Question 1. Naïve Duck

Your friend *Howard the Duck* has constructed a highly-biased probabilistic model of the animal kingdom, consisting of the joint probability distribution $P(W, Q, D)$, where W stands for "walks," Q stands for "quacks," and D stands for "is-a-duck."

	$W=0$		$W=1$	
	$Q=0$	$Q=1$	$Q=0$	$Q=1$
$D=0$	0.15	0.08	0.05	0.02
$D=1$	0.05	0.10	0.10	0.45

(a) (1 pt) Suppose that you are drawing random samples from this probabilistic model. What is the likelihood that a randomly-sampled animal will *walk*, *quack*, and *be a duck*?

0.45

(b) (4 pt) Calculate the following joint probability tables through marginalization (i.e. "summing out"): $P(W, Q)$, $P(D, Q)$, and $P(D)$. Note: we have provided the table $P(D, W)$ as an example.

	$W=0$	$W=1$
$D=0$	0.23	0.07
$D=1$	0.15	0.55

	$Q=0$	$Q=1$
$W=0$	0.2	0.18
$W=1$	0.15	0.47

	$Q=0$	$Q=1$
$D=0$	0.2	0.1
$D=1$	0.15	0.55

$D=0$	0.3
$D=1$	0.7

(c) (3 pt) If you are observing a duck, what is the probability that it can quack (i.e. $P(Q = 1|D = 1)$)? What is the probability that it can both quack and walk?

$$P(Q=1|D=1) = \frac{P(Q=1, D=1)}{P(D=1)} = \frac{0.55}{0.7} = 0.786$$

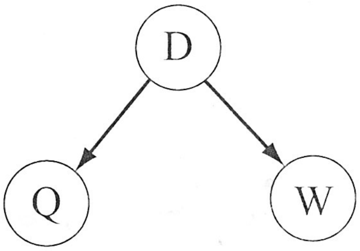
$$P(Q=1, W=1|D=1) = \frac{P(Q=1, W=1, D=1)}{P(D=1)} = \frac{0.45}{0.7} = 0.643$$

(c) (2 pt) You observe that a particular animal walks and quacks. What is the probability that it is a duck? Compare this answer to your answer in part (a). Are the probabilities the same or different? Explain why.

$$P(D=1 \mid Q=1, W=1) = \frac{P(D=1, Q=1, W=1)}{P(Q=1, W=1)} = \frac{0.45}{0.47} = 0.957$$

They are different because in one case you have evidence and in one you do not. More precisely, the first case (a) counts all animals that walk, quack, and are ducks. In (c) we consider the prob. that animal is a duck out of all animals that walk and quack

(d) (2 pt) Howard claims that Bayesian networks can provide a more compact and elegant model of the animal kingdom, and proposes the model below. Demonstrate that his model is incorrect. (Hint: Consider the case where all attributes are true, since a single counter-example is sufficient to disprove an assertion).



$$Q \perp W \mid D \Leftrightarrow P(Q=1, W=1 \mid D=1) = P(Q=1 \mid D=1)\,\underline{P(W=1 \mid D=1)}$$

$$0.643 \stackrel{?}{=} 0.786 \times 0.786$$

$$0.643 \neq 0.617$$

DISPROVED

$$\frac{P(W=1, D=1)}{P(D=1)} \rightarrow \frac{0.55}{0.7} \rightarrow 0.786$$

ALTERNATIVELY:

$$P(Q,W,D) \stackrel{?}{=} P(D)\,P(Q \mid D)\,P(W \mid D)$$

$$P(Q=1, W=1, D=1) \stackrel{?}{=} \frac{P(Q=1, D=1)\,P(W=1, D=1)\,P(\cancel{D=1})}{P(D=1)\,P(\cancel{D=1})}$$

$$0.45 \neq \frac{0.55}{0.7} \times 0.55 = 0.4321$$

ALTERNATIVELY:

$$P(Q \mid D, W) \stackrel{?}{=} P(Q \mid D)$$

$$\frac{0.45}{0.55} \quad \frac{P(Q=1, D=1, W=1)}{P(D=1, W=1)} \stackrel{?}{=} P(Q=1 \mid D=1)$$

$$0.818 \neq 0.786$$

Question 2. Markov Decision Process

Consider the following grid world for an MDP, where the states are $\{S_1, S_2, S_3\}$ and there are four goal terminals $\{G_1, G_2, G_3, G_4\}$. The reward for being in each state is -0.1. The reward for reaching G_1 is 10, while the reward for reaching each of the other goals is 1. The agent starts in S_1 and has *two* possible actions: *Right* and *Up*. The probabilistic transition model is as follows:

- When moving *Right*, the agent will go *Up* by accident with probability 0.2
- When moving *Up*, the agent will go *Right* by accident with probability 0.2.

