

L11 - MAC0315

4.1

$$\begin{array}{ll} \text{maximize} & 3p_2 + 6p_3 \\ \text{subject to} & p_1 \leq 0, \\ & p_2 \geq 0, \\ & p_3 \text{ free,} \\ & 2p_1 + 3p_2 - p_3 \geq 1, \\ & 3p_1 + p_2 - p_3 \leq -1, \\ & -p_1 + 4p_2 + 2p_3 \leq 0, \\ & p_1 - 2p_2 + p_3 = 0 \end{array}$$

4.2

maximize

$$p'b$$

subject to

$$p \geq 0,$$

$$p'A \leq c$$

Transformando em um problema de minimização

$$\text{minimize } -x'b$$

subject to

$$x \geq 0,$$

~~$$-x'A \leq -c$$~~

$$-x'A \geq -c$$

Queremos que seja idêntico ao problema primal, então

$$\bullet \quad c'x = -x'b \Rightarrow c'x + x'b = 0$$

$$\Rightarrow c'x + b'x = 0$$

$$\Rightarrow (c' + b')x = 0$$

$$\text{mas } x \geq 0 \Rightarrow \underline{(c+b)' = 0} \quad (\Rightarrow b = -c)$$

• queremos que $Ax \geq b$ e $-x'A \geq -c$
determinem as mesmas restrições

$$x'A \geq -C \Rightarrow x'A \leq C$$

$$\Rightarrow \sum a_i x_i \leq C,$$

a_i as linhas de A

$$Ax \geq b \Rightarrow \sum A_i x_i \geq b,$$

A_i as colunas de A .

$$\begin{aligned} \sum a_i x_i &\leq c \\ \sum A_i x_i &\geq b \end{aligned} \quad \Rightarrow \quad \begin{aligned} \sum a_i x_i - c &\leq 0 \\ \sum A_i x_i - b &\geq 0 \end{aligned}$$

$$\Rightarrow \sum A_i x_i - b - (\sum a_i x_i - c) \geq 0$$

$$\Rightarrow \sum A_i x_i - \sum a_i x_i - b + c \geq 0$$

$$\Rightarrow \sum A_i x_i - a_i x_i \geq b - c$$

$$\Rightarrow \sum (A_i - a_i) x_i \geq b - c = 2b$$

$$\Rightarrow \sum (A_i - a_i) x_i \geq 2b \Rightarrow \sum (A_i - a_i) \geq |2b| \Rightarrow A_i - a_i \geq |2b|, \forall i$$

$\hookrightarrow x \geq 0$

Ex:

$$A = \begin{pmatrix} 5 & 2 & 3 \\ 2 & 5 & 2 \\ 3 & 2 & 5 \\ 3 & 3 & 2 \end{pmatrix}$$

$$b = (-1, -1, -1, 1) \quad c = (1, 1, 1, 1)$$

Primal:

min ~~max~~ $x_1 + x_2 + x_3 + x_4$

subj. to $5x_1 + 2x_2 + 3x_3 \geq -1$

$$2x_1 + 5x_2 + 2x_3 \geq -1$$

$$3x_1 + 2x_2 + 5x_3 \geq -1$$

$$3x_1 + 3x_2 + 2x_3 \geq -1$$

$$x_1, \dots, x_4 \geq 0$$

Dual:

min $x_1 + x_2 + x_3 + x_4$

subj. to $-5x_1 + 2x_2 - 3x_3 - 3x_4 \geq 1$

$$-2x_1 + 5x_2 - 2x_3 - 3x_4 \geq 1$$

$$-3x_1 - 2x_2 - 5x_3 - 2x_4 \geq 1$$