Text Fingerprinting

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- $S = s_1..s_N$ string of length n
- alphabet Σ of size $|\Sigma|$, not fixed (possibly O(n))

A fingerprint f : set of character(s) of a substring $s_i.. s_j$

General problem:

Compute and represent the set of all fingerprints of S

Examples:

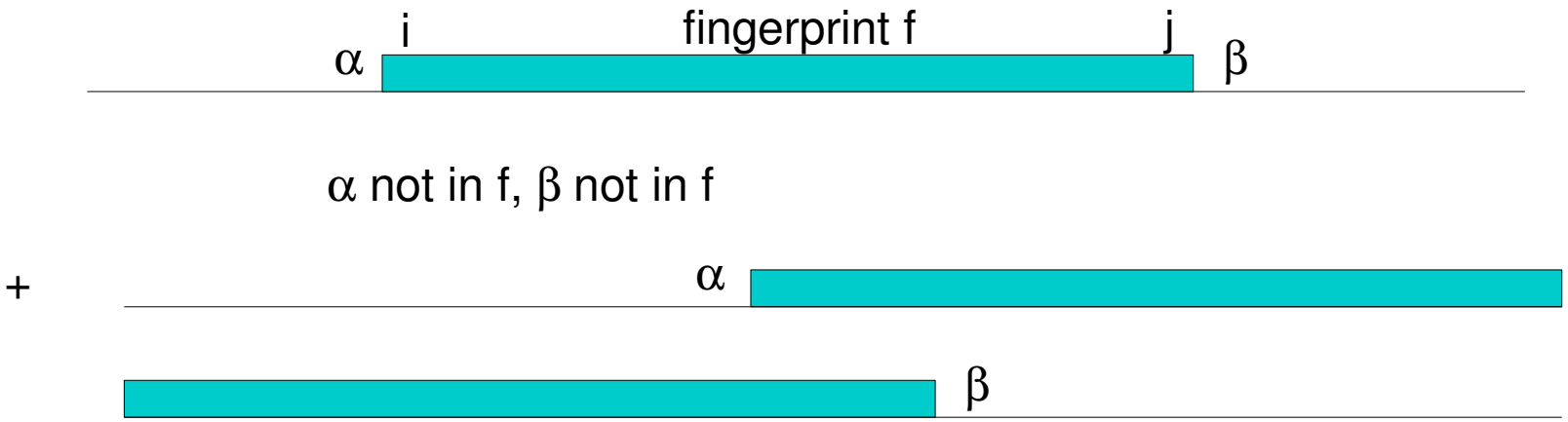
dccbcbabbbc

{a} {b} {c} {d} {c,d} {b,c} {a,b} {b,c,d} {a,b,c} {a,b,c,d}

acbdcadad

{a} {b} {c} {d} {a,c} {a,d} {b,c} {b,d} {c,d} {a,b,c} {a,c,d} {b,c,d} {a,b,c,d}

Maximal location <i,j> of f



Number of maximal locations: $L <= n|\Sigma|$ Complexity of the bound easily reached

But is usually much less

$\Sigma_k = \{a_1,a_2,..,a_k\}$ $w_1 = a_1$, $w_k = w_{k-1}\, a_k\, w_{k-1}$

$w_2=a_1(a_2)a_1$, $w_3=(a_1a_2a_1)a_3(a_1a_2a_3)$, ...

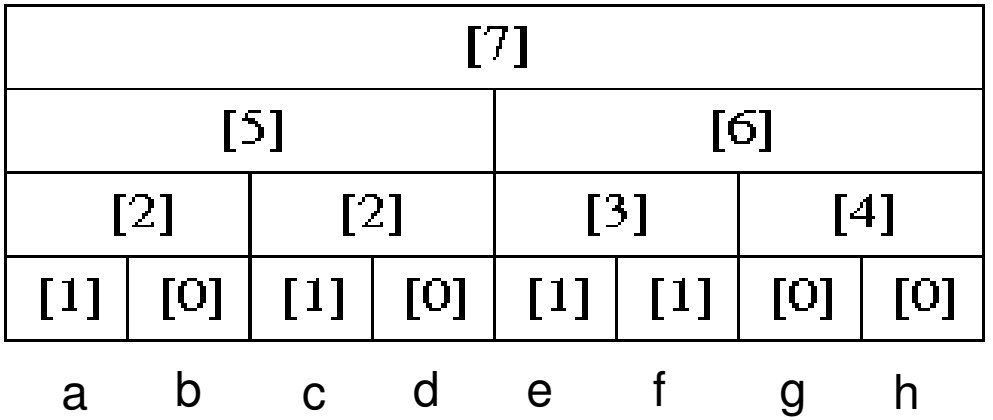
$|w_k| \,.\, |L_k| = k \,.\, (2^k - 1)$ $|L|_k = 2^{k+1}-(k+2)$ → $|L|_k=o(|w_k| \,.\, |L_k|)$

