (登船地點) 因為比例不高，所以隨意挑一種方式補上就可以了：

```
# 'Embarked' 登船地點，補上與下一筆相同值，若最後一筆是 na 則補上與前一筆相同值
df['Embarked'].fillna(method='bfill', inplace=True)
df['Embarked'].fillna(method='pad', inplace=True)
```

2. 資料清理 & 刪減

將 SibSp (兄弟姐妹/配偶) 和 Parch (父母/孩子) 合併為 FamilyNum (家人數)：

```
# 合併 Sibsp & Parch = FamilyNum
df['FamilyNum'] = df['SibSp'] + df['Parch']
```

將不需要的資料丟棄：

```
# 刪除 PassengerId, Name, SibSp, Parch, Ticket, Title
col_drop = ['PassengerId', 'Name', 'SibSp', 'Parch', 'Ticket', 'Title']
df = df.drop(col_drop, axis = 1)
```

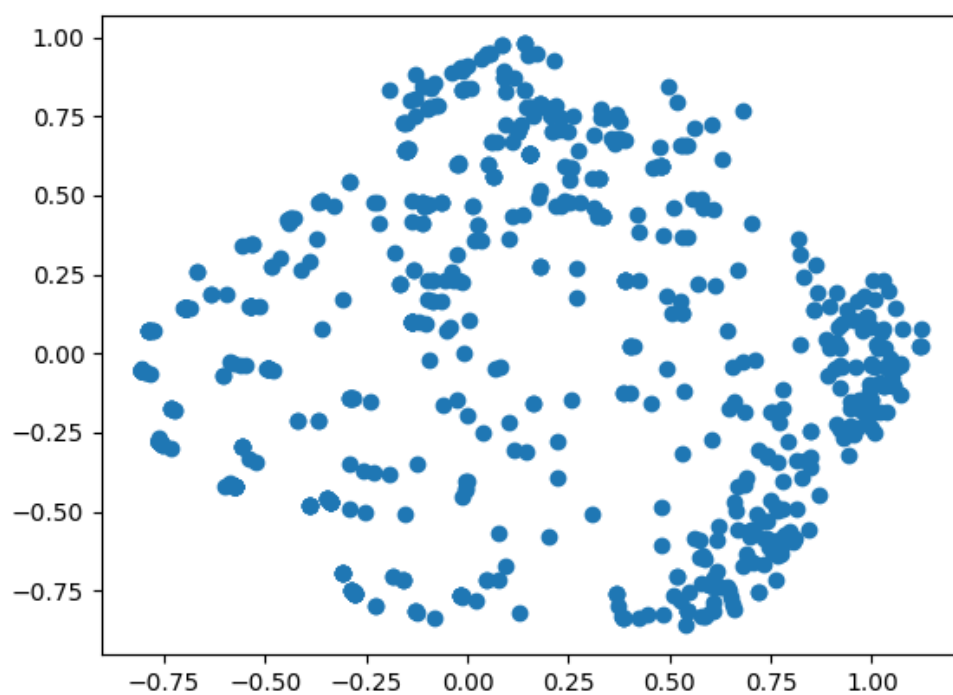
整理後的樣子：

```
*****
   Survived  Pclass  Sex  Age   Fare  Cabin  Embarked  FamilyNum
0         0      3    0  2.0   7.2500    6.0         0         1
1         1      1    1  3.0  71.2833    3.0         1         1
2         1      3    1  2.0   7.9250    6.0         0         0
3         1      1    1  3.0  53.1000    3.0         0         1
4         0      3    0  3.0   8.0500    6.0         0         0
..      ...     ...   ...   ...     ...     ...       ...     ...
886        0      2    0  2.0  13.0000    5.5         0         0
887        1      1    1  1.0  30.0000    2.0         0         0
888        0      3    1  2.0  23.4500    6.0         0         3
889        1      1    0  2.0  30.0000    3.0         1         0
890        0      3    0  3.0   7.7500    6.0         2         0

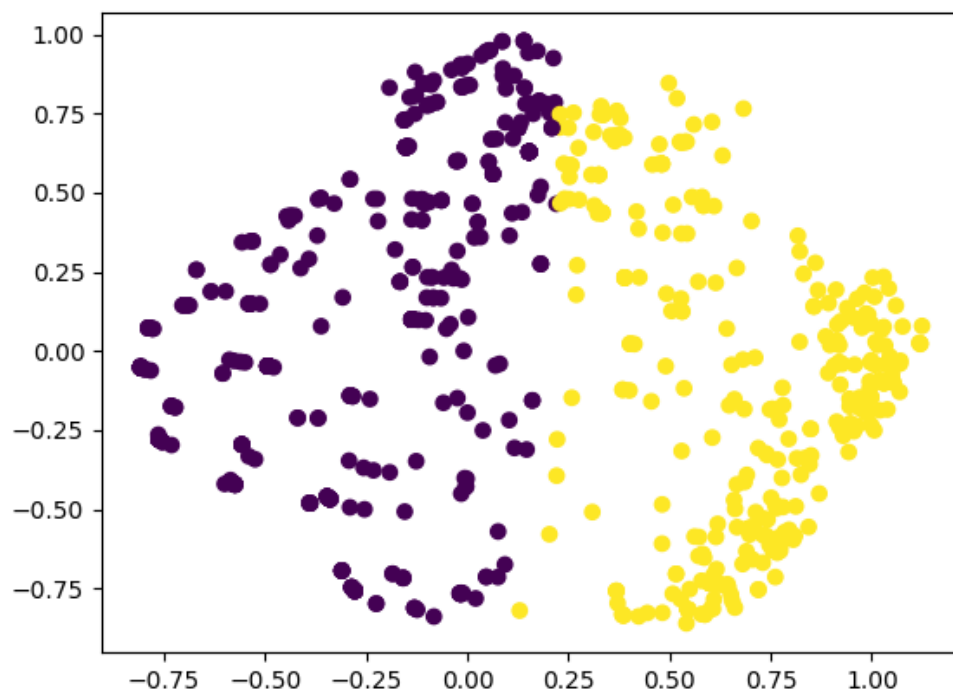
[891 rows x 8 columns]
*****
```

