Interplay between Topology and Edge Weights in Real-World Graphs: Supplementary Materials

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1. Full results w.r.t the observations

In this section, we show the full results w.r.t our observations which we cannot put all in the main text due to the space limit.

1.1. Observation 1: adjacency and strongness

In Figures 1 to 11, for each dataset and each $1 \leq i \leq 5$, we report how (a) the fraction of adjacent pairs within each group of pairs (i.e., $|E_{c;i}|/|R_{c;i}|$) and (b) the fraction of strong edges within each group of edges (i.e., $f_{c;i}$) depend on the number of CNs, and include the Pearson correlation coefficient between the two sequences.

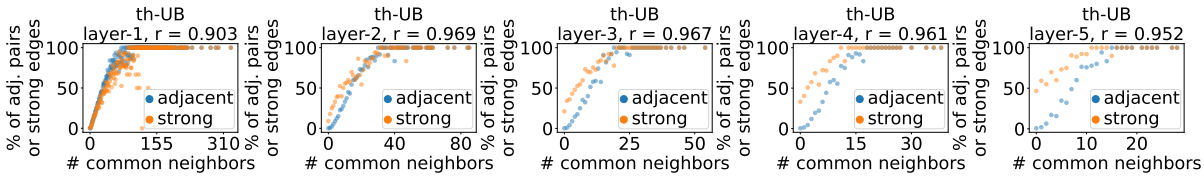

Fig. 1. The full results w.r.t Observation 1 on *th-UB*.

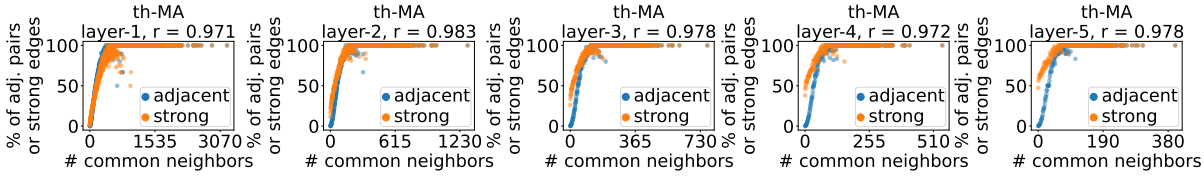

Fig. 2. The full results w.r.t Observation 1 on *th-MA*.

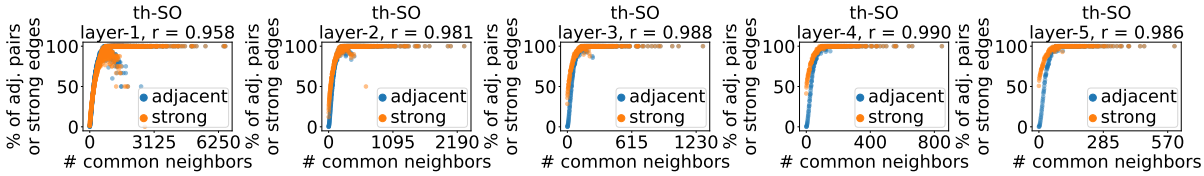

Fig. 3. The full results w.r.t Observation 1 on *th-SO*.

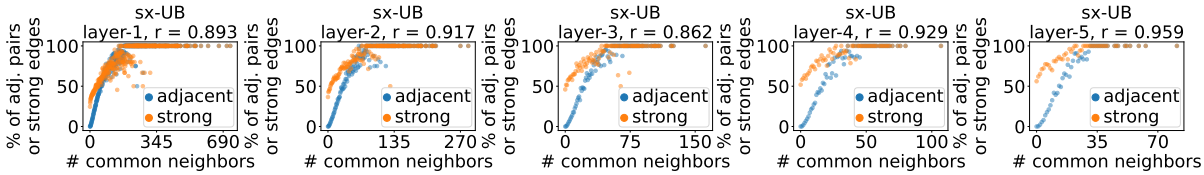

Fig. 4. The full results w.r.t Observation 1 on *sx-UB*.

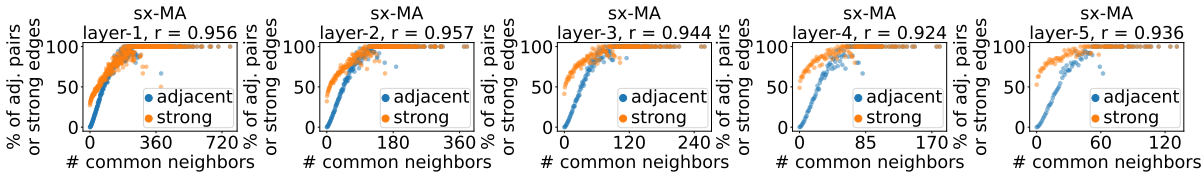

Fig. 5. The full results w.r.t Observation 1 on *sx-MA*.

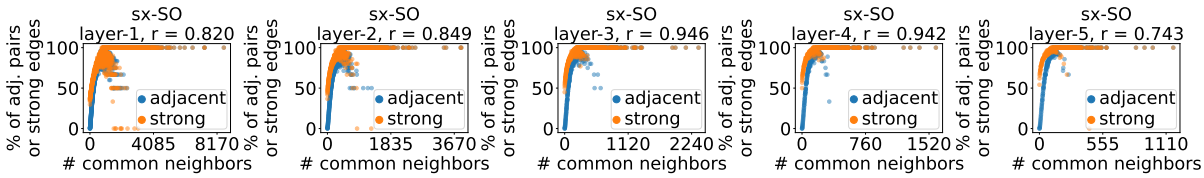

Fig. 6. The full results w.r.t Observation 1 on *sx-SO*.

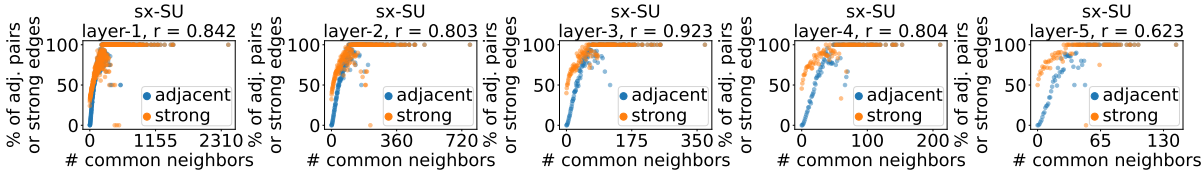

Fig. 7. The full results w.r.t Observation 1 on *sx-SU*.

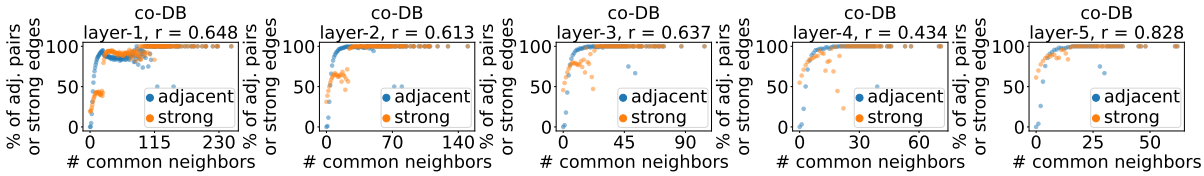

Fig. 8. The full results w.r.t Observation 1 on *co-DB*.

