RULES OF THE GAME:

- Solutions need to be submitted through Gradescope.
- If you would like to learn the LaTeX typesettting system and use it for assignment submissions, then please take a look at this template for what it can do.
- If you plan to write your answers on paper, then you will need to scan your work:

App	Platform	Description
Adobe Scan	iOS, Android	Great quality, but requires an Adobe account
Microsoft Office Lens	iOS, Android, Windows 10	Works on laptops too
Evernote Scannable	iOS	Recommended by Gradescope
Genius Scan	iOS, Android	Recommended by Gradescope

- Please consult Gradescope's Scanning Work on a Mobile Device page and watch the Submitting PDF Homework video.

OUR LATENESS POLICY:

- You have one slip day for **each** assignment you can use without having to notify us. **Work submitted beyond the slip day without an academically valid excuse will not earn any points.**
- The deadline to submit documents regarding academically valid excuses is the original deadline of the assignment (i.e. not that of the slip day).
- In order to reward complying with deadlines, anyone who uses only one slip day will have a 1% added to their final score and anyone who uses zero slip days will a 2% added to their final score.

REGARDING ACADEMIC INTEGRITY:

- You may collaborate with one or two of your peers in the course to brainstorm for solving the assigned problems. However, your solution must be written up completely on your own.
- If you collaborate with others during the brainstorming part of the assignment, you must list their names at the beginning of your submission.
- You cannot use any online resources, any solutions from past semesters, etc. You are not allowed to share digital or written notes or images of your work in any form.
- Even when a question feels overwhelming, you should not compromise your academic integrity. Since the exam questions will be similar to the assigned exercises/problems, not struggling with these questions (in the context of assignments) will be counter productive with respect to grades.

QUESTIONS/PROBLEMS:

While we have not officially talked about sets in CS-2110, it is beneficial to pick up one notation from that branch of mathmematics. A set is a container of elements that neither permit repetitions nor preserves order. The number of element in a set s is referred to as the **cardinality** of that set and is notated as $|s|$. In our study of graphs, we use this notation often since a graph G is made up of the vertex set V and the edge set E. And when we refer to algorithms on sets, the asymptotic complexity of these algorithms are usually framed as functions of $|V|$ and $|E|$.

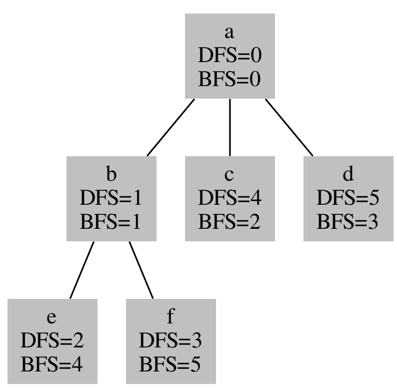
1. The following table lists nine functions as row headers and five asymptotic complexity symbols as column headers. Each cell makes a proposition: if the row-header function belongs to the column-header asymptotic complexity class, then we put a TRUE in that call; otherwise we put a FALSE. For example, we put a TRUE in the top-left cell because $3\lg n + 8$ is asymptotically upper-bounded by n^2; that is, $3\lg n + 8 = O(n^2)$:

	$a(n) \stackrel{?}{=} O(n^2)$	$a(n) \stackrel{?}{=} o(n^2)$	$a(n) \stackrel{?}{=} \Omega(n^2)$	$a(n) \stackrel{?}{=} \omega(n^2)$	$a(n) \stackrel{?}{=} \Theta(n^2)$
$a(n) = 3\lg n + 8$					
$a(n) = 4n^2$					
$a(n) = 4n^3 + 3n^2$					
$a(n) = 5n + 7$					
$a(n) = 6n^2 + 9$					
$a(n) = 6n^6 + n^4$					
$a(n) = 2n\lg n$					
$a(n) = 5n^2 + 2n$					
$a(n) = 2^n + 4n$					

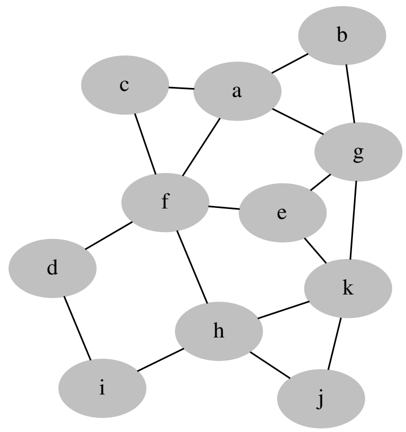
If you put the correct entries in all 45 entries, then you will see a pattern among the TRUE/FALSE values that will make it easy to remember the asymptotic complexity classifications.