First Part

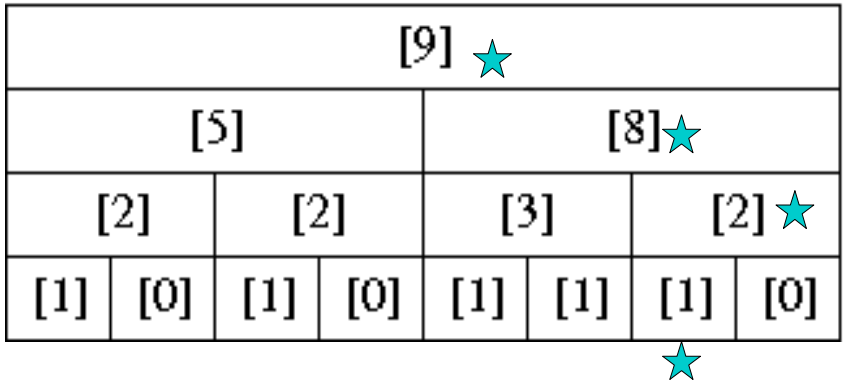
Compute the set of all fingerprints

Naming technique

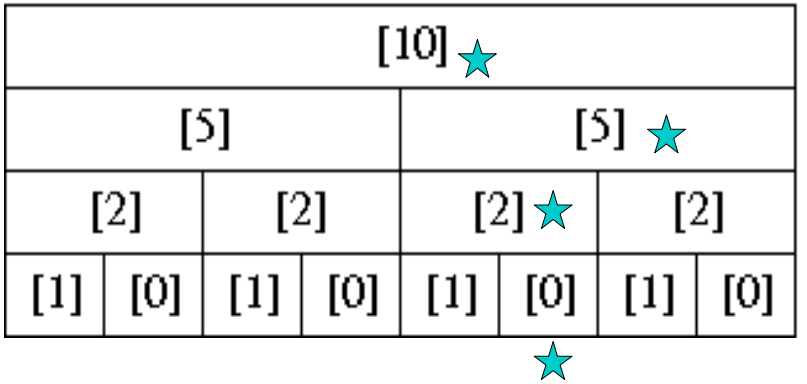
{a,c,e,f} $\Sigma = \{a,b,c,d,e,f,g,h\}$



$\log |\Sigma| + 1$



{a,c,e,f,g}

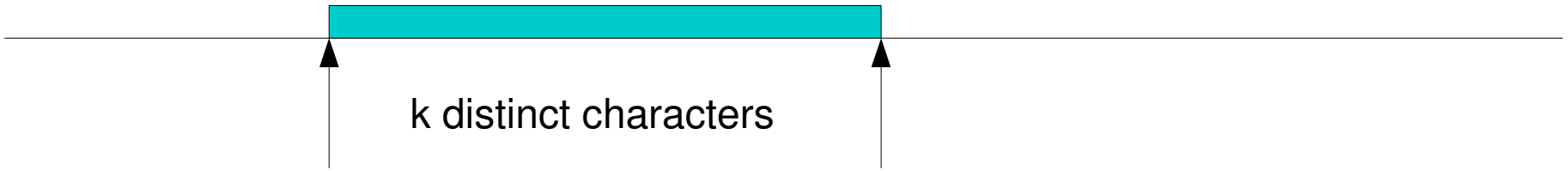


{a,c,e,g}

Names = {[1],[2],[3],[4],[5],[6],[7],[8],[9],[10]}

Fingerprints ={[7],[9],[10]}

Amir, Apostolico, Landau, Satta 2003

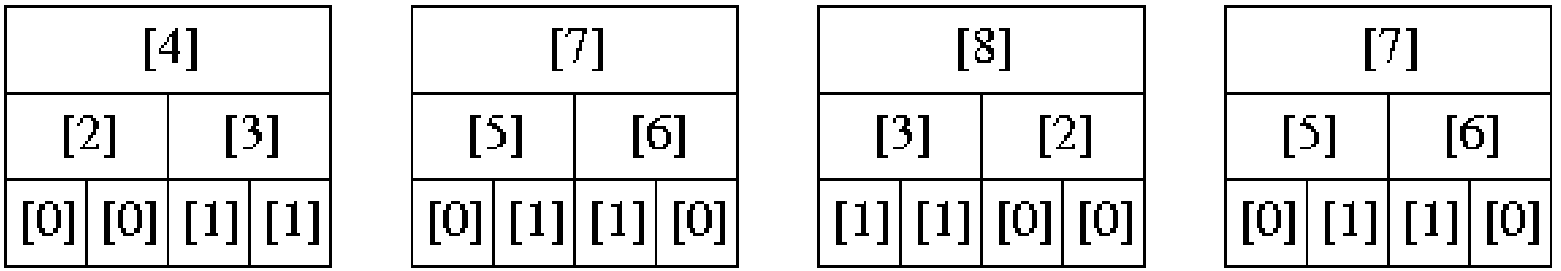


Changing a character: O(log |Σ| log n) (n new names maximum by level)

