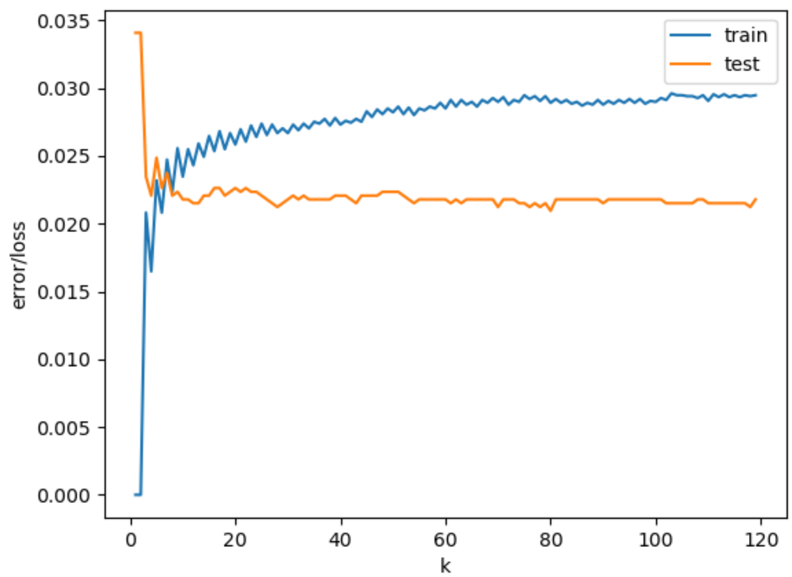
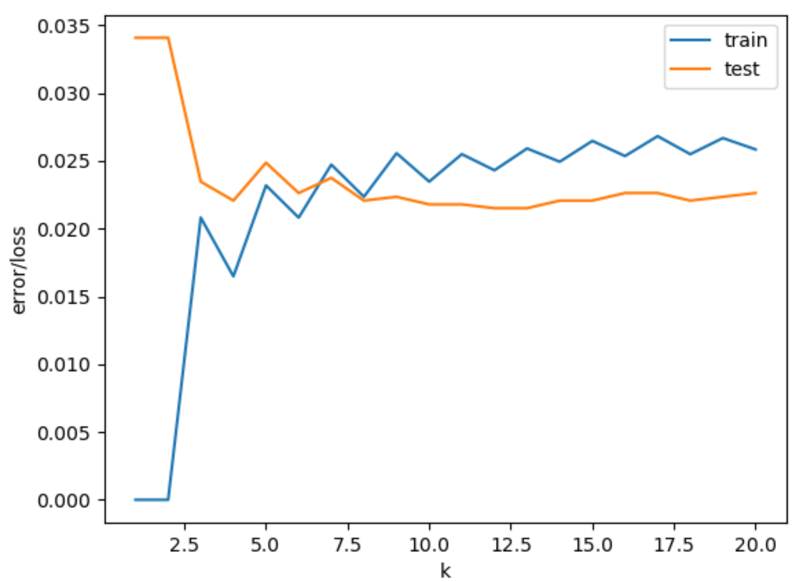
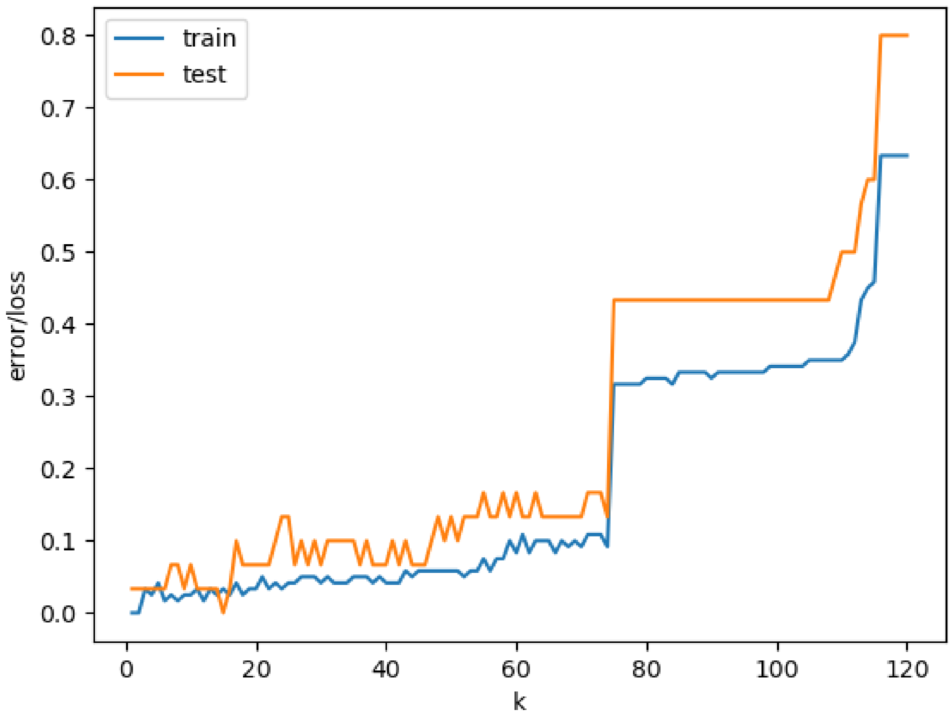
CS 4641 Machine Learning
Bojun Yang — Section B Homework 2 Writeup

