HANIF AHMAD SYAUQI

Catatan MSD - Hanif A S - 1612116 1

- Ruang hasil kali dalam

Contoh:

- Diketahui $\bar{u} = 2i$ dan $\bar{v} = 3i + 4j$ tentukan $\bar{u} \cdot \bar{v}$?

jawab:

$\bar{u} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \bar{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\bar{u} \cdot \bar{v} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\bar{u} \cdot \bar{v} = (2 \cdot 3) + (3 \cdot 4)$

$\bar{u} \cdot \bar{v} = 6 + 12 = 18$

- Menyelesaikan Sudut Antar dua vektor

Contoh:

Dik $\bar{u} = 2i$ dan $\bar{v} = 2i + 2j$ tentukan besar sudut yang terbentuk?

jawab:

$\|\bar{u}\| = \sqrt{2^2 + 0^2} = 2$

$\bar{u} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \bar{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad \|\bar{v}\| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$$\cos\theta = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| \cdot \|\bar{v}\|} = \frac{\begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix}}{2 \cdot 2\sqrt{2}} = \frac{(2 \cdot 2) + (0 \cdot 2)}{2 \cdot 2\sqrt{2}}$$

$$= \frac{4+0}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2} \rightarrow \theta = \frac{\pi}{4} = 45^\circ$$

Hasil kali silang

jika $\bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$, $\bar{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ adalah vektor pada 3D

maka hasil kali silang

$$\bar{u} \times \bar{v} = \begin{bmatrix} \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \\ -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \\ \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \end{bmatrix}$$

Contoh

Carilah $\bar{u} \times \bar{v}$ dimana $\bar{u} = [1, 2, -2]$ $\bar{v} = [3, 0, 1]$.

jawab :

$$\bar{u} \times \bar{v} = \begin{bmatrix} 1 & 2 & -2 \\ 3 & 0 & 1 \end{bmatrix} = \begin{vmatrix} \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \\ -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \\ \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \end{vmatrix} = \begin{vmatrix} \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} \end{vmatrix}$$

$$= \begin{bmatrix} (2.1) - (-2.0) \\ -((1.1) - (-2.3)) \\ (1.0) - (2.3) \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ -6 \end{bmatrix}$$

Definisi hasil kali dalam dinotasikan $\langle \cdot , \cdot \rangle$

Simetris : $\langle \bar{u}, \bar{v} \rangle = \langle \bar{v}, \bar{u} \rangle$

Aditivitas : $\langle \bar{u} + \bar{v}, \bar{w} \rangle = \langle \bar{u}, \bar{w} \rangle + \langle \bar{v}, \bar{w} \rangle$

Homogenitas : $\langle a\bar{u}, \bar{v} \rangle = a \langle \bar{u}, \bar{v} \rangle$

Positifitas : $\langle \bar{u}, \bar{u} \rangle \geq 0$ dan $\langle \bar{u}, \bar{u} \rangle = 0 \Leftrightarrow \bar{u} = \bar{0}$

Ruang vektor yang dilengkapi dengan hasil kali dalam disebut ruang hasil kali dalam.

Contoh

Diketahui $\langle \bar{u}, \bar{v} \rangle = 2$, $\langle \bar{v}, \bar{w} \rangle = -3$,
$\langle \bar{u}, \bar{w} \rangle = 5$, $\|\bar{u}\| = 1$, $\|\bar{v}\| = 2$
$\|\bar{w}\| = 4$. Hitunglah $\langle \bar{u} + \bar{v}, \bar{v} + \bar{w} \rangle$?

jawab:

$$\begin{aligned}
\langle \bar{u} + \bar{v}, \bar{v} + \bar{w} \rangle &= \langle \bar{u}, \bar{v} + \bar{w} \rangle + \langle \bar{v}, \bar{v} + \bar{w} \rangle \\
&= \langle \bar{u}, \bar{v} \rangle + \langle \bar{u}, \bar{w} \rangle + \langle \bar{v}, \bar{v} \rangle + \langle \bar{v}, \bar{w} \rangle \\
&= 2 + 5 + \|\bar{v}\|^2 + (-3) \\
&= 4 + 2^2 \\
&= 8
\end{aligned}$$

