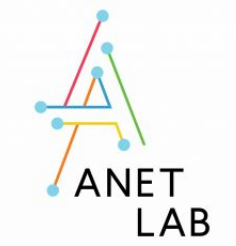
Network and Spatial Analyses
22. April 2020

Lecture:
Urban scaling

Seminar:
Urban scaling fits on OSM data

Course Github page: https://github.com/bokae/anet_course

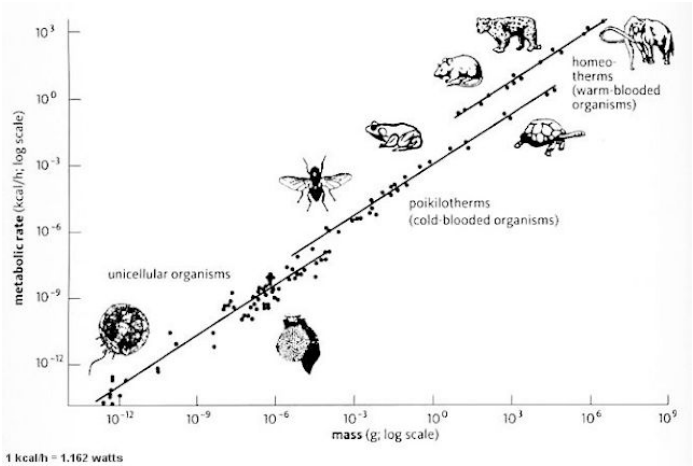


Motivation

Concepts from previous lectures

- Spiky world
- Clustering in global cities
- Spatial concentration -> positive externalities
- Returns to scale
- Urbanization economies
    - Advantages of region size
    - Size-efficient public services

Kleiber's law, first scaling relationships



$$R \propto M^{\frac{3}{4}}$$

$$\downarrow$$

$$Y \propto N^{\beta}$$

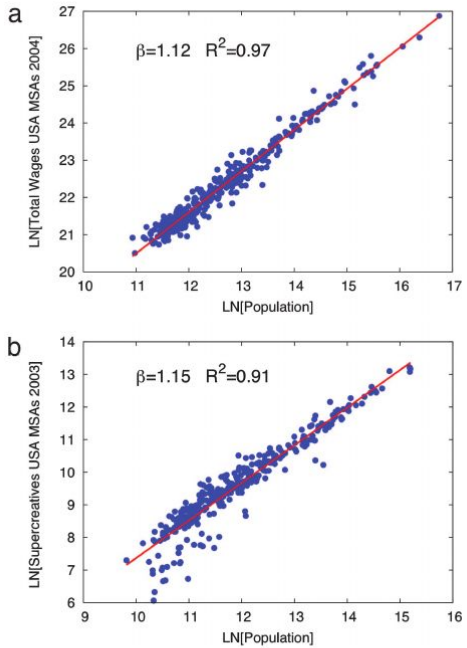


Fig. 1. Examples of scaling relationships. (*a*) Total wages per MSA in 2004 for the U.S. (blue points) vs. metropolitan population. (*b*) Supercreative employment per MSA in 2003, for the U.S. (blue points) vs. metropolitan population. Best-fit scaling relations are shown as solid lines.

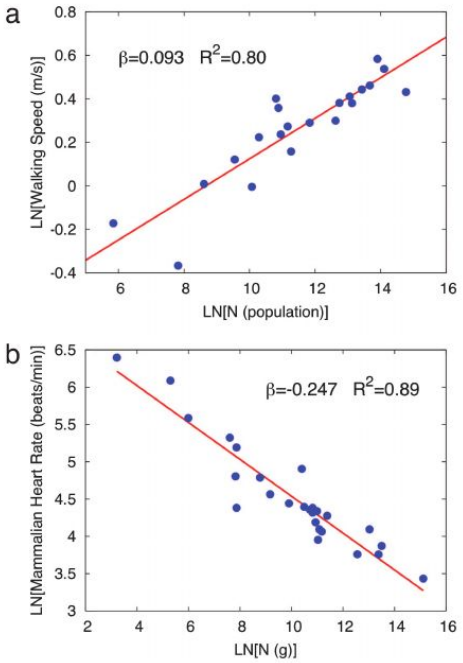
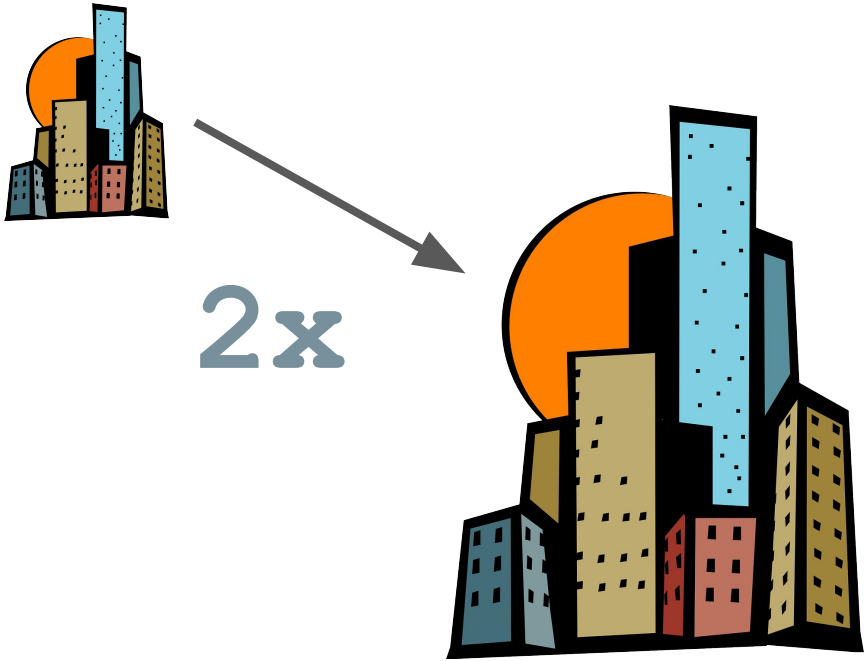


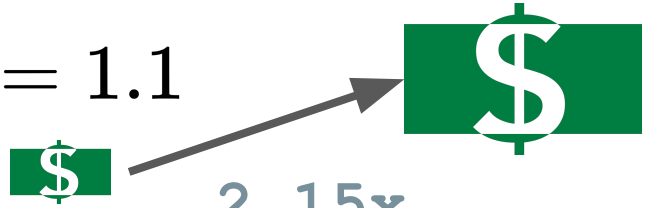
Fig. 2. The pace of urban life increases with city size in contrast to the pace of biological life, which decreases with organism size. (*a*) Scaling of walking speed vs. population for cities around the world. (*b*) Heart rate vs. the size (mass) of organisms.

Bettencourt (2007)

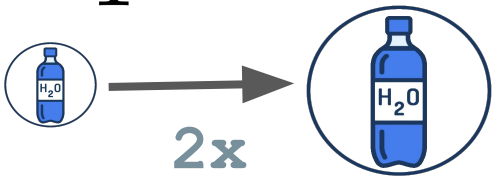
What do exponents mean?



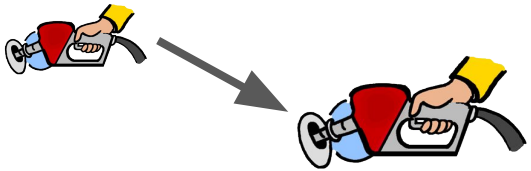
$\beta = 1.1$



$\beta = 1$



$\beta = 0.9$



Sample exponents

Table 1. Scaling exponents for urban indicators vs. city size

Y	β	95% CI	Adj-R^2	Observations	Country–year
New patents	1.27	[1.25,1.29]	0.72	331	U.S. 2001
Inventors	1.25	[1.22,1.27]	0.76	331	U.S. 2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	U.S. 2002
"Supercreative" employment	1.15	[1.11,1.18]	0.89	287	U.S. 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	U.S. 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
Total wages	1.12	[1.09,1.13]	0.96	361	U.S. 2002
Total bank deposits	1.08	[1.03,1.11]	0.91	267	U.S. 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999–2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
Total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
New AIDS cases	1.23	[1.18,1.29]	0.76	93	U.S. 2002–2003
Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003
Total housing	1.00	[0.99,1.01]	0.99	316	U.S. 1990
Total employment	1.01	[0.99,1.02]	0.98	331	U.S. 2001
Household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	U.S. 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	U.S. 2001
Length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
Road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

Data sources are shown in *SI Text*. CI, confidence interval; Adj-R^2, adjusted R^2; GDP, gross domestic product.

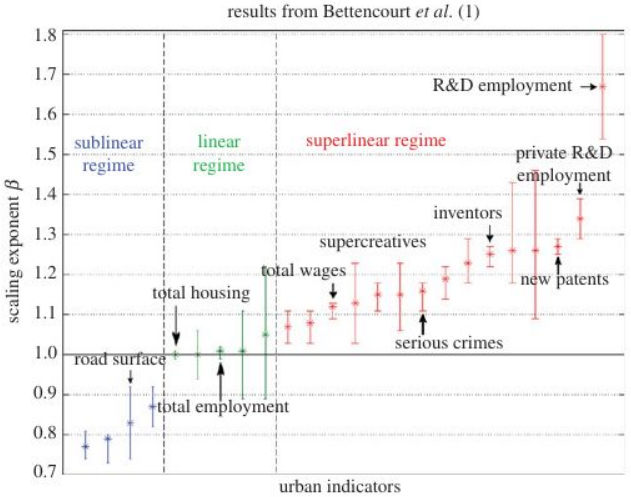


Figure 1. Exponents with 95% CI for different urban indicators found for the USA, Germany and China in reference [14]. These are colour-coded according to their regime. (Online version in colour.)

Bettencourt (2007)
Arcaute (2016)

Implications on urban growth processes

$$\frac{dN(t)}{dt} = \left(\frac{Y_0}{E}\right) N(t)^{\beta} - \left(\frac{R}{E}\right) N(t).$$

Solutions for different exponents -> different growth scenarios

Table 2. Classification of scaling exponents for urban properties and their implications for growth

Scaling exponent	Driving force	Organization	Growth
$\beta < 1$	Optimization, efficiency	Biological	Sigmoidal: long-term population limit
$\beta > 1$	Creation of information, wealth and resources	Sociological	Boom/collapse: finite-time singularity/unbounded growth; accelerating growth rates/discontinuities
$\beta = 1$	Individual maintenance	Individual	Exponential

Bettencourt (2007)

Scale-adjusted metropolitan indicators

$$D_{Y_i}(t) = \log Y_i(t) - [\mathcal{A}_i + \beta_i \log N(t)] .$$

"Detrending" per-capita indicators

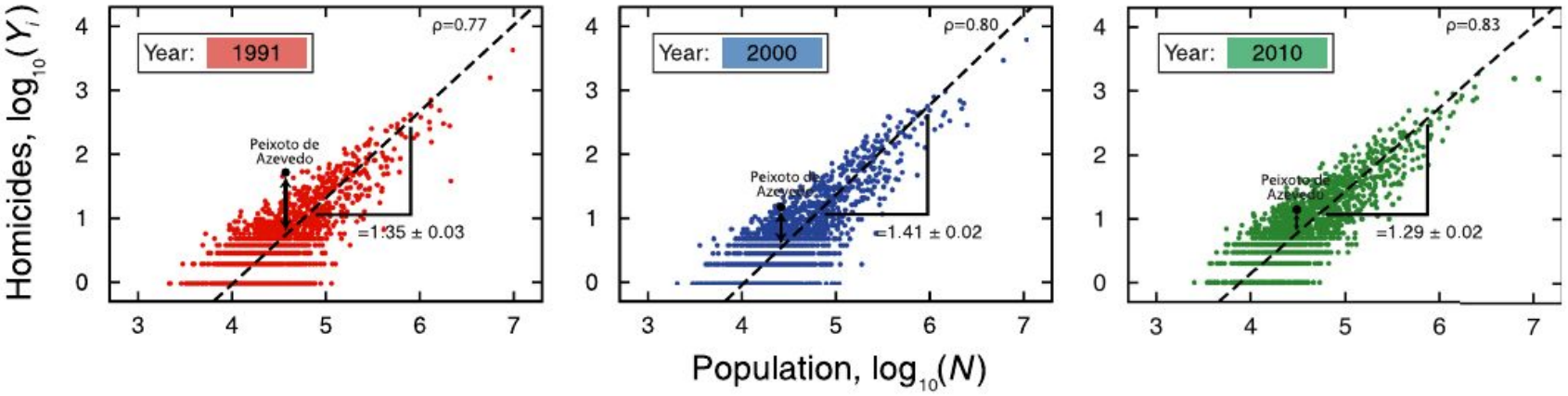
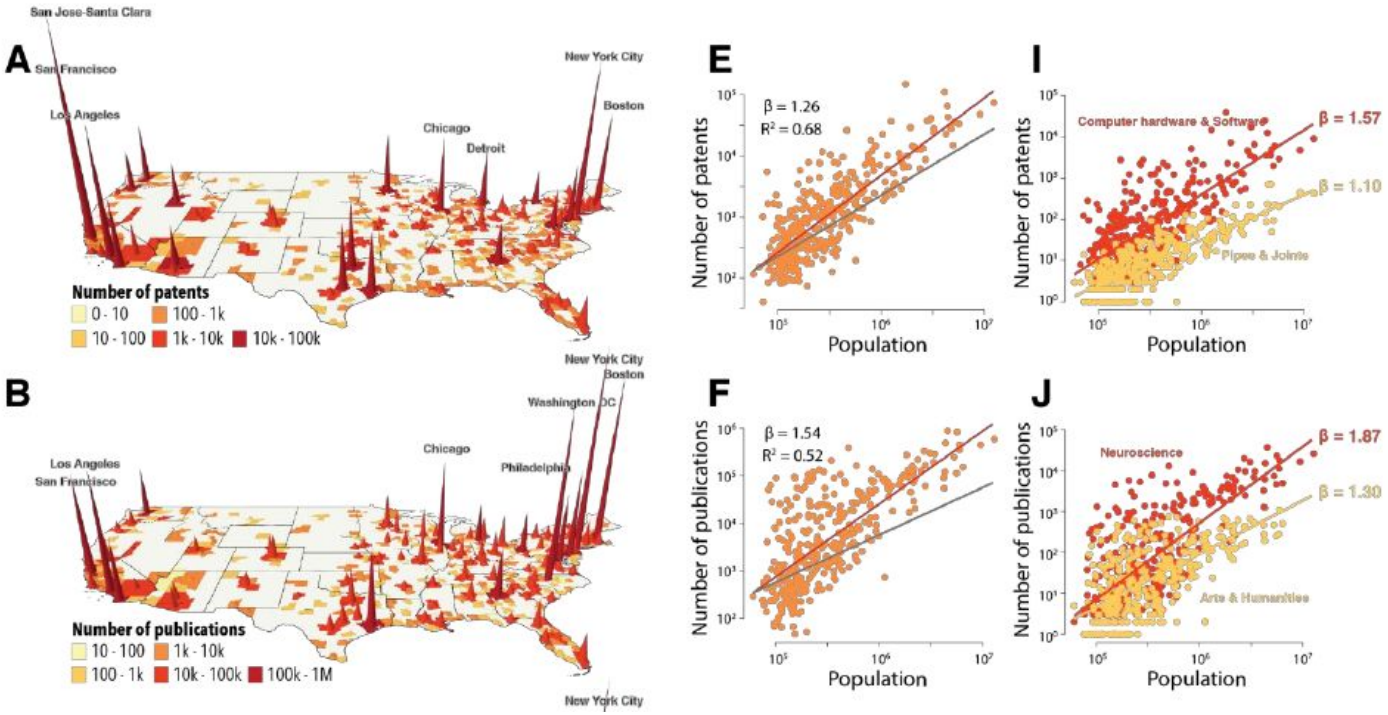


