Lecture 6

Stereo Systems
Multi-view geometry

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Computational Vision and Geometry Lab

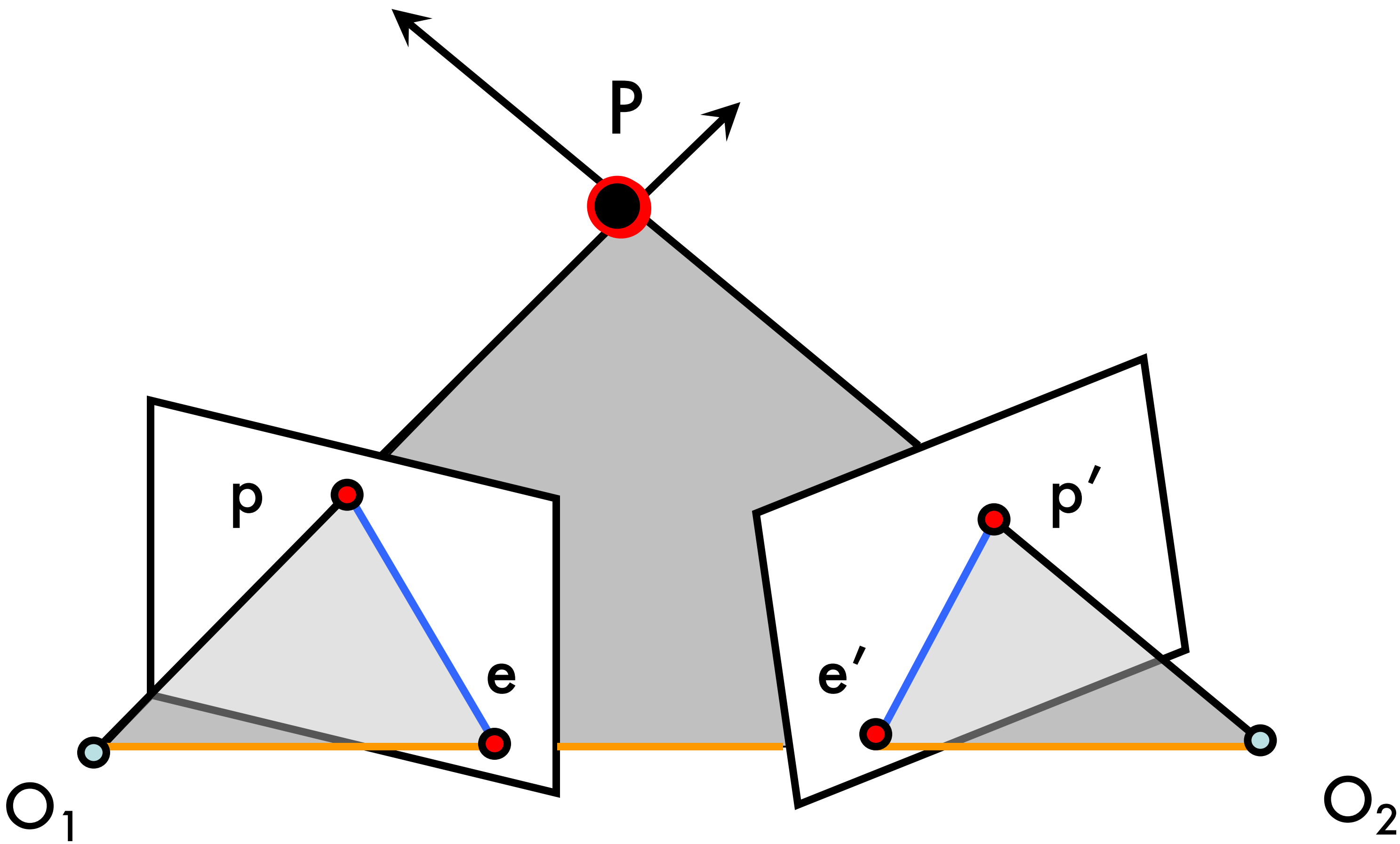
Lecture 6

Stereo Systems
Multi-view geometry

- Stereo systems
  - Rectification
  - Correspondence problem
- Multi-view geometry
  - The SFM problem
  - Affine SFM

Reading: [AZ] Chapter: 9 "Epip. Geom. and the Fundam. Matrix Transf."
[AZ] Chapter: 18 "N view computational methods"
[FP] Chapters: 7 "Stereopsis"
[FP] Chapters: 8 "Structure from Motion"

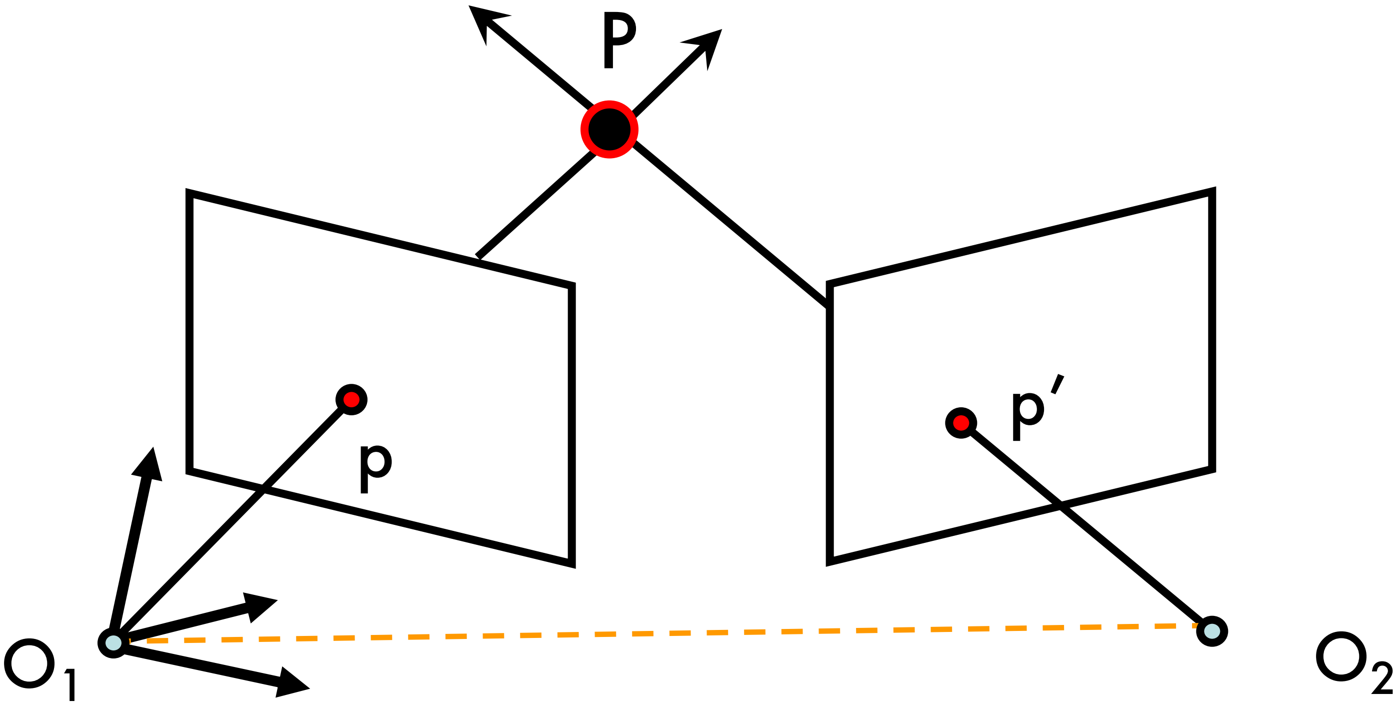
Epipolar geometry



- Epipolar Plane
- Baseline
- Epipolar Lines
- Epipoles e, e′

= intersections of baseline with image planes
= projections of the other camera center