- Himpunan Orthogonal

Diketahui V adalah ruang hasil kali dalam dan $\bar{V}_1, \bar{V}_2, \ldots, \bar{V}_n, \in V$. $H = \{\bar{V}_1, V_2, \ldots, V_n\}$ disebut himpunan orthogonal jika untuk setiap vector dalam V saling tegak lurus berikut

$$\langle \bar{V}_i, \bar{V}_j = 0 \quad i \neq j \quad \text{dan } i, j = 1, 2, \ldots, n$$

Himpunan Orthonormal

Diketahui V adalah ruang hasil kali dalam

Diketahui $\bar{U} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \bar{V} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \bar{w} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \in R^3$

dan R^3 adalah ruang hasil kali dalam.

Tunjukkan bahwa $\bar{u}, \bar{V}, \bar{w}$ orthonormal.
untuk membuktikan orthogonal maka perlu ditunjukkan

$$\langle \bar{U}, \bar{V} \rangle = \langle \bar{V}, \bar{w} \rangle = \langle \bar{u}, \bar{w} \rangle = 0$$

$$\langle \bar{u}, \bar{V} \rangle = \bar{u} \cdot \bar{V} = u_1 V_1 + u_2 V_2 + u_3 V_3$$
$$= 0 \cdot 1 + 1 \cdot 0 + 0 \cdot 1 = 0$$

$$\langle \bar{V}, \bar{w} \rangle = \bar{V} \cdot \bar{w} = V_1 w_1 + V_2 w_2 + V_3 w_3$$
$$= 1 \cdot 1 + 0 \cdot 0 + 1 \cdot (-1) = 1 - 1 = 0$$

$$\langle \vec{u}, \vec{w} \rangle = \bar{u} \cdot \bar{w} = u_1 w_1 + u_2 w_2 + u_3 w_3$$
$$= 0 \cdot 1 + 1 \cdot 0 + 0 \cdot (-1) = 0$$

jadi $\bar{u}_1, \bar{u}_2, \bar{u}_3$ adalah vektor² orthogonal

- Diketahui $\bar{u}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\bar{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \in R^3$ dan R^3 adalah ruang hasil kali dalam. Tunjukkan bahwa $\bar{u}, \bar{v}, \bar{w}$ orthonormal.

Untuk membuktikan orthonormal maka perlu ditunjukkan $\|\bar{u}\| = \|\bar{v}\| = \|\bar{w}\| = 1$

$$\|\bar{u}\| = \sqrt{0^2 + 1^2 + 0^2} = 1$$
$$\|\bar{v}\| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$$
$$\|\bar{w}\| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2}$$

karena $\|\bar{u}\| = \|\bar{v}\| = \|\bar{w}\| \neq 1$

Jadi $\bar{u}, \vec{v}, \bar{w}$ adalah bukan vektor-vektor orthonormal

Metode Gram - Schmidt

Diketahui $H = \{ \bar{V}_1, \bar{V}_2, \bar{V}_3 \}$ dengan $\bar{V}_1 = [1,1,1]^T$, $\bar{V}_2 = [1,2,1]^T$, $\bar{V}_3 = [-1,1,0]^T$.

$H = \{ \bar{V}_1, \bar{V}_2, \bar{V}_3 \}$ adalah basis

a.) Ubahlah $\{ \bar{V}_1, \bar{V}_2, \bar{V}_3 \}$ menjadi basis - basis ortogonal
b.) Ubahlah $\{ \bar{V}_1, \bar{V}_2, \bar{V}_3 \}$ menjadi basis - basis ortonormal