Fig 2. Allometric laws and the definition of the scale-adjusted metric $D_{Y_i}(t)$. The scatter plots show the allometric relationships between number of homicides and population size for the years t = 1991, 2000 and 2010 in log-log scale (see S1 and S2 Figs for all other indicators). The allometric exponents β_i (see Methods Section for details on the calculation of β_i) and Pearson correlation coefficient ρ are shown in the figures. We highlight a particular city (Peixoto de Azevedo) in the three years to illustrate the definition and the evolution $D_{Y_i}(t)$. For this city, the number homicides was quite above the allometric law in the year t = 1991; however, it has approached the expected value by the allometric law over the years.

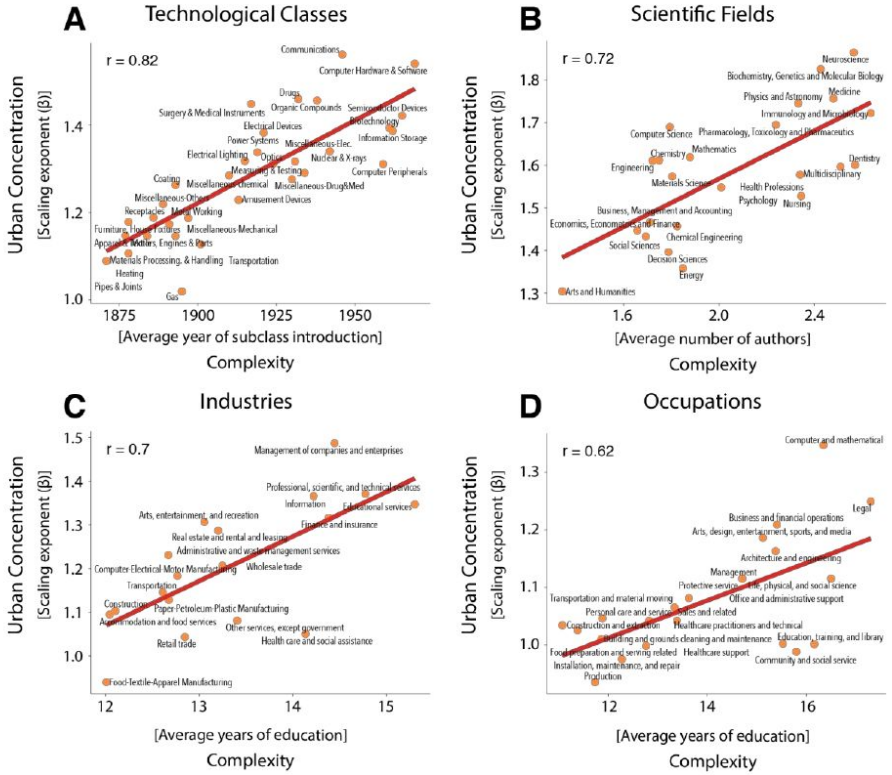
Alves (2015)

Concentration of complex economic activities



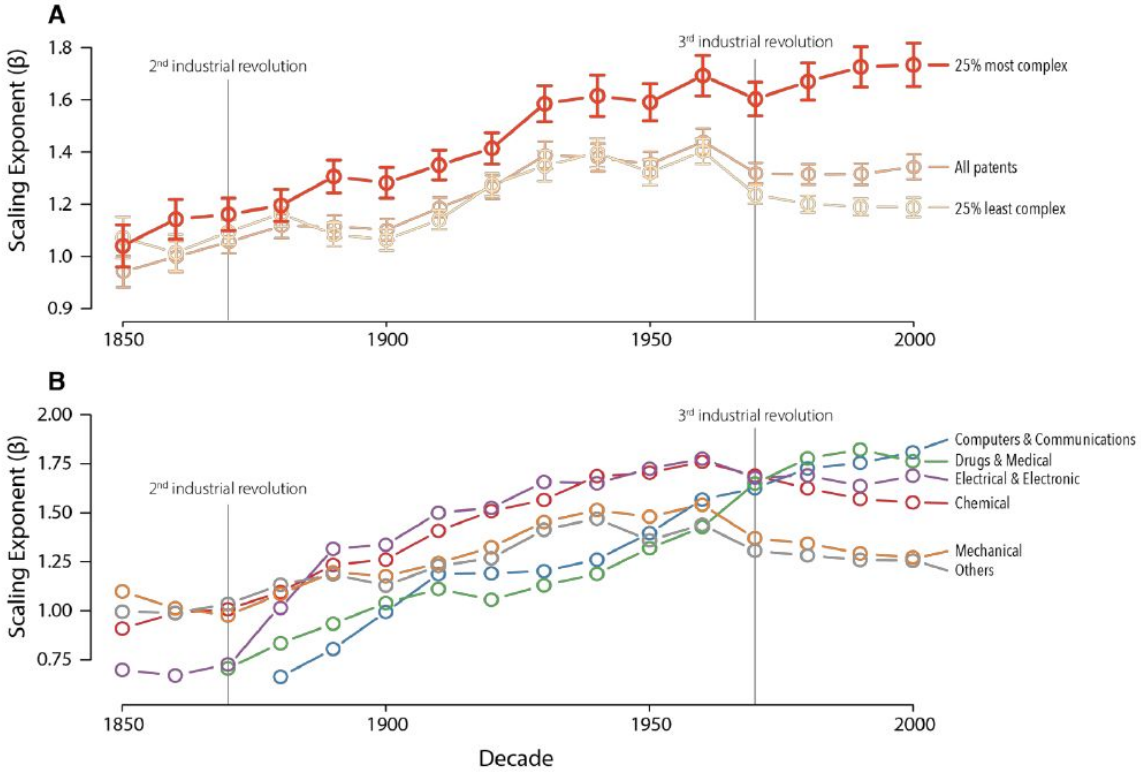
Balland (2018)

Concentration of complex economic activities



Balland (2018)

Time evolution of scaling exponents



Balland (2018)

Criticisms I. - MAUP

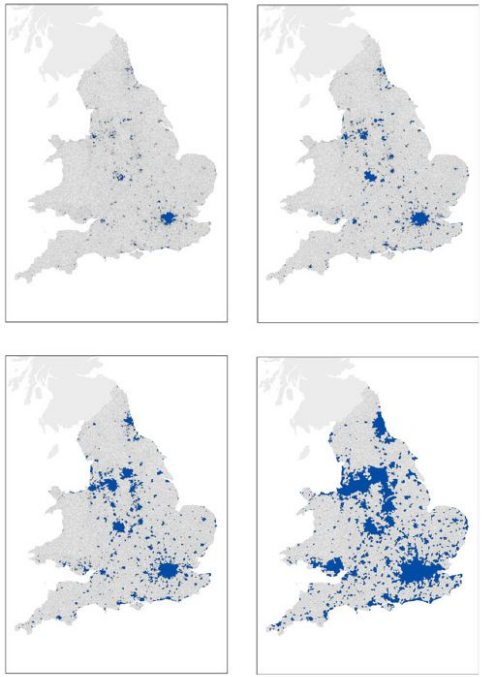


Figure 2. Sample of configurations of cities for four different density cut-offs. From top left to bottom right: $\rho = 40$, $\rho = 24$, $\rho = 10$ and $\rho = 2$ prs ha^{-1}. (Online version in colour.)

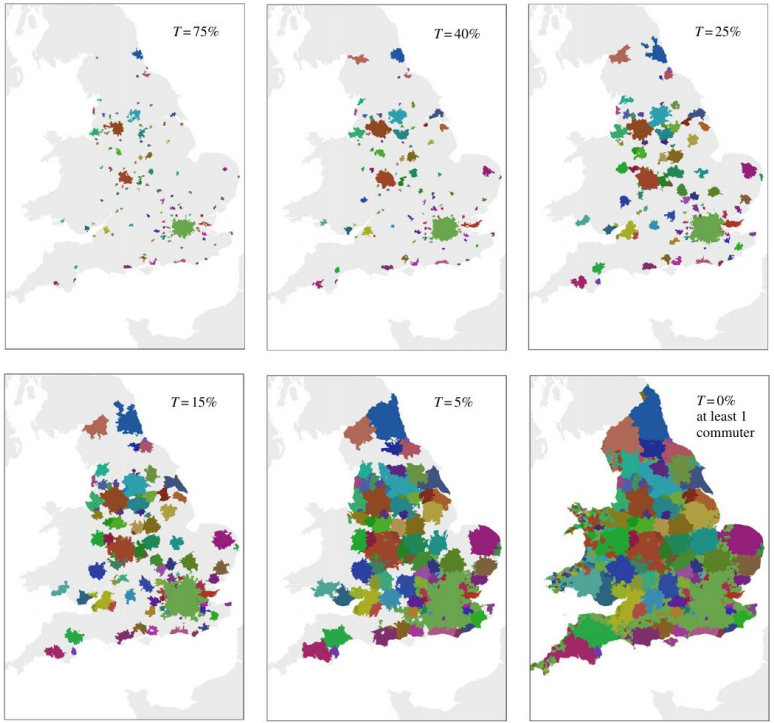


Figure 4. Realizations of metropolitan areas at fixed density cut-off of $\rho_c = 14$ prs ha^{-1} and a minimum population size of 5×10^4 individuals for a selection of several commuting flow thresholds τ.

Arcaute (2016)

Criticisms I. - MAUP

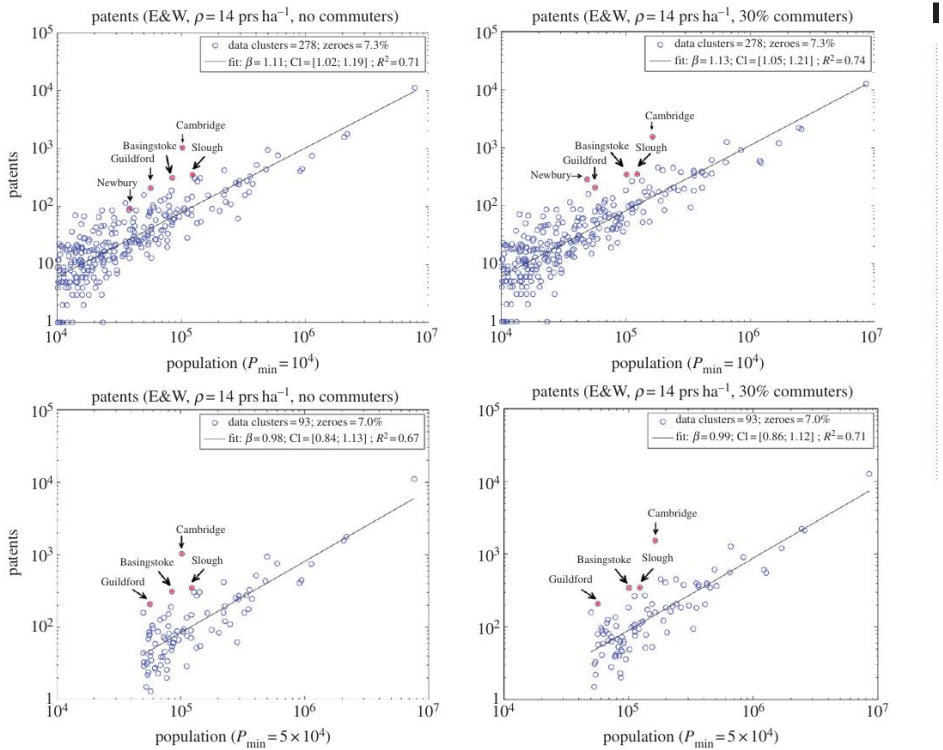
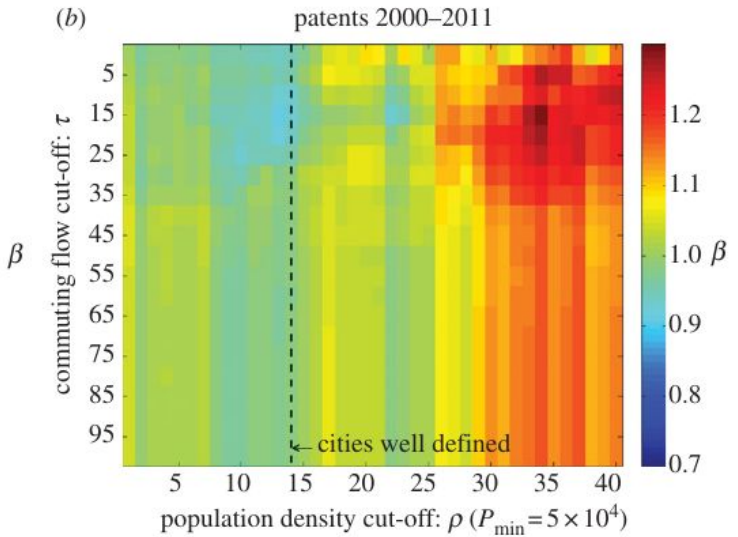


Figure 6. Scatter plots of patents for two different population size cut-offs. The top plots have a minimum population size of 10^4 individuals, whereas the bottom ones have a cut-off of 5×10^4 individuals. (Online version in colour.)