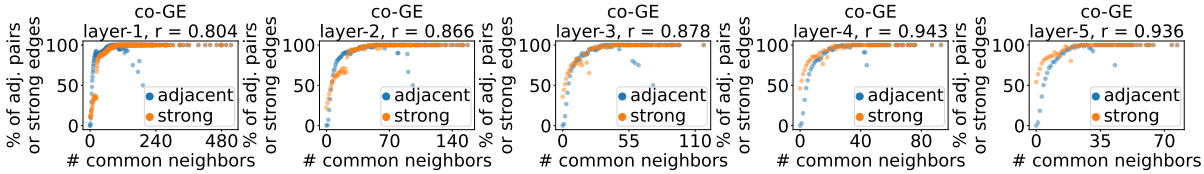

Fig. 9. The full results w.r.t Observation 1 on *co-GE*.

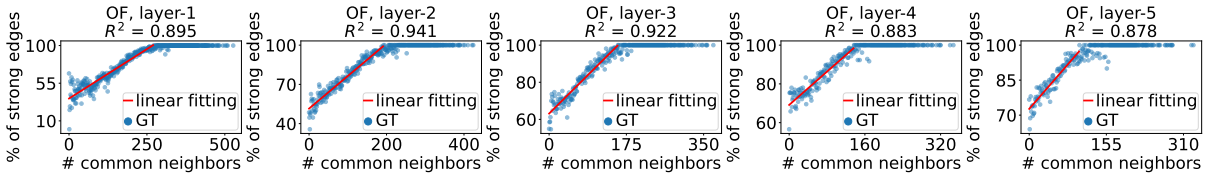

Fig. 10. The full results w.r.t Observation 2 on *OF*.

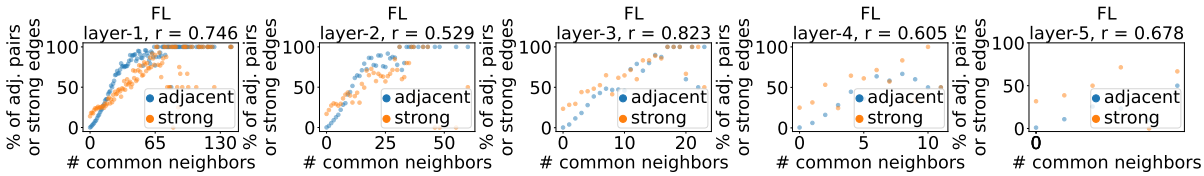

Fig. 11. The full results w.r.t Observation 1 on *FL*.

1.2. *Observation 2: the fractions of strong edges*

In Figures 12 to 22, for each dataset and each $1 \leq i \leq 5$, we plot the the fractions of strong edges (FoSEs) with the results of the linear fitting for the points truncated before the saturation point (SP), and include the R^2 value of the linear fitting.

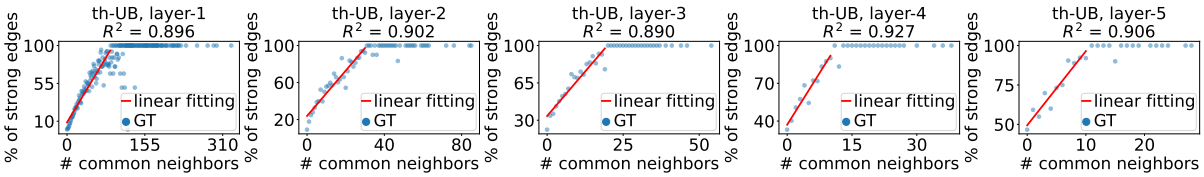

Fig. 12. The full results w.r.t Observation 2 on *th-UB*.

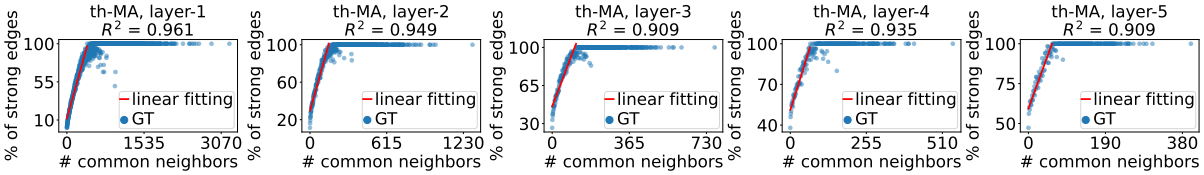

Fig. 13. The full results w.r.t Observation 2 on *th-MA*.

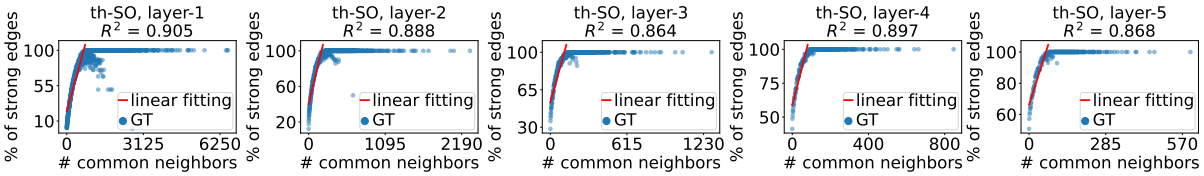

Fig. 14. The full results w.r.t Observation 2 on *th-SO*.

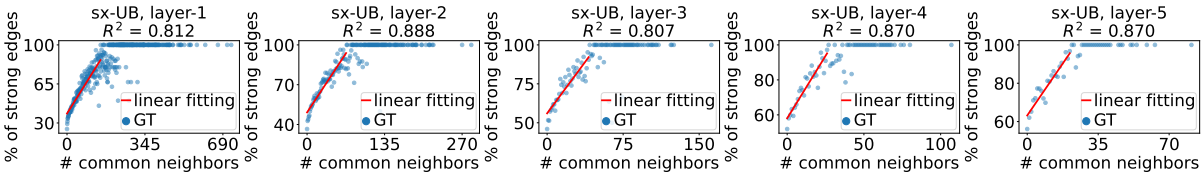

Fig. 15. The full results w.r.t Observation 2 on *sx-UB*.

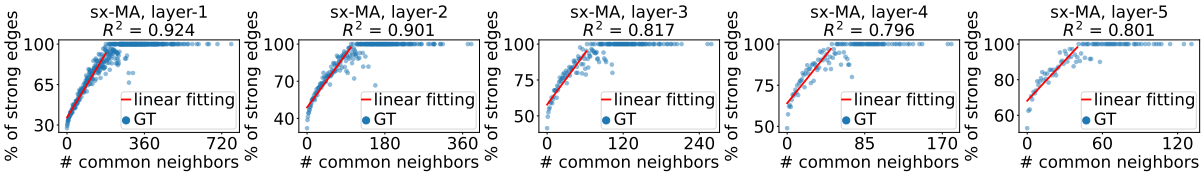

Fig. 16. The full results w.r.t Observation 2 on *sx-MA*.