1. KNN Analysis
   (a) Euclidean Distance
      i. HTRU2 — Best individual: 0.02094972067039106,(k=80)
      A good range for k is [6,15]. The testing data loss within this range has reached it's lowest and we are not overfitting the training data yet.

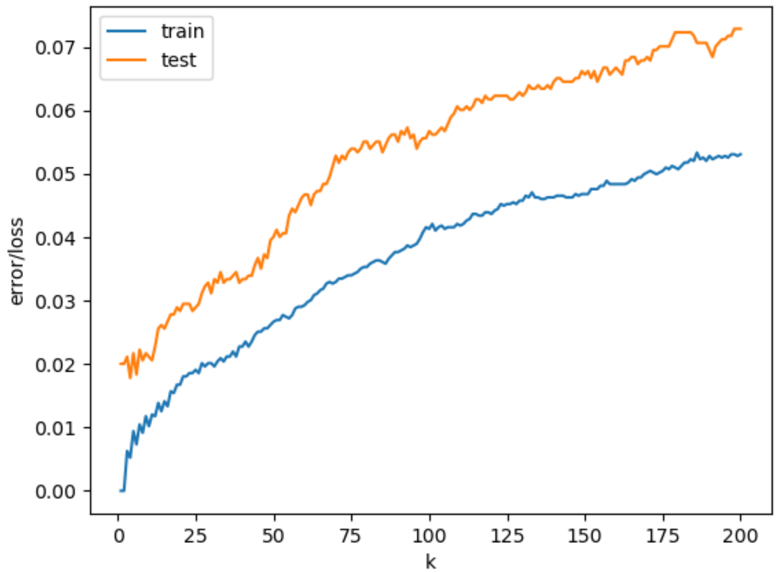


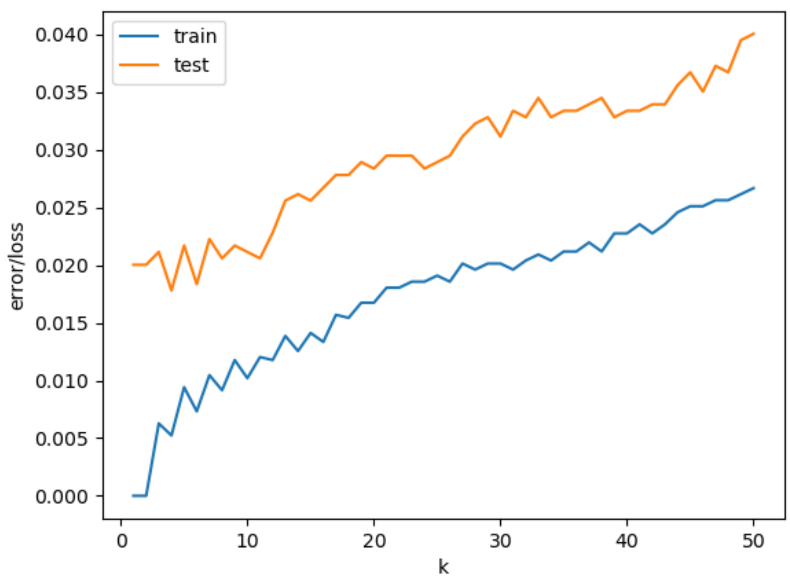


ii. iris — Best individual: 0.03333333333333333,(k=1)
A good range for k seems to be around k = 15 or k = 1. There is a significant decrease in loss for the testing data set here. It is difficult to choose a k for this dataset since there isn't a clear decreasing loss trend for the testing data set. We know that k values above 70 are definitely unideal because the loss increases dramatically and plateaus.