Arcaute (2016)

Criticisms II. - Unsophisticated fitting method

The distribution of city populations has heavy tails (Zipf's law)

There is heteroskedasticity

Fitting methods accounting for error behaviour: Maximum Likelihood Estimators
Allows for hypothesis testing as well!

		City model				Person model
		Log-normal		Gaussian		
	database	$\delta = 2$ (min. sq. fit)	$\delta \in [1,3]$	$\delta = 1$	$\delta \in [1,2]$	
UK	Income	**0.97**(0.04) $\rightarrow^*$	0.97(0.04) $\rightarrow^*$	1.07(0.15) $\nearrow$	0.97(0.05) $\rightarrow$	1.06(0.15) $\nearrow$
UK	Railroads	**1.04**(0.13) $\rightarrow^*$	1.03(0.10) $\rightarrow^*$	1.21(0.10) $\nearrow$	1.05(0.12) $\rightarrow$	1.19(0.16) $\nearrow$
UK	Patents	**0.99**(0.12) $\rightarrow$	0.97(0.13) $\rightarrow$	1.07(0.26) $\rightarrow$	0.96(0.27) $\rightarrow$	1.04(0.25) $\nearrow$
USA	GDP	**1.11**(0.03) $\nearrow$	1.11(0.02) $\nearrow$	1.14(0.03) $\nearrow$	1.12(0.02) $\nearrow$	1.12(0.02) $\nearrow$
USA	Roads	0.85(0.03) $\searrow$	**0.82**(0.03) $\searrow$	0.82(0.04) $\searrow$	0.81(0.04) $\searrow$	0.81(0.04) $\searrow$
EU	Cinema Cap.	1.00(0.09) $\rightarrow$	**0.90**(0.06) $\circ$	0.95(0.07) $\rightarrow$	0.92(0.06) $\circ$	0.93(0.08) $\searrow$
EU	Cinema Usa.	1.46(0.19) $\nearrow$	1.00(0.30) $\rightarrow$	1.13(0.13) $\nearrow$	**1.13**(0.11) $\circ$	1.06(0.09) $\nearrow$
EU	Museums Usa.	**1.42**(0.12) $\nearrow$	1.35(0.15) $\nearrow$	1.80(0.34) $\nearrow$	1.39(0.22) $\nearrow$	1.46(0.21) $\nearrow$
EU	Theaters	0.91(0.09) $\rightarrow$	**0.92**(0.11) $\rightarrow$	1.47(0.30) $\nearrow$	1.14(0.19) $\nearrow$	1.09(0.16) $\nearrow$
EU	Libraries	0.80(0.10) $\searrow$	0.81(0.09) $\searrow$	0.87(0.14) $\searrow$	**0.78**(0.08) $\searrow$	0.76(0.12) $\searrow$
OECD	GDP	1.12(0.06) $\nearrow$	1.11(0.05) $\nearrow$	1.09(0.08) $\nearrow$	**1.13**(0.05) $\nearrow^*$	1.05(0.07) $\nearrow$
OECD	Patents	**1.29**(0.27) $\rightarrow$	1.12(0.21) $\rightarrow$	1.28(0.33) $\nearrow$	1.13(0.21) $\rightarrow$	1.08(0.17) $\nearrow$
Brazil	GDP	1.04(0.02) $\nearrow$	**1.04**(0.02) $\nearrow$	2.00(0.44) $\nearrow$	1.09(0.05) $\nearrow$	1.17(0.04) $\nearrow$
Brazil	Aids	0.74(0.03) $\searrow$	**0.61**(0.05) $\searrow$	1.13(0.11) $\nearrow$	0.81(0.04) $\searrow$	1.04(0.05) $\nearrow$
Brazil	External	1.03(0.01) $\nearrow$	**1.02**(0.01) $\circ$	1.04(0.04) $\nearrow$	1.02(0.01) $\nearrow$	1.01(0.03) $\nearrow$

1st col is OLS fitting
Error bars: bootstrapping
*: p>0.05
Delta BIC compared to beta=1:
Linear, inconclusive, sub/superlinear

Leitao (2016)

Criticisms II. - Unsophisticated fitting method

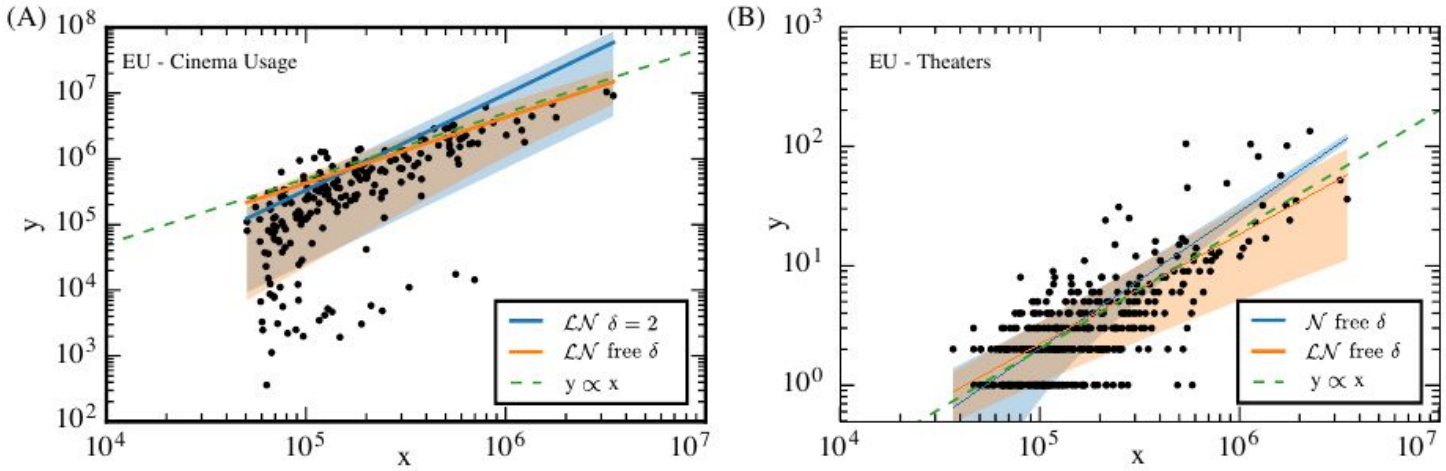


FIG. 2: Effect of fluctuations on the estimation of β. (A) In the "EU Cinema Usage" database, the log-normal model with $\delta = 2$ yields $\beta = 1.46$, while free δ yields $\beta = 1.00$. (B) In the "EU Theaters" database, the log-normal with free δ yields $\beta = 0.92$, a lower value than $\beta = 1.14$ obtained in the Gaussian model with free δ. Shaded areas represent the 68th-percentile (± 1 standard deviations) of $P(y \mid x)$.

Leitao (2016)

Criticisms III. - Unaccounted factors, bad theory

"Re-analysis of the gross economic production and personal income for cities in the United States, however, shows that the data cannot distinguish between power laws and other functional forms, including logarithmic growth, and that size predicts relatively little of the variation between cities. The striking appearance of scaling in previous work is largely artifact of using extensive quantities (city-wide totals) rather than intensive ones (per-capita rates). The remaining dependence of productivity on city size is explained by concentration of specialist service industries, with high value-added per worker, in larger cities, in accordance with the long-standing economic notion of the "hierarchy of central places"."

$$\ln y = \ln c + b \ln N + \sum_{j=1}^{4} f_j(x_j) + \epsilon,$$

R^2 is not a good metric for the goodness of logarithmic $\frac{\mathrm{Var}[\ln N]}{\mathrm{Var}[\ln N] + \mathrm{Var}[\ln y]}$

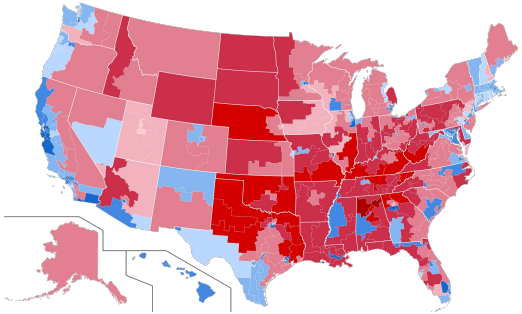
Shalizi (2011)

Urban scaling in metro area election results

Bokányi, E., Szállási, Z., & Vattay, G. (2018). Universal scaling laws in metro area election results. PLOS ONE, 13(2), e0192913. https://doi.org/10.1371/journal.pone.0192913

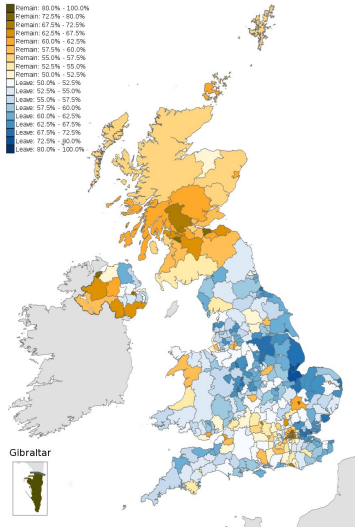
Conjecture I.

November 2016
Trump / Clinton?

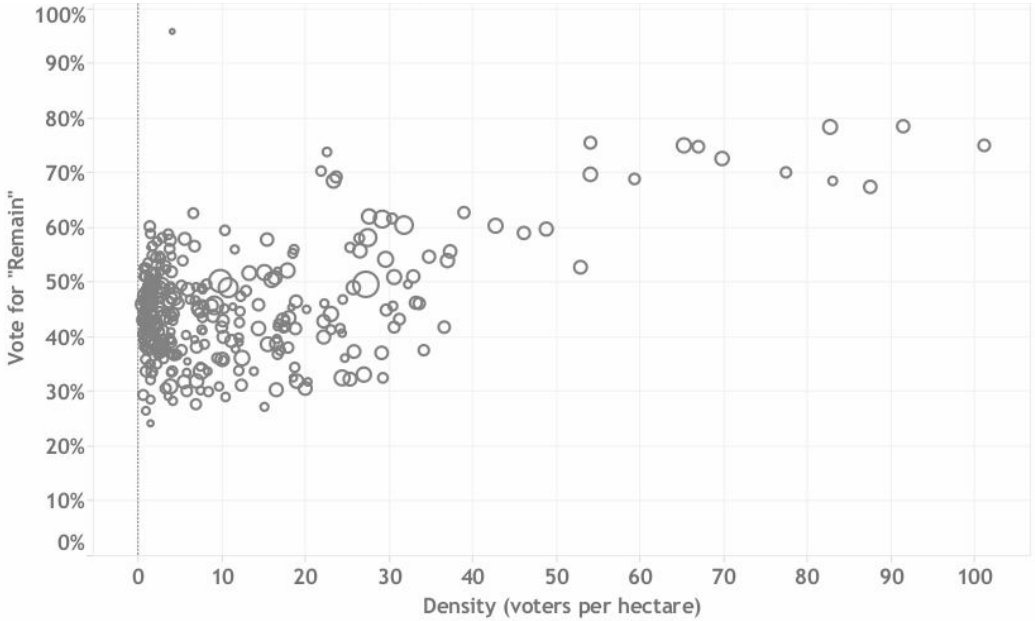


Conjecture II.

