

# EE2101-Control Systems

## Assignment-1

### Problem-4

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## 1 Problem-4 Question

## 2 Solution

- Part 'a'
- Part 'b'
- Part 'c'

# Question

Solve the following differential equations with the given initial conditions using Laplace transforms.

a.  $\frac{dx}{dt} + 7x = 5\cos 2t$  where  $x(0) = 4$ ,  $x'(0) = -4$  ;

b.  $\frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 8x = 5\sin 3t$  where  $x(0) = 4$ ,  $x'(0) = 1$ ;

c.  $\frac{d^2x}{dt^2} + 8 \frac{dx}{dt} + 25x = 10u(t)$  where  $x(0) = 2$ ,  $x'(0) = 3$ ;  
where  $x'(0) = \frac{dx}{dt}(0)$ .

Assume that the forcing functions are zero prior to  $t = 0^-$ .

## Part 'a'

- In the given problem statement, the part 'a' is a differential equation of order=1 and 2 initial conditions are given.
- But only one initial condition  $x(0) = 4$  is needed for a differential equation of order 1.
- So, I am modifying the problem assuming 2 different ways:
  - 1) Only  $x(0) = 4$  condition is given for the given first order differential equation.
  - 2) Considering  $\frac{d^2x}{dt^2}$  instead of  $\frac{dx}{dt}$  with conditions  $x(0) = 4$ ,  $x'(0) = -4$  because 2 initial conditions are needed for a differential equation of order = 2.

## Part 'a' 1st way

$$\frac{dx}{dt} + 7x = 5\cos 2t \text{ where } x(0) = 4$$

Applying Laplace transforms to the differential equation,

$$\mathcal{L}\{x'(t)\} = sX(s) - x(0)$$

$$sX(s) - x(0) + 7X(s) = \frac{5s}{s^2+4}$$

$$sX(s) - 4 + 7X(s) = \frac{5s}{s^2+4}$$

$$(s+7)X(s) = \frac{5s}{s^2+4} + 4$$

$$X(s) = \frac{4s^2+5s+16}{(s^2+4)(s+7)}$$

Now solve it using partial fractions,

$$X(s) = \frac{As+B}{s^2+4} + \frac{C}{s+7}$$

By comparing the numerators,

We get  $A = \frac{35}{53}$ ,  $B = \frac{20}{53}$ ,  $C = \frac{177}{53}$

So,

$$X(s) = \frac{35s}{53(s^2+4)} + \frac{20}{53(s^2+4)} + \frac{177}{53(s+7)}$$

Applying inverse Laplace transform, we get  $x(t)$ .

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at, \quad \mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at, \quad \mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

$$x(t) = \frac{35}{53} \cos 2t + \frac{10}{53} \sin 2t + \frac{177}{53} e^{-7t}$$

## Part 'a' 2nd way

$$\frac{d^2x}{dt^2} + 7x = 5\cos 2t \text{ where } x(0) = 4, x'(0) = -4$$

Applying Laplace transforms to the differential equation,

$$\mathcal{L}\{x''(t)\} = s^2X(s) - sx(0) - x'(0)$$

$$s^2X(s) - sx(0) - x'(0) + 7X(s) = \frac{5s}{s^2+4}$$

$$s^2X(s) - 4s + 4 + 7X(s) = \frac{5s}{s^2+4}$$

$$(s^2 + 7)X(s) = \frac{5s}{s^2+4} + 4s - 4$$

$$X(s) = \frac{4s^3 - 4s^2 + 21s - 16}{(s^2+4)(s^2+7)}$$

Now solve it using partial fractions,

$$X(s) = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+7}$$

By comparing the numerators,

We get  $A = \frac{5}{3}$ ,  $B = 0$ ,  $C = \frac{7}{3}$ ,  $D = -4$

So,

$$X(s) = \frac{5s}{3(s^2+4)} + \frac{0}{s^2+4} + \frac{7s}{3(s^2+7)} - \frac{4}{s^2+7}$$

Applying inverse Laplace transform, we get  $x(t)$ .