Epipolar Constraint



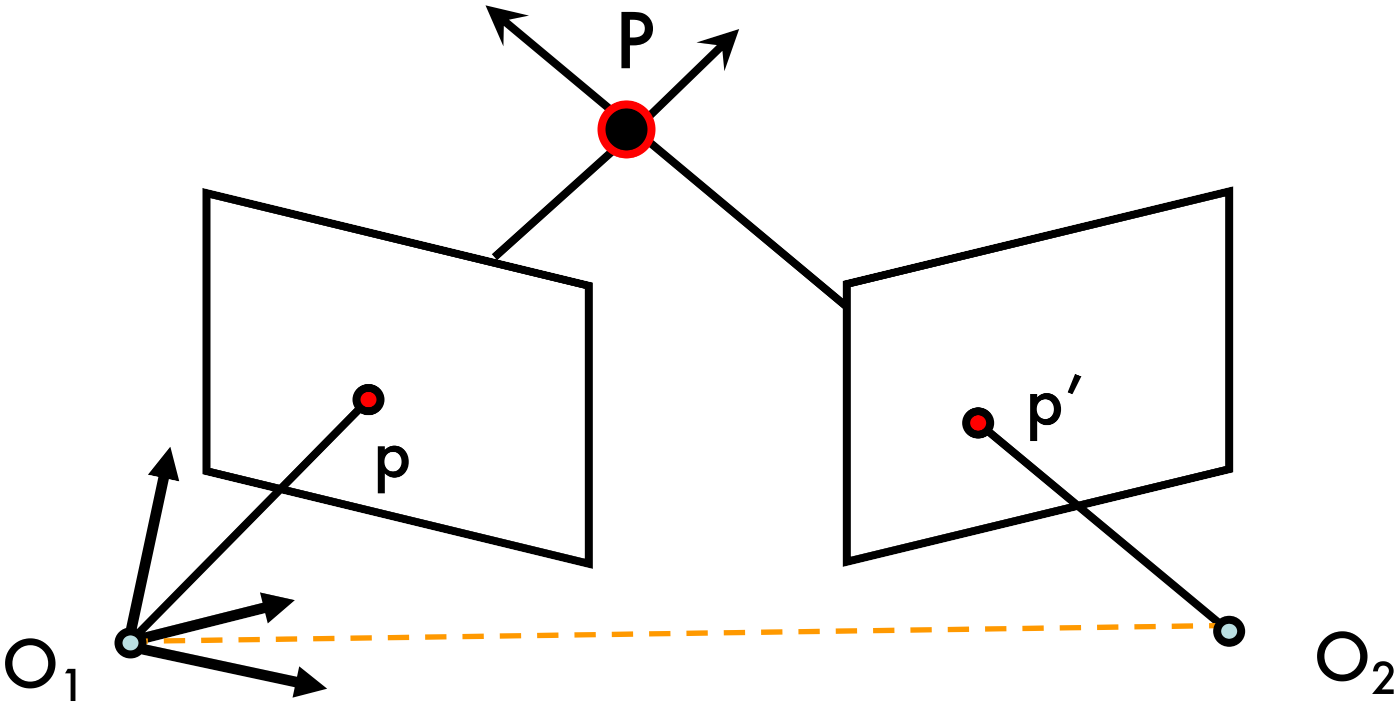
$$\boxed{p^T \; E \; p' = 0}$$

$$E = [T_\times] \cdot R$$

E = Essential Matrix

(Longuet-Higgins, 1981)

Epipolar Constraint



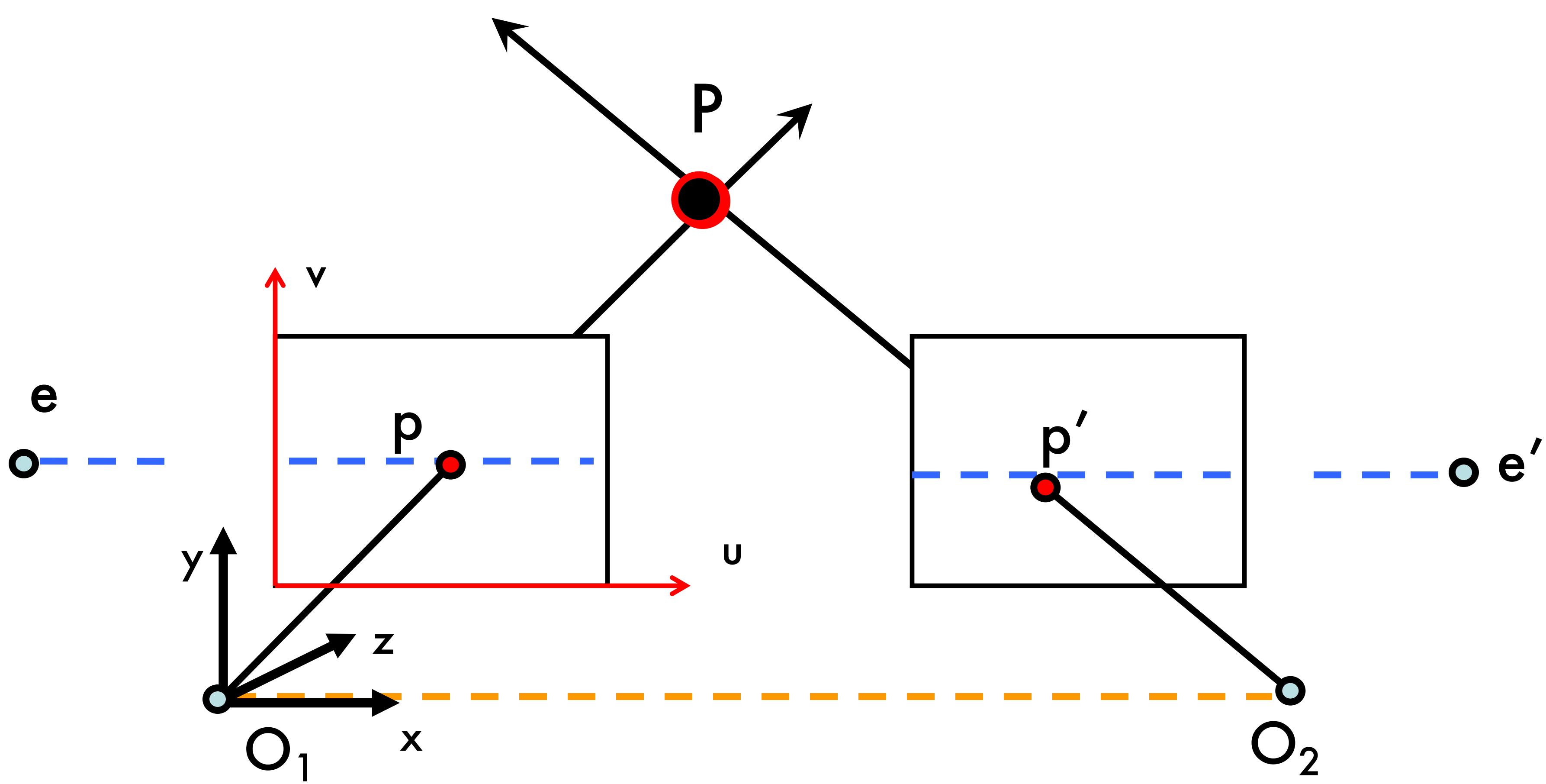
$$\boxed{p^T F p' = 0}$$

$$F = K^{-T} \cdot [T_\times] \cdot R K'^{-1}$$

F = Fundamental Matrix

(Faugeras and Luong, 1992)

