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CS 3630 Quiz 2

1. • The inverse of a rotation matrix is its transpose : ${}^{a}R_{b} = ({}^{b}R_{a})^{-1} = ({}^{b}R_{a})^{T}$

• The determinant of a rotation matrix is 1 : $\det(R) = 1$

2. angle θ alone is not a good way to describe Frame 1 w.r.t. frame 0. This approach will not generalize to rotations in 3 dimensions.
A better way is to specify the directions of x_1 and y_1 w.r.t. Frame 0 by projecting onto x_0 and y_0. This can be represented as a rotation matrix.

3 - a) $^1g = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $^2h \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$^0g = {}^0R_1 \, {}^1g = \begin{bmatrix} \cos \pi/4 & -\sin \pi/4 \\ \sin \pi/4 & \cos \pi/4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$^2g = {}^2R_0 \, {}^0g = \begin{bmatrix} \cos \pi/2 & -\sin \pi/2 \\ \sin \pi/2 & \cos \pi/2 \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$^cg = {}^cR_0 \, {}^0g = \begin{bmatrix} \cos 3\pi/4 & -\sin 3\pi/4 \\ \sin 3\pi/4 & \cos 3\pi/4 \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

b) $$^0h = {}^0R_2 \, {}^2h = \begin{bmatrix} \cos -\pi/2 & -\sin -\pi/2 \\ \sin -\pi/2 & \cos -\pi/2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$^1h = {}^1R_2 \, {}^2h = \begin{bmatrix} \cos -3\pi/4 & -\sin -3\pi/4 \\ \sin -3\pi/4 & \cos -3\pi/4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$^ch = {}^cR_2 \, {}^2h = \begin{bmatrix} \cos \pi/4 & -\sin \pi/4 \\ \sin \pi/4 & \cos \pi/4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

4. a) $^{2}cube = {}^{2}T_{0}\,{}^{0}cube = \begin{bmatrix} \cos \pi/2 & -\sin \pi/2 & -5 \\ \sin \pi/2 & \cos \pi/2 & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

b) $^{1}cube = {}^{1}T_{2}\,{}^{2}cube = \begin{bmatrix} \cos -3\pi/4 & -\sin -3\pi/4 & \\ \sin -3\pi/4 & \cos -3\pi/4 & \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

$^{1}T_{2} = ({}^{2}T_{1})^{-1}$

$\Rightarrow {}^{1}cube = ({}^{2}T_{1})^{-1}\,{}^{2}cube = \begin{bmatrix} \cos -3\pi/4 & \sin -3\pi/4 & 4 \\ -\sin -3\pi/4 & \cos -3\pi/4 & 3 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

$$^{1}T_{2} = \begin{bmatrix} \cos -3\pi/4 & \sin -3\pi/4 & 4 \\ -\sin 3\pi/4 & \cos -3\pi/4 & 3 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

c) $^{1}cube = {}^{1}T_{2}\,{}^{2}cube$

$$= \begin{bmatrix} \cos -3\pi/4 & \sin -3\pi/4 & 4 \\ -\sin -3\pi/4 & \cos -3\pi/4 & 3 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

5.

$$^{1}cube = ({}^{1}T_c)({}^{c}cube)$$

$\uparrow$

don't know

we know ${}^{c}T_2$ and ${}^{1}T_2$

$$^{2}T_c = ({}^{c}T_2)^{-1}$$

$$^{1}T_c = {}^{1}T_2\,{}^{2}T_c = {}^{1}T_2\,({}^{c}T_2)^{-1}$$

$$\therefore \boxed{{}^{1}cube = {}^{1}T_2\,({}^{c}T_2)^{-1}\,({}^{c}cube)}$$