3. 標準化 & 數據轉 2 維 (如 code 所示)

數據圖：



經過 Spectral Clustering 分類的圖 ($n = 2$)：



4. 計算 eigenvalues & eigenvectors，並將 eigenvalues 由大到小排序：

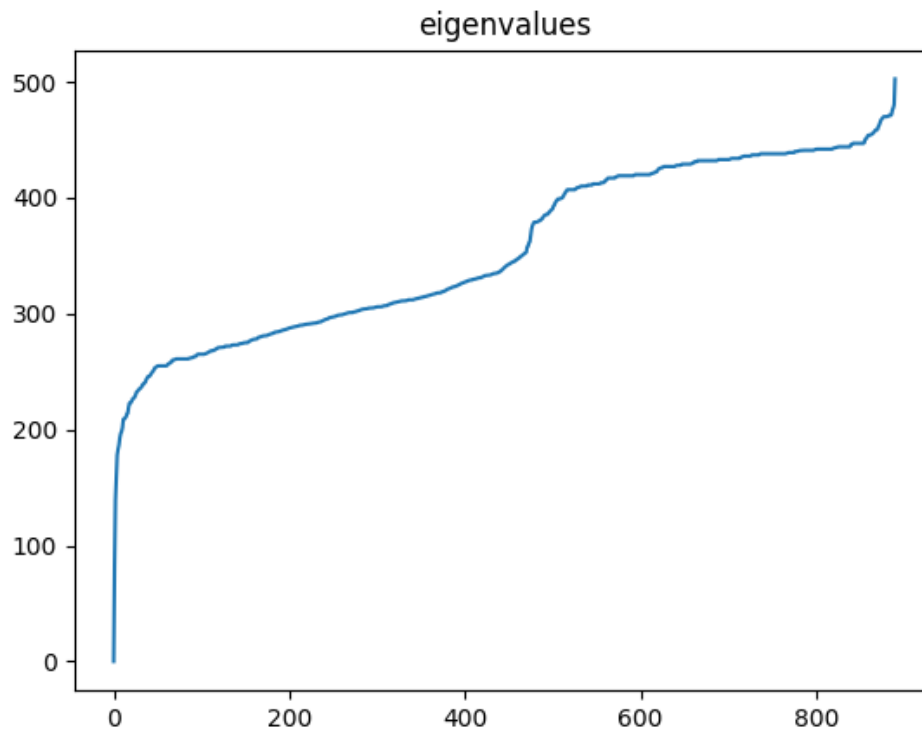
```
eigval: [ 0.      82.86428 139.1262 160.78013 178.01018 183.      186.61471
192.82337 196.54681 198.33294 201.6839 209.      209.      209.95922
210.83402 213.47722 214.      219.13769 223.      223.      223.6951
225.29404 226.90413 227.26602 228.20247 230.      232.47352 232.96793
233.84589 234.97919 235.41982 236.089 237.16837 239.      239.03986
240.13362 240.80882 243.03831 244.22536 245.74786 245.95856 246.11667
247.30779 248.29597 249.70971 250.40671 252.      253.42884 253.84752
254.18614 254.93088 255.      255.      255.      255.      255.
255.      255.      255.      255.      255.00458 255.50744 256.57179
257.09425 257.43352 257.58892 258.94292 260.      260.      260.1709
260.68006 261.      261.      261.      261.      261.      261.
261.      261.      261.      261.      261.      261.      261.
261.      261.      261.48176 261.66116 262.      262.      262.15309
262.77259 262.99949 263.      263.5909 264.52728 265.      265.
265.      265.      265.      265.      265.      265.01622 265.26572
265.67857 265.90045 266.29985 266.62422 267.15654 267.798 267.97827
268.      268.      268.57023 268.72292 269.13462 269.86789 270.09087
270.68976 270.80268 271.      271.      271.      271.      271.09513
271.17269 272.      272.      272.      272.      272.      272.
272.00501 272.48757 273.      273.      273.      273.      273.
273.      273.14564 273.34339 273.88781 274.10385 274.12915 274.27429
274.4166 274.68907 275.      275.      275.      275.      275.20585
276.12668 276.34546 276.84748 277.0917 277.60319 277.68489 278.
278.      278.17048 278.22578 278.89448 279.64503 279.71812 280.