1 G_2	1 G_3	1 G_4	
-0.1 S_1	-0.1 S_2	-0.1 S_3	10 G_1

6 pts
(2 per utility, 1 per action)

(a) Assume that all of the *utilities are initialized to 0.1* and there is *no discounting* (i.e. $\gamma = 1$). Calculate the updated utilities for states S_1, S_2, and S_3 over *three rounds* of value iteration, by filling in the missing elements below. (Note that $U^i(S_j)$ denotes the utility for state j in iteration number i.)

$$U^1(S_3) = -0.1 + \max_{R,U}\{ \begin{array}{ll} 0.8 \times (10) + 0.2 \times (1) = 8.2 & R \\ 0.8 \times (1) + 0.2 \times (10) = 2.8 & U \end{array} = 8.1$$

$$U^2(S_2) = -0.1 + \max_{R,U}\{ \begin{array}{ll} 0.8 \times (8.1) + 0.2 \times (1) = 6.68 & R \\ 0.8 \times (1) + 0.2 \times (8.1) = 2.42 & U \end{array} = 6.58$$

$$U^3(S_1) = -0.1 + \max_{R,U}\{ \begin{array}{ll} 0.8 \times (6.58) + 0.2 \times (1) = 5.464 & R \\ 0.8 \times (1) + 0.2 \times (6.58) = 2.116 & U \end{array} = 5.364$$

3 pts.
2 for grid policy
1 for explaining

(b) When ~~value~~ iteration has converged, indicate on the grid world figure from the previous page what you believe the final (optimal) policy will be. How many iterations will be required for ~~policy~~ Value iteration to converge (in the sense that the policy no longer changes)? Explain your answer.

By $i=3$ the policy won't change

Each utility is coupled to its neighbor to right, and

$U(S_3) = 8.1 \; \forall i$, so utilities to right trump fixed reward of 1

3 pts.
1.5 for utilities
1.5 for actions

(c) Demonstrate that your policy in part (b) is optimal using *policy iteration*. Policy iteration has converged when we compute the expected utility $U(\pi^*)$ for a policy π^*, and then demonstrate that π^* is optimal under $U(\pi^*)$.

Under π^*, the utilities from part (a) are already correct

$U_3 = 8.1$ $\qquad$ $U_i = U^{\pi^*}(s_i)$

$U_2 = 6.58$ $\qquad$ $\pi^* = \{a_1 = R, a_2 = R, a_3 = R\}$

$U_1 = 5.364$

$$a_3^* = \underset{R,U}{\arg\max} \begin{Bmatrix} 8.2 & R \\ 2.8 & U \end{Bmatrix} = R$$

$$a_2^* = \underset{R,U}{\arg\max} \begin{Bmatrix} 0.8\,U_3 + 0.2(1) = 6.68 \\ 0.8(1) + 0.2\,U_3 = 2.42 \end{Bmatrix} = R$$

$$a_1^* = \underset{R,U}{\arg\max} \begin{Bmatrix} 0.8\,U_2 + 0.1(1) = 5.464 \\ 0.8(1) + 0.2\,U_2 = 2.116 \end{Bmatrix} = R$$

✓

Question 3. First Order Logic

(a) Represent the following sentences in first order logic, using a consistent vocabulary (which you must define):

- Some students took Finnish in fall 2014

$\exists x, y \; student(x) \wedge Finnish(y) \wedge takes(x,y) \wedge offered(y, Fall2014)$

- Every student who takes Finnish passes it

$\forall x, y \; (student(x) \wedge Finnish(y) \wedge takes(x,y)) \Rightarrow passes(x,y)$

- Only one student took Quenya in fall 2014

$\forall y \; (Quenya(y) \wedge offered(y, Fall14)) \Rightarrow (((\exists x \; student(x) \wedge takes(x,y)) \wedge (\exists z \; student(z) \wedge takes(z,y))) \Rightarrow x = z)$