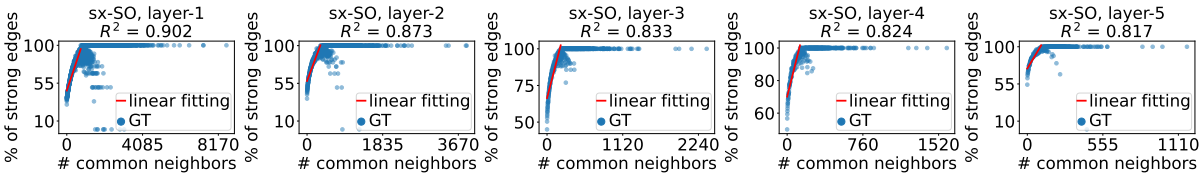

Fig. 17. The full results w.r.t Observation 2 on *sx-SO*.

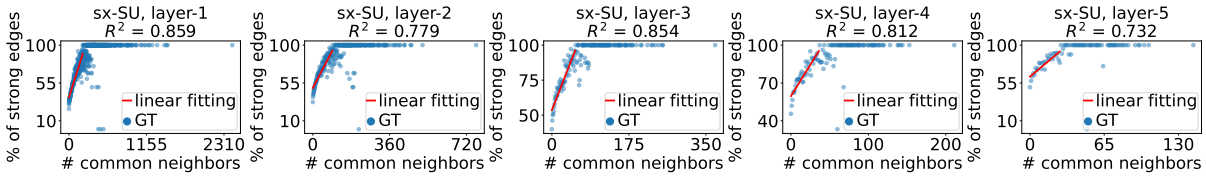

Fig. 18. The full results w.r.t Observation 2 on *sx-SU*.

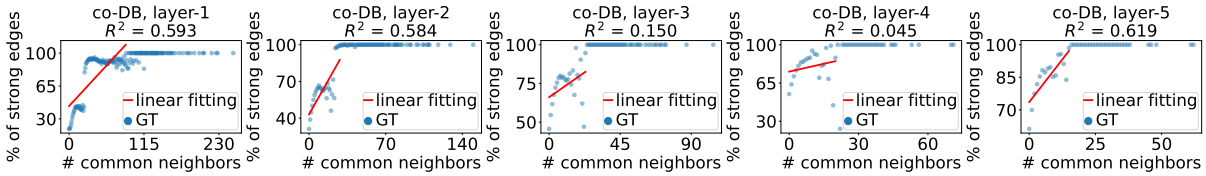

Fig. 19. The full results w.r.t Observation 2 on *co-DB*.

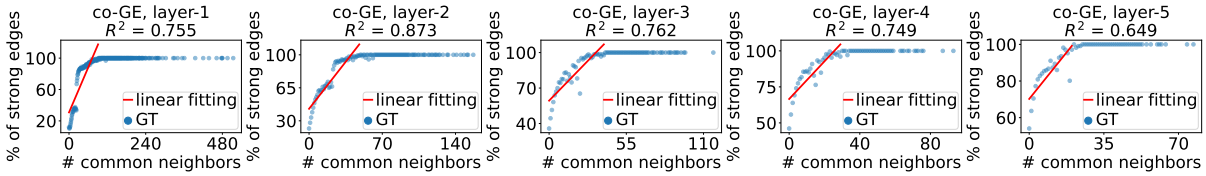

Fig. 20. The full results w.r.t Observation 2 on *co-GE*.

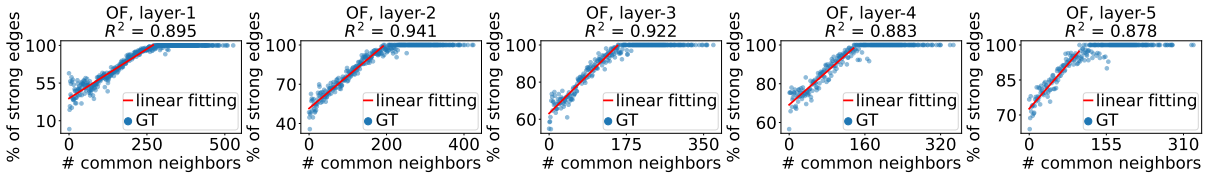

Fig. 21. The full results w.r.t Observation 2 on *OF*.

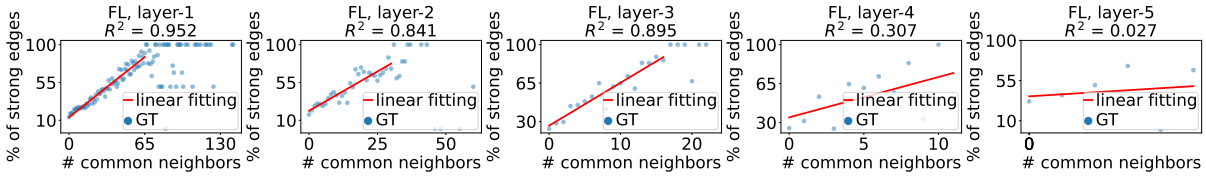

Fig. 22. The full results w.r.t Observation 2 on *FL*.

1.3. *Observation 3: a power law across layers*

In Figures 23 to 33, for each dataset, we plot (a) the point $(f_{overall;i}, f_{0;i})$ for each $1 \leq i \leq 10$ [1] in the log-log scale and (b) the power-law fitting line, which is linear in the log-log scale. We include the formula and the R^2 value of each power-law fitting line.

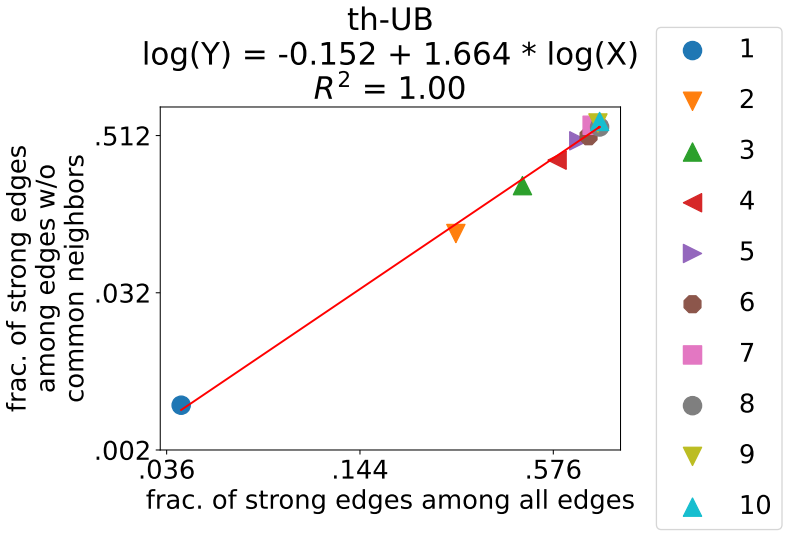

Fig. 23. The full results w.r.t Observation 3 on *th-UB*.