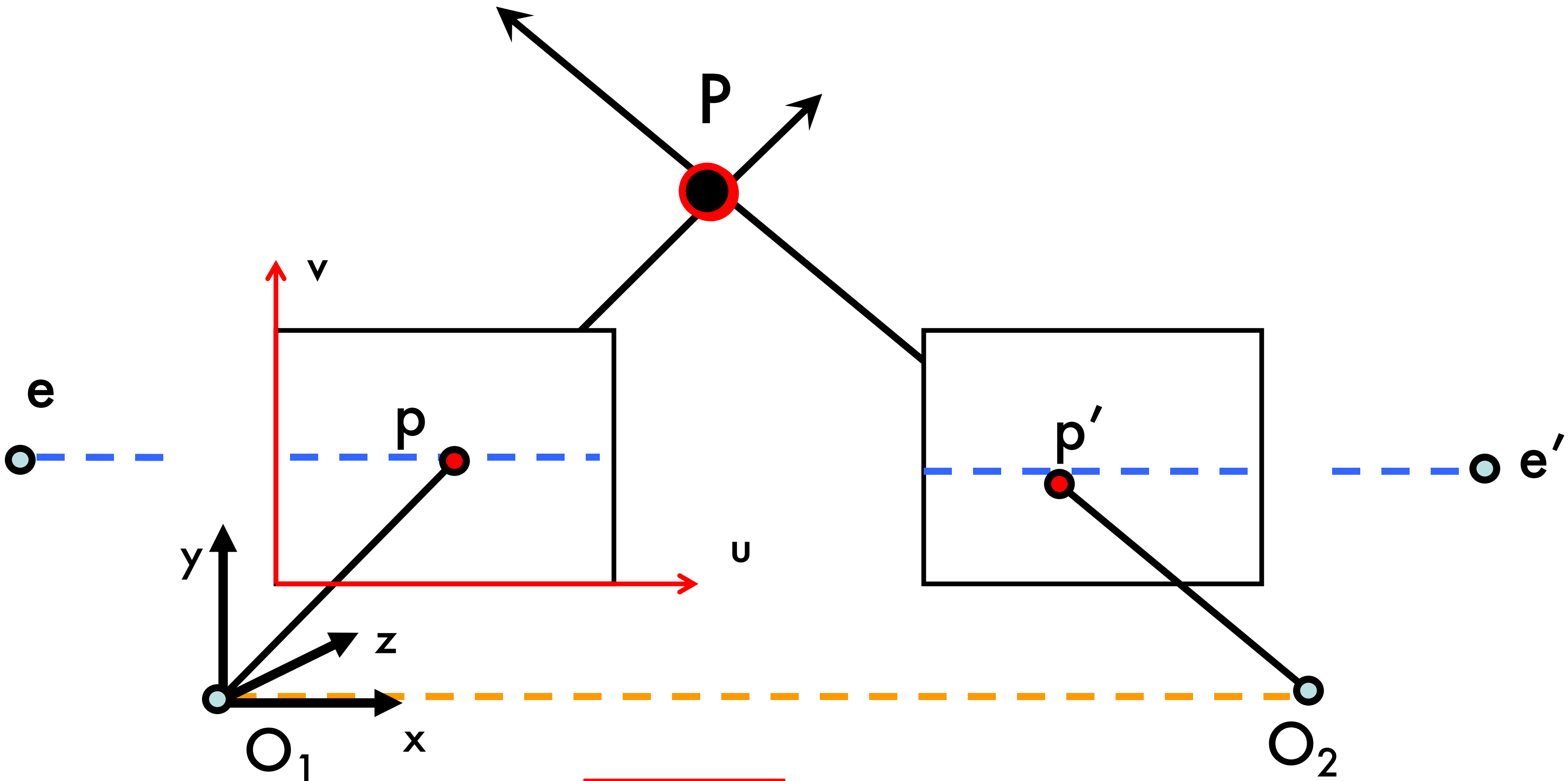
Parallel image planes



- Epipolar lines are horizontal
- Epipoles go to infinity
- v-coordinates are equal

$$p = \begin{bmatrix} p_u \\ p_v \\ 1 \end{bmatrix} \qquad p' = \begin{bmatrix} p'_u \\ p_v \\ 1 \end{bmatrix}$$

Parallel image planes



$\boxed{E = ?}$

Hint :

$\boldsymbol{R} = I \qquad \boldsymbol{T} = (T, 0, 0)$

$K_1 = K_2$ = known

x parallel to O_1O_2

Essential matrix for parallel images

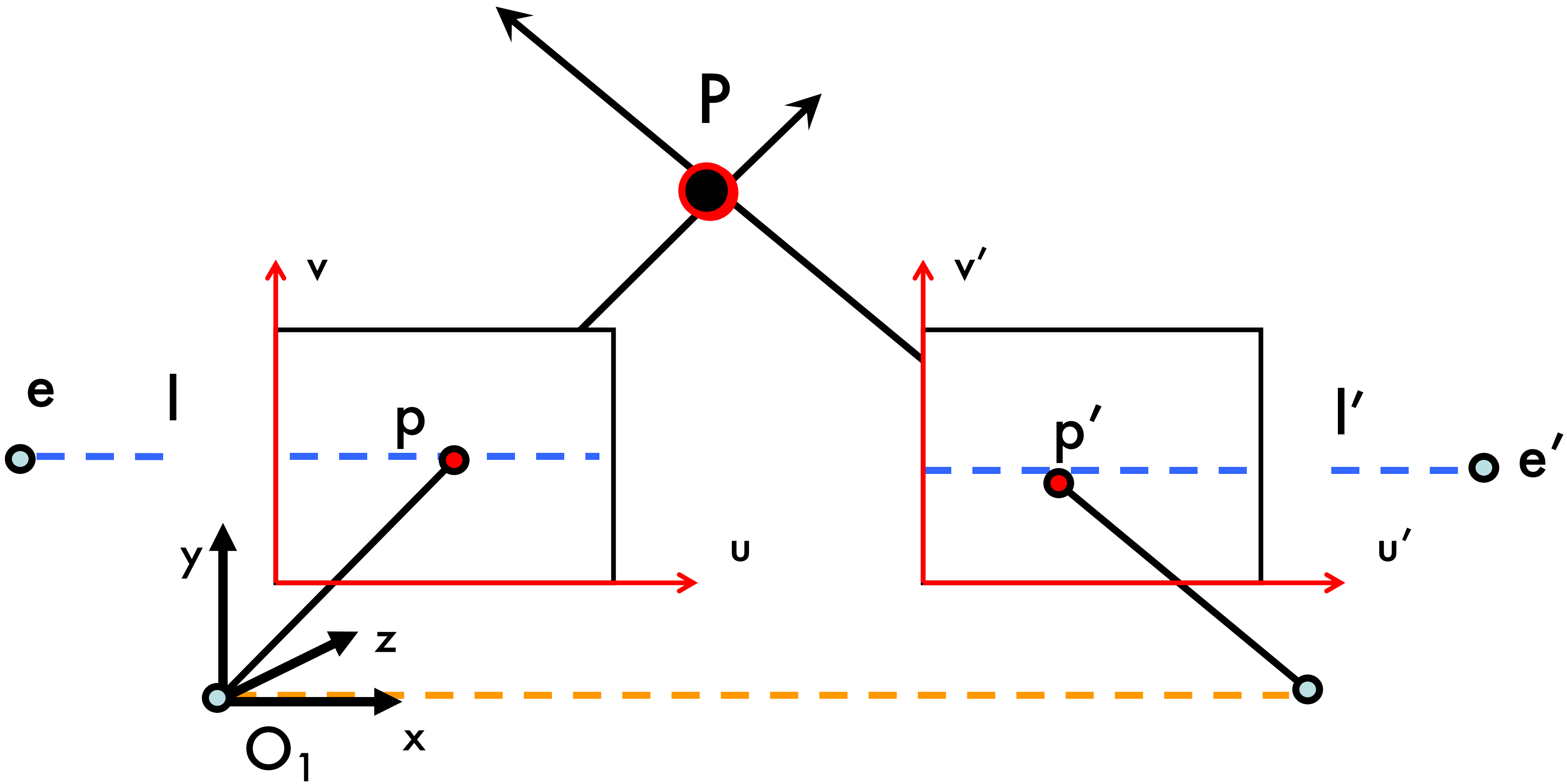
$$\mathbf{E} = [\mathbf{T}_{\times}] \cdot \mathbf{R}$$

$$\mathbf{E} = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix} \mathbf{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$\mathbf{T} = [\ \mathrm{T}\ \ 0\ \ 0\ ]$

