```
function BACK-PROP-LEARNING(examples, network) returns a neural network
  inputs: examples, a set of examples, each with input vector x and output vector y
          network, a multilayer network with L layers, weights w_{i,j}, activation function g
  local variables: Δ, a vector of errors, indexed by network node

  repeat
    for each weight w_{i,j} in network do
      w_{i,j} ← a small random number
    for each example (x, y) in examples do
      /* Propagate the inputs forward to compute the outputs */
      for each node i in the input layer do
        a_i ← x_i
      for ℓ = 2 to L do
        for each node j in layer ℓ do
          in_j ← Σ_i w_{i,j} a_i
          a_j ← g(in_j)
      /* Propagate deltas backward from output layer to input layer */
      for each node j in the output layer do
        Δ[j] ← g'(in_j) × (y_j − a_j)
      for ℓ = L − 1 to 1 do
        for each node i in layer ℓ do
          Δ[i] ← g'(in_i) Σ_j w_{i,j} Δ[j]
      /* Update every weight in network using deltas */
      for each weight w_{i,j} in network do
        w_{i,j} ← w_{i,j} + α × a_i × Δ[j]
  until some stopping criterion is satisfied
  return network
```

Figure 18.24 The back-propagation algorithm for learning in multilayer networks.

Now the weight-update rule for the weights between the inputs and the hidden layer is essentially identical to the update rule for the output layer:

$$w_{i,j} \leftarrow w_{i,j} + \alpha \times a_i \times \Delta_j \ .$$

The back-propagation process can be summarized as follows:

- Compute the Δ values for the output units, using the observed error.
- Starting with output layer, repeat the following for each layer in the network, until the earliest hidden layer is reached:
  - Propagate the Δ values back to the previous layer.
  - Update the weights between the two layers.

The detailed algorithm is shown in Figure 18.24.

For the mathematically inclined, we will now derive the back-propagation equations from first principles. The derivation is quite similar to the gradient calculation for logistic