One iteration: n log |Σ| log n

|Σ| iterations: |Σ| n log |Σ| log n

Important: different set of names for each iteration

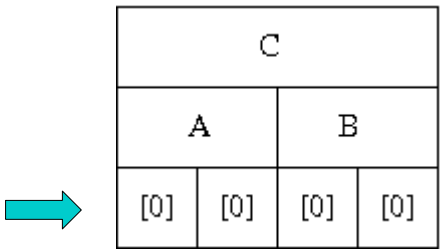


k=2 d c c b c b a b b b c

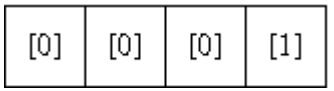
d c c b c b a b b b c

Tsur 2005

List of fingerprints: d c a d^{-1} b {d}, {c,d}, {a,c,d}, {a,c}, {a,b,c}

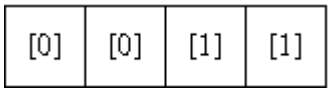


d

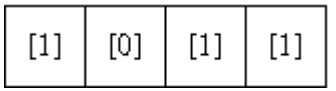


{([0],[1]), B}

d c

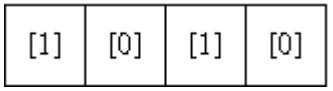


{([1],[1]), B}



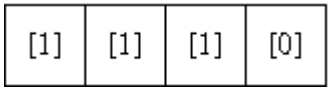
{([1],[0]), A}

d c a d^{-1}



{([1],[0]), B}

d c a d^{-1} b



{([1],[1]), A}

List of changes:

{([0],[0]), A} {([0,0]), B} | {([0],[1]), B} {([1],[1], B} {([1],[0]), A} {([1],[0]), B} {([1],[1]), A}

Radix sort on the pairs + unique -> new names

Tsur 2005

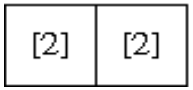
List of changes:

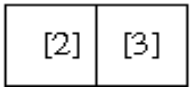
{([0],[0]), A} {([0],[0]), B} | {([0],[1]), B} {([1],[1], B} {([1],[0]), A} {([1],[0]), B} {([1],[1]), A}

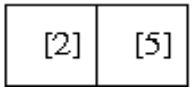
[2] -> ([0],[0])
[3] -> ([0],[1])
[4] -> ([1],[0])
[5] -> ([1],[1])

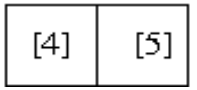
New list:

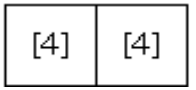
{[2], A} {[2], B} | {[3], B} {[5], B} {[4], A} {[4], B} {[5], A}

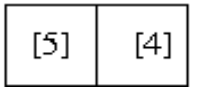












{([2],[2]), C} {([2],[3]),C}

New list: {([2],[2]),C} | {([2],[3]),C} {([2],[5]),C} {([4],[5]),C} {([4],[4]),C} {([5],[4]),C}

Radix sort, ...

Tsur 2005

Radix sort: O(n) (bounded integers)

One iteration : n log $|\Sigma|$ No more name search !

→ $|\Sigma|$ iterations: $|\Sigma|$ n log $|\Sigma|$

Problems

- does not depend of L
- distinct names at each iteration

Our approach (2006) Simple sequence: no repeated character

lfo(i) a b a c e a b a c d a b a c e a b a c d

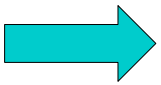
lfo(4)=ceab lfo(2) = bace

Concatenate # to the sequence

Bijection L / proper prefixes of lfo(i)

cea a b a c e a b a c d #

bac a b a c e a b a c d #



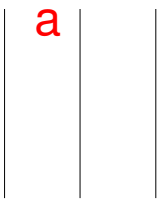
Compute all lfo(i) of S#

Our approach (2006)

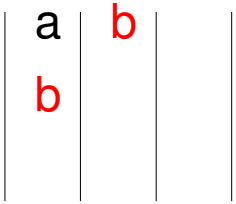
How to calculate all lfo(i) ?