Jawab

a) $\bar{S}_1 = \bar{V}_1 = (1, 1, 1)^T$

$$\bar{S}_2 = \bar{V}_2 - \frac{\langle \bar{V}_2, \bar{S}_1 \rangle}{\| \bar{S}_1 \|^2} \bar{S}_1 = \begin{vmatrix} 1 \\ 2 \\ 1 \end{vmatrix} - \frac{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{(\sqrt{1^2 + 1^2 + 1^2})^2} \cdot \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 \\ 2 \\ 1 \end{vmatrix} - \frac{4}{3} \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 - \frac{4}{3} \\ 2 - \frac{4}{3} \\ 1 - \frac{4}{3} \end{vmatrix} = \begin{vmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{vmatrix}$$

Experience is the best teacher

$$\overline{S_3} = \overline{V_3} - \frac{\langle \overline{V_3}, \overline{S_1} \rangle}{\|S_1\|^2}\overline{S_2} = \begin{vmatrix} -1 \\ 1 \\ 6 \end{vmatrix} - \frac{\begin{vmatrix} -1 \\ 1 \\ 0 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}}{1^2+1^2+1^2} - \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

$$- \frac{\begin{vmatrix} -1 \\ 1 \\ 0 \end{vmatrix} \cdot \begin{vmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{vmatrix}}{\left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2} \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$\|S_1\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

$$\|S_2\| = \sqrt{\left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2} = \sqrt{\frac{6}{9}} = \frac{1}{3}\sqrt{6}$$

$$\|S_3\| = \sqrt{\left(-\frac{1}{2}\right)^2 + (0)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{2}{4}} = \frac{1}{2}\sqrt{2}$$

$$w = \{\overline{W_1}, \overline{W_2}, \ldots, \overline{W_n}\} = \left\{\frac{\overline{S_1}}{\|S_1\|}, \frac{\overline{S_2}}{\|S_2\|}, \ldots, \frac{\overline{S_n}}{\|S_n\|}\right\}$$

$$= \left\{ \frac{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\sqrt{3}}, \frac{\begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}}{\frac{1}{3}\sqrt{6}}, \frac{\begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}}{\frac{1}{2}\sqrt{2}} \right\}$$

• Transformasi Linear

Kernel dan Range

Misalkan $T : P_2 \to R^2$ didefinisikan oleh

$$T(a + bx + cx^2) = \begin{bmatrix} a-b \\ a-c \end{bmatrix}$$

Manakah yg merupakan ker(T)?

a) $S_1 = 1 + x + x^2$

b) $S_2 = 1 + 2x + x^2$

Jawab

a) $T(S_1) = T(1 + x + x^2) = \begin{bmatrix} a-b \\ a-c \end{bmatrix} = \begin{bmatrix} 1-1 \\ 1-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \bar{0} \Rightarrow S_1 \in \ker(T)$

b) $T(S_2) = T(1 + 2x + x^2) = \begin{bmatrix} a-b \\ a-c \end{bmatrix} = \begin{bmatrix} 1-2 \\ 1-1 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 0 \end{bmatrix} \neq \bar{0} \Rightarrow S_1 \notin \ker(T)$

Rank dan Nullitas

Tentukan basis & dimensi dari $Ker(T)$ dan $R(T)$ dari transformasi linear $T: R^4 \to R$ dengan

$$T\left[\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}\right] = \begin{bmatrix} a+b \\ c-2d \\ -a-b+c-2d \end{bmatrix}$$

jawab

$$T\left[\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}\right] = \begin{bmatrix} a+b \\ c-2d \\ -a-b+c-2d \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ -1 & -1 & 1 & 0 \end{bmatrix}$$

Maka Basis Jangkauan $R(T) = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ Rank = 2

Basis $ker(T)$ dan dimensi Nulitas

Berdasarkan hasil OBE sebelumnya

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

berdasarkan OBE sebelumnya kolom yang tidak terdapat 1 utama ada pada kolom c dan d

maka, dimisalkan kes dan dat dimana $s, t \in R$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow a+b = 0 \to a = -b$
$\quad c-2d = 0 \quad c = 2d$

$\Rightarrow a = -s$
$\quad c = 2t$

Pisahkan vector dengan parameter

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -s \\ s \\ 2t \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} t$$