June 2016
EU Referendum, UK

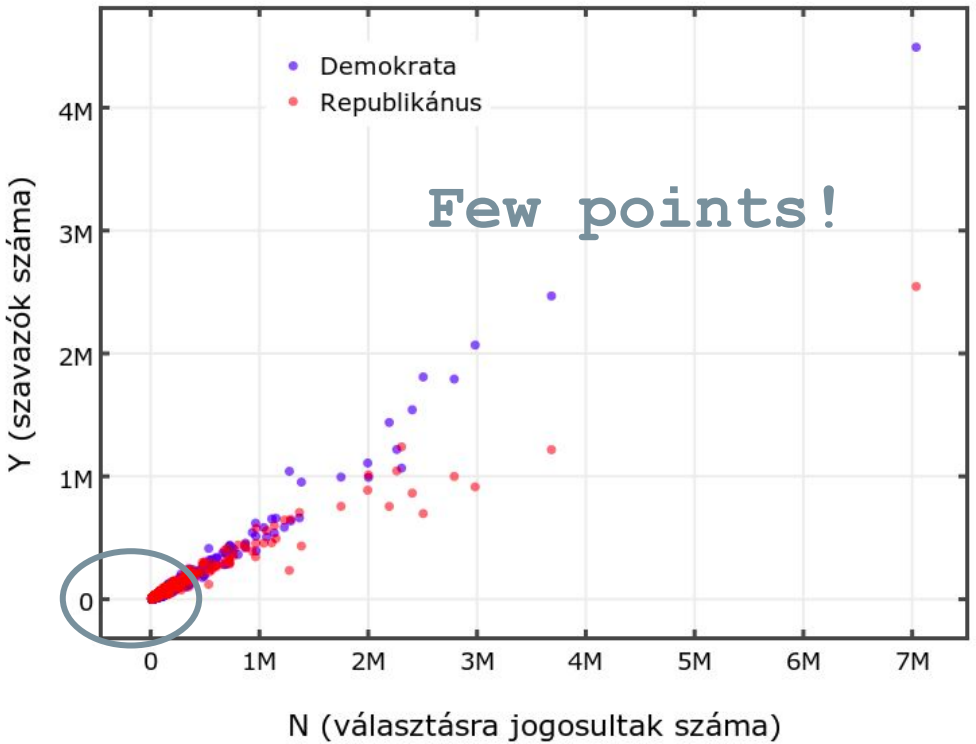


> Cities may dominate our culture, but a backlash against liberal values and multiculturalism has been led by rural and small-town voters.
>
> Amy Beckett - The Guardian



USA 2016

$$Y = Y_0 \cdot N^{\beta}$$



USA 2016

$$Y = Y_0 \cdot N^{\beta}$$

USA 2016

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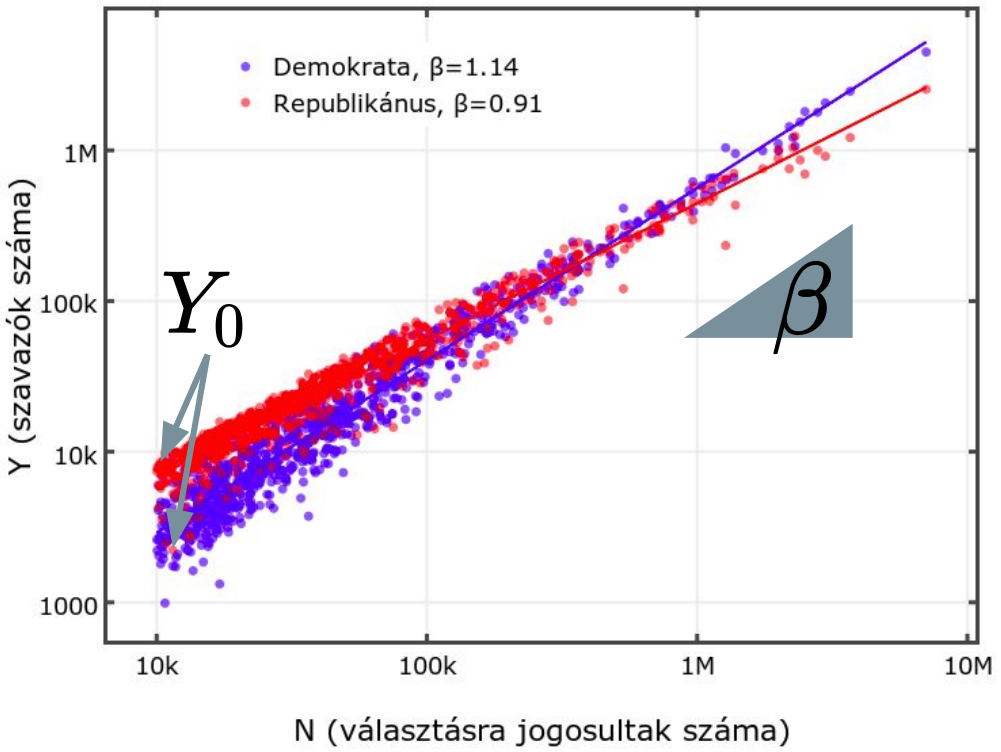
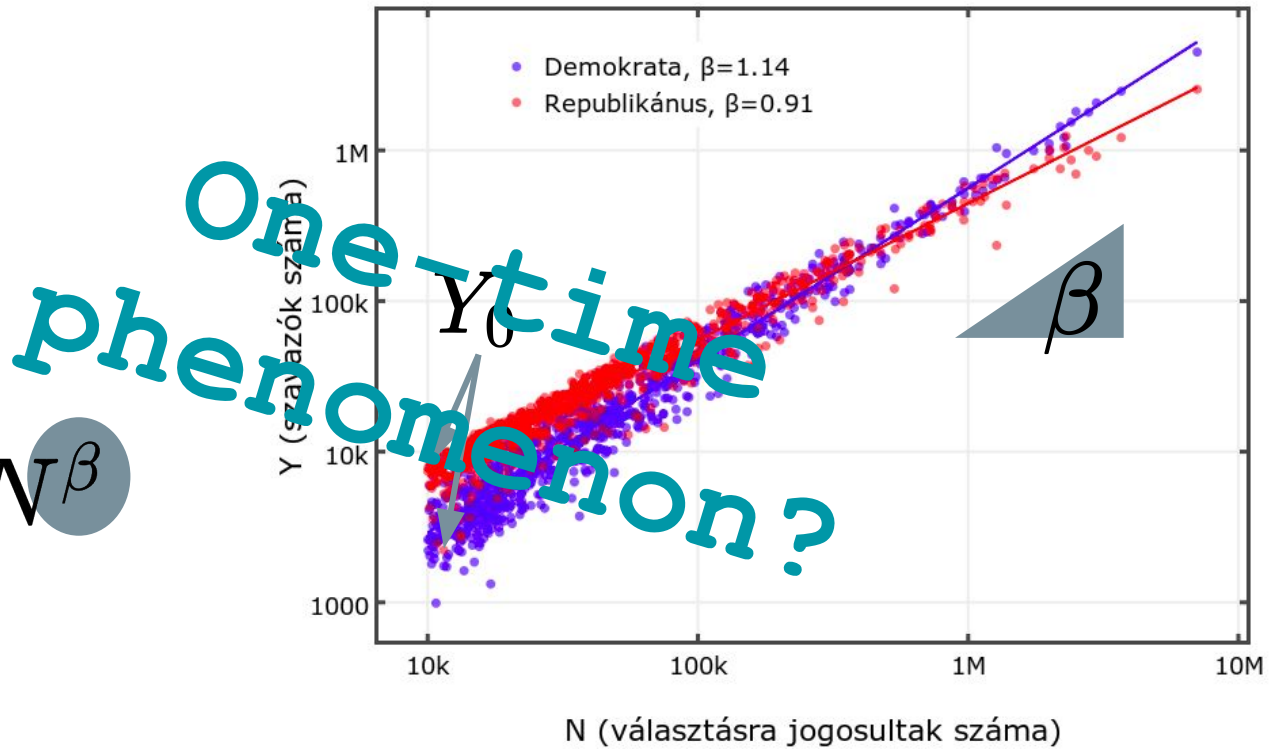
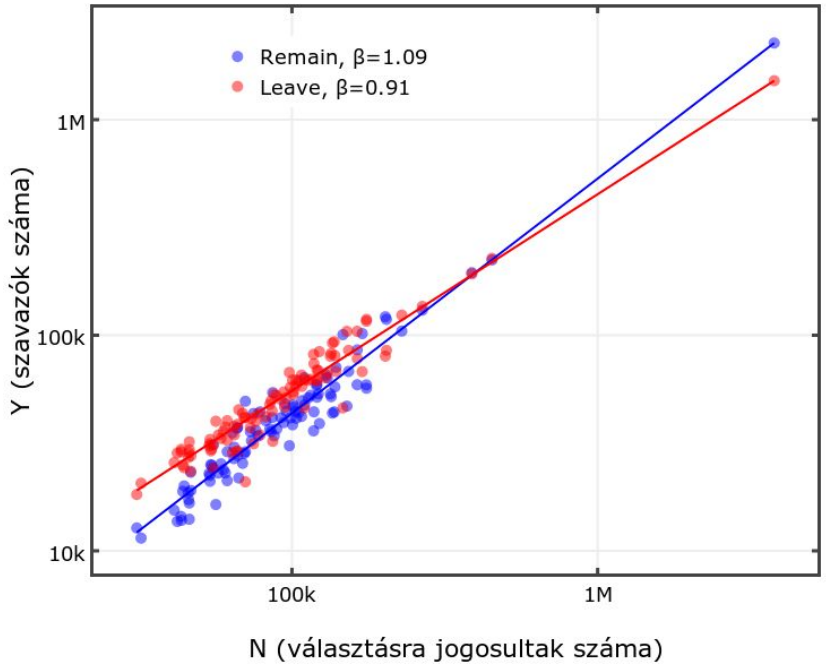


USA 2016

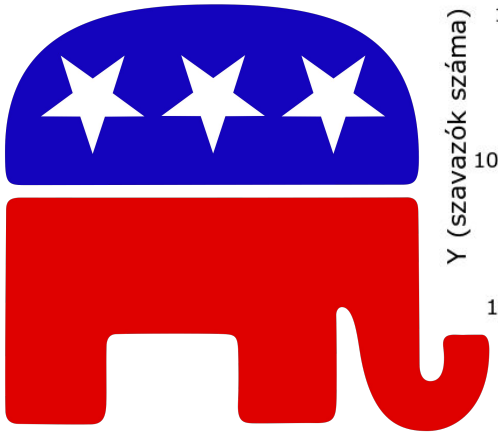
$$Y = Y_0 \cdot N^{\beta}$$

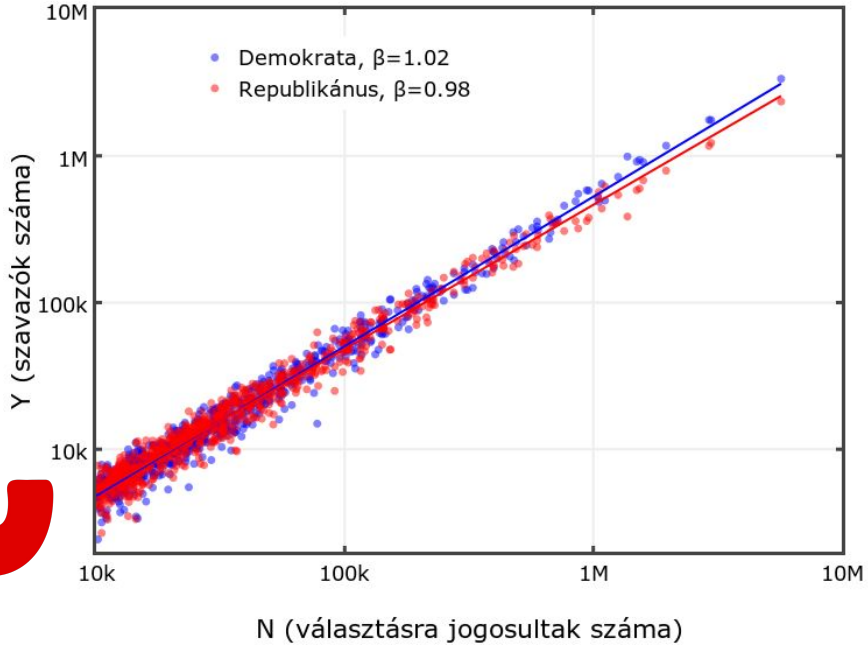


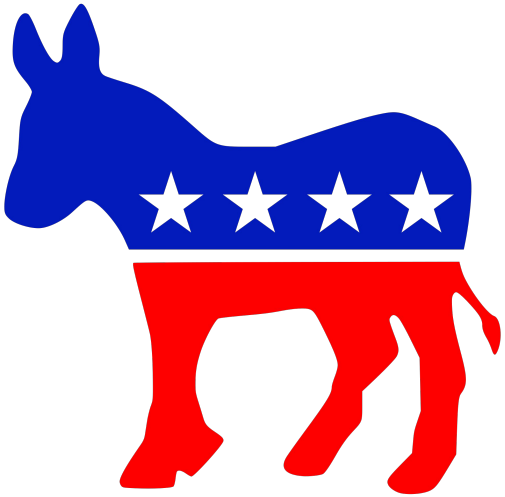
UK 2016 (térben máshol)



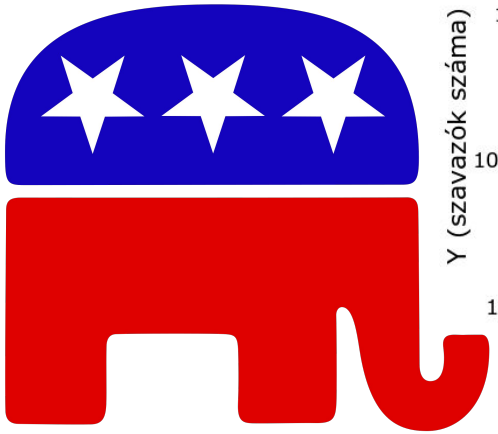
USA 1992 (different time)