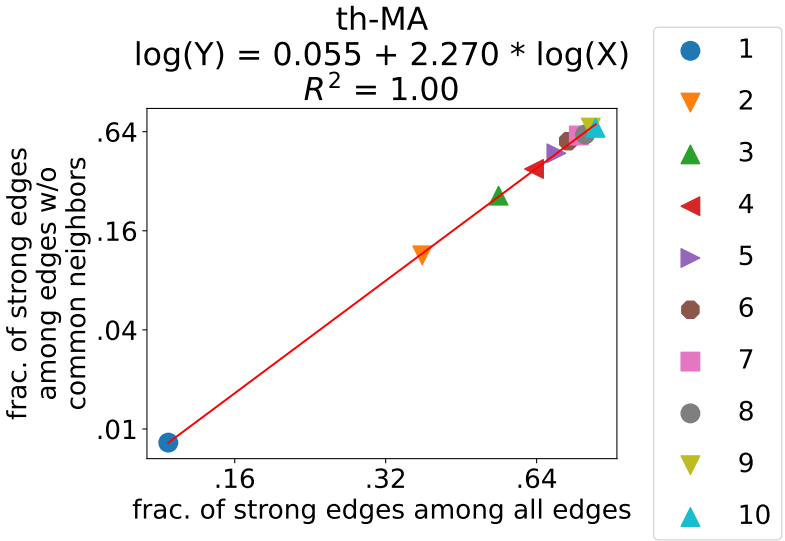

Fig. 24. The full results w.r.t Observation 3 on *th-MA*.

[1] We only include the first four layers of *FL* since the layer-5 is too sparse and small and we skip the first layer of *OF* since the corresponding point is an outlier.

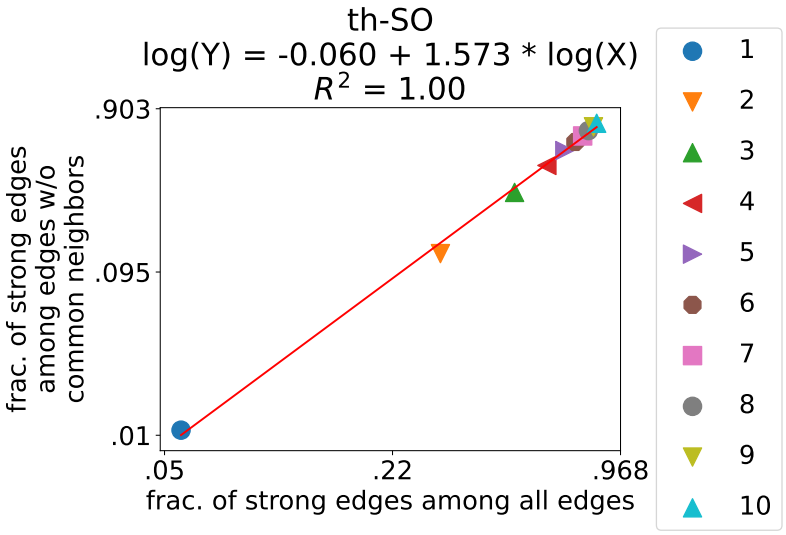

Fig. 25. The full results w.r.t Observation 3 on *th-SO*.

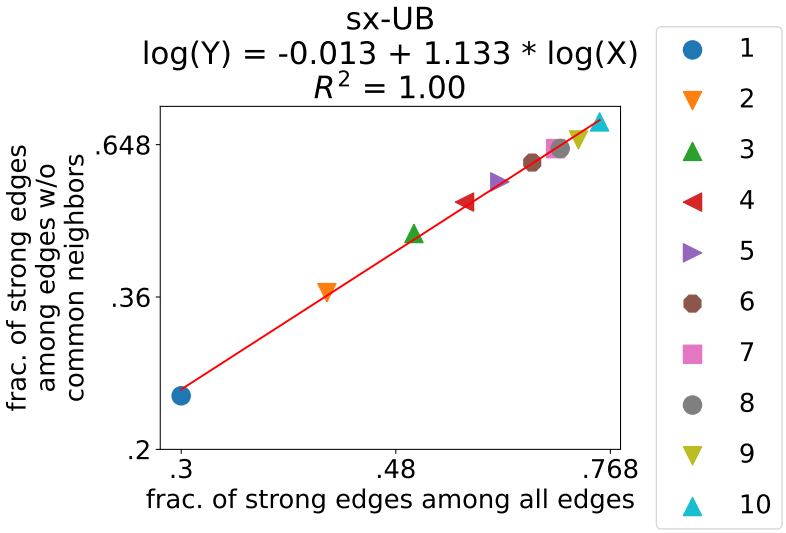

Fig. 26. The full results w.r.t Observation 3 on *sx-UB*.

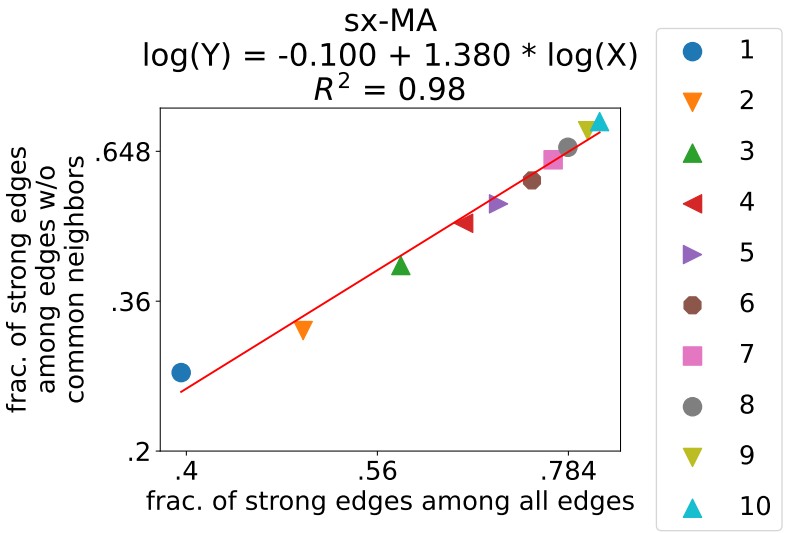

Fig. 27. The full results w.r.t Observation 3 on *sx-MA*.