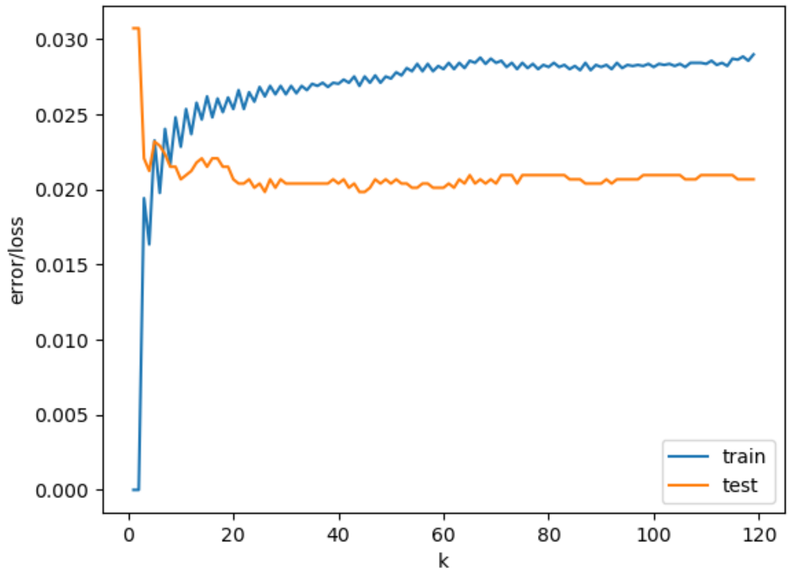
iii. optdigits — Best individual: 0.017807456872565387,(k=4)
A good range for k is [4,10]. Looking at the first graph, we see that the loss for testing data is lowest from k = (0,20]. Zooming into the graph, we can see that the loss for testing data starts to increase at k =10.