280.20932 280.42294 280.76355 280.92126 281.      281.      281.17398
281.27775 281.73238 281.95004 282.06412 282.52434 282.79859 283.
283.12431 283.90038 284.      284.10561 284.19281 284.34474 284.5163
284.76346 285.02783 285.1985 285.55019 285.92892 285.98305 286.21953
286.40839 286.67953 286.90755 287.3486 287.52464 287.74959 287.79774
288.      288.35132 288.58447 288.74108 288.89525 289.      289.26017
289.35095 289.68411 290.      290.      290.      290.17899 290.3176
290.58891 290.70529 290.82324 291.      291.      291.07593 291.25718
291.38057 291.40178 291.50723 291.73871 291.79128 291.93672 291.99436
292.04572 292.42295 292.53326 292.88936 292.94725 293.02299 293.44967
293.79255 294.32225 294.50789 295.      295.24878 295.40062 295.62712
296.24502 296.55522 296.73432 296.88968 297.      297.13624 297.48765
297.57904 297.94078 298.10769 298.54081 298.61999 298.82137 298.98052
299.      299.04738 299.29759 299.6077 299.94638 300.15129 300.20833
300.56343 300.74046 300.94382 301.      301.      301.      301.
301.37364 301.52814 301.77337 301.83592 302.22577 302.4053 302.68019
302.80672 303.25133 303.58918 303.59735 303.93686 304.      304.
304.      304.07959 304.3301 304.71711 304.76899 304.95924 305.
305.      305.      305.08151 305.30288 305.60251 305.84907 305.91953
305.99517 306.      306.      306.      306.      306.26629 306.61188
306.78969 306.8593 306.99487 307.      307.29043 307.86046 308.05315
308.18682 308.57347 308.89227 309.23157 309.47013 309.56434 309.73744
309.99607 310.0949 310.21494 310.41915 310.66437 310.77398 311.
311.      311.00892 311.09023 311.24849 311.25769 311.35907 311.77181
311.98812 312.      312.      312.      312.      312.      312.32314
312.58716 312.76045 312.97537 313.21504 313.2765 313.4699 313.77252
313.85198 314.1778 314.19427 314.52131 314.73001 314.82506 314.99611
```