$\mathbf{R} = \mathbf{I}$

Parallel image planes

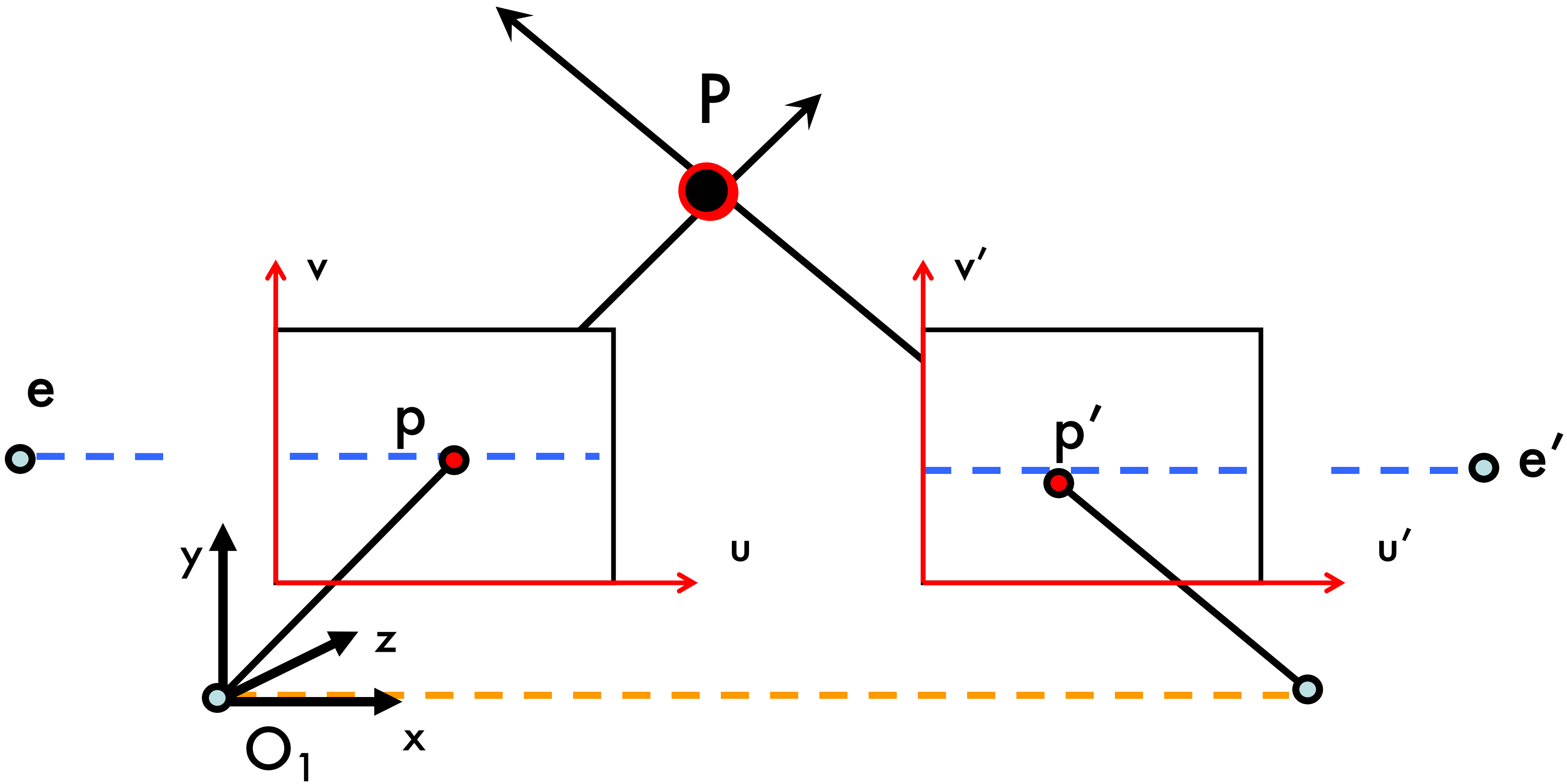


What are the directions of epipolar lines?

$$l = \mathrm{E}\, p' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -T \\ T v' \end{bmatrix}$$

horizontal!

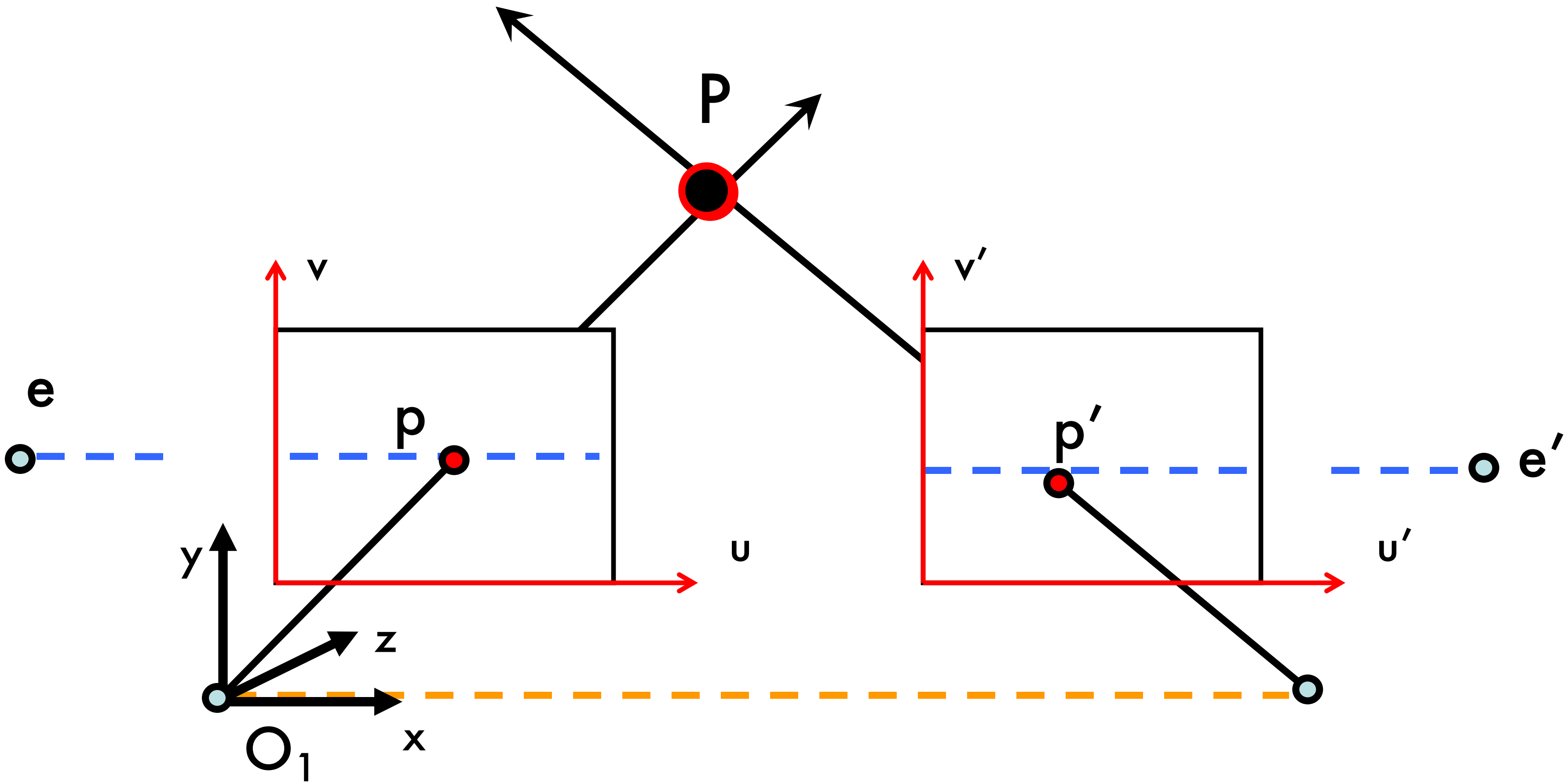
Parallel image planes



How are p and p′ related?

$$p^T \cdot E\, p' = 0$$

Parallel image planes

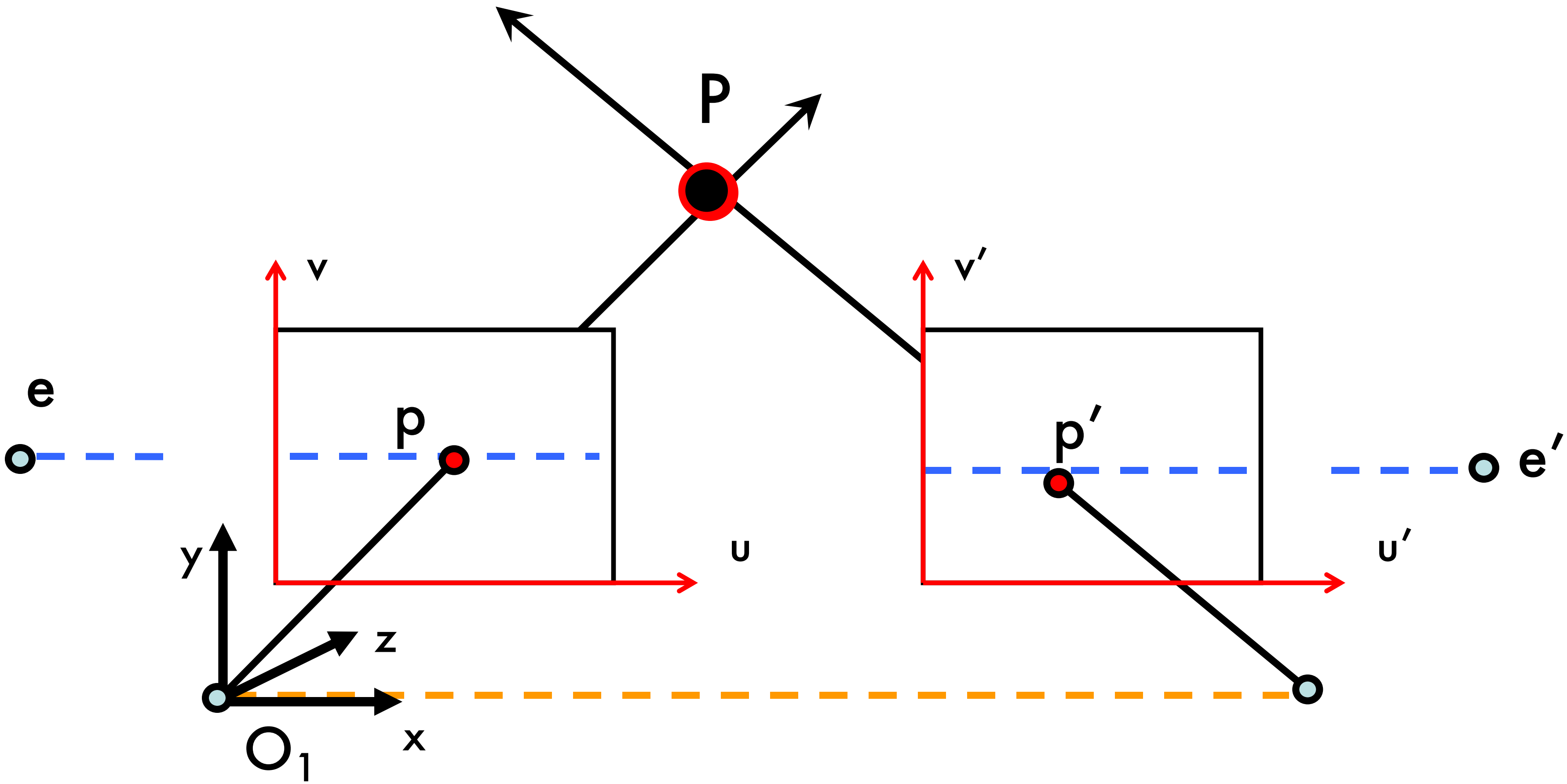


How are p and p′ related?