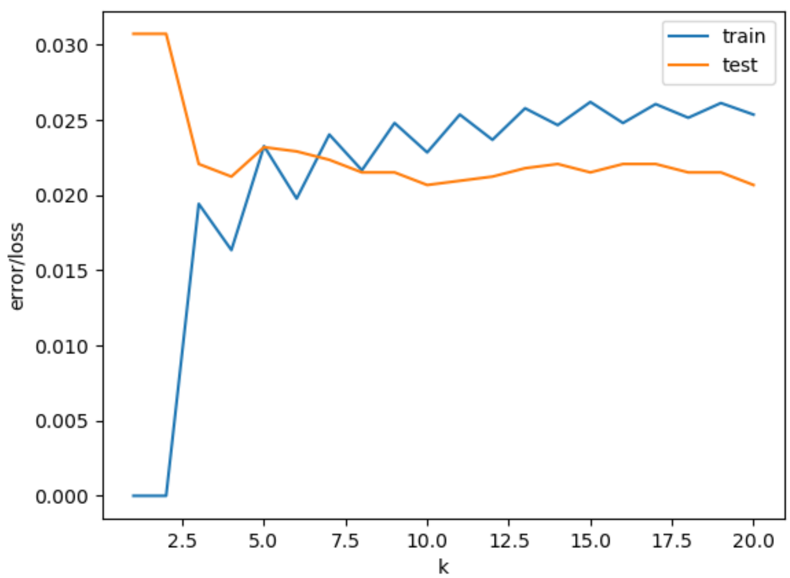




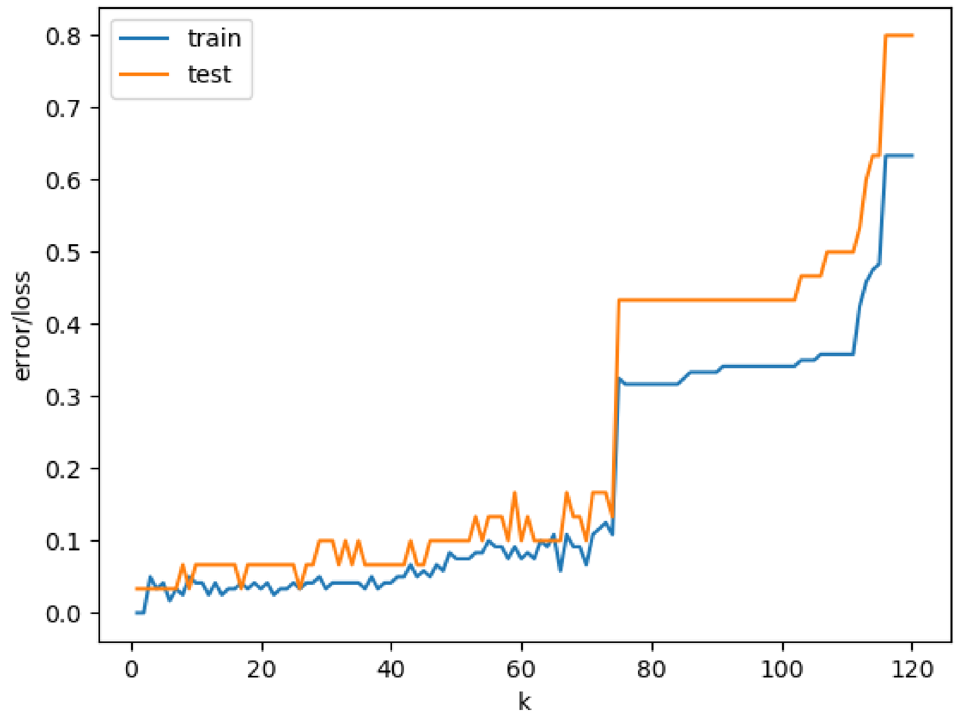
(b) Manhattan Distance

i. HTRU2 — Best individual: 0.019832402234636987,(k=26)
A good range for k is [20, 30]. The testing data loss within this range has reached it's lowest and we are not overfitting the training data yet.