315.3037	315.54164	315.74562	315.84297	316.02168	316.44018	316.66706
316.75924	317.	317.15025	317.58891	317.8408	317.99572	318.
318.	318.	318.4427	318.88203	319.	319.36419	319.49195
320.05178	320.53172	321.041	321.25847	321.42359	321.85684	322.30014
322.76386	323.00198	323.03207	323.39233	323.49796	323.80566	323.99414
324.49048	324.93311	325.1414	325.78176	326.10725	326.45343	326.63628
326.88651	327.37332	327.57398	327.70464	328.37264	328.41022	328.69377
329.	329.	329.07383	329.54173	329.62953	329.82851	330.
330.1363	330.54897	330.66499	330.96925	331.	331.	331.3791
331.78568	332.17098	332.32685	332.89042	332.95238	333.	333.
333.08427	333.19822	333.37798	333.65422	334.02182	334.48139	334.60108
334.84764	335.	335.	335.07984	335.67943	336.	336.16342
337.12635	337.4702	338.31315	339.2645	339.79545	340.46352	341.01191
341.72759	342.17415	342.59614	342.82647	343.63132	343.97006	344.33245
345.	345.	345.26948	346.38608	346.54522	347.22842	347.6195
348.08096	349.25026	349.6041	349.96131	350.71789	351.45172	351.84584
352.5495	353.10518	357.64109	358.15824	361.09038	362.	367.85868
373.05237	376.	377.72755	379.	379.	379.	379.
379.50247	380.	380.15989	380.51535	381.54242	381.8437	383.19971
384.24585	385.01	385.43793	385.50638	385.58056	386.61896	387.23031
388.58653	389.02792	389.91748	390.46841	393.	393.67234	395.81122
397.	398.	399.	399.	399.03133	399.43944	399.59845
399.9482	401.15287	402.70438	404.06497	404.9432	406.51387	407.
407.	407.	407.	407.	407.	407.	407.
407.	407.39312	408.44129	408.64344	409.	409.	409.33385
410.	410.	410.	410.	410.	410.	410.
410.15831	411.	411.	411.	411.	411.	411.
412.	412.	412.	412.	412.	412.	412.
412.01364	412.36377	413.	413.	413.	413.16751	413.6376
414.98094	415.15338	416.18324	417.	417.	417.	417.
417.	417.	417.	417.03889	417.11678	418.01752	418.2796
418.59854	419.	419.	419.	419.	419.	419.
419.	419.	419.	419.	419.	419.	419.
419.	419.	419.	419.	419.	419.51984	420.
420.	420.	420.	420.	420.	420.	420.
420.	420.	420.	420.	420.	420.	420.
420.	420.	420.17031	420.72971	421.	421.10585	421.42474
422.	422.	422.28862	422.72359	424.53419	425.	425.34441
425.75968	426.	426.03855	426.63543	426.99003	427.	427.
427.	427.	427.	427.	427.	427.	427.
427.	427.	427.	427.45977	428.	428.	428.
428.	428.	428.11263	429.	429.	429.	429.
429.	429.	429.	429.	429.	429.	429.08012
429.36331	429.72742	430.02938	431.	431.	431.04623	431.12707
432.	432.	432.	432.	432.	432.	432.
432.	432.	432.	432.	432.	432.	432.
432.	432.	432.	432.	432.	432.	432.
432.	432.04679	432.59243	433.	433.	433.	433.
433.	433.	433.	433.	433.	433.	433.
433.	433.	433.42769	433.69939	433.98115	434.	434.
434.	434.	434.	434.	434.	434.	434.

```

435.      435.      435.34746 435.62755 435.98809 436.      436.
436.      436.      436.      436.      436.      436.39259 437.
437.      437.      437.      437.      437.      437.      437.
437.      437.61857 438.      438.      438.      438.      438.
438.      438.      438.      438.      438.      438.      438.
438.      438.      438.      438.      438.      438.      438.
438.      438.      438.      438.      438.      438.      438.
438.      438.      438.      438.19277 438.31514 438.74915 439.
439.      439.      439.      439.      439.      439.      440.
440.      440.      440.07661 440.45461 440.70627 440.78258 441.
441.      441.      441.      441.      441.      441.      441.
441.      441.      441.      441.      441.      441.      441.
441.29965 441.50536 441.90151 442.      442.      442.      442.
442.      442.      442.      442.      442.      442.      442.
442.      442.      442.      442.      442.      442.      442.
442.45297 442.78334 442.98185 443.28518 443.38454 443.69806 443.88413
444.      444.      444.      444.      444.      444.      444.
444.      444.      444.      444.      444.      444.      444.26378
446.1036 446.14319 447.      447.      447.      447.      447.
447.      447.      447.      447.      447.      447.      447.
447.      447.58907 450.47866 451.      452.      453.73646 454.
454.      454.41896 454.5775 455.11143 455.91609 456.52873 457.79625
458.47711 458.77132 458.92301 461.37697 462.87217 464.94384 467.10166
468.      469.14203 469.97903 470.      470.      470.      470.07679
470.44035 471.      471.      471.01137 471.91919 475.72019 477.51784
480.44976 502.55623]