- The best score in Quenya is always higher than the best score in Finnish

$\forall x, y \; (Finnish(x) \wedge Quenya(y)) \Rightarrow (\exists s, t \; student(s) \wedge student(t) \wedge topper(x,s) \wedge topper(y,t) \wedge (grade(x,s) < grade(y,t)))$

could add NatNum

- Everyone who lives on-campus buys a meal-plan

$\forall x \; (\exists y \; lives(x,y) \wedge (oncampus(y) \Rightarrow (\exists z \; mealplan(z) \wedge buys(x,z))))$

- No student buys an expensive meal-plan $\Rightarrow$ anyone who buys expensive mP is not student (need this later)

R3:

$\forall z (\exists x \; (mealplan(z) \wedge buys(x,z) \wedge expensive(z)) \Rightarrow \neg student(x))$

- There is an office on-campus that sells meal-plans only to students

$\exists b \; (\forall x, z \; (oncampus(b) \wedge mealplan(z) \wedge sells(b,z,x) \Rightarrow student(x)))$

- If a faculty-member buys a meal-plan, then it must be expensive

R2:

$\forall z (\exists x \; (mealplan(z) \wedge buys(x,z) \wedge faculty(x) \Rightarrow expensive(z)))$

- Bud Peterson's meal-plan is the most expensive

R1:

$\exists z \; mealplan(z) \wedge buys(BudPeterson, z) \wedge (\forall y \; mealplan(y) \Rightarrow (cost(y) < cost(z)))$

(b) Use unification to prove that Bud Peterson is not a student. What can we conclude about whether Bud Peterson lives on campus?

(see scratch)

Here is some extra space. **Show all of your work on the questions!** If you need more paper just ask. Good luck!!

Q3 (b)

Bud Peterson meal plan

E.I. for R1 gives: mealplan(BPM)

buys(BP, BPM)

Bud Peterson

Add: faculty(BP) ← You must add this to KB

Apply Gen mod Pon to R2:

$$\frac{\text{mealplan}(BPM),\ \text{buys}(BP, BPM),\ \text{faculty}(BP),\ (\text{mealplan}(z) \wedge \text{buys}(x,z) \wedge \text{faculty}(x) \Rightarrow \text{expensive}(z))}{\text{expensive}(BPM)}$$

$\theta = \{z/BPM,\ x/BP\}$

Apply Gen. mod. Pon. again to R3:

$$\frac{\text{mealplan}(BPM),\ \text{buys}(BP, BPM),\ \text{expensive}(BPM),\ (\text{mealplan}(z) \wedge \text{buys}(x,z) \wedge \text{expensive}(z) \Rightarrow \neg\text{student}(x))}{\neg\text{student}(BP)}$$

$\theta = \{z/BPM,\ x/BP\}$

we can't say anything about BP on campus

we have on campus ⇒ buy

we would need buy ⇒ on campus

Question 4. Decision Tree Learning

Sarah organizes a local science fiction book club. She plans to build an agent that will decide whether to buy a book for the club. It will eventually search the internet. But first, she has to learn a decision tree classifier based on books the club has already read. The classifier will learn to identify good books by their attributes:

- Space - the story is not set on Earth
- Time - the story involves someone traveling through time
- Heroine - the protagonist is a female

	Space	Time	Heroine	Good?
1	1	1	0	0
2	1	1	1	1
3	1	0	1	1
4	0	1	1	1
5	1	1	0	0
6	0	0	0	0
7	0	0	1	0
8	0	1	1	1

TRUE:

space: $\frac{2}{5}B(1) + \frac{3}{5}B(\frac{2}{3}) = 0.551$

time: $\frac{3}{5}B(1) + \frac{2}{5}B(\frac{1}{2}) = 0.4$ ✓

$H(Y|\text{hero}=1) = B(\frac{4}{5}) = 0.729$ SPLIT

FALSE:

space: $\frac{2}{3}B(0) + \frac{1}{3}B(0) = 0$

time: $\frac{2}{3}B(0) + \frac{1}{3}B(0) = 0$

$H(Y|\text{hero}=F) = B(0) = 0$ NO SPLIT

When determining the root for her decision tree, Susan calculates that Gain(Heroine)=0.54, Gain(Space)=0.00, and Gain(Time)=0.05. She picks **heroine** to be the root.