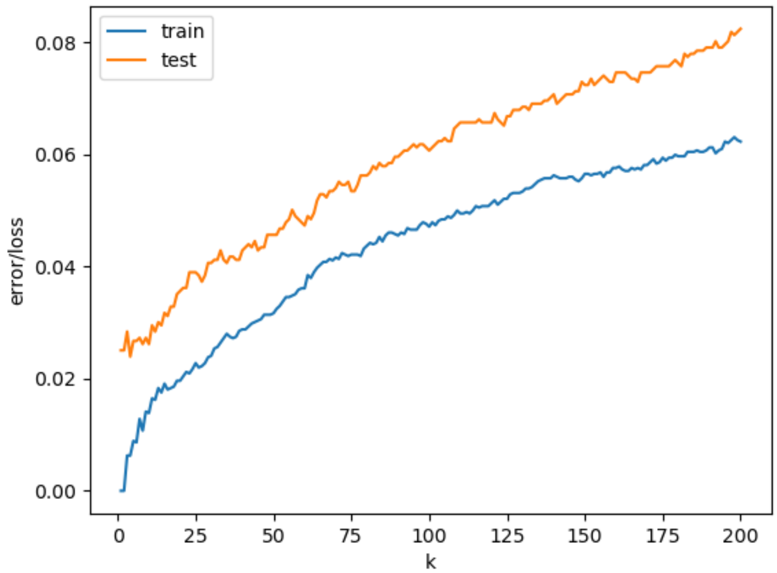


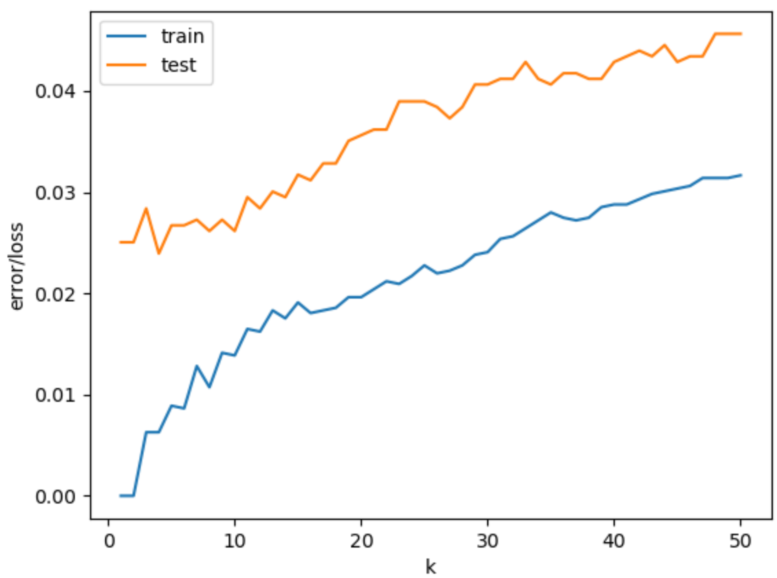


ii. iris — Best individual: 0.03333333333333333,(k=1)
A good range for k seems to be [1,13]. It is difficult to choose a k for this dataset since there isn't a clear decreasing loss trend for the testing data set. We know that k values above 70 are definitely unideal because the loss increases dramatically and plateaus.



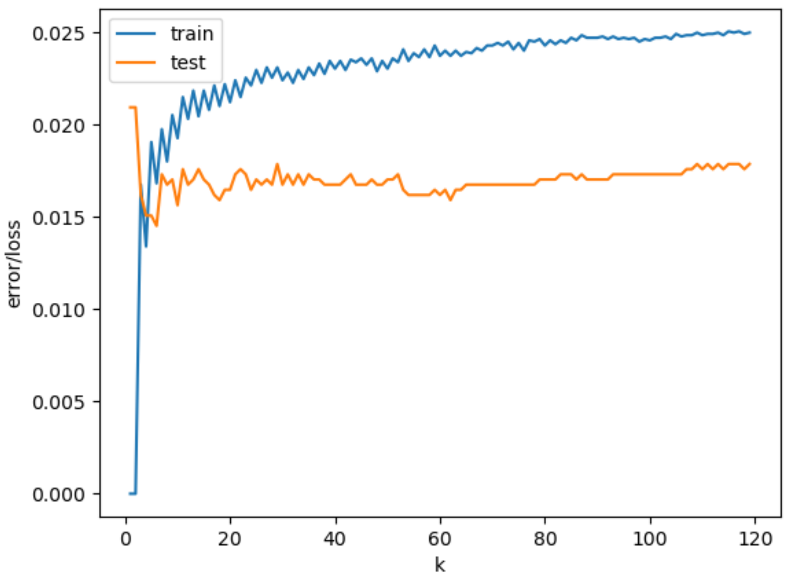
iii. optdigits — Best individual: 0.023928770172509738,(k=4)
A good range for k is [2,8]. Looking at the first graph, we see that the loss for testing data is lowest from k = (0,20]. Zooming into the graph, we can see that the loss for testing data starts to increase at k =10.