$$\Rightarrow \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \Rightarrow Tv = Tv'$$

$$\Rightarrow v = v'$$

Parallel image planes



Rectification: making two images "parallel"

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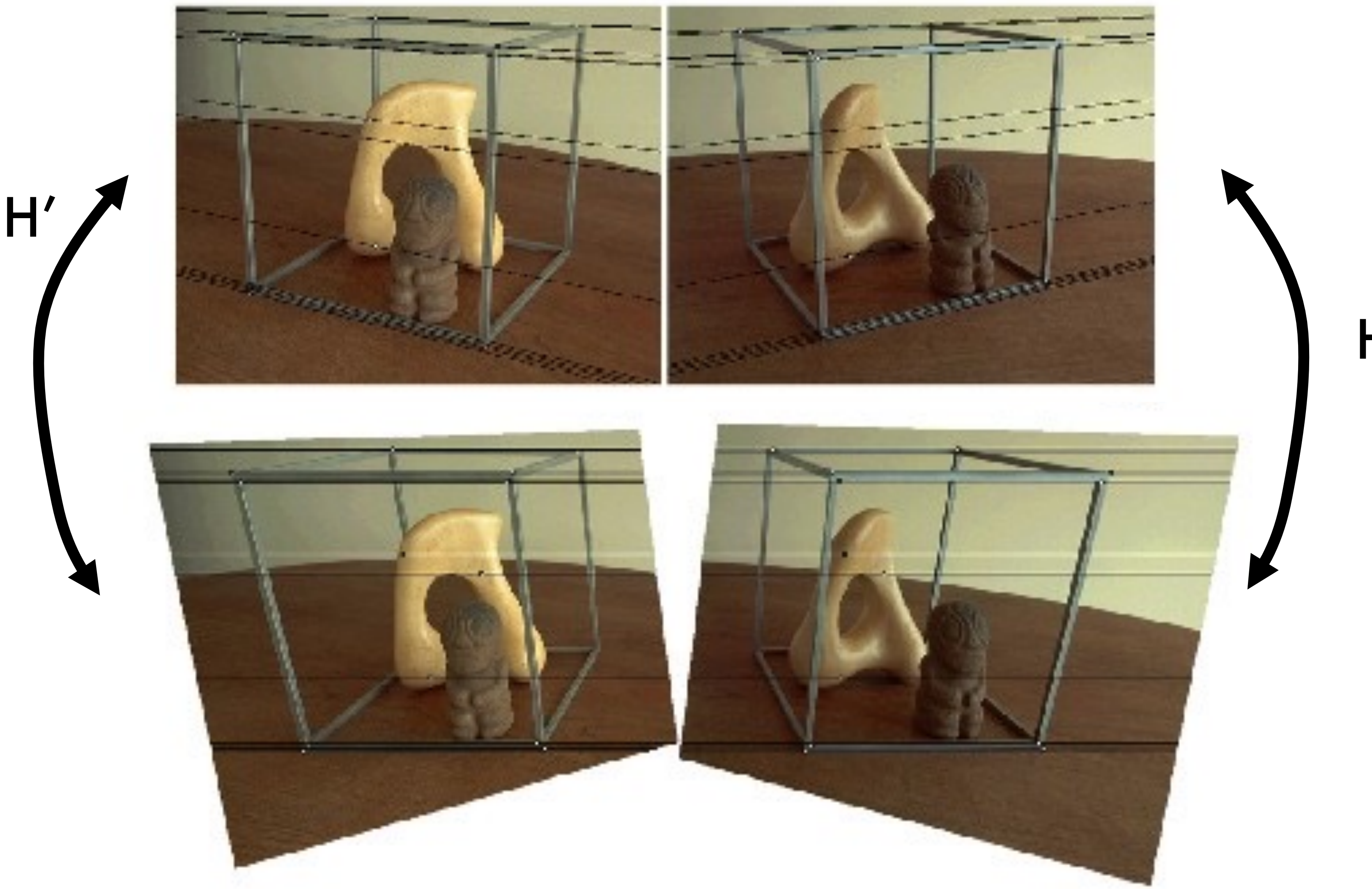
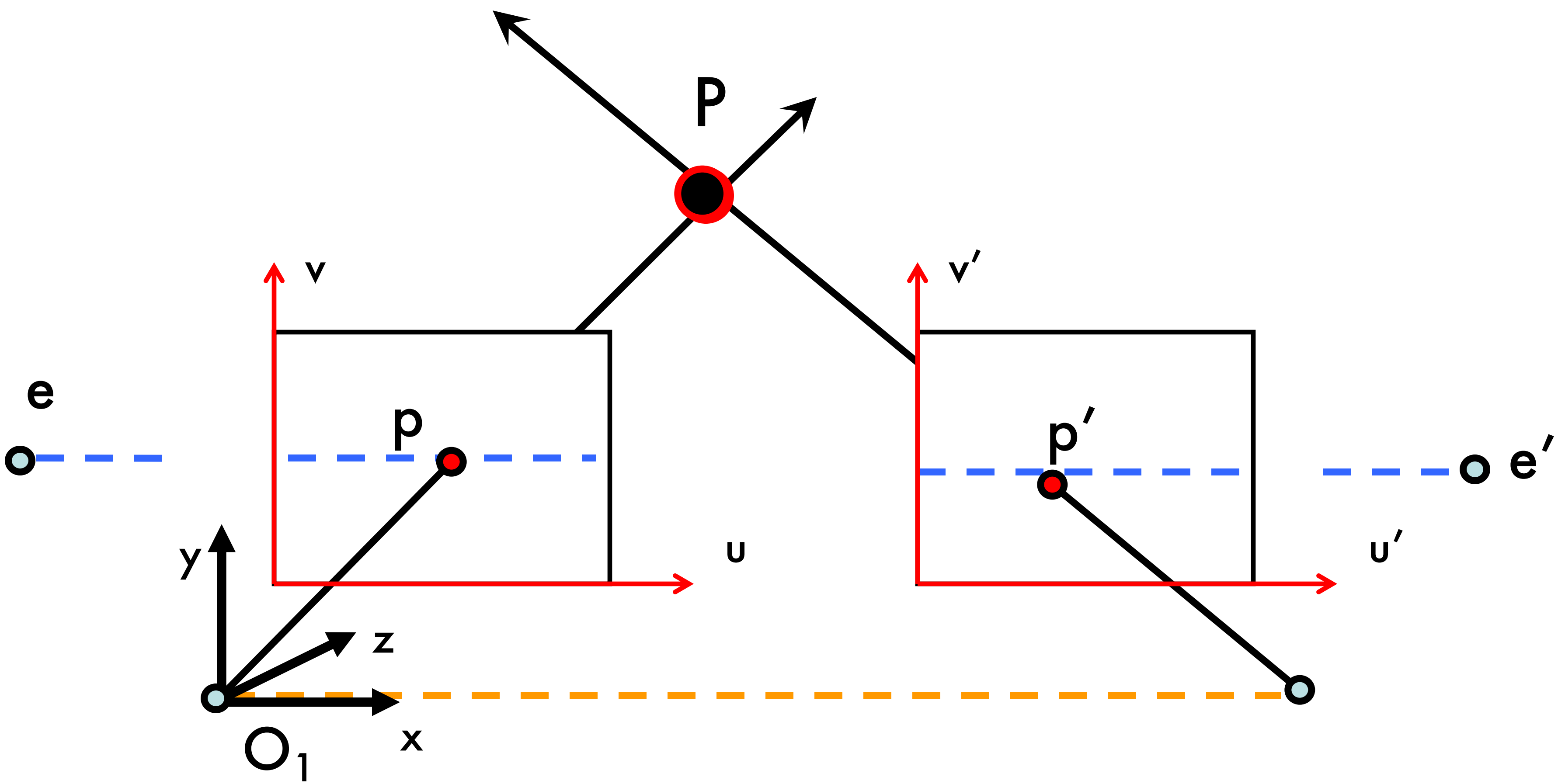