Maka basis $Ker(T) = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$ Nulitas = 2

Determinan

Determinan metode ekspansi kofaktor (Minor dan kofaktor)

$$A = \begin{bmatrix} 1 & 3 \\ -4 & 3 \end{bmatrix} \quad M_{23} = \begin{vmatrix} 1 & 3 \\ -4 & 3 \end{vmatrix} = 5$$

$$C_{23} = (-1)^{2+3} \cdot M_{23} = -15$$

Ekspansi kofaktor

$$\det(A) = \begin{vmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{vmatrix} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$= a_{11}(-1)^{1+1} M_{11} + a_{12}(-1)^{1+2} M_{12} + a_{13}(-1)^{1+3} M_{13}$$

$$= 3 \cdot (-1)^2 \begin{vmatrix} -4 & 3 \\ 4 & -2 \end{vmatrix} + 1 \cdot (-1)^3 \begin{vmatrix} -2 & 3 \\ 5 & -2 \end{vmatrix} + 0 \cdot (-1)^4 \begin{vmatrix} -2 & \cancel{-4} \\ 5 & 4 \end{vmatrix}$$

$$= 3(-4) + (-1)(-11) + 0 = 1$$

You'll never know till you have tried

Aturan Cramer

$x_1 + 2x_3 = 6$
$-3x_1 + 4x_2 + 6x_3 = 30$
$-x_1 - 2x_2 + 3x_3 = 8$

Tentukan Penyelesaian dari SPL diatas dengan aturan cramer

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{pmatrix} \quad A_1 = \begin{pmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{pmatrix} \quad A_3 = \begin{pmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{pmatrix}$$

Sehingga diperoleh

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{-40}{44} = \frac{-10}{11}, \quad x_2 = \frac{\det(A_2)}{\det(A)}$$

$$= \frac{72}{44} = \frac{18}{11}, \quad x_3 = \frac{\det(A_3)}{\det(A)} = \frac{152}{44} = \frac{38}{11}$$

• Ruang Eigen

Diketahui $A = \begin{bmatrix} 6 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ Tentukan : Nilai dan vektor eigen serta ruang eigen

Penyelesaian :

$\det(\lambda I - A) = 0$ maka

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}\right)$$

$$= \det\left(\begin{bmatrix} \lambda & 0 & 2 \\ -1 & \lambda-2 & -1 \\ -1 & 0 & \lambda-3 \end{bmatrix}\right)$$

$$= \lambda(\lambda-2)(\lambda-3) + 2(0+\lambda-2)$$

$$= \lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

dengan memfaktorkan diperoleh $(\lambda-2)(\lambda-2)(\lambda-1) = 0$ maka nilai eigen adalah 2 dan 1

Diketahui $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ Tentukan Nilai dan Vektor eigen serta ruang eigen

Penyelesaian:

untuk mendapatkan vektor eigen maka di substitusikan nilai-nilai eigen ke persamaan $(A - \lambda I)\bar{x} = \bar{0}$ yaitu:

$$\begin{bmatrix} \lambda & 0 & 2 \\ -1 & \lambda-2 & -1 \\ -1 & 0 & \lambda-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ diperoleh untuk $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

diperoleh dengan OBE

$$\begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Sehingga solusi $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

maka Vektor eigen adalah $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Diketahui $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ Tentukan nilai dan vektor eigen serta ruang eigen

Penyelesaian :

- untuk $\lambda = 1$ $\begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

- Dengan OBE diperoleh $\begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ -1 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

- Sehingga solusi $\bar{x} = \begin{bmatrix} x^1 \\ x^2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ dan

vektor eigen adalah $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$.

Diketahui $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ Tentukan nilai dan vektor eigen serta ruang eigen

penyelesaian :

Ruang eigen yang berkaitan dengan nilai $\lambda = 2$ adalah

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$ dengan basis

$$= \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$ dan ruang eigen yang berkaitan

dengan nilai $\lambda = 1$ adalah

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$ dengan basis $= \left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$

Experience is the best teacher