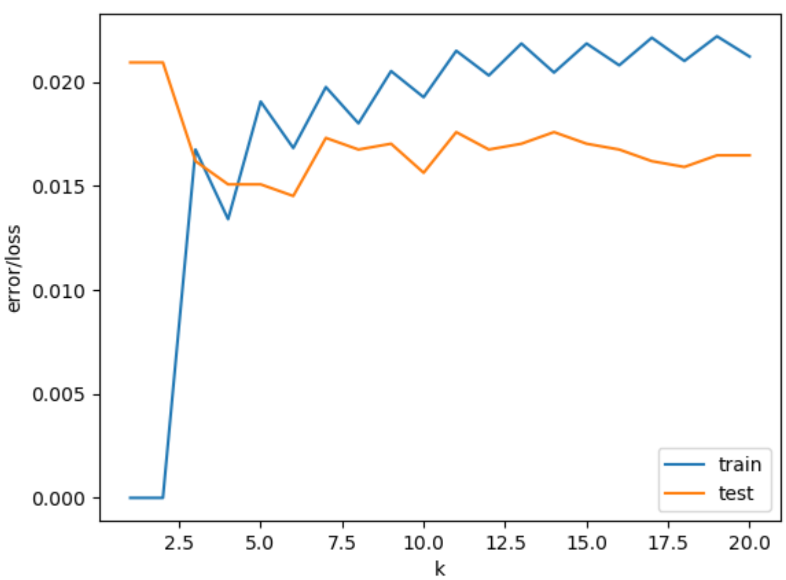




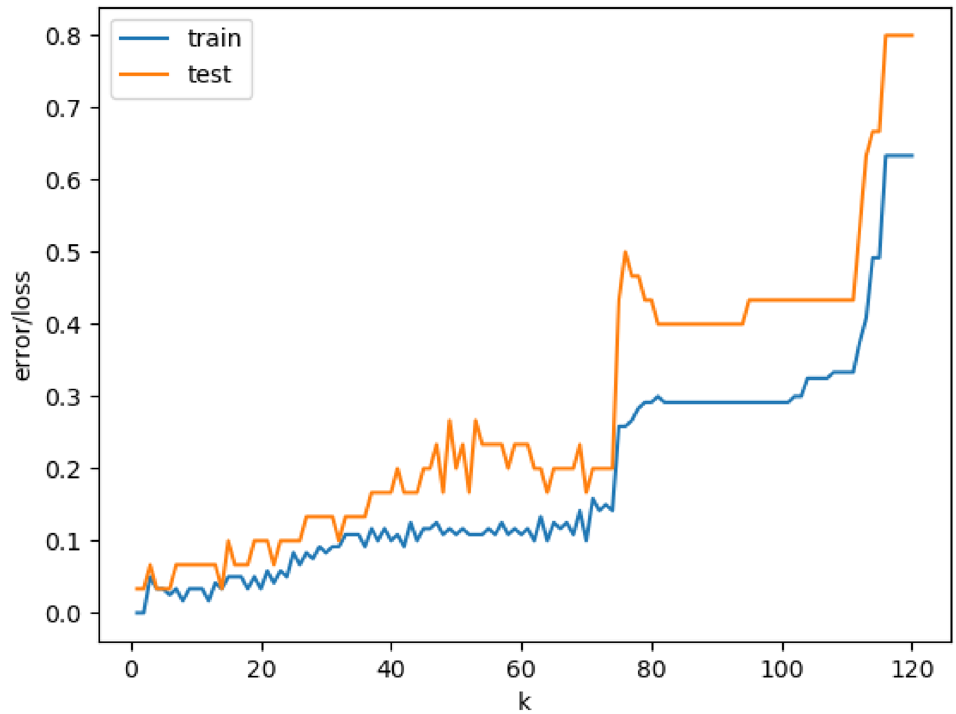
(c) Mahalanobis Distance

i. HTRU2 — Best individual: 0.01452513966480447,(k=6)
A good range for k is [4,10]. The testing data loss within this range has reached it's lowest and we are not overfitting the training data yet.