The questions start on the next page, break any ties between attributes alphabetically.

You may want to use the following table of logarithms:

$$\log_2(1/3) = -1.585$$
$$\log_2(2/3) = -0.585$$
$$\log_2(1/5) = -2.322$$
$$\log_2(2/5) = -1.322$$
$$\log_2(3/5) = -0.737$$
$$\log_2(4/5) = -0.322$$
$$\log_2(3/8) = -1.415$$
$$\log_2(5/8) = -0.678$$

6 pts

4(a) Consider the **TRUE** branch of the heroine node.

3 pts

Is another attribute necessary for this branch? $H = B(4/5) = 0.722$ YES

4 pts

If so, calculate Gain(Space) and Gain(Time). Which attribute should go next?

2 pts/gain

Show your calculations to be eligible for partial credit.

		GAIN
SPACE:	$2/5\, B(1) + 3/5\, B(2/3) = 0.551$	$0.722 - 0.551 = \underline{0.171}$
✓ time:	$3/5\, B(1) + 2/5\, B(1/2) = 0.4$	$0.722 - 0.4 = \underline{0.322}$

space: $\frac{2}{5}[0] + \frac{3}{5}\underbrace{\left[-\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3}\right]}_{0.9183} \doteq 0.551$

−1pt wrong entropy ⇒ 0.722

time: $\frac{3}{5}(0) + \frac{2}{5}[1] = 2/5 = 0.4$

6 pts total

4(b) Consider the **FALSE** branch of the heroine node.

2 pts Is another attribute necessary for this branch? $H = B(0) = 0$ No

4 pts If so, calculate Gain(Space) and Gain(Time). Which attribute should go next?

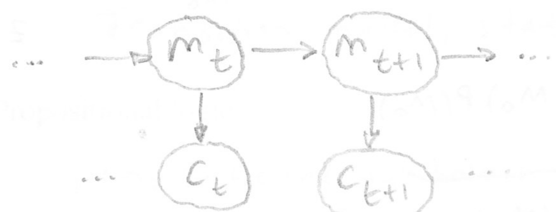
Show your calculations to be eligible for partial credit. No

Question 5. Dynamic Bayesian Networks

The AI faculty decide to build an agent to guess the mood of Dean Zvi Galil from hour to hour (so visits to the Dean's office can be carefully timed). After detailed observation, they determine that when the Dean is in a good mood, there is a 75% chance he will be in a good mood an hour later. But when he is in a bad mood, there is a 50% chance he will be in a bad mood an hour later. Unfortunately the Dean's mood can't be observed directly. However, through careful study the faculty have determined that when the Dean is in a good mode, the likelihood that he will send a reminder email about CIOS is 80%, but there is only a 60% chance of sending a CIOS email when he is in a bad mood.

2 pts

(a) Draw a dynamic Bayesian network model for this problem. Be sure to show at least two time slices. Label the nodes in your graph. Use the random variable M to indicate the Dean's mood. M can take on the values "good" or "bad." Use the random variable C to indicate whether a CIOS email was sent. C can take on the values "true" or "false." Use subscripts on M and C to indicate time slices.



4 pts

(b) Fill in the conditional probability tables for the transition model and sensor model elements of the DBN. Be sure the columns and rows are labeled properly in the spaces provided.

Transition Model: $P(M_{t+1} \mid M_t)$

	M_t = bad	M_t = good
M_{t+1} = bad	0.5	0.25
M_{t+1} = good	0.5	0.75

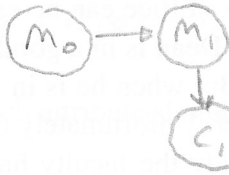
Sensor Model: $P(C_t \mid M_t)$

	M_t = bad	M_t = good
C_t = F	0.4	0.2
C_t = T	0.6	0.8

2 pts **(c)** The faculty have no knowledge of the Dean's mood prior to the first day of Finals Week. Give the distribution for the Dean's mood.

$P(M_0) =$ 0.5

$P(\neg M_1) = 0.5$



4 pts. **(d)** On the first day of Finals Week, the Dean sends a CIOS reminder email. Compute the normalized probability distribution for the Dean's mood on the first day of Finals Week, given that $C = 1$. Show your work to be eligible for partial credit.