abcbadca

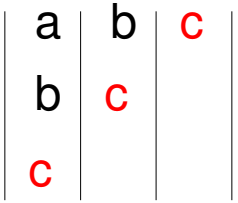
a | bcbadca#



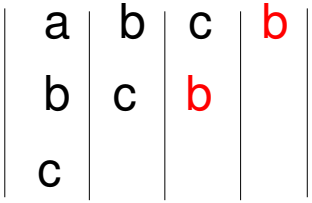
ab | cbadca#



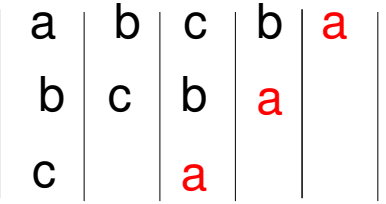
abc | badca#



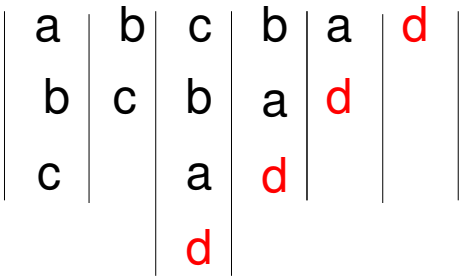
abcb | adca#



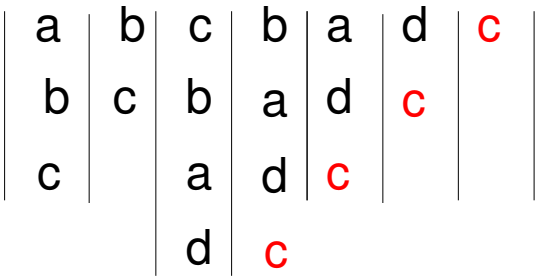
abcba | dca#



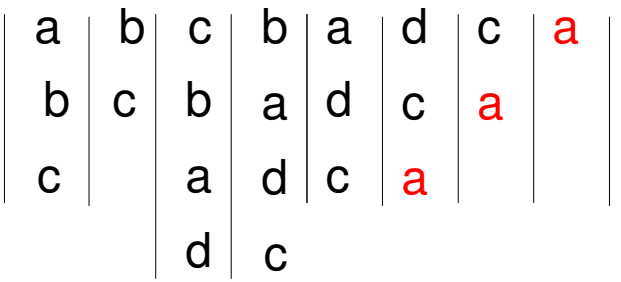
abcbad | ca#



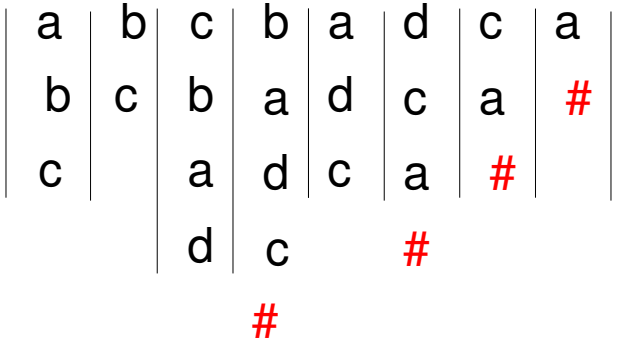
abcbadc | a#



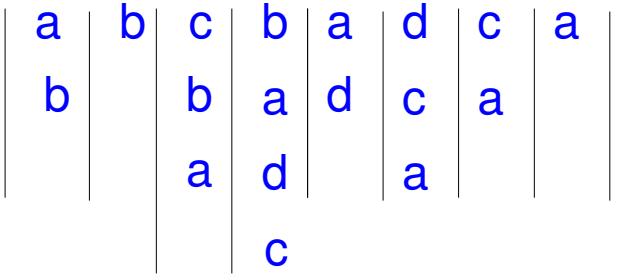
abcbadca | #



abcbadca#

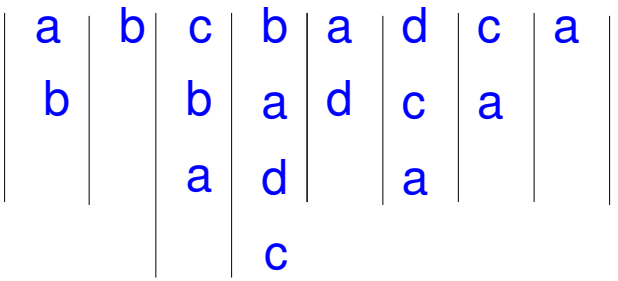


lfo(i)



Our approach (2006)

Naming all proper prefixes of lfo(i)



n lists:

- Tsur algorithm
- Common names

Simple sequence: O(|L| log |Σ|)

General sequence: O(n+|L| log |Σ|)

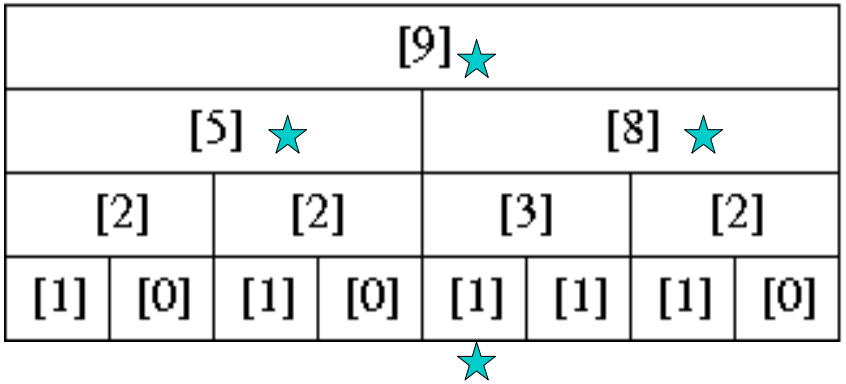
|L|<= n |Σ| ⇒ Faster or as fast as that of Tsur