Courtesy figure S. Lazebnik

Parallel image planes



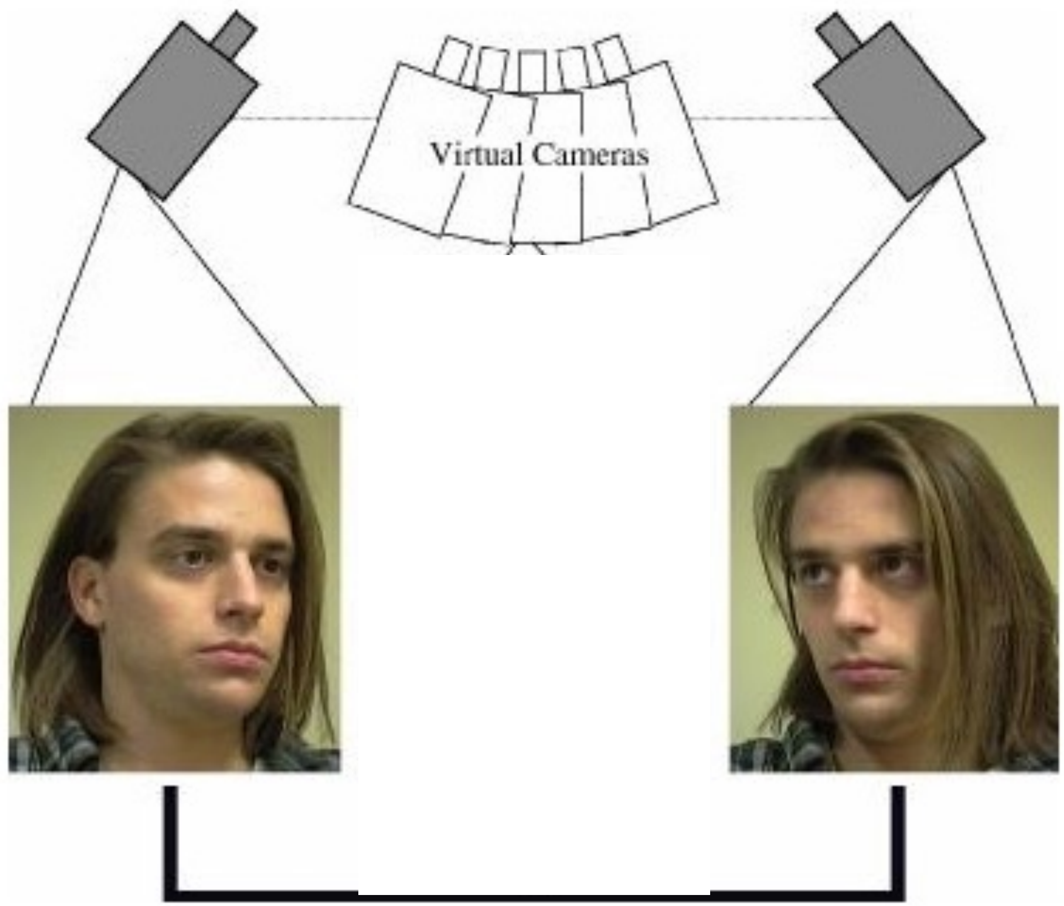
Rectification: making two images "parallel"

Why it is useful?

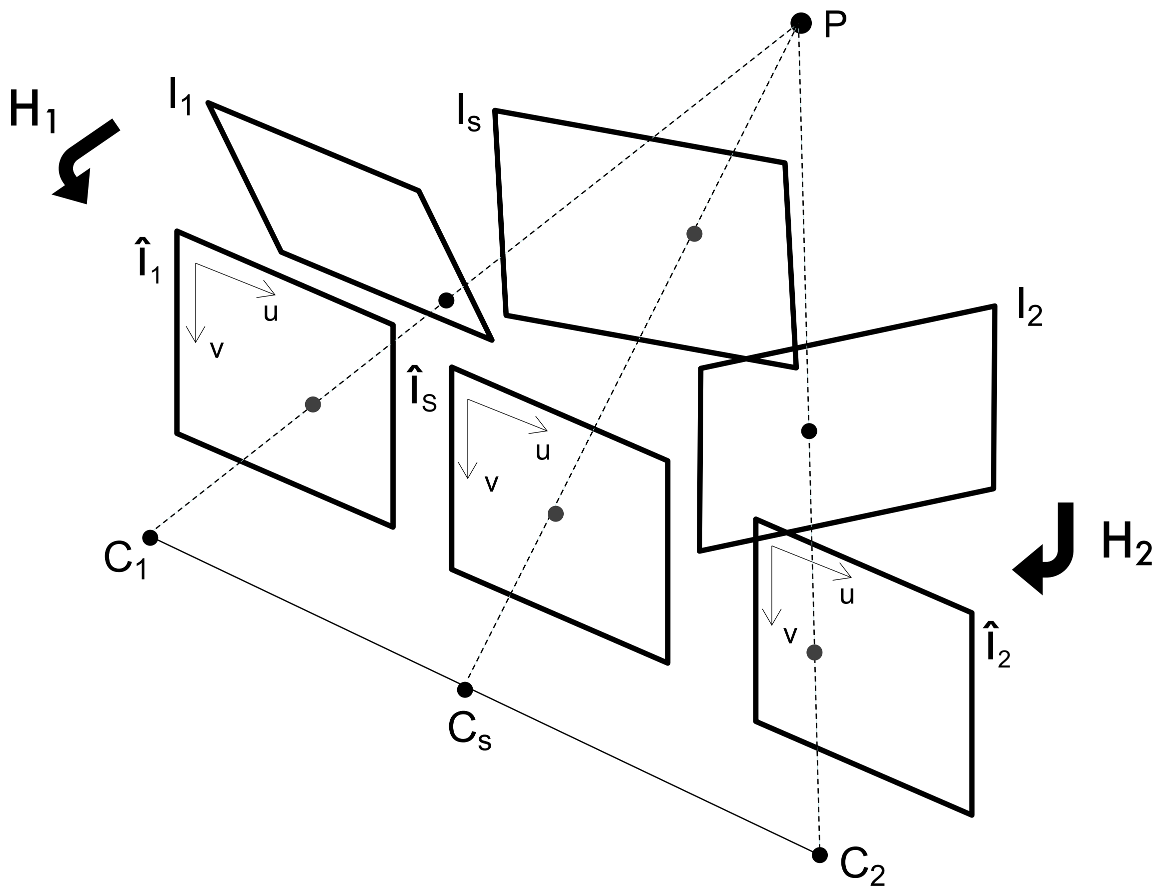
- Epipolar constraint $\rightarrow$ v = v′
- New views can be synthesized by linear interpolation

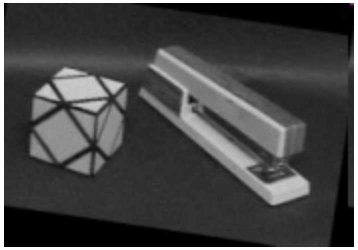
Application: view morphing

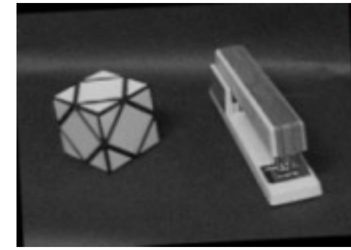
S. M. Seitz and C. R. Dyer, *Proc. SIGGRAPH 96*, 1996, 21-30