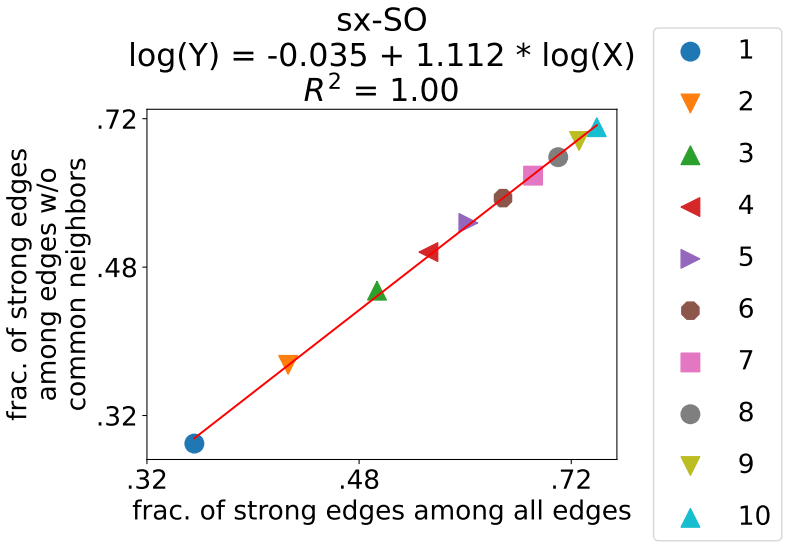

Fig. 28. The full results w.r.t Observation 3 on *sx-SO*.

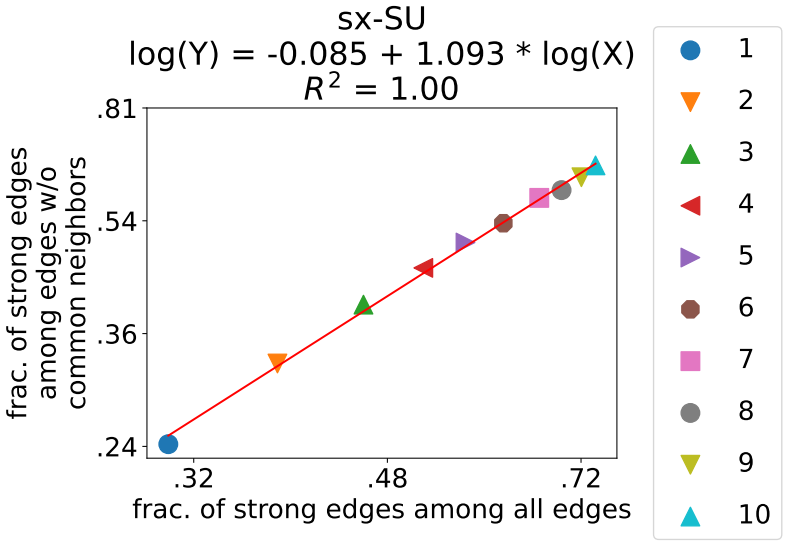

Fig. 29. The full results w.r.t Observation 3 on *sx-SU*.

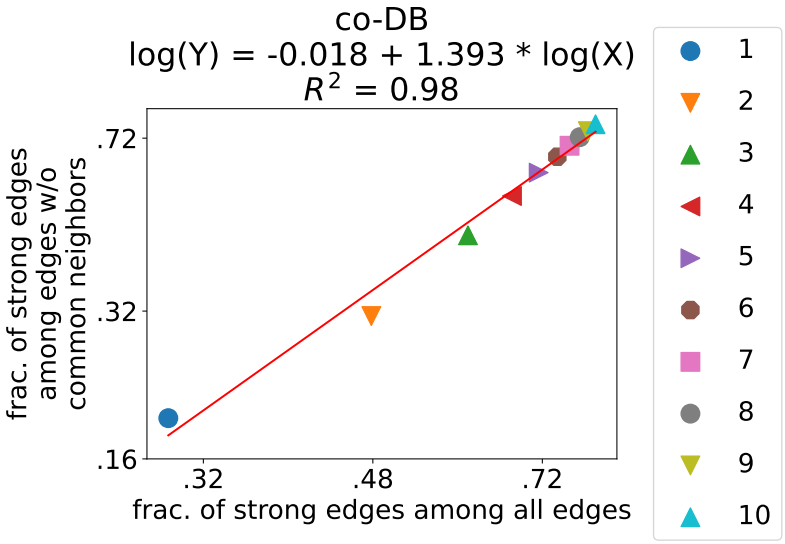

Fig. 30. The full results w.r.t Observation 3 on *co-DB*.

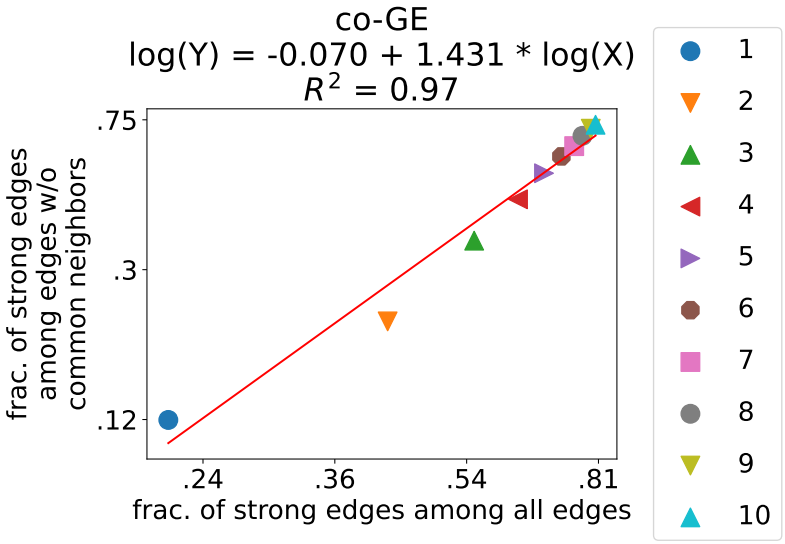

Fig. 31. The full results w.r.t Observation 3 on *co-GE*.

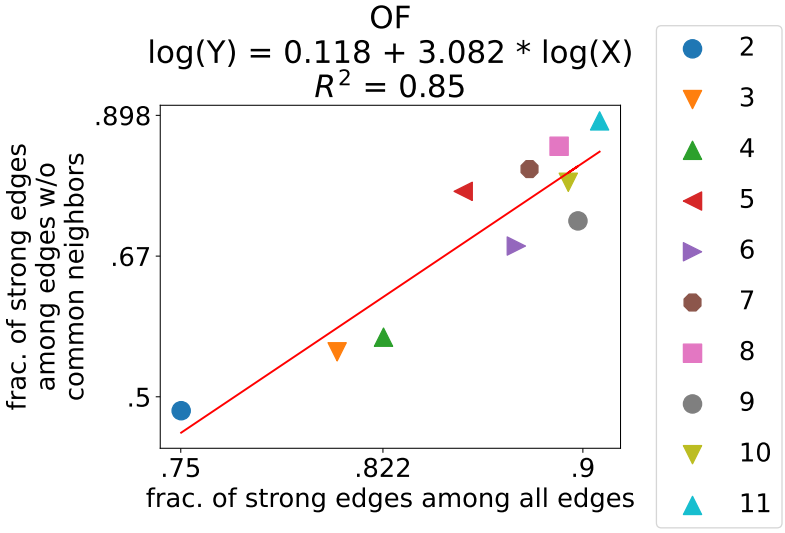

Fig. 32. The full results w.r.t Observation 3 on *OF*.