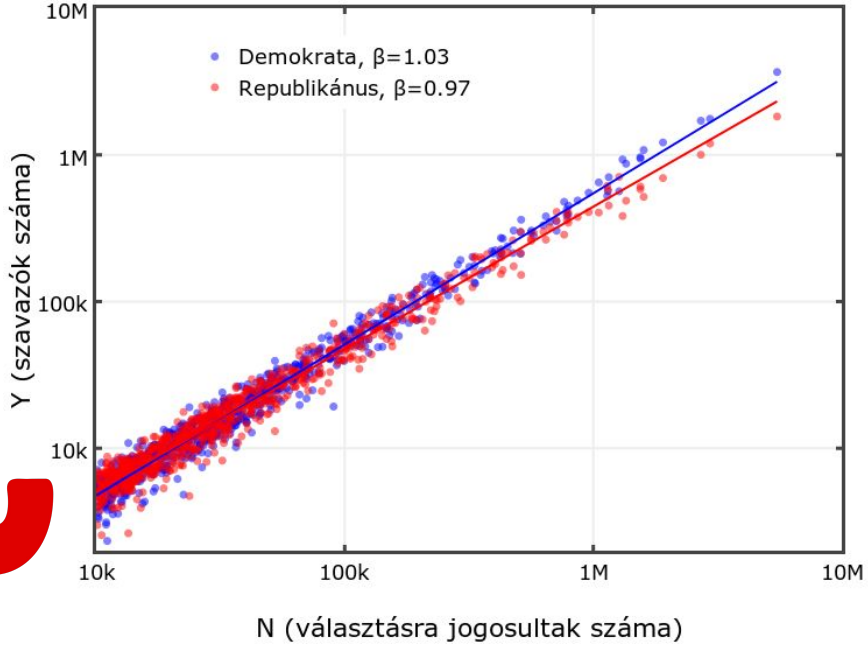


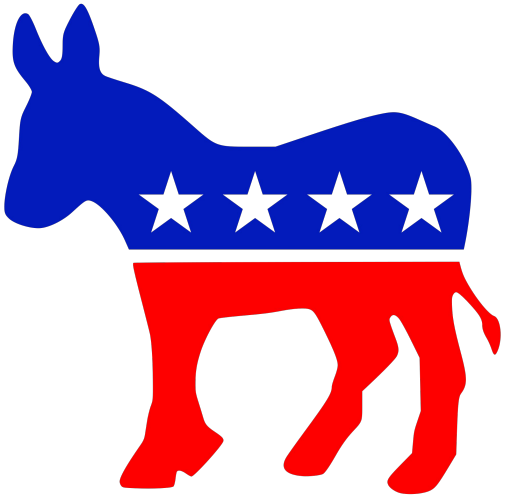




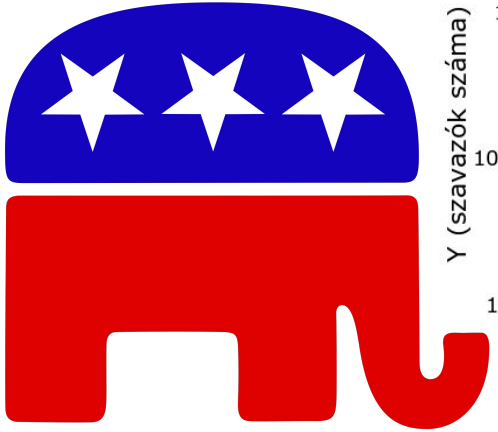
USA 1996

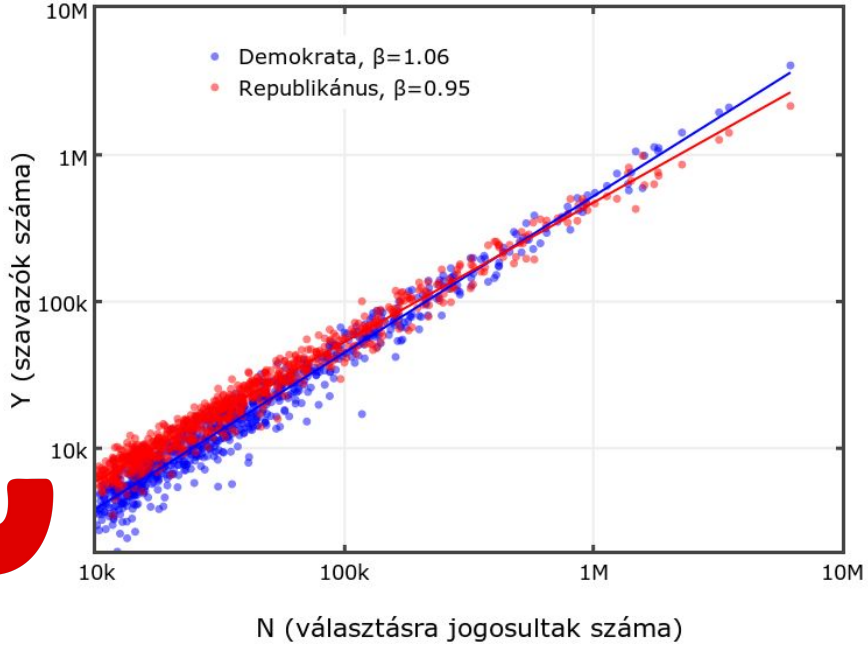


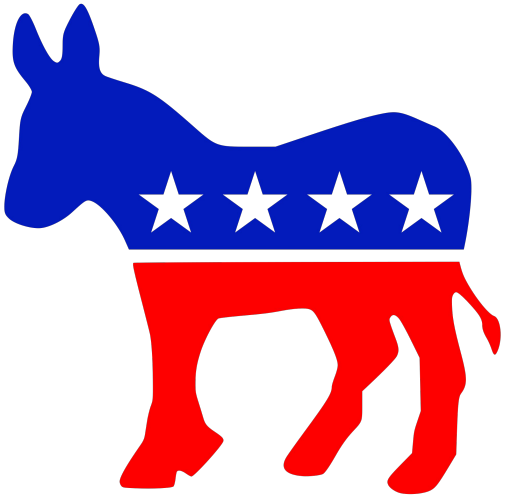

Demokrata, β=1.03
Republikánus, β=0.97
10M
1M
100k
10k
Y (szavazók száma)
10k
100k
1M
10M
N (választásra jogosultak száma)



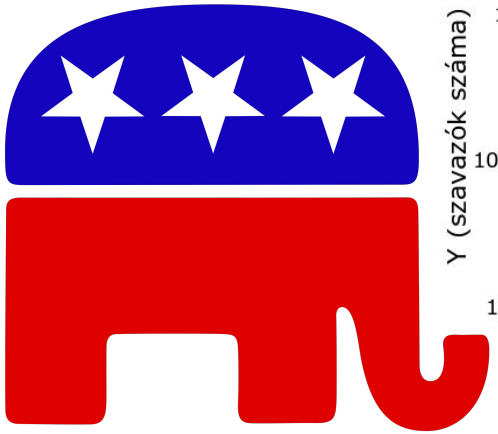
USA 2000

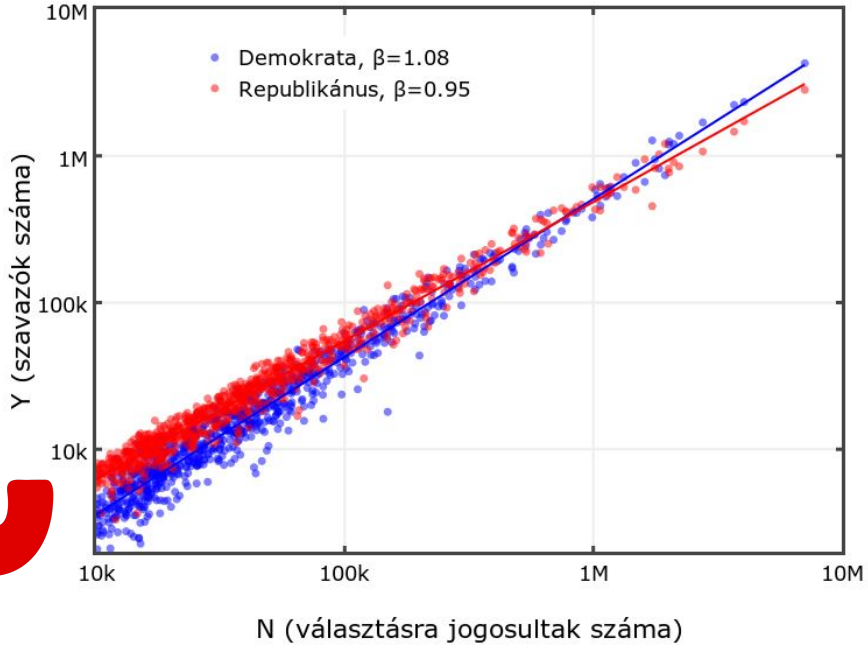


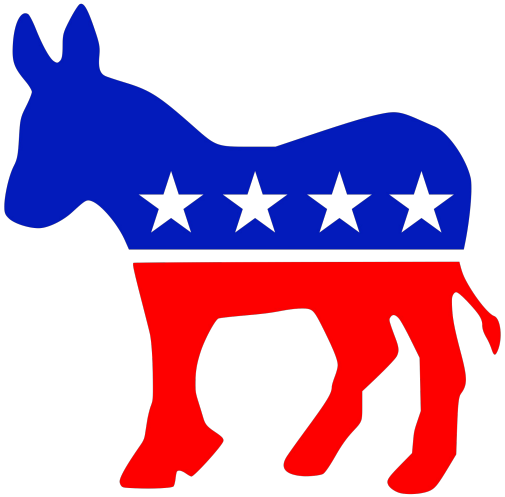

Demokrata, β=1.06
Republikánus, β=0.95
10M
1M
100k
10k
Y (szavazók száma)
10k
100k
1M
10M
N (választásra jogosultak száma)



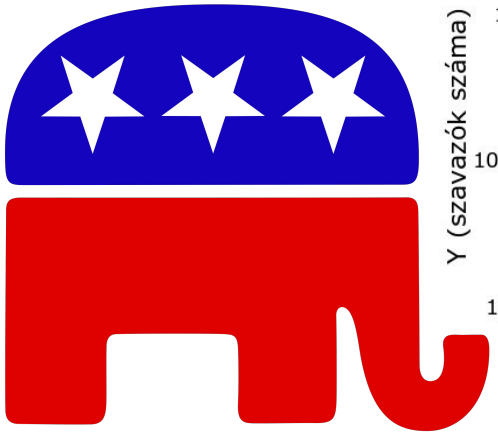
USA 2004

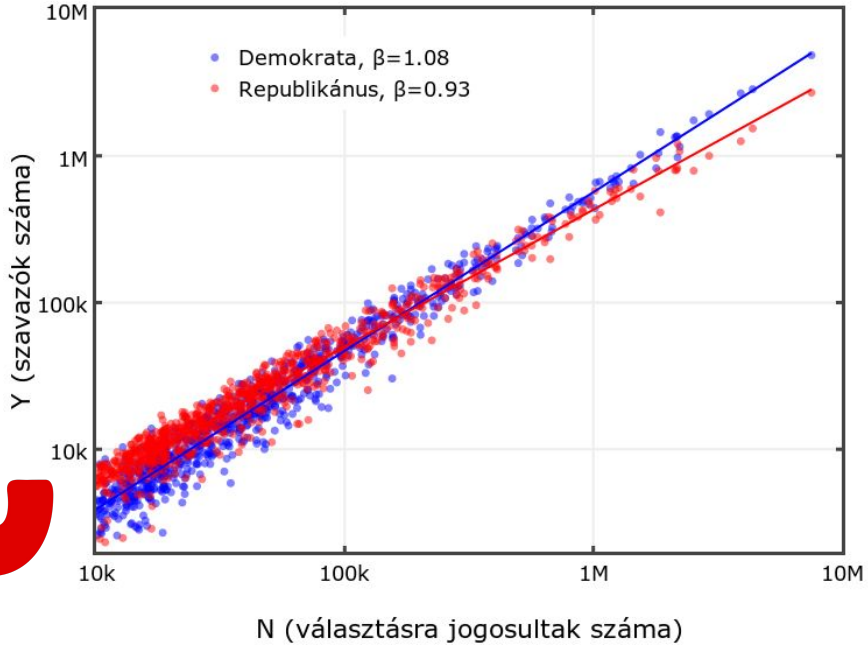


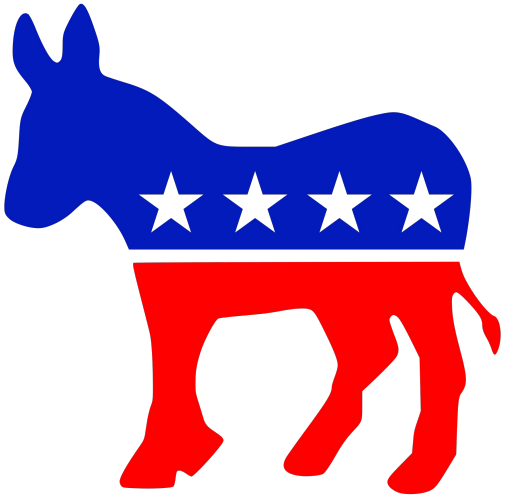

Demokrata, β=1.08
Republikánus, β=0.95
10M
1M
100k
10k
Y (szavazók száma)
10k
100k
1M
10M
N (választásra jogosultak száma)