$P(M_1|C_1 = 1) =$

$$P(M_1|C_1) = \sum_{M_0} P(M_1, M_0 | C_1) = \frac{1}{Z}\sum_{M_0} P(M_1, M_0, C_1) = \frac{1}{Z}\sum_{M_0} \overset{P(C_1|M_1)}{P(C_1|M_1\cancel{, M_0})} P(M_1|M_0) P(M_0)$$

$$Z = P(C_1)$$

$$= \frac{1}{P(C_1)} P(C_1|M_1) \underbrace{\sum_{M_0} P(M_1|M_0) P(M_0)}_{P(M_1)}$$

		$P(M_1)$
$M_1 = b:$	$0.5 \times 0.5 + 0.25 \times 0.5 =$	0.375
$M_1 = g:$	$0.5 \times 0.5 + 0.75 \times 0.5 =$	0.625

$$= \frac{1}{P(C_1)} \underbrace{P(C_1|M_1) P(M_1)}_{P(C_1, M_1)}$$

$M_1 \backslash C_1$	0	1
b	0.15	0.225
g	0.125	0.5

$P(M_1, C_1 = 1)$

$P(C_1 = 1|M_1 = b) P(M_1 = b) = 0.6 \times 0.375 = 0.225$

$P(C_1 = 1|M_1 = g) P(M_1 = g) = 0.8 \times 0.625 = 0.5$

$\Downarrow$

$P(C_1 = 1) = 0.725 = Z$

$$P(M_1|C_1 = 1) = \boxed{\begin{array}{ll} M_1 = b & \frac{0.225}{0.725} = 0.31 \\ M_1 = g & \frac{0.5}{0.725} = 0.69 \end{array}}$$

6 pts

Question 6. Knowledge Representation

Some people define AI as knowledge representation plus efficient algorithms. In each of the AI techniques below, very briefly describe how knowledge is represented. [Hint: you should just need a couple words or terms each.]

1pt per bullet

- Informed Search:

  Heuristic function + state/action model

- Markov decision processes:

  (NOT utilities)

  Transition model, reward function, state model

- Dynamic Bayesian network:

  Transition model, state model, sensor model

- Propositional logic:

  propositional symbols + sentences in KB

- Neural network:

  Network weights + training set plays a role

- Constraint satisfaction:

  Variables + constraints

12 pts

2 pts per production rule drawn correctly

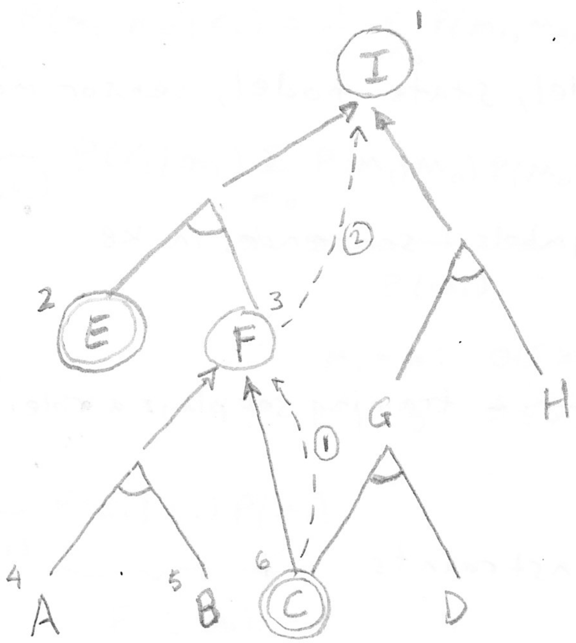
2 pts for back-chaining

Question 7. Horn Clauses

Consider the following set of Horn clauses which define a knowledge base:

$A \wedge B \Rightarrow F$
$C \Rightarrow F$
$C \wedge D \Rightarrow G$
$G \wedge H \Rightarrow I$
$E \wedge F \Rightarrow I$

(a) Draw the AND-OR tree associated with this set of clauses.



Input: $E \wedge C$

Output: $F \wedge I$

(b) Suppose that E and C are true (ie asserted in our knowledge base KB). Use backward chaining in the above AND-OR graph to demonstrate that $KB \models I$. Show your work by indicating the order in which nodes are visited and by indicating the nodes that are instantiated.

Visited: I, E, F, A, B, C

Instantiated: E, C, F, I