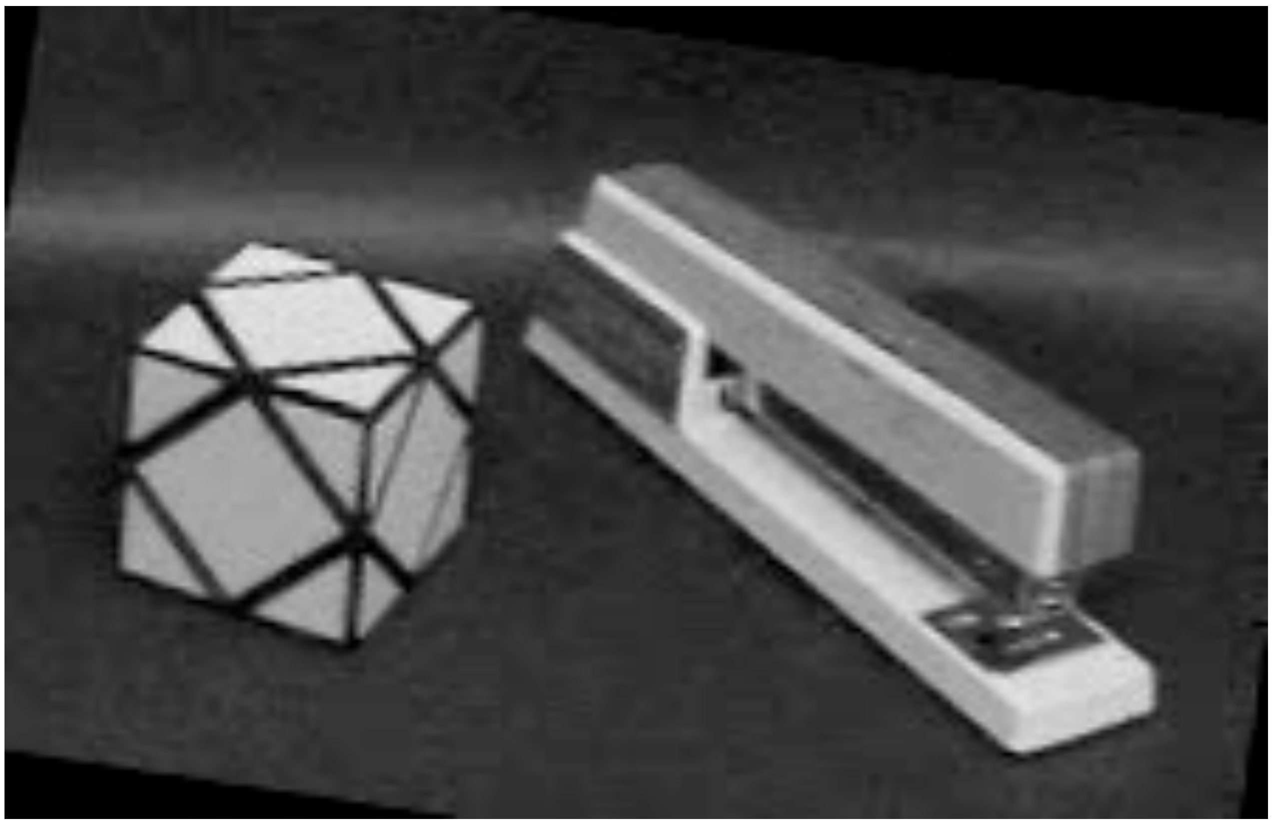


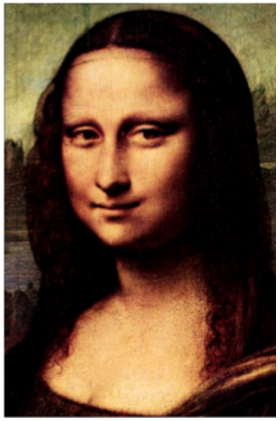
Rectification

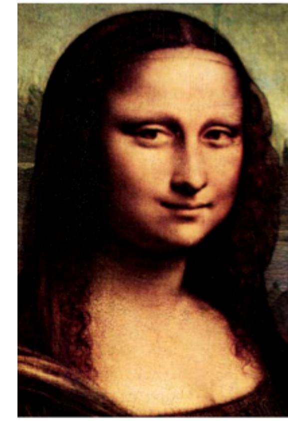








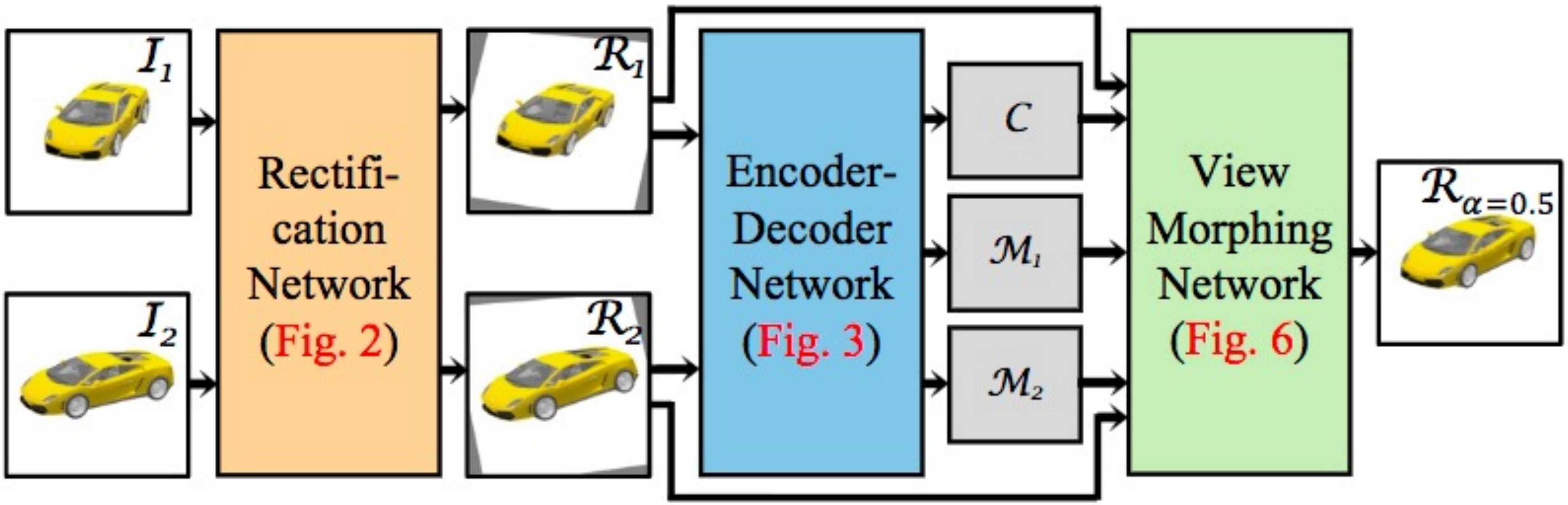




From its reflection!

Deep view morphing

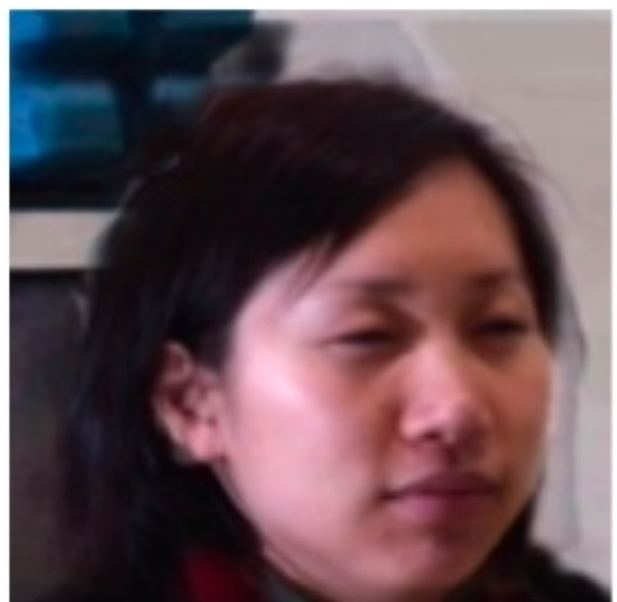
D. Ji, J. Kwon, M. McFarland, S. Savarese, CVPR 2017