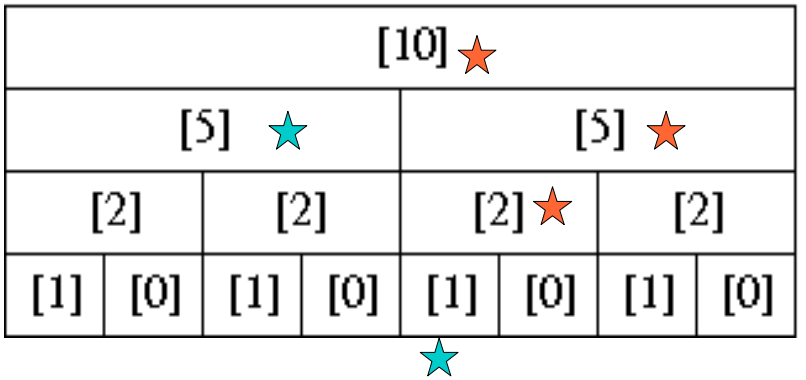
Our approach (2006)

Properties and operations on our names

- a unique set of names

→ Compute the LCP of two fingerprints in log |Σ|



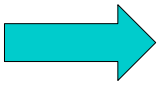


- names sorted by lexicographic order of fingerprints

Second Part

Represent the set of all fingerprints

Table of names

A fingerprint f  Fingerprint table of size $|\Sigma|$

Bottom up name

Complexity: $O(|\Sigma| \log n)$

Perfect hashing

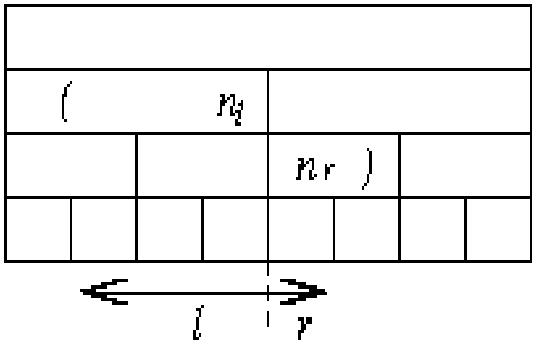
	Search	Preprocessing
Expected time:	$O(|\Sigma|)$	$O(|F|\log|\Sigma|)$

Preprocessing worst case time and space: $O(|F|^2\log|\Sigma|)$

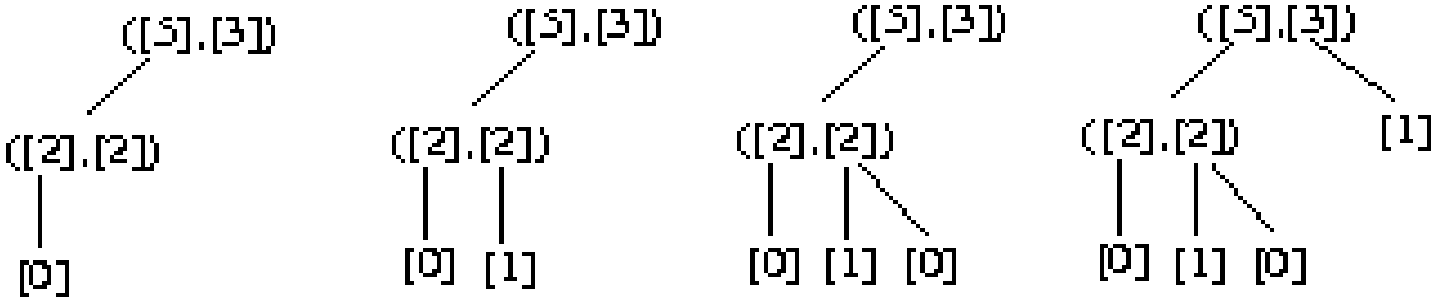
Fingerprint tree - names in lexicographic order