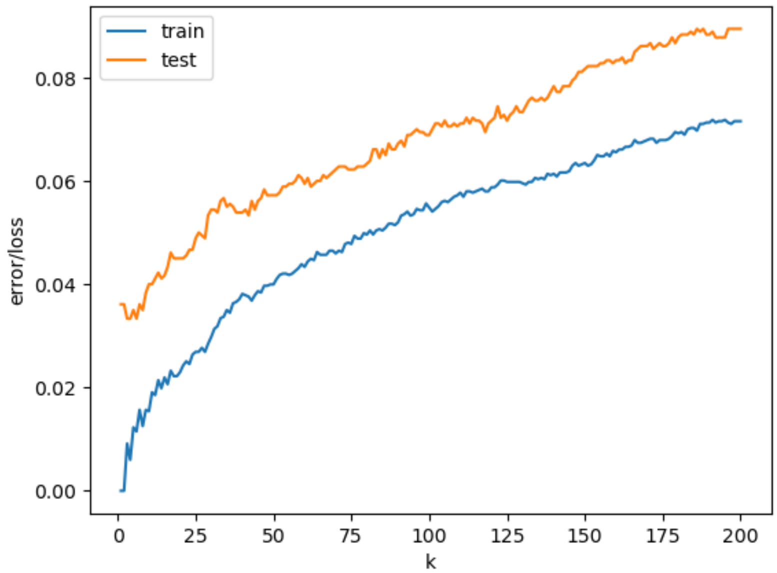


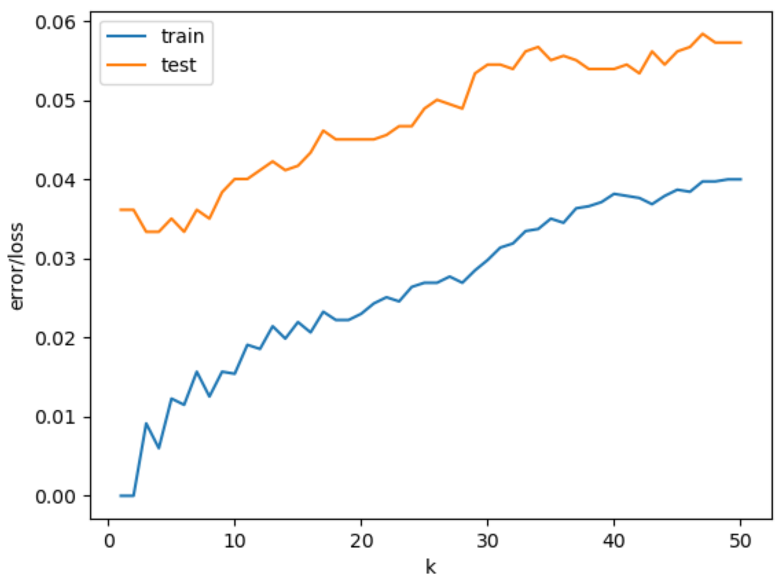


ii. iris — Best individual: 0.03333333333333333,(k=1)
A good range for k seems to be [1,15]. It is difficult to choose a k for this dataset since there isn't a clear decreasing loss trend for the testing data set. We know that k values above 70 are definitely unideal because the loss increases dramatically and plateaus.



iii. optdigits — Best individual: 0.0333889816360601,(k=3)
A good range for k is [2,7]. Looking at the first graph, we see that the loss for testing data is lowest from k = (0,20]. Zooming into the graph, we can see that the loss for testing data starts to increase at k =10.





2. Logistic Regression Analysis
   (a) HTRU2
   Converged, iteration 1109
   HTRU2 LogReg Test score: 0.976536312849162, HTRU2 LogReg Train score: 0.9698281882944545
   Compared to the lowest loss of KNN on HRTU2 (0.0145), Logistic regression falls short with about a 0.03 loss. KNN had better performance.
   (b) iris
   learner 0: Converged, iteration 14577
   learner 1: Converged, iteration 8816
   learner 2: Converged, iteration 28537
   iris LogReg Test score: 0.9666666666666667, iris LogReg Train score: 0.9666666666666667
   Compared to the lowest loss of KNN on iris (0.03), Logistic regression has a similar performance with about a 0.033 loss. However, we should note that the best KNN performances all had k=1, which is overfitting.
   (c) optdigits
   learner 0: Converged, iteration 2502
   learner 1: Converged, iteration 3376
   learner 2: Converged, iteration 3161
   learner 3: Converged, iteration 3426
   learner 4: Converged, iteration 3790
   learner 5: Converged, iteration 3948
   learner 6: Converged, iteration 2706
   learner 7: Converged, iteration 2894
   learner 8: Converged, iteration 4033
   learner 9: Converged, iteration 4410
   digits LogReg Test score: 0.9365609348914858, digits LogReg Train score: 0.9641642688987706
   Compared to the lowest loss of KNN on optdigits (0.0178), Logistic regression falls short with about a 0.07 loss. KNN had better performance.
   (d) Analysis
   Logistic regression peroforms well when the data can be linearly separated, a good learning rate is chosen, and a good enough initial theta is chosen. HTRU2 is a 8 dimensional dataset with binary class labels. Iris is a 4 dimensional dataset with multi-class labels. Optdigits is a 64 dimensional dataset with multi-class labels. logistic regression seems to perform well with HTRU2 and iris. It makes sense for HTRU2 because the line is able to use 8 dimensions to fit 2 labels. Similarly with iris (3 unique class labels), logistic regression performs well. With optdigits, logistic regression performs the worst possibly due to the increased number of labels (10). Based on these reasonings, I don't think that the optdigits dataset is easily linearly separable while HTRU2 and iris datasets are. Our learning rate was small enough so that logistic regression converged within a reasonable amount of runs and our thetas were initialized to 0, which is good enough.