```



5. 列出最小的 nonzero eigenvalue & corresponding eigenvector :

The smallest nonzero eigenvalue and the corresponding eigenvector:

eigenvalue:

82.86428

eigenvector:

```
[[ -0.034  0.053 -0.011  0.013  0.009  0.005 -0.017  0.    0.003 -0.
  0.007  0.009 -0.    -0.006 -0.001 -0.001 -0.    0.001 -0.    -0.
  0.    -0.    -0.001  0.    -0.    0.    -0.    0.    0.001 -0.001
 -0.    0.    0.001 -0.001  0.001 -0.001 -0.    -0.    0.001 -0.
 -0.003  0.004 -0.001  0.005 -0.007  0.    0.    0.    -0.007  0.003
  0.    0.    0.    -0.003 -0.004 -0.001 -0.    -0.    -0.005  0.005
  0.    -0.    -0.001  0.    -0.    -0.001  0.002  0.    0.    -0.001
  0.    0.    0.001  0.    -0.    -0.    0.    -0.    0.    0.
  0.001 -0.    -0.    0.    0.    -0.    -0.001 -0.001 -0.    -0.
  0.001 -0.    -0.002 -0.    -0.    -0.002  0.    0.    -0.001  0.002
 -0.002  0.002  0.    -0.001  0.001 -0.001  0.    0.001 -0.001 -0.
  0.001 -0.    0.    0.    0.    0.    0.    -0.    -0.    -0.
  0.    -0.    -0.    0.    -0.    -0.    -0.    0.    0.001  0.004
  0.001  0.001  0.013  0.009 -0.    0.003 -0.007 -0.002  0.003  0.007
 -0.    -0.008 -0.003 -0.001  0.005  0.004 -0.001 -0.    0.011 -0.
  0.    0.001 -0.005  0.004 -0.    0.005  0.005 -0.002 -0.002  0.001
  0.    0.002 -0.    -0.001  0.002  0.007 -0.    -0.    -0.001  0.003
  0.    0.    0.    0.001 -0.    -0.002  0.001 -0.001  0.    0.
 -0.    -0.    -0.    -0.    0.    -0.    0.    -0.    -0.    0.
  0.    0.    0.    0.003 -0.    0.    0.    0.    -0.    0.
 -0.001  0.    0.    0.001 -0.    0.    -0.002 -0.004  0.016 -0.001
  0.    -0.004  0.005  0.011 -0.    -0.    -0.003  0.    -0.    0.
 -0.    0.    -0.    0.    0.    0.    -0.008 -0.004 -0.014  0.009
  0.021 -0.024  0.    -0.    -0.003  0.005 -0.007  0.002 -0.    0.
 -0.004 -0.    -0.    -0.    0.    0.    0.    -0.    -0.    0.002
 -0.015  0.    -0.001 -0.008  0.002  0.003  0.007 -0.02  0.018 -0.004
  0.016 -0.011  0.007  0.    -0.    0.    -0.    -0.    0.    -0.
 -0.    0.    -0.    0.002  0.003  0.01  0.007  0.006 -0.007 -0.007
  0.011  0.015 -0.007 -0.    0.012  0.013  0.007  0.007 -0.003 -0.
 -0.001 -0.    0.    0.    0.    0.    -0.    0.    -0.    0.
 -0.    -0.    -0.001  0.005  0.005 -0.003 -0.017  0.004  0.009  0.001
 -0.008  0.001 -0.006  0.009 -0.021  0.    0.    0.    0.003 -0.08
 -0.019 -0.    0.    -0.    0.    0.    0.    0.    -0.    0.
 -0.    -0.    -0.    -0.    0.    -0.005  0.008  0.004 -0.001  0.
  0.007  0.    -0.001 -0.02  -0.001  0.011 -0.    0.031 -0.007  0.032