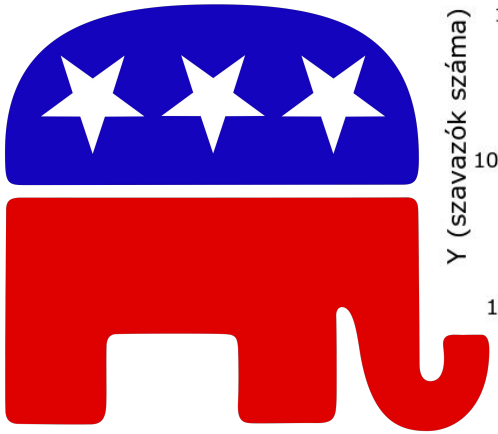
USA 2008

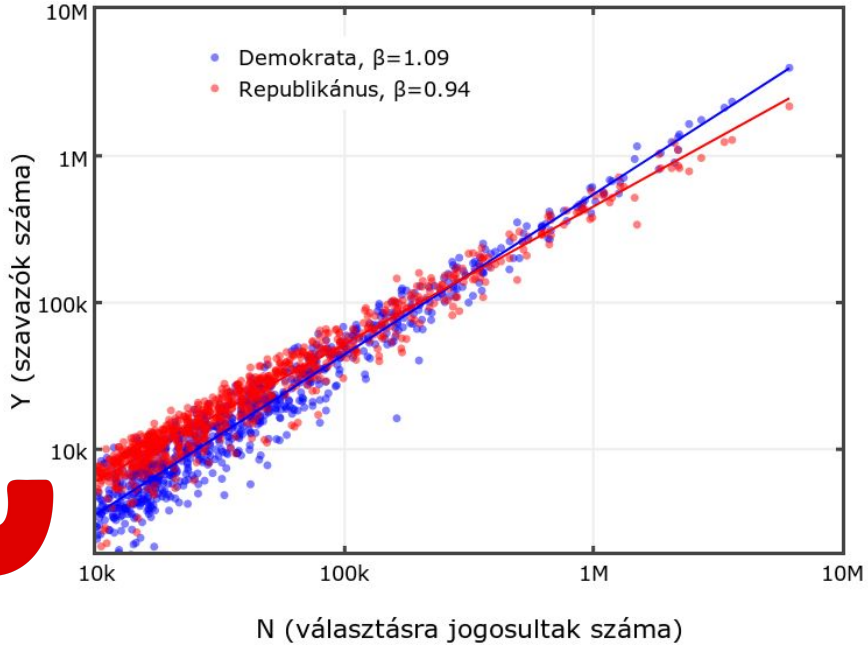


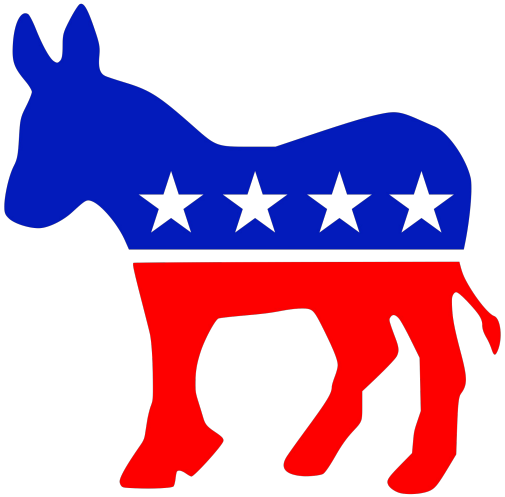

Demokrata, β=1.08
Republikánus, β=0.93
10M
1M
100k
10k
Y (szavazók száma)
10k
100k
1M
10M
N (választásra jogosultak száma)



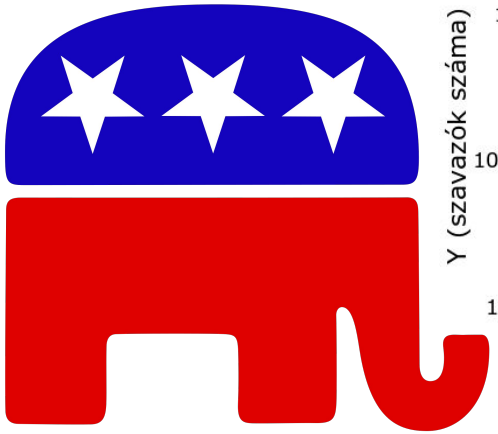
USA 2012

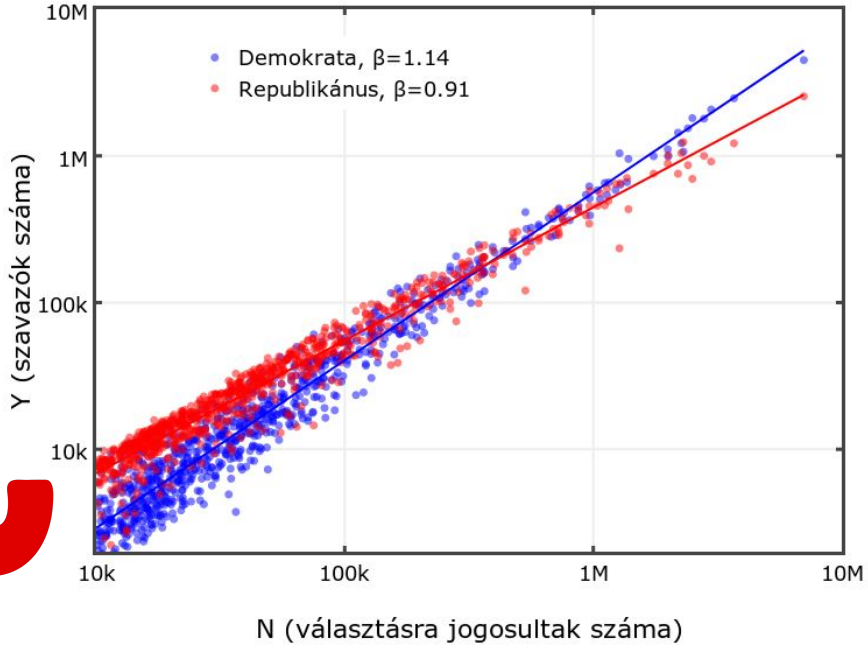


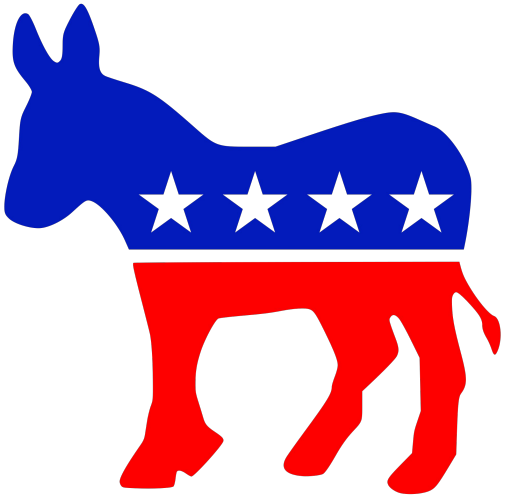

Demokrata, β=1.09
Republikánus, β=0.94
10M
1M
100k
10k
Y (szavazók száma)
10k
100k
1M
10M
N (választásra jogosultak száma)



USA 2016



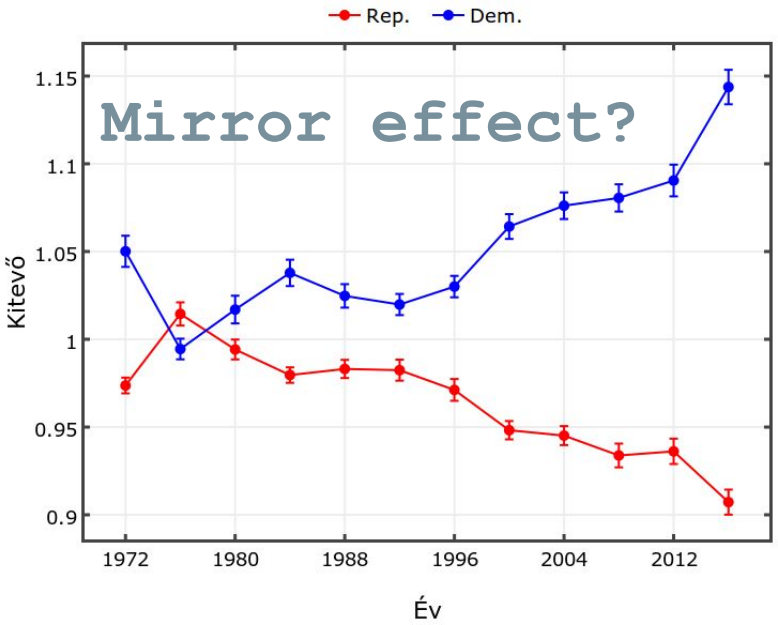

Demokrata, β=1.14
Republikánus, β=0.91
10M
1M
100k
10k
Y (szavazók száma)
10k
100k
1M
10M
N (választásra jogosultak száma)

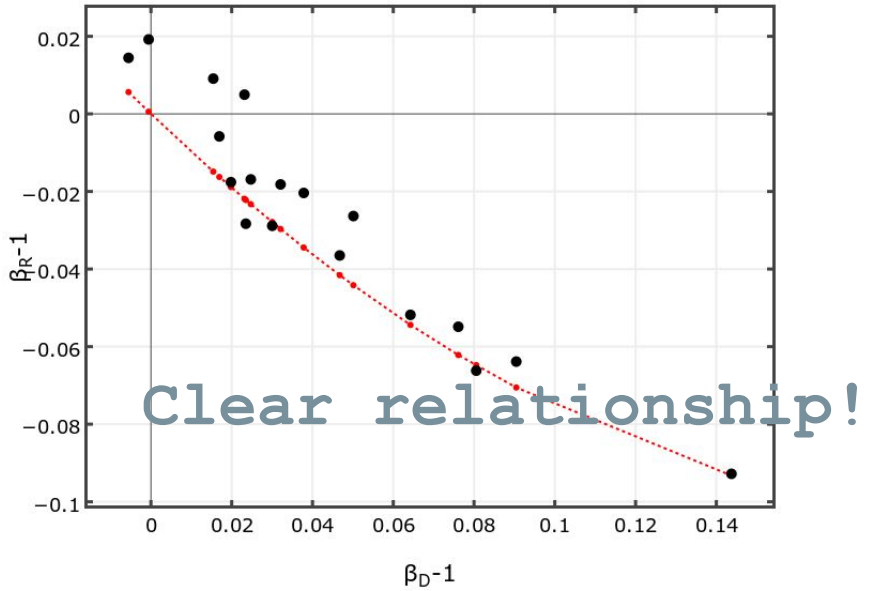


Reducing parameters I.

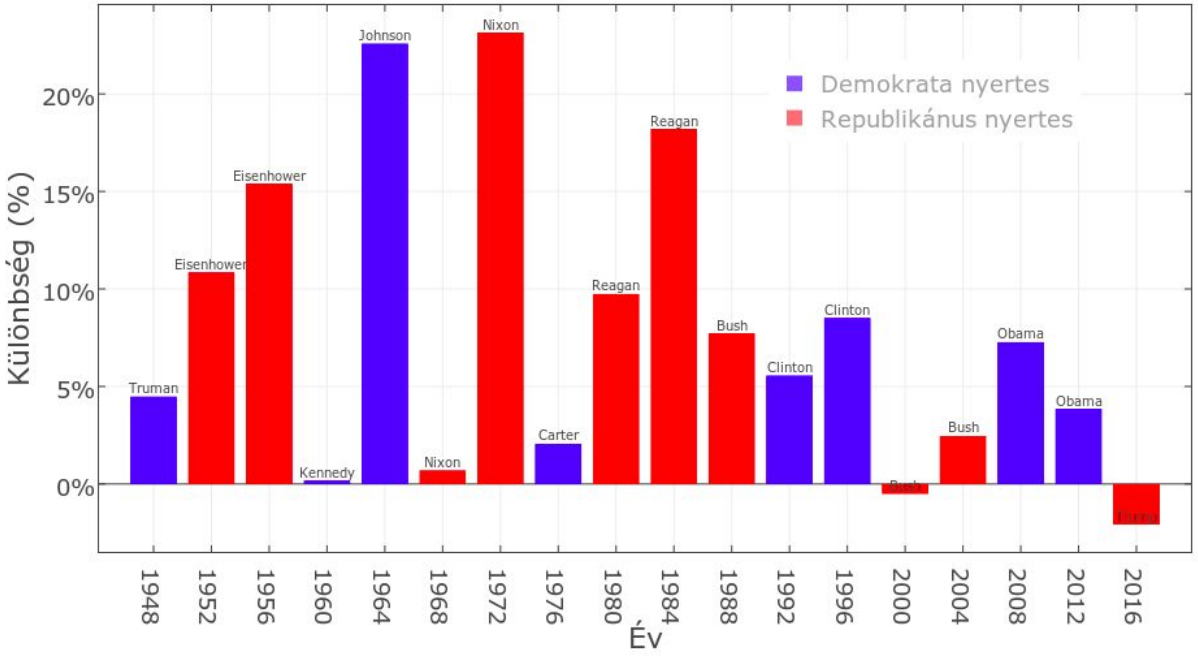
$N_{Dem}+N_{Rep} \cong = N$ roughly for every city!

One exponent is enough: β_{Dem}.