3. Support Vector Machines

(a) $h_{\mathbf{w},w_0}(\mathbf{x}) = \text{sgn}\left\{w_0 + \phi(\mathbf{x})^\top \mathbf{w}\right\}$
$w = \Sigma_{i-=1}^{N} \alpha^{(i)} y^{(i)} x^{(i)}$
Kernal trick: $K(x^{(i)}, x) = \phi(x^{(i)})\phi(\mathbf{x})^\top$
Apply feature transform to x: $w = \Sigma_{i=1}^{N} \alpha^{(i)} y^{(i)} \phi(x^{(i)})$
$\phi(\mathbf{x})^\top \mathbf{w} = \phi(\mathbf{x})^\top \Sigma_{i=1}^{N} \alpha^{(i)} y^{(i)} \phi(x^{(i)}) = \Sigma_{i=1}^{N} \alpha^{(i)} y^{(i)} \phi(x^{(i)})\phi(\mathbf{x})^\top = \Sigma_{i=1}^{N} \alpha^{(i)} y^{(i)} K(x^{(i)}, x)$
The Kernel computes the inner product of $x^T x'$ with a function k. Using the kernelized version of SVMs as seen above, we don't have to explicitly calculate $\phi(x)\phi(x')^T$. This allows us to operate in higher dimensions by using an easy to compute kernel function instead of computing a potentially high-dimensional transform. $\alpha^{(i)}$ determine which points are the support vectors that determine the decision boundary. Support vectors have an $\alpha^{(i)} > 0$, which means if a point is not a support vector, it's $\alpha^{(i)} = 0$. This effectively removes this point from the calculation of $h_{\mathbf{w},w_0}(\mathbf{x})$.

(b) $k_{lin}(\mathbf{x}, \mathbf{x}') : D$ dimensions
The vectors in the kernel are the original vectors. The dimensions will stay the same in ϕ because a linear transform does not increase the number dimensions.

$k_{poly}(\mathbf{x}, \mathbf{x}') : \binom{(D+M)}{M}$ dimensions
e.g. $(a + b + c + d + 1)^2, D = 4, M = 2$
squaring a polynomial with 4 variables results in a new polynomial with $\binom{6}{2}$ terms. Generalizing to the binomial theorem: a polynomial with D terms raised to the M'th power results in a new polynomial with $\binom{(M+D)}{M}$ terms.

$k_{exp}(\mathbf{x}, \mathbf{x}')$: infinite dimensions
The taylor series expansion of $e^z = 1 + \frac{1}{1!}z + \frac{1}{2!}z^2 + \frac{1}{3!}z^3 + ... + \frac{1}{\infty!}z^\infty$
Seeing how the power of z goes to infinity, this kernel is similar to a polynomial kernel with D = ∞. Therefore, the number of dimensions this feature transform calculates is infinity.

(c) Find vector v parallel to $\mathbf{w}$

$x_1 = 0, x_2 = \sqrt{2}, y_1 = -1, y_2 = 1$
$\phi(x) = [1, \sqrt{2}x, x^2]$
$\phi(x_1) = [1, 0, 0], \phi(x_2) = [0, 2, 2], v = \phi(x_2) - \phi(x_1) = [0, 2, 2]$

(d) find r, value of the margin achieved by $\mathbf{w}$
$r = \sqrt{0^2 + 2^2 + 2^2} = \sqrt{8}$

(e) find $\mathbf{w}$
$||\mathbf{w}|| = \frac{2}{r} = \frac{2}{\sqrt{8}}$
$\mathbf{w} = \frac{||\mathbf{w}||}{r} v = \frac{\frac{2}{\sqrt{8}}}{\sqrt{8}} v = \frac{1}{4} < 0, 2, 2 > = < 0, \frac{1}{2}, \frac{1}{2} >$

(f) find w_0

I don't writeout $\mathbf{w}^T$ and $\phi(x)$ because they are defined in my part c and e.
$-1(\mathbf{w}^T\phi(x_1) + w_0) = -1(0 + w_0) = -w_0 \geq 1 \rightarrow w_0 \leq -1$
$1(\mathbf{w}^T\phi(x_2) + w_0) = 1(2 + w_0) = 2 + w_0 \geq 1 \rightarrow w_0 \geq -1$
Therefore $w_0 = -1$

(g) Write down discriminant function
$f(x) = w_0 + \mathbf{w}^T\phi(x) = -1 + \frac{\sqrt{2}}{2}x + \frac{x^2}{2}$