  0.027  0.007  0.002 -0.    0.004 -0.011  0.    0.    -0.    0.
 -0.    -0.    0.    -0.    0.    0.    -0.    -0.    -0.    -0.
  0.    0.002  0.004  0.007 -0.    0.    -0.    -0.018 -0.009  0.012
  0.008 -0.029 -0.031 -0.01 -0.022 -0.009  0.014 -0.006  0.016  0.
 -0.    -0.    -0.    0.    0.    0.    -0.    -0.    -0.    0.
  0.    -0.    -0.    -0.    0.    0.004  0.    -0.    -0.004  0.001
  0.009 -0.007 -0.004 -0.001 -0.013 -0.002  0.006 -0.008 -0.002  0.002
  0.    0.    -0.065 -0.02  0.046 -0.005  0.011  0.003  0.013  0.006
 -0.027  0.001 -0.002 -0.018  0.003 -0.    0.    0.    0.003 -0.012
  0.008  0.001 -0.004  0.005  0.001  0.003  0.    -0.032 -0.009  0.023
  0.    0.    -0.    0.    -0.    0.018  0.    -0.    -0.    0.
 -0.    0.    -0.    -0.    0.    0.    0.    -0.    -0.001 -0.011
```

```
-0.038 0.005 -0.001 -0.006 -0.009 0.001 0.002 -0. 0.007 -0.
-0. -0. 0. -0. 0. 0.085 0. -0. -0. 0.029
-0.031 0.04 -0.015 -0. 0.002 0.032 0.015 -0.012 0.004 0.007
0.033 -0.009 -0.003 -0. 0.005 -0.004 0. -0.004 0.002 -0.001
0.007 0.006 -0.008 -0.002 0.008 0.001 -0.001 0. 0. 0.
-0.017 -0.003 0. 0.009 -0.355 -0. -0. -0. 0.099 -0.
-0.005 -0.175 -0. 0. -0.003 0.009 0.005 0. 0. -0.
-0. -0.186 0.172 0. 0. 0.26 -0. -0.706 0.238 -0.181
0.112 -0.042 0.004 0.02 0.026 -0.036 -0.054 -0. -0.001 -0.001
0.007 -0.009 -0.003 -0.004 -0.015 -0.004 -0.007 0.002 0.01 -0.004
-0.01 0.003 -0.015 0.011 -0.01 0.013 -0.009 -0.004 0.012 -0.005
0.009 0.012 0.007 -0.006 -0.013 0.01 -0.011 0.002 0.004 -0.
0. -0.004 -0.003 -0.009 0.011 0.004 0. 0. 0. 0.071
0.044 -0.024 0.058 0.017 -0.004 0.006 0.007 0.005 0. 0.
0. -0. -0.042 -0.002 0.032 0.011 -0.001 0.01 -0.033 -0.024
-0.022 -0.007 0.003 0.06 0.027 -0.02 -0.005 -0.005 -0.037 -0.001
0.002 -0.007 0.005 0.014 0.005 -0.003 -0.015 0.01 0.006 -0.002
0.011 -0.011 -0.001 -0.007 -0.017 -0. 0.002 -0. 0. 0.026
0.011 -0.007 0.009 -0.005 0.042 0. -0. -0. -0. -0.
0. 0. -0. -0. 0. -0. -0. -0. 0. 0.
-0. -0. -0. -0. -0.009 0. -0.054 -0. -0. 0.048
-0.017 0.006 0.004 0.023 0.005 0. 0.003 -0.003 -0.032 -0.001
0.014 0. 0. -0. 0. -0. 0. -0. -0. -0.
-0. -0. -0. -0.004 0.034 0.003 0.009 -0.007 0.003 -0.005
0.03 -0. 0. 0.007 0.005 -0.002 -0.006 -0.013 -0.001 -0.024
-0.008 -0.023 0.004 -0.035 0. 0. 0. 0. 0. -0.
-0. 0. 0. 0. 0.023 -0.003 0. 0. -0. 0.
-0. -0. -0. -0. 0. 0. 0. 0. -0. 0.
-0. -0. 0. -0. 0. -0. 0. -0. -0. 0.
0. -0. 0. 0. -0. 0. -0. 0. -0. -0. -0.
-0. 0. -0. 0. -0. 0. -0. -0. -0. -0.
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]]
```