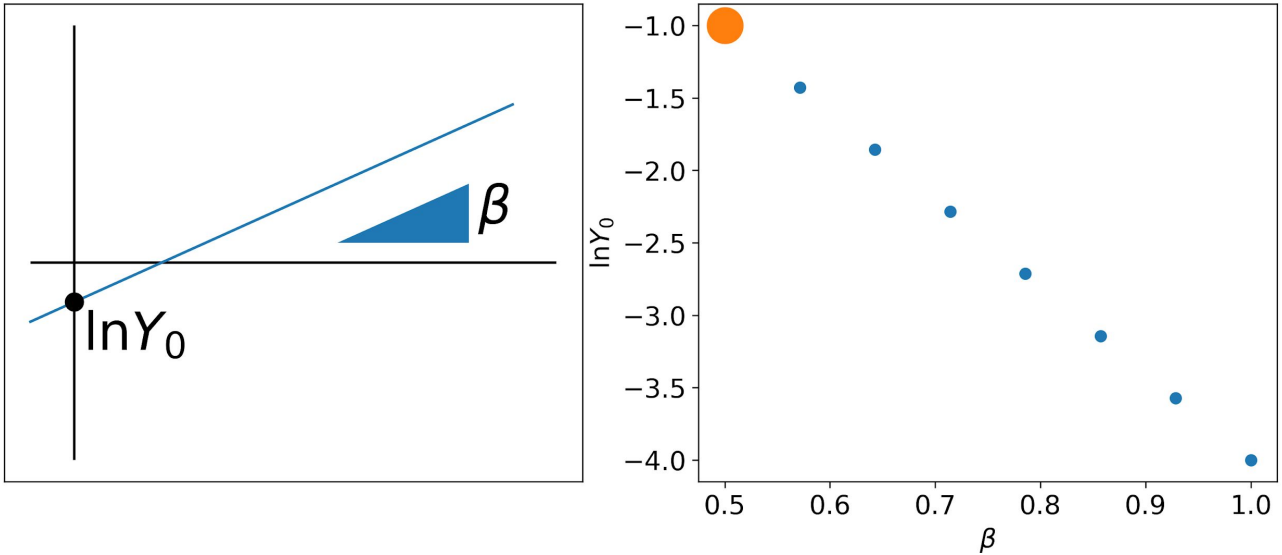




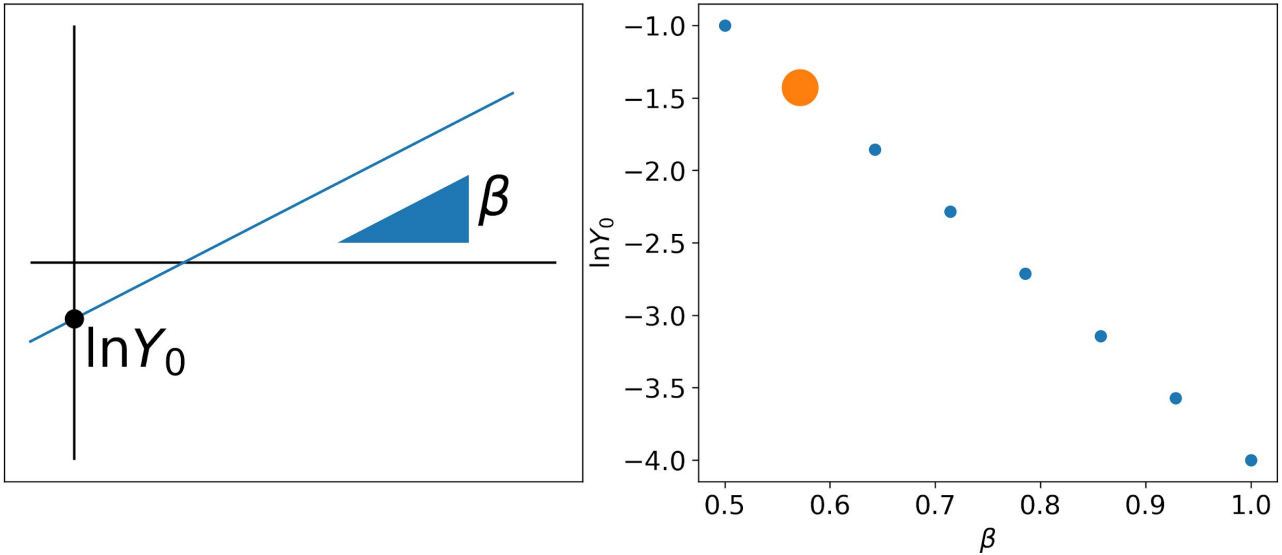
Questions, ideas



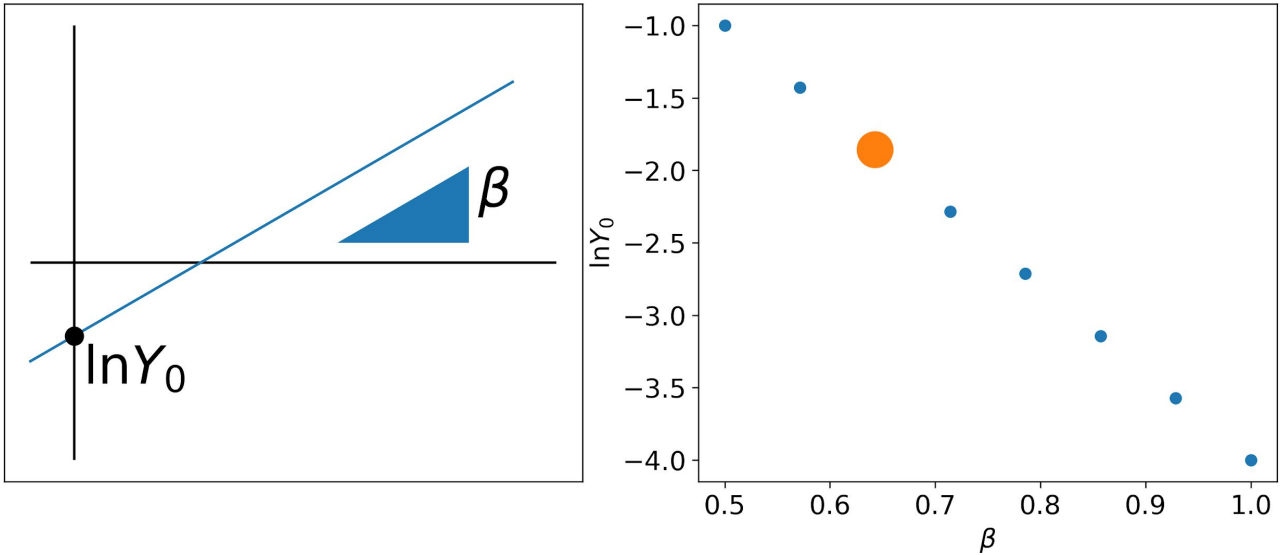
Parameter reduction II.



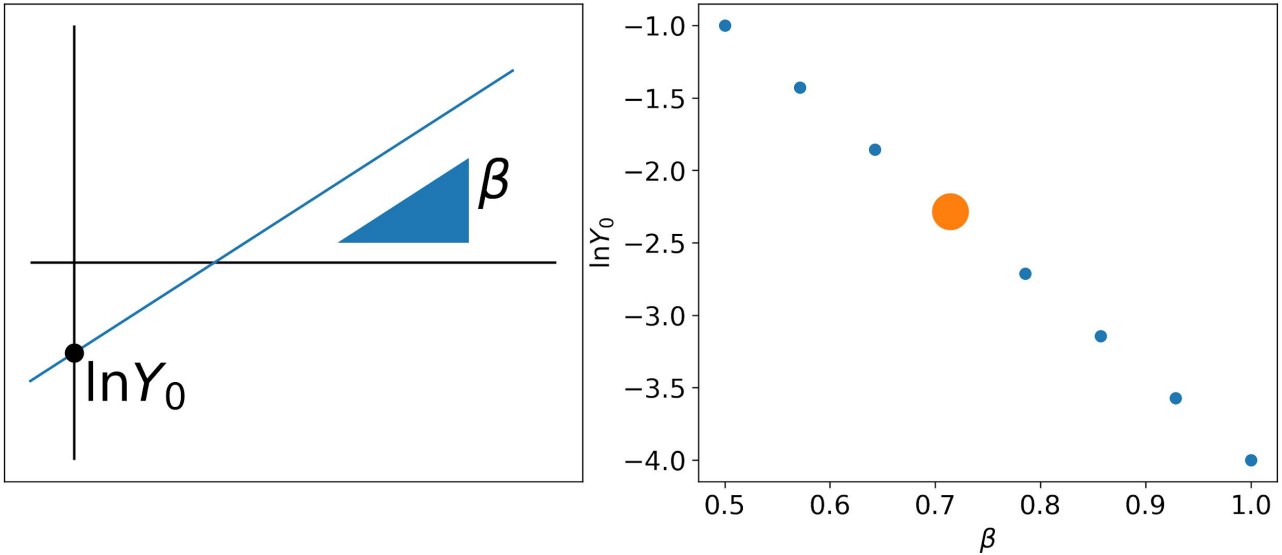
Parameter reduction II.



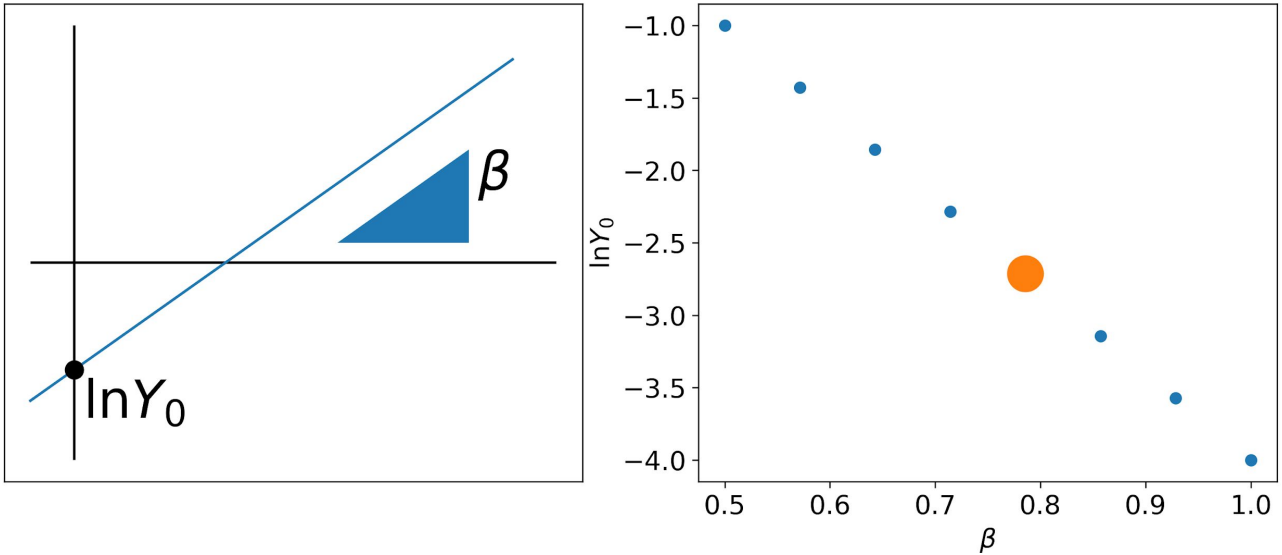
Parameter reduction II.



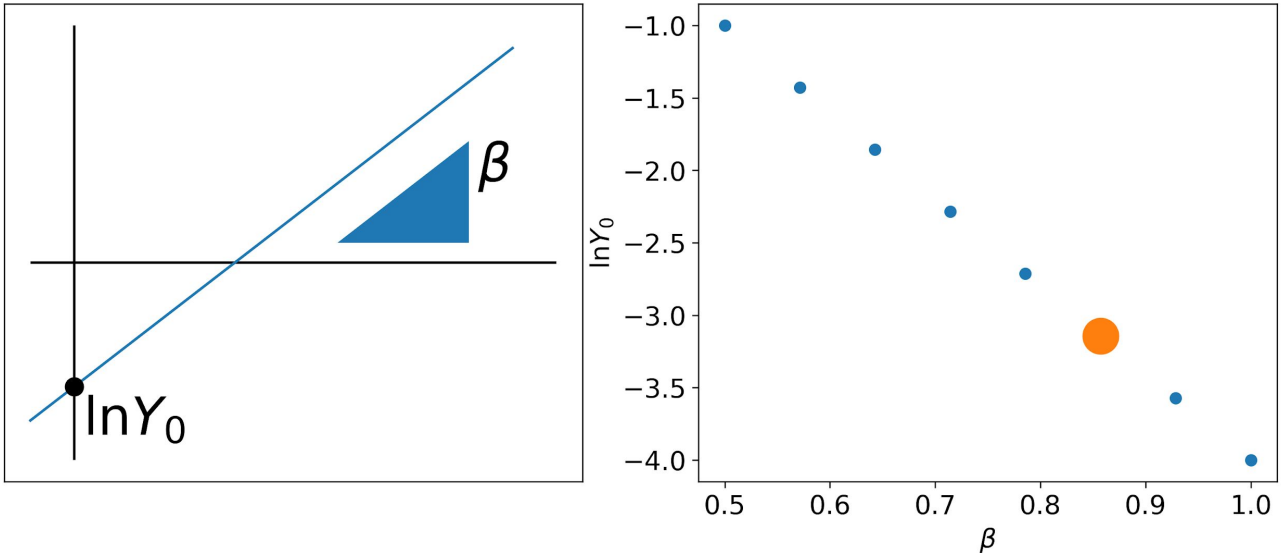
Parameter reduction II.