2. Suppose we have an undirected simple graph $G = (V, E)$. What is the maximum value of $|E|$ as a function of $|V|$? If you cannot immediately think of an answer, please do a few examples. For example, what is the maximum values of $|E|$ if $|V| = 2$? If $|V| = 3$? if $|V| = 4$? If $|V| = 5$? Based on the answers to these questions, what is the general answer?

3. At the end of class on 7/27 (Tuesday) we used the following example to define what it means to label the nodes of a tree (and thus a graph) according to the BFS and DFS traversal patterns:



Suppose we have the following graph G:



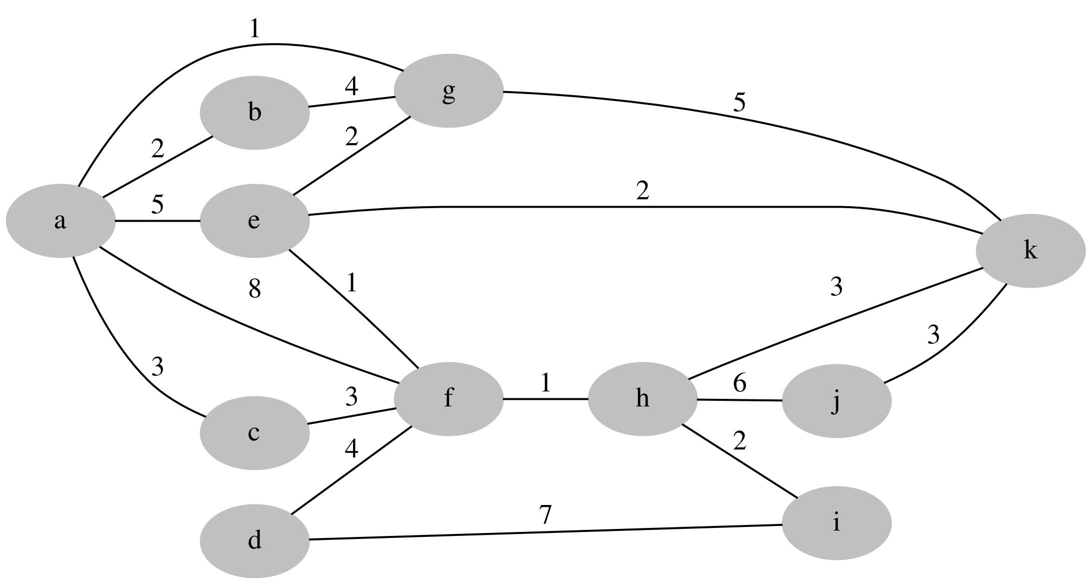
(a) Draw the adjacency list of G.

(b) Draw the adjacency matrix of G.

(c) Number the nodes of G by a DFS traversal, starting at node a. In order to have consistent answers within our class, process the vertexes alphabetically whenever order does not make a difference in the operation of the algorithm.

(d) Number the nodes of G by a BFS traversal, starting at node a. In order to have consistent answers within our class, process the vertexes alphabetically whenever order does not make a difference in the operation of the algorithm.

4. Suppose we start Prim's algorithm on node c of the following graph:



(a) Which edges are chosen for the spanning tree?

(b) What is the order in which the edges are chosen for the spanning tree?

(c) What is the asymptotic complexity of Prim's algorithm?