- LCP

Missing: Edge coding



Decode an edge

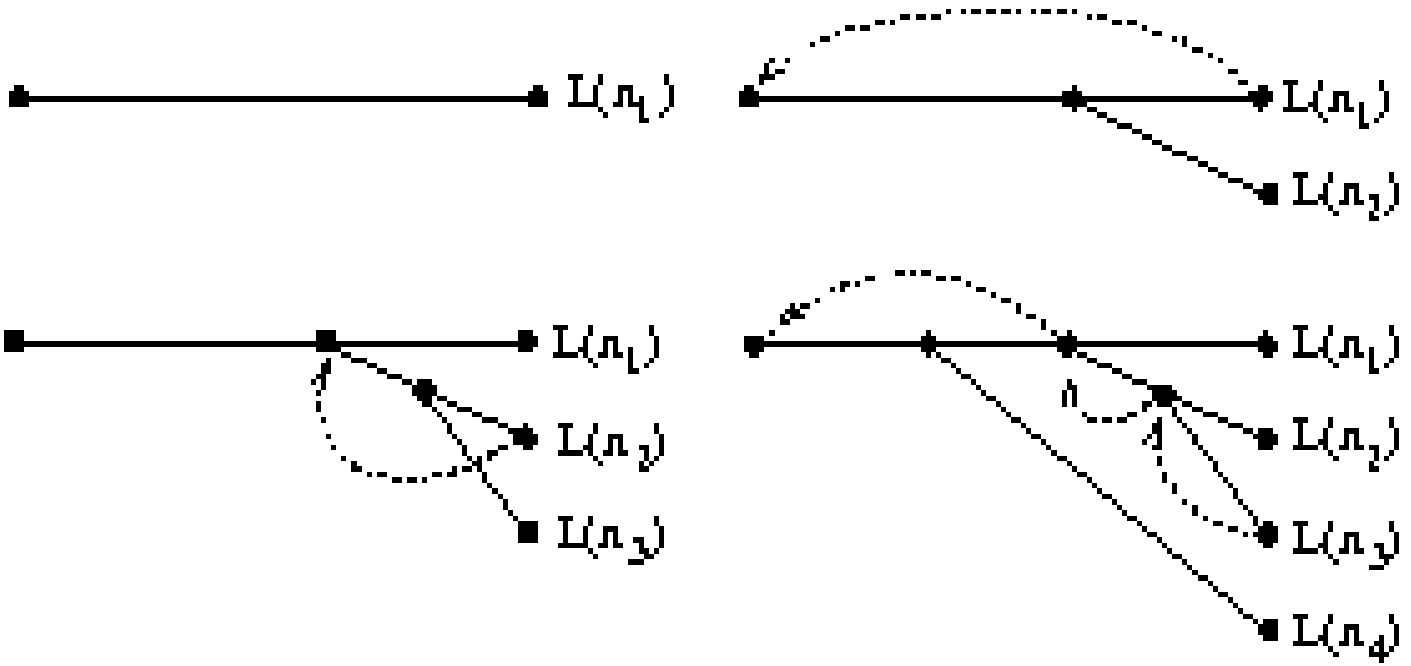


Decoding is linear in the size of the edge !

Fingerprint tree

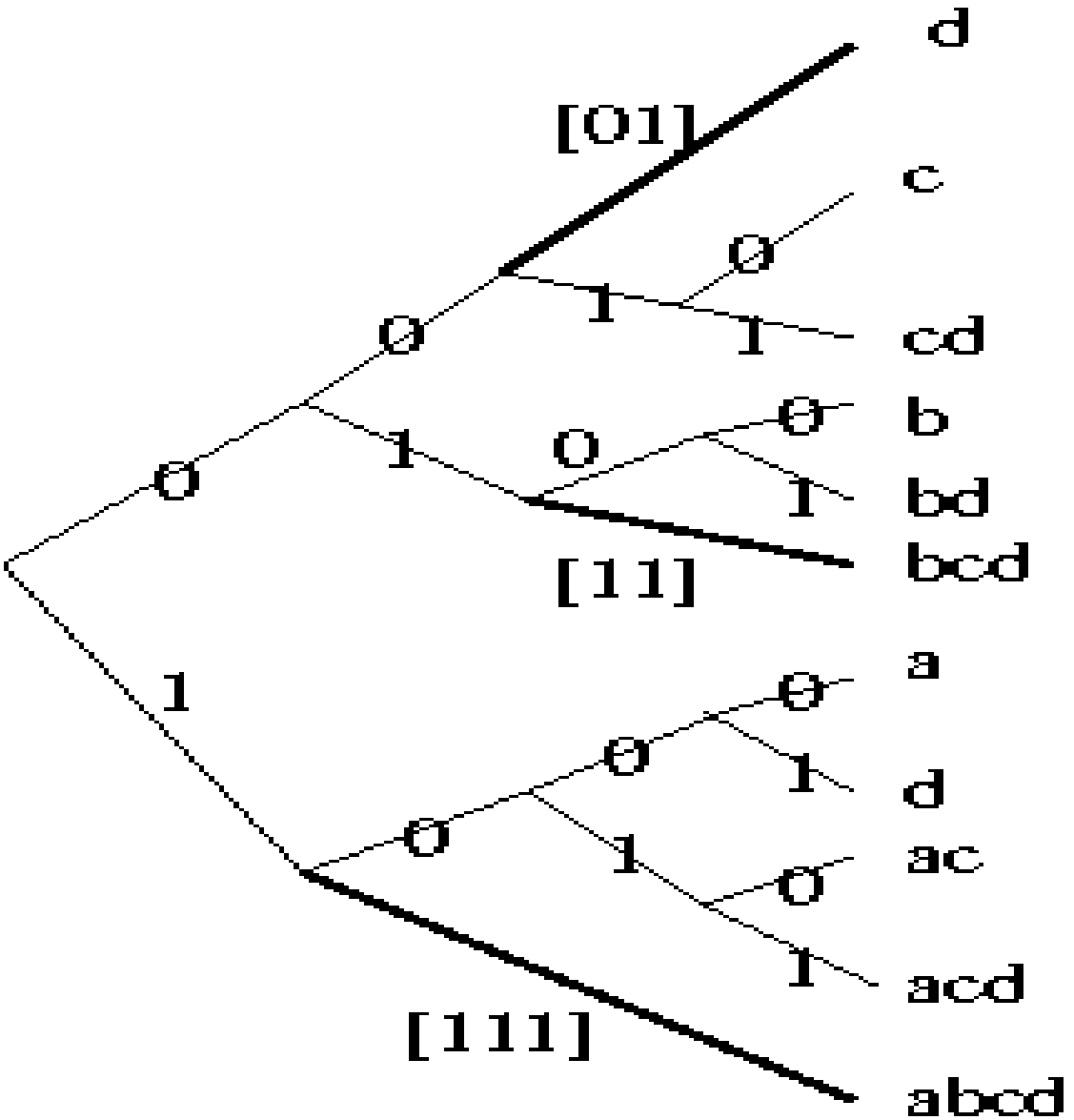
3 steps:

- → $LCP[n_i,n_{i+1}]$ — $O(|F| \log|\Sigma|)$
- → Structure of the tree — $O(|F|)$
- → Compute each edge code — $O(|F| \log|\Sigma|)$



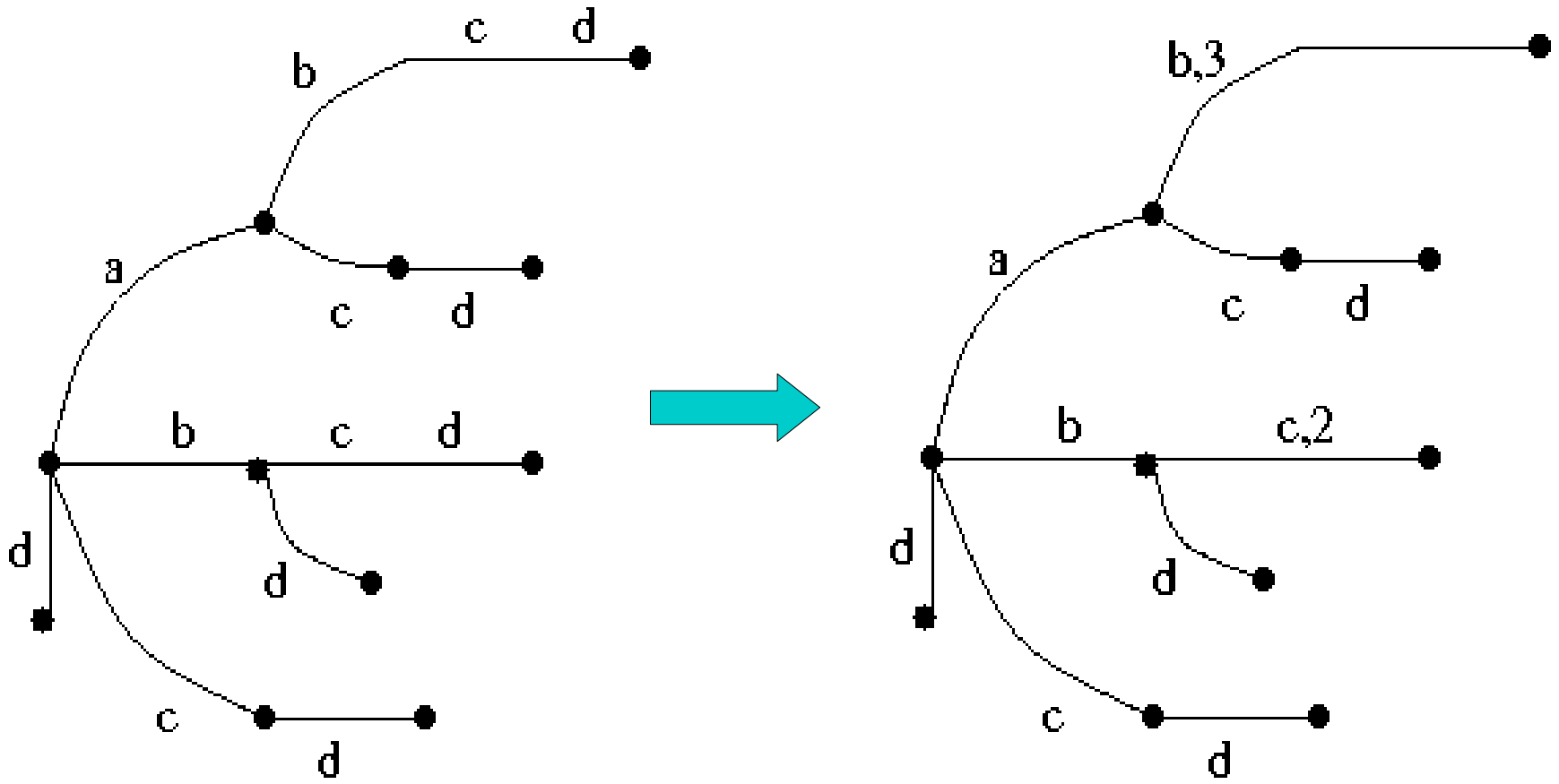
Fingerprint tree example

bdcad



Search: O(|Σ|)

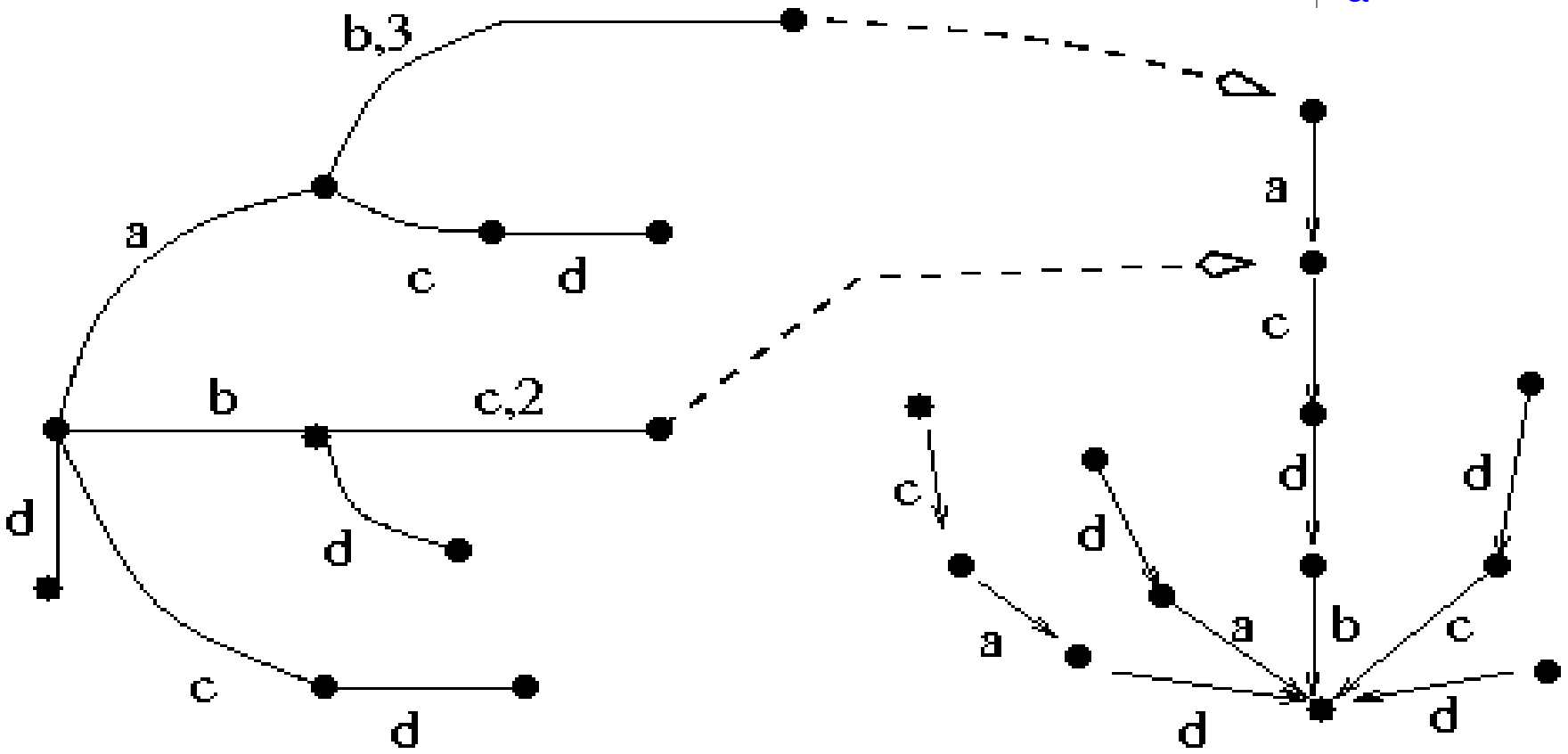
Fingerprint trie bdcad Chan *et al*, ESA 2007



Fingerprint trie

bdcad

b	d	c	a	d
d	c	a	d	
c		d		
a				



O(|F|) space

O(|F|log|Σ|) time

Search in O(|f|log(|f|/|Σ|))

Open problems

- Memory space reduction
- Order ?
- Approximate fingerprint
- Distance by fingerprints
- 2D fingerprints