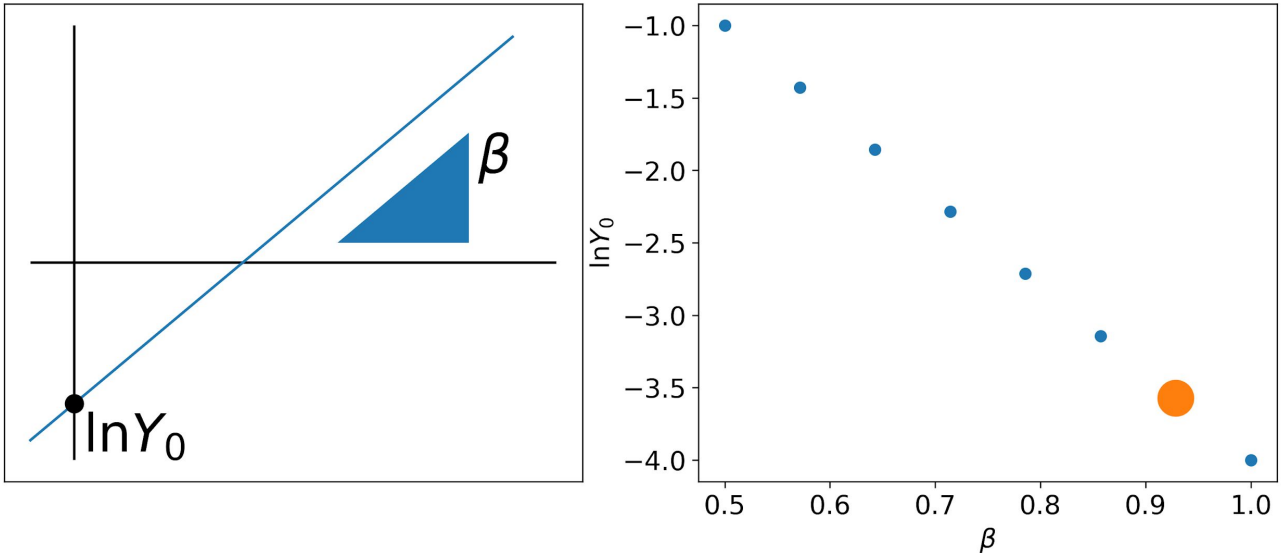
Parameter reduction II.



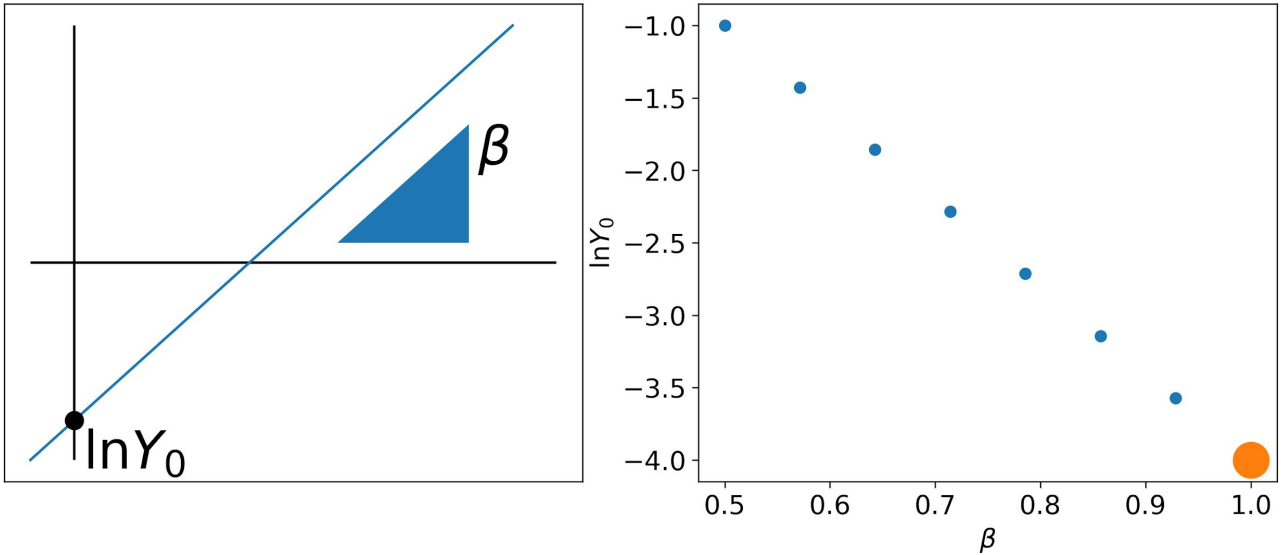
Parameter reduction II.



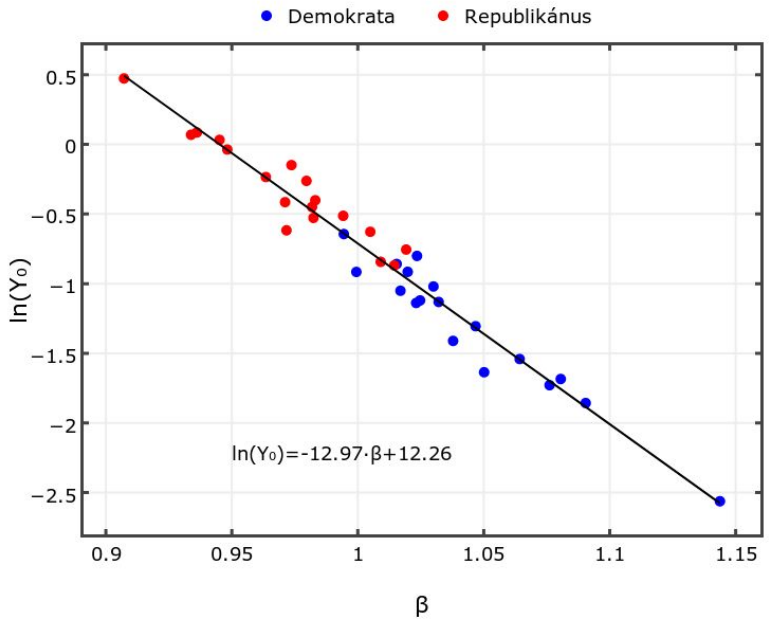
Parameter reduction II.



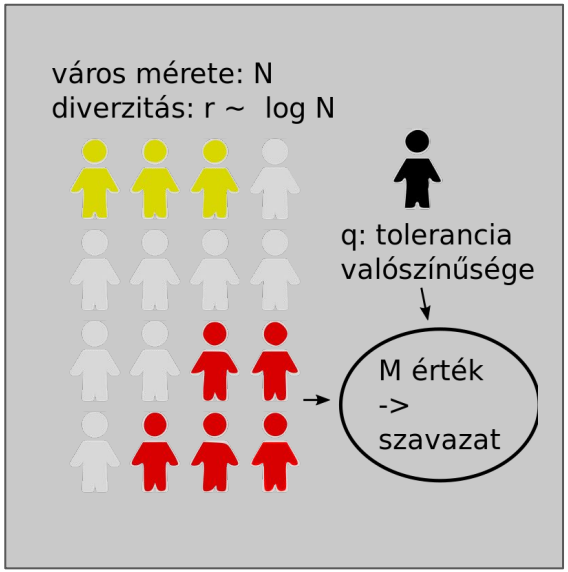
Parameter reduction II.



Parameter reduction II.



Model

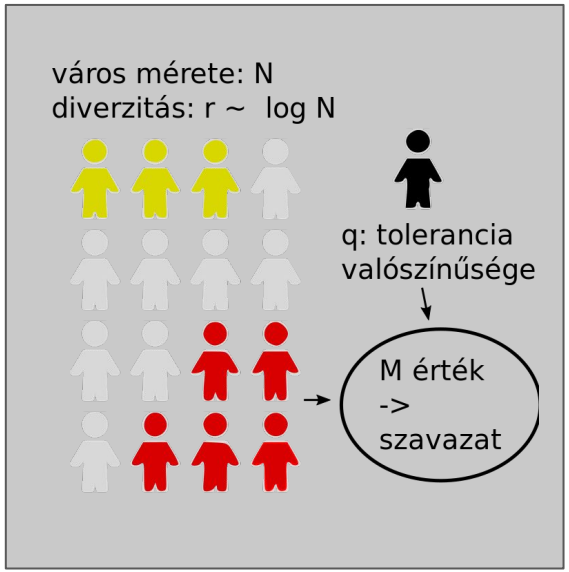


"The voting data suggest that people don't make cities liberal -- cities make people liberal."

Josh Kron - The Atlantic

$$p = (1-q)^{M-m} \rightarrow Y = N(p)_N$$

Model



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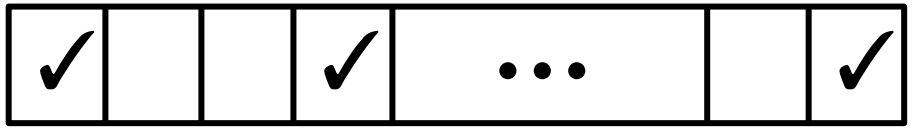
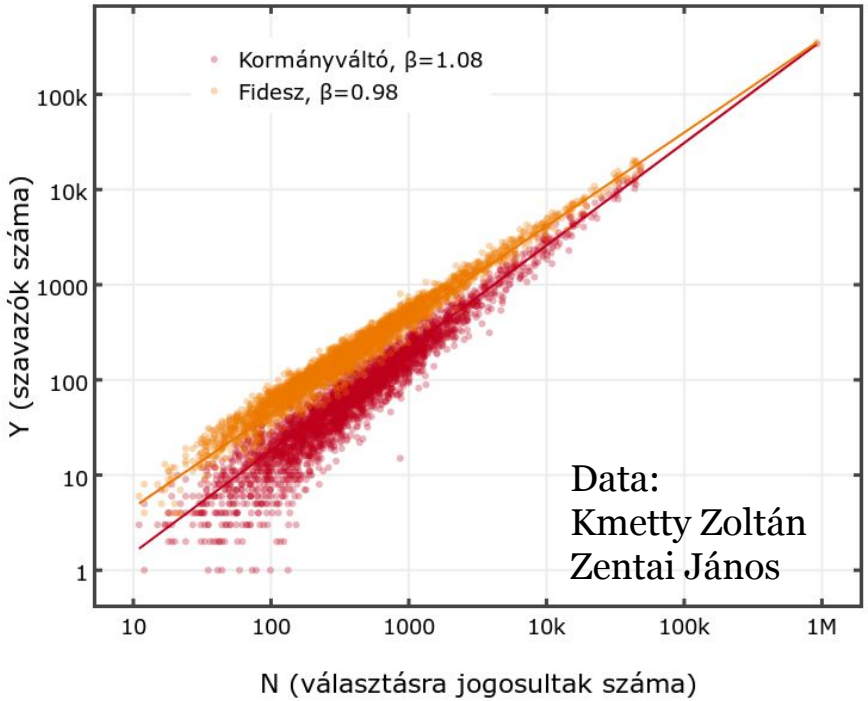


$$p = (1-q)^{M-m} \rightarrow Y = N\langle p \rangle_N$$

"mikroszkopikus" jelenségek összegzése

$$Y \approx N \cdot e^{qM(1-r(N))} = Y_0 N^{\beta}$$

Hazai terepen



Small settlements?
Multi-party system?

Hungary

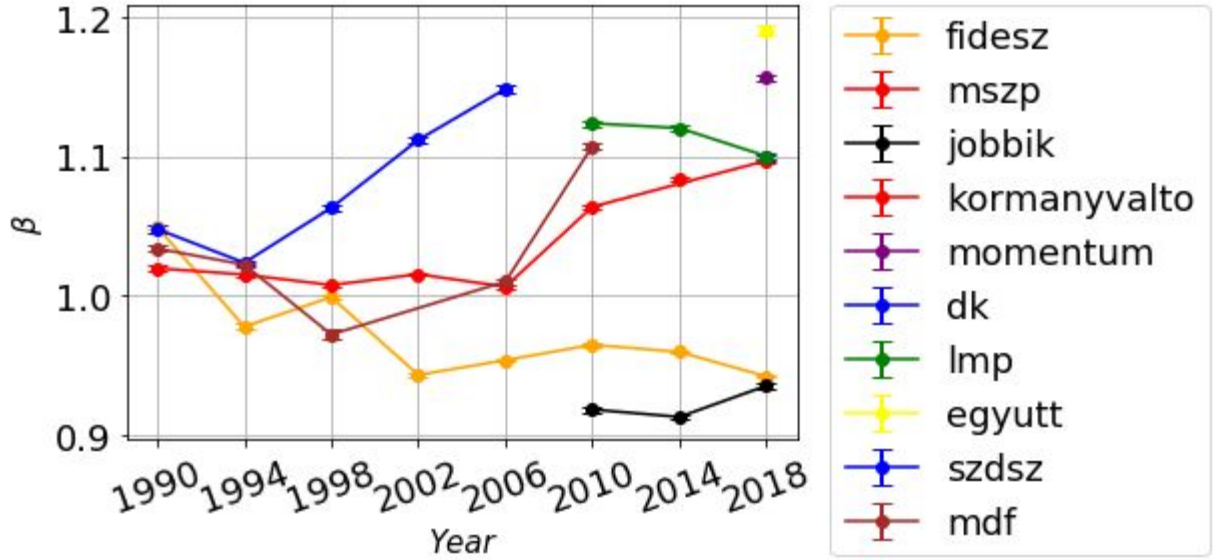


Figure:
Zentai János

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