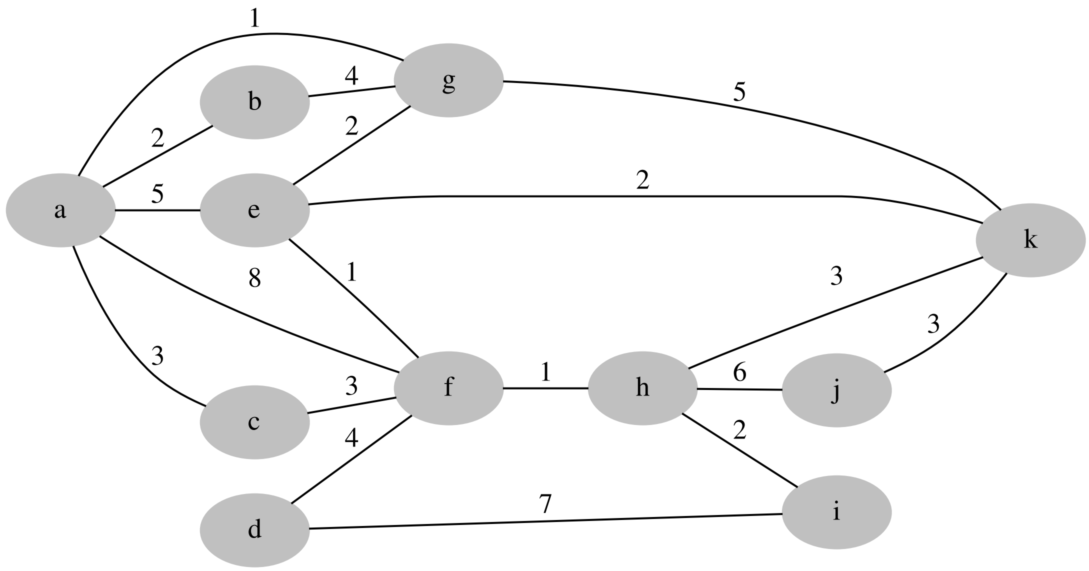
5. Suppose we start Dijkstra's algorithm on the following graph on vertex g:

(a) For each vertex v, what is the cost of the cheapest path from g to v?

(b) What is the order in which the cheapest path costs are determined in part 5a?

(c) The edges selected by Dijkstra's algoritm form a spanning tree as well. How does the sum of the costs of these edges compare to the sum of the costs of the edges select by Prim's algoritm in question 4?

(d) What is the asymptotic complexity of Dikstra's algorithm?

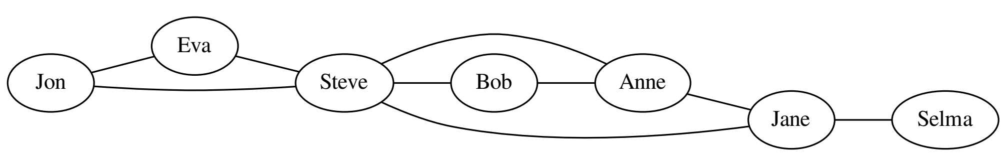


6. As you all know, Facebook is based on the symmetric "friending" model such that when user x friends user y, x ends up being a friend of y and y ends up being a friend with x. Instagram, on the other hand, is based on the asymmetric "following" model such that when x decides to follow y, y does not (automatically) follow x.

   Suppose we have a group of N Instagram users. If there is a user in this group who follows none of the remaining $N-1$ users but is followed by every one of them, then that user is called an influencer. Create a $o(N^2)$ algorithm (i.e. one that is strictly more efficient than $O(N^2)$) to determine which one of a group of N users if an influencer; needless to say, your algorithm must accommodate the fact that there may be no influencers. Asking whether x is following y should be your fundamental $O(1)$ operation; you may not assume the existence of any batch-oriented (i.e. unrealistically time-saving) $O(1)$ operation.

   Please provide your algorithm is pseudo-code; please have a complete mathematical argument to justify the $o(N^2)$ time complexity.

7. In question 6, we explored an efficient algorithm to find influencers[1]; we now explore a different societal role: fragmenter[2] A fragmenter is someone whose removeal from a social network breaks that social network into multiple connected components. For example, in the following graph, we have two fragmenters: Steve and Jane.



   Create an algorithm (in pseudo-code) that runs in $O(|V|+|E|)$ time to determine all fragmenters in the graph version of a social network.

[1] Just for the record, it actually annoys me to have to write his word (influencer)... I think it's sad that social media has constructed an environment where we have to define (and worse) use such words. What is up with deferring to someone else's decisions? What happened to the seductiveness of doing something <u>only</u> because no one else is doing it?? OK, I am done.

[2] It's not a thing, I made it up.