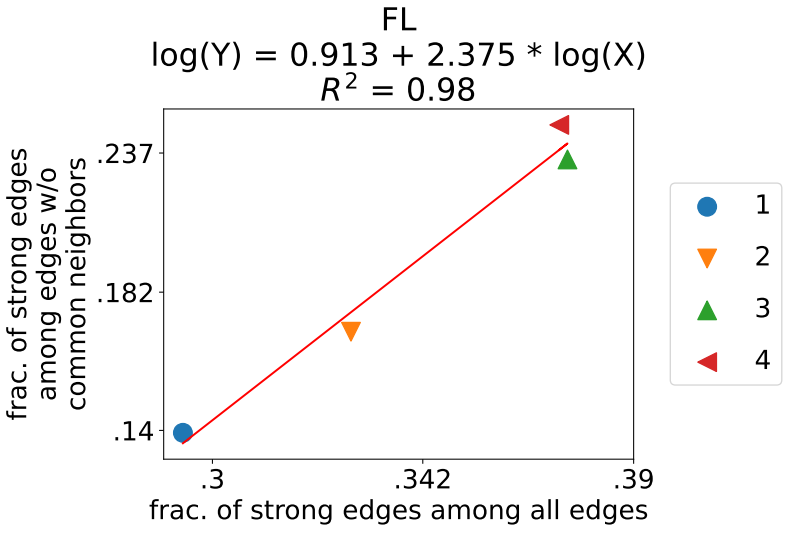

Fig. 33. The full results w.r.t Observation 3 on *FL*.

2. Full experimental results

In this section, we show the full results of our experiments which we cannot put all in the main text due to the space limit. In Tables 1 to 11, for each dataset, we report the detailed results w.r.t each metric, each method, and each layer.

Table 1. Full experimental results on *th-UB*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.621	0.715	0.763	0.790
	SCN	0.606	0.552	0.551	0.426
	SEB	0.467	0.588	0.657	0.660
	PEB	0.619	0.640	0.692	0.684
	RFF	0.313	0.369	0.352	0.351
	NEB	0.609	0.715	0.763	0.790
	PEAR (ours)	0.189±0.002	0.111±0.008	0.169±0.009	0.236±0.025
KSND	PRD	0.203	0.298	0.384	0.467
	SCN	0.501	0.499	0.360	0.268
	SEB	0.211	0.336	0.423	0.477
	PEB	0.200	0.267	0.323	0.363
	RFF	0.144	0.350	0.298	0.275
	NEB	0.270	0.375	0.443	0.469
	PEAR (ours)	0.114±0.003	0.173±0.007	0.185±0.016	0.185±0.004
DACC	PRD	0.143	0.209	0.276	0.328
	SCN	0.501	0.520	0.396	0.338
	SEB	0.063	0.146	0.223	0.264
	PEB	0.143	0.198	0.256	0.284
	RFF	0.138	0.374	0.330	0.273
	NEB	0.136	0.209	0.276	0.328
	PEAR (ours)	0.100±0.002	0.127±0.007	0.149±0.011	0.154±0.004
NetSimile	PRD	20.831	19.712	22.016	23.930
	SCN	21.145	20.147	18.546	14.948
	SEB	20.496	21.960	25.279	26.451
	PEB	20.879	20.882	19.066	17.793
	RFF	17.313	18.154	15.549	15.041
	NEB	22.501	20.131	20.839	21.778
	PEAR (ours)	10.105±0.047	9.027±0.476	9.045±0.624	8.016±0.683

Table 2. Full experimental results on *th-MA*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.735	0.844	0.886	0.895
	SCN	0.585	0.594	0.579	0.565
	SEB	0.796	0.813	0.804	0.797
	PEB	0.742	0.672	0.714	0.737
	RFF	0.342	0.339	0.374	0.415
	NEB	0.887	0.903	0.905	0.902
	PEAR (ours)	0.166±0.002	0.143±0.002	0.182±0.007	0.266±0.013
KSND	PRD	0.132	0.200	0.256	0.278
	SCN	0.588	0.602	0.588	0.560
	SEB	0.245	0.312	0.352	0.382
	PEB	0.140	0.238	0.291	0.320
	RFF	0.405	0.390	0.383	0.373
	NEB	0.424	0.425	0.442	0.431
	PEAR (ours)	0.136±0.001	0.191±0.002	0.204±0.006	0.175±0.019
DACC	PRD	0.304	0.369	0.403	0.409
	SCN	0.464	0.459	0.447	0.436
	SEB	0.274	0.325	0.350	0.353
	PEB	0.306	0.300	0.339	0.323
	RFF	0.473	0.421	0.391	0.386
	NEB	0.321	0.374	0.404	0.410
	PEAR (ours)	0.215±0.001	0.279±0.002	0.291±0.004	0.266±0.005
NetSimile	PRD	20.073	24.033	25.065	25.760
	SCN	21.234	22.287	21.987	21.612
	SEB	20.939	24.163	26.942	28.533
	PEB	20.307	18.017	19.384	20.416
	RFF	19.096	17.926	17.502	17.892
	NEB	24.681	21.923	22.156	22.753
	PEAR (ours)	10.012±0.027	14.906±0.082	16.850±0.021	17.361±0.032

Table 3. Full experimental results on *th-SO*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.707	0.775	0.792	0.800
	SCN	0.650	0.647	0.632	0.609
	SEB	0.587	0.650	0.675	0.692
	PEB	0.709	0.523	0.574	0.621
	RFF	0.451	0.421	0.425	0.439
	NEB	0.474	0.579	0.571	0.614
	PEAR (ours)	0.189±0.001	0.112±0.001	0.107±0.004	0.176±0.010
KSND	PRD	0.074	0.134	0.190	0.242
	SCN	0.453	0.460	0.439	0.423
	SEB	0.184	0.254	0.316	0.358
	PEB	0.075	0.126	0.145	0.184
	RFF	0.312	0.327	0.327	0.320
	NEB	0.329	0.322	0.321	0.331
	PEAR (ours)	0.111±0.000	0.164±0.003	0.142±0.001	0.113±0.003
DACC	PRD	0.149	0.205	0.235	0.257
	SCN	0.528	0.501	0.467	0.450
	SEB	0.088	0.153	0.190	0.217
	PEB	0.150	0.177	0.204	0.231
	RFF	0.459	0.406	0.376	0.354
	NEB	0.092	0.155	0.181	0.218
	PEAR (ours)	0.123±0.000	0.154±0.001	0.155±0.001	0.149±0.002
NetSimile	PRD	20.673	22.355	23.016	23.601
	SCN	23.682	23.860	23.492	22.647
	SEB	19.771	21.526	23.288	24.151
	PEB	20.670	14.175	16.402	19.143
	RFF	20.500	18.956	18.409	17.991
	NEB	22.043	20.275	19.527	20.103
	PEAR (ours)	11.229±0.096	12.772±0.087	13.894±0.123	15.288±0.162