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

$$x(t) = \frac{5}{3} \cos 2t + \frac{7}{3} \cos \sqrt{7}t - \frac{4}{\sqrt{7}} \sin \sqrt{7}t$$



## Part 'b'

$$\frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 8x = 5\sin 3t \text{ where } x(0) = 4, x'(0) = 1$$

Applying Laplace transforms to the differential equation,

$$s^2X(s) - sx(0) - x'(0) + 6(sX(s) - x(0)) + 8X(s) = \frac{15}{s^2+9}$$

$$s^2X(s) - 4s - 1 + 6(sX(s) - 4) + 8X(s) = \frac{15}{s^2+9}$$

$$X(s)(s^2 + 6s + 8) = \frac{15}{s^2+9} + 4s + 25$$

$$X(s) = \frac{4s^3+25s^2+36s+240}{(s^2+9)(s^2+6s+8)}$$

$$X(s) = \frac{4s^3+25s^2+36s+240}{(s^2+9)(s+2)(s+4)}$$

Now solve it using partial fractions,

$$X(s) = \frac{As+B}{s^2+9} + \frac{C}{s+2} + \frac{D}{s+4}$$

By comparing the numerators,

We get  $A = -\frac{18}{65}$ ,  $B = -\frac{3}{65}$ ,  $C = \frac{118}{13}$ ,  $D = -\frac{24}{5}$

So,

$$X(s) = -\frac{18s}{65(s^2+9)} - \frac{3}{65(s^2+9)} + \frac{118}{13(s+2)} - \frac{24}{5(s+4)}$$

Applying inverse Laplace transform, we get  $x(t)$ .

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

$$x(t) = -\frac{18}{65} \cos 3t - \frac{1}{65} \sin 3t + \frac{118}{13} e^{-2t} - \frac{24}{5} e^{-4t}$$

## Part 'c'

$$\frac{d^2x}{dt^2} + 8 \frac{dx}{dt} + 25x = 10u(t) \text{ where } x(0) = 2, x'(0) = 3$$

Applying Laplace transforms to the differential equation,

$$s^2X(s) - sx(0) - x'(0) + 8(sX(s) - x(0)) + 25X(s) = \frac{10}{s}$$

$$s^2X(s) - 2s - 3 + 8(sX(s) - 2) + 25X(s) = \frac{10}{s}$$

$$(s^2 + 8s + 25)X(s) - 2s - 19 = \frac{10}{s}$$

$$(s^2 + 8s + 25)X(s) = \frac{10}{s} + 2s + 19$$

$$X(s) = \frac{2s^2 + 19s + 10}{s(s^2 + 8s + 25)}$$

Now solve it using partial fractions,

$$X(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 8s + 25}$$

By comparing the numerators,

We get  $A = \frac{2}{5}$ ,  $B = \frac{8}{5}$ ,  $C = \frac{79}{5}$

So,

$$X(s) = \frac{2}{5s} + \frac{\frac{8}{5}s + \frac{79}{5}}{s^2 + 8s + 25}$$

Now, again separate  $\frac{\frac{8}{5}s + \frac{79}{5}}{s^2 + 8s + 25}$  as partial fractions

$$\frac{\frac{8s+79}{5}}{s^2+8s+25} = \frac{A}{(s+4)^2+9} + \frac{B(s+4)}{(s+4)^2+9}$$

Comparing numerators, we get  $A = \frac{47}{5}$ ,  $B = \frac{8}{5}$

$$X(s) = \frac{2}{5s} + \frac{47}{5((s+4)^2+9)} + \frac{8(s+4)}{5((s+4)^2+9)}$$

Applying inverse Laplace transform, we get  $x(t)$ .

$$x(t) = \mathcal{L}^{-1}\{X(s)\}$$

$$x(t) = \frac{2}{5} + \frac{47}{15}e^{-4t} \sin 3t + \frac{8}{5}e^{-4t} \cos 3t$$