Code 與註解：

```
import pandas as pd
import numpy as np
from sklearn.cluster import SpectralClustering
from sklearn.preprocessing import StandardScaler, normalize
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA

df = pd.read_csv('titanic/train.csv')

print(df.isnull().sum())

# 取出稱謂 Mr., Mrs., Miss.: '空格' + 字母 + '.'
df['Title'] = df['Name'].str.extract(' ([A-Za-z]+)\.', expand = False)

def title_map(x):  # title mapping 'Mr' = 0 , 'Miss' = 1 , 'Mrs' = 2
    and other = 3
    title = 0
    if x == 'Mr':
        title = 0
    elif x == 'Miss' :
        title = 1
    elif x == 'Mrs':
        title = 2
    else:
        title = 3
    return title
df['Title'] = df['Title'].map(title_map)
# Name 已轉為 Title 紀錄，其餘部分與模型訓練無關
df.drop('Name', axis = 1 , inplace = True)

# Sex Mapping
sex_mapping = { 'male': 0 , 'female': 1 }
df['Sex'] = df['Sex'].map(sex_mapping)

# 計算個別 Title 的年齡中位數，補值
df['Age'].fillna(df.groupby('Title')['Age'].transform('median'),
inplace = True)
```

```

# Age Mapping, 取十分位
# print(df.Age.max()) # MAX 值為 80
df.loc[ df['Age'] < 10 , 'Age' ] = 0
df.loc[ (df['Age'] >= 10) & (df['Age'] < 20) , 'Age' ] = 1
df.loc[ (df['Age'] >= 20) & (df['Age'] < 30) , 'Age' ] = 2
df.loc[ (df['Age'] >= 30) & (df['Age'] < 40) , 'Age' ] = 3
df.loc[ (df['Age'] >= 40) & (df['Age'] < 50) , 'Age' ] = 4
df.loc[ (df['Age'] >= 50) & (df['Age'] < 60) , 'Age' ] = 5
df.loc[ (df['Age'] >= 60) & (df['Age'] < 70) , 'Age' ] = 6
df.loc[ (df['Age'] >= 70) & (df['Age'] < 80) , 'Age' ] = 7
df.loc[ (df['Age'] >= 80) , 'Age' ] = 8

# 'Embarked' 登船地點, 補上與下一筆相同值, 若最後一筆是 na 則補上與前一筆相同值
df['Embarked'].fillna(method='bfill', inplace=True)
df['Embarked'].fillna(method='pad', inplace=True)

# Embarked Mapping
embarked_mapping = { 'S': 0 , 'C': 1 , 'Q': 2 }
df['Embarked'] = df['Embarked'].map(embarked_mapping)

# 取出船艙的第一個字母, 表示船艙所在的區域
df['Cabin'] = df['Cabin'].str[:1]

# Cabin Mapping
cabin_mapping = { 'A': 0 , 'B': 2 , 'C': 3 , 'D': 4 , 'E': 5 ,
                  'F': 6 , 'G': 7 , 'T': 8 }
df['Cabin'] = df['Cabin'].map(cabin_mapping)

# 計算個別 Pclass 的船艙區域中位數, 補值
df['Cabin'].fillna(df.groupby('Pclass')['Cabin'].transform('median'),
inplace = True)

# 合併 Sibsp & Parch = FamilyNum
df['FamilyNum'] = df['SibSp'] + df['Parch']

# 刪除 PassengerId, SibSp, Parch, Ticket, Title