Table 4. Full experimental results on *sx-UB*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.191	0.256	0.263	0.262
	SCN	0.677	0.802	0.828	0.836
	SEB	0.291	0.262	0.253	0.248
	PEB	0.215	0.089	0.063	0.073
	RFF	0.789	0.806	0.813	0.810
	NEB	0.315	0.269	0.252	0.258
	PEAR (ours)	0.050±0.002	0.115±0.003	0.186±0.001	0.239±0.008
KSND	PRD	0.020	0.040	0.043	0.048
	SCN	0.398	0.562	0.612	0.634
	SEB	0.070	0.092	0.086	0.087
	PEB	0.033	0.024	0.023	0.002
	RFF	0.558	0.594	0.620	0.635
	NEB	0.161	0.125	0.116	0.109
	PEAR (ours)	0.142±0.002	0.126±0.003	0.097±0.002	0.061±0.003
DACC	PRD	0.040	0.038	0.034	0.031
	SCN	0.432	0.581	0.640	0.668
	SEB	0.036	0.028	0.027	0.025
	PEB	0.043	0.020	0.005	0.023
	RFF	0.628	0.645	0.651	0.655
	NEB	0.042	0.032	0.027	0.028
	PEAR (ours)	0.033±0.000	0.021±0.000	0.009±0.001	0.002±0.000
NetSimile	PRD	8.944	15.477	17.934	19.287
	SCN	20.511	25.139	26.715	27.552
	SEB	14.502	15.823	16.326	16.925
	PEB	10.860	9.088	11.983	15.766
	RFF	25.865	25.415	25.597	25.555
	NEB	17.233	17.121	16.549	17.552
	PEAR (ours)	9.737±0.055	14.221±0.074	14.829±0.201	14.531±0.089

Table 5. Full experimental results on *sx-MA*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.264	0.474	0.579	0.654
	SCN	0.565	0.700	0.725	0.745
	SEB	0.216	0.331	0.389	0.410
	PEB	0.331	0.281	0.290	0.342
	RFF	0.511	0.549	0.578	0.604
	NEB	0.458	0.431	0.403	0.371
	PEAR (ours)	0.056±0.001	0.127±0.006	0.186±0.006	0.209±0.006
KSND	PRD	0.023	0.042	0.075	0.103
	SCN	0.549	0.577	0.557	0.546
	SEB	0.081	0.104	0.151	0.187
	PEB	0.024	0.060	0.107	0.130
	RFF	0.407	0.424	0.419	0.414
	NEB	0.158	0.157	0.165	0.170
	PEAR (ours)	0.012±0.002	0.046±0.006	0.087±0.004	0.128±0.005
DACC	PRD	0.115	0.136	0.147	0.163
	SCN	0.437	0.506	0.576	0.562
	SEB	0.030	0.050	0.065	0.084
	PEB	0.134	0.074	0.037	0.042
	RFF	0.442	0.475	0.482	0.472
	NEB	0.129	0.106	0.090	0.087
	PEAR (ours)	0.043±0.002	0.080±0.003	0.102±0.002	0.126±0.002
NetSimile	PRD	7.190	11.749	14.830	16.485
	SCN	20.297	22.964	23.864	24.096
	SEB	5.748	10.396	13.164	14.084
	PEB	9.541	10.110	13.732	16.323
	RFF	20.941	20.592	20.722	21.213
	NEB	15.878	15.291	13.433	12.214
	PEAR (ours)	1.929±0.023	4.674±0.173	6.817±0.599	9.235±0.335

Table 6. Full experimental results on *sx-SO*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.142	0.273	0.305	0.314
	SCN	0.678	0.815	0.841	0.846
	SEB	0.261	0.246	0.208	0.195
	PEB	0.193	0.055	0.077	0.063
	RFF	0.690	0.736	0.753	0.757
	NEB	0.402	0.295	0.266	0.254
	PEAR (ours)	0.186±0.001	0.073±0.001	0.137±0.005	0.187±0.004
KSND	PRD	0.011	0.030	0.033	0.034
	SCN	0.282	0.359	0.410	0.446
	SEB	0.109	0.110	0.144	0.158
	PEB	0.030	0.088	0.096	0.093
	RFF	0.160	0.226	0.277	0.320
	NEB	0.376	0.373	0.335	0.299
	PEAR (ours)	0.238±0.000	0.326±0.001	0.323±0.003	0.294±0.003
DACC	PRD	0.034	0.038	0.036	0.036
	SCN	0.385	0.511	0.572	0.603
	SEB	0.017	0.012	0.009	0.009
	PEB	0.039	0.018	0.001	0.002
	RFF	0.535	0.549	0.554	0.555
	NEB	0.036	0.021	0.020	0.020
	PEAR (ours)	0.046±0.000	0.032±0.000	0.023±0.001	0.018±0.001

Table 7. Full experimental results on *sx-SU*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.201	0.313	0.333	0.348
	SCN	0.684	0.792	0.807	0.800
	SEB	0.387	0.352	0.334	0.342
	PEB	0.237	0.122	0.103	0.138
	RFF	0.698	0.721	0.721	0.713
	NEB	0.416	0.355	0.338	0.346
	PEAR (ours)	0.087±0.001	0.099±0.002	0.161±0.004	0.210±0.004
KSND	PRD	0.025	0.047	0.051	0.059
	SCN	0.404	0.506	0.562	0.587
	SEB	0.076	0.122	0.121	0.120
	PEB	0.038	0.027	0.026	0.017
	RFF	0.339	0.401	0.440	0.461
	NEB	0.241	0.203	0.171	0.162
	PEAR (ours)	0.149±0.001	0.157±0.001	0.127±0.002	0.106±0.001
DACC	PRD	0.047	0.048	0.046	0.048
	SCN	0.451	0.598	0.637	0.668
	SEB	0.053	0.045	0.041	0.043
	PEB	0.050	0.031	0.015	0.013
	RFF	0.580	0.597	0.602	0.601
	NEB	0.061	0.046	0.043	0.044
	PEAR (ours)	0.039±0.000	0.031±0.001	0.023±0.001	0.018±0.001
NetSimile	PRD	8.667	15.314	17.992	19.083
	SCN	21.993	25.658	26.895	27.346
	SEB	14.717	17.429	18.461	18.908
	PEB	10.409	8.972	10.637	12.054
	RFF	24.993	25.270	25.487	25.930
	NEB	19.497	19.244	19.118	19.210
	PEAR (ours)	10.736±0.043	14.656±0.082	14.685±0.152	14.303±0.239