Deep view morphing

D. Ji, J. Kwon, M. McFarland, S. Savarese, CVPR 2017

I_1 GT I_2



Deep view morphing

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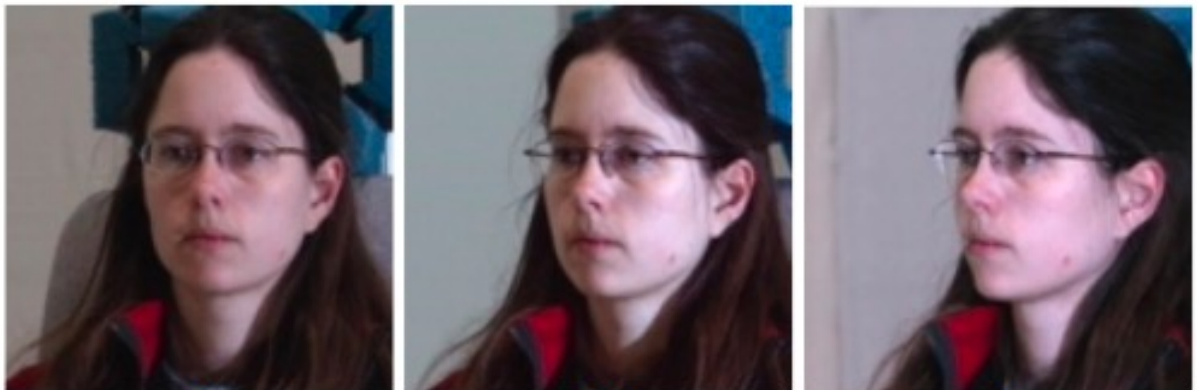
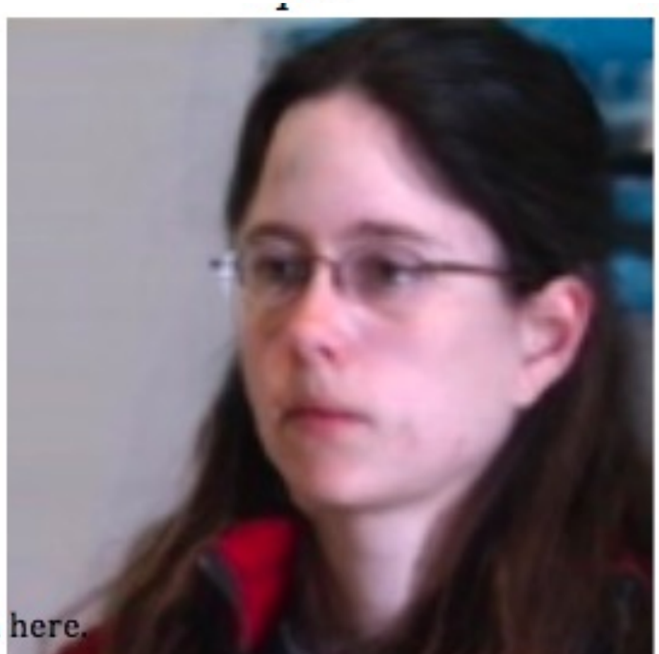
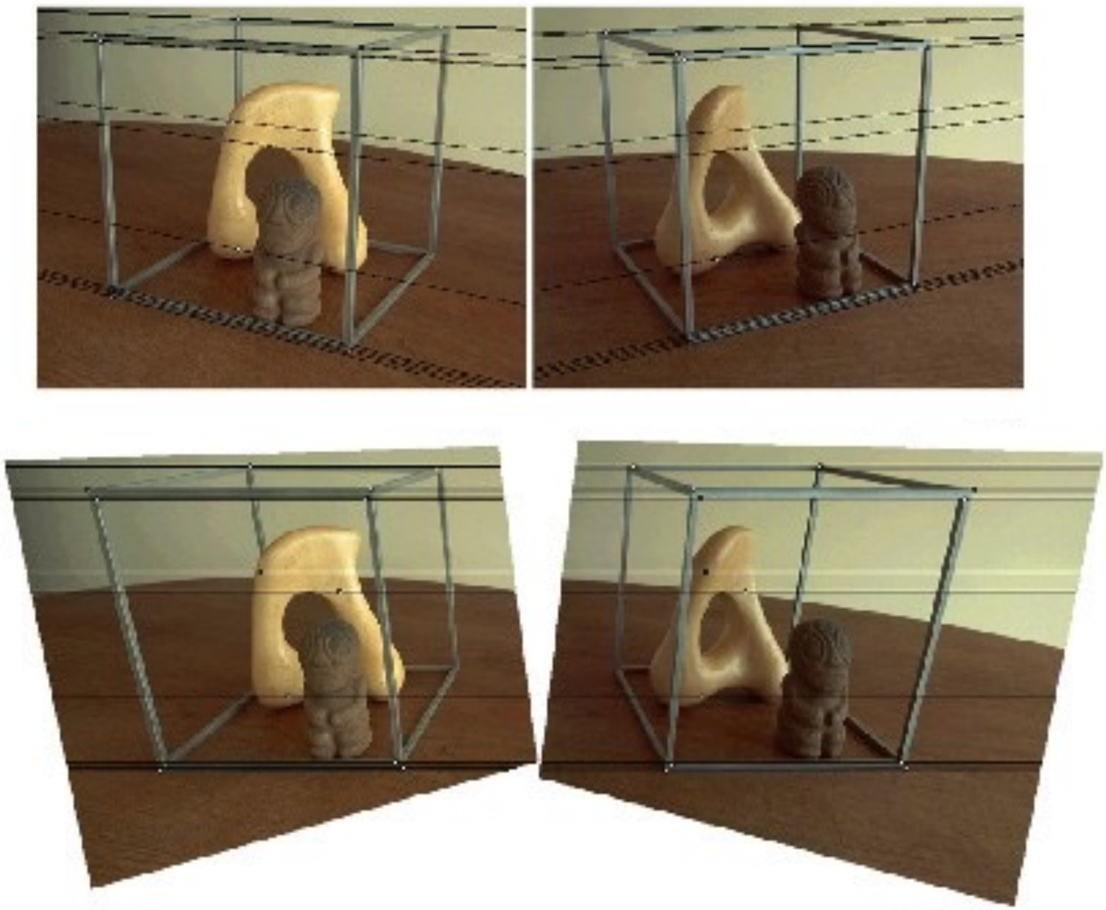


I_1 GT I_2

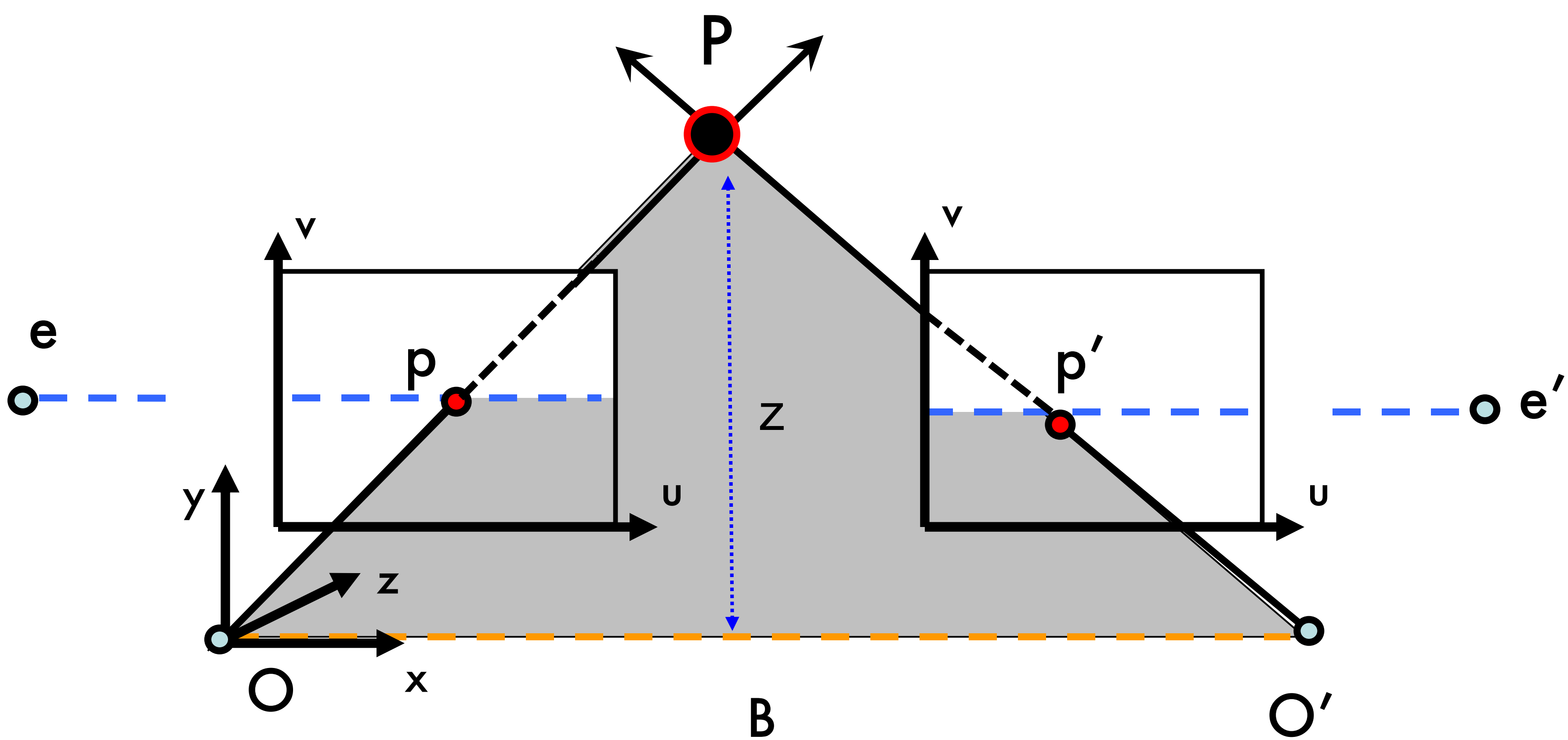


Why are parallel