```

```

col_drop = ['PassengerId', 'SibSp', 'Parch', 'Ticket', 'Title']
df = df.drop(col_drop, axis = 1)

print('*****')
print(df.head(3))
print('*****')

target = df['Survived']
dataset = df.drop('Survived', axis = 1)

X = dataset.values
Y = target.values

# 平均 & 變異數標準化
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)
X_normalized = normalize(X_scaled)

# 主成分分析，數據轉 2 維
pca = PCA(n_components = 2)
X_2 = pca.fit_transform(X_normalized)

# 座標圖
plt.scatter(X_2[:,0], X_2[:,1])
plt.show()

# 建立 SpectralClustering 模型
sc = SpectralClustering(n_clusters=2)
sc.fit(X_2, Y)
labels = sc.labels_
plt.scatter(X_2[:,0], X_2[:,1], c=labels)
plt.show()

from sklearn.metrics import pairwise_distances

# 計算各點之間的距離，距離 < mean 才視為相連，生成 A 矩陣
A = pairwise_distances(X_normalized, metric='euclidean')

```

```

A_mean = A.mean()
print('\nA mean:', A_mean)
vectorizer = np.vectorize(lambda x: 1 if (x > 0) & (x < A_mean) else 0)
A = np.vectorize(vectorizer)(A)
print('A:', A)

# Laplacian Matrix
from scipy.sparse import csgraph
L = csgraph.laplacian(A, normed=False)
print('L: ', L)

# 計算 eigenvalues & eigenvectors
eigval, eigvec = np.linalg.eig(L)
eigval = eigval.astype(float).round(5)
eigvec = eigvec.astype(float).round(5)
eigval = np.sort(eigval)
print('eigval: ', eigval)
# print('eigvec: ', eigvec)

# 取得最小的 nonzero eigenvalue
minval = np.min(eigval[np.nonzero(eigval)])

# 取得 corresponding eigenvector
def near(a, b, rtol = 1e-5, atol = 1e-8):
    return np.abs(a - b) < (atol + rtol * np.abs(b))

nonzero_eigvec = eigvec[near(eigval, minval)].astype(float).round(3)

print('\nThe smallest nonzero eigenvalue and the corresponding
eigenvector:')
print('eigenvalue: \n', minval)
print('eigenvector: \n', nonzero_eigvec)

plt.plot(eigval)
plt.title('eigenvalues')
plt.show()

```