Table 8. Full experimental results on *co-DB*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.577	0.696	0.694	0.657
	SCN	0.586	0.675	0.691	0.691
	SEB	0.023	0.109	0.178	0.237
	PEB	0.610	0.536	0.504	0.465
	RFF	0.431	0.310	0.245	0.188
	NEB	0.091	0.037	0.103	0.152
	PEAR (ours)	0.034±0.000	0.043±0.000	0.056±0.000	0.056±0.000
KSND	PRD	0.183	0.225	0.259	0.279
	SCN	0.342	0.337	0.294	0.256
	SEB	0.060	0.020	0.058	0.089
	PEB	0.213	0.286	0.303	0.298
	RFF	0.328	0.227	0.177	0.135
	NEB	0.109	0.050	0.029	0.032
	PEAR (ours)	0.069±0.000	0.121±0.000	0.141±0.000	0.145±0.000
DACC	PRD	0.398	0.391	0.361	0.329
	SCN	0.152	0.158	0.166	0.164
	SEB	0.005	0.070	0.119	0.154
	PEB	0.420	0.360	0.315	0.280
	RFF	0.287	0.212	0.163	0.126
	NEB	0.043	0.018	0.055	0.087
	PEAR (ours)	0.077±0.000	0.062±0.000	0.068±0.000	0.077±0.000
NetSimile	PRD	16.278	19.594	21.209	21.816
	SCN	10.284	12.121	13.374	14.101
	SEB	9.491	8.140	8.460	8.619
	PEB	17.200	17.399	19.779	19.544
	RFF	20.067	15.482	12.799	10.487
	NEB	11.914	9.378	8.875	8.742
	PEAR (ours)	9.302±0.019	12.828±0.051	14.699±0.020	16.396±0.012

Table 9. Full experimental results on *co-GE*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.684	0.765	0.753	0.722
	SCN	0.618	0.619	0.563	0.489
	SEB	0.125	0.018	0.079	0.138
	PEB	0.700	0.566	0.534	0.503
	RFF	0.263	0.126	0.063	0.072
	NEB	0.154	0.064	0.048	0.075
	PEAR (ours)	0.105±0.000	0.153±0.000	0.163±0.001	0.169±0.001
KSND	PRD	0.229	0.283	0.314	0.346
	SCN	0.411	0.306	0.230	0.185
	SEB	0.139	0.096	0.053	0.034
	PEB	0.250	0.291	0.311	0.322
	RFF	0.254	0.116	0.071	0.043
	NEB	0.178	0.122	0.087	0.063
	PEAR (ours)	0.157±0.000	0.203±0.000	0.217±0.000	0.222±0.000
DACC	PRD	0.427	0.392	0.355	0.330
	SCN	0.134	0.142	0.129	0.103
	SEB	0.046	0.076	0.123	0.161
	PEB	0.437	0.351	0.303	0.273
	RFF	0.220	0.129	0.080	0.052
	NEB	0.047	0.008	0.037	0.055
	PEAR (ours)	0.066±0.000	0.080±0.000	0.068±0.000	0.052±0.000
NetSimile	PRD	16.969	19.851	20.997	21.678
	SCN	12.608	13.692	13.505	12.561
	SEB	14.545	10.927	8.750	9.050
	PEB	17.527	15.852	17.860	19.044
	RFF	14.022	7.388	4.724	4.033
	NEB	11.076	8.974	7.700	7.819
	PEAR (ours)	9.227±0.026	12.025±0.046	12.815±0.059	13.306±0.048

Table 10. Full experimental results on *OF*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.353	0.542	0.627	0.665
	SCN	0.390	0.498	0.518	0.524
	SEB	0.100	0.109	0.089	0.078
	PEB	0.658	0.593	0.615	0.636
	RFF	0.060	0.226	0.340	0.433
	NEB	0.223	0.157	0.119	0.091
	PEAR (ours)	0.048±0.001	0.052±0.004	0.076±0.007	0.101±0.008
KSND	PRD	0.079	0.099	0.102	0.106
	SCN	0.353	0.360	0.361	0.367
	SEB	0.058	0.090	0.082	0.072
	PEB	0.180	0.207	0.265	0.269
	RFF	0.061	0.084	0.129	0.186
	NEB	0.223	0.171	0.115	0.083
	PEAR (ours)	0.055±0.004	0.035±0.004	0.065±0.005	0.120±0.007
DACC	PRD	0.183	0.285	0.342	0.381
	SCN	0.099	0.161	0.181	0.209
	SEB	0.040	0.049	0.050	0.047
	PEB	0.310	0.250	0.218	0.195
	RFF	0.162	0.189	0.196	0.200
	NEB	0.138	0.125	0.120	0.119
	PEAR (ours)	0.061±0.003	0.123±0.004	0.188±0.006	0.233±0.008
NetSimile	PRD	8.003	12.500	13.252	14.472
	SCN	14.097	14.815	15.661	16.568
	SEB	8.826	12.128	11.639	11.238
	PEB	12.205	12.613	12.045	11.441
	RFF	10.795	11.329	12.851	14.079
	NEB	12.730	11.113	10.429	10.134
	PEAR (ours)	5.576±0.338	6.625±0.401	7.983±0.392	10.411±0.613

Table 11. Full experimental results on *FL*.

metric	method	layer-2	layer-3	layer-4	layer-5
KSCN	PRD	0.396	0.614	0.597	0.490
	SCN	0.605	0.689	0.627	0.665
	SEB	0.150	0.166	0.086	0.091
	PEB	0.454	0.430	0.355	0.332
	RFF	0.335	0.283	0.147	0.178
	NEB	0.361	0.345	0.373	0.278
	PEAR (ours)	0.104±0.004	0.246±0.009	0.286±0.018	0.409±0.017
KSND	PRD	0.051	0.137	0.184	0.275
	SCN	0.513	0.431	0.448	0.353
	SEB	0.098	0.109	0.090	0.105
	PEB	0.070	0.152	0.226	0.245
	RFF	0.245	0.166	0.089	0.084
	NEB	0.104	0.118	0.119	0.092
	PEAR (ours)	0.041±0.006	0.048±0.002	0.066±0.002	0.101±0.006
DACC	PRD	0.222	0.206	0.186	0.139
	SCN	0.355	0.444	0.480	0.404
	SEB	0.072	0.082	0.138	0.162
	PEB	0.240	0.163	0.090	0.081
	RFF	0.175	0.033	0.082	0.135
	NEB	0.169	0.138	0.152	0.110
	PEAR (ours)	0.047±0.007	0.024±0.010	0.044±0.008	0.028±0.010
NetSimile	PRD	10.978	17.282	20.712	19.354
	SCN	20.332	21.599	20.375	19.367
	SEB	9.777	11.062	10.961	12.394
	PEB	13.123	12.505	15.163	15.991
	RFF	14.230	16.560	14.197	10.233
	NEB	13.182	12.921	13.251	12.441
	PEAR (ours)	7.382±0.413	11.649±0.128	13.662±0.139	16.680±0.264