ECON381 Fall 2024

Homework Assignment 4

Explanation of Questions (Brief and Clear)

1. Minimum Spanning Tree (MST)

An **MST** is a subgraph of a graph that:

- Includes all nodes.
- Connects them with the minimum total edge weight.
- Does not form cycles.

Using **Kruskal's algorithm**, you sort edges by weight and add them one by one, avoiding cycles. For the given graph, the MST includes edges:
(C-E, 1), (B-C, 3), (D-F, 3), (D-E, 4), and (A-B, 4) with a total weight of **15**.

2. Uniqueness of the MST

An MST is **unique** if all edge weights in the graph are distinct.
In this graph, some edges share the same weight (e.g., (B-C, 3) and (D-F, 3)), so the MST is **not unique**.

3. Shortest Path with Dijkstra's Algorithm

Dijkstra's algorithm calculates the shortest paths from a starting node (A) to all others:

1. Start from A with distance 0.
2. Visit neighbors and update distances if a shorter path is found.
3. Repeat for the next closest node.
   For this graph:
   dist(A) = 0, dist(B) = 4, dist(C) = 7, dist(D) = 7, dist(E) = 8, dist(F) = 10.

4. Critical Edges

A **critical edge** disconnects the graph if removed.
In this graph, removing any edge does not disconnect it. So, there are **no critical edges**.

5. Articulation Points

An **articulation point** is a node whose removal disconnects the graph.
In this graph, removing any node still leaves the graph connected. Therefore, there are **no articulation points**.

6. Path When a Node is Unavailable

If the path A-B-C-E is disrupted because C becomes unavailable, you can be sure another path (e.g., A-D-E) exists because there are no critical edges or articulation points.

7. Graph Robustness

Graph robustness measures how resilient a graph is to disruptions. This graph is robust because:

- It has no critical edges.
- It has no articulation points.
  Removing one edge or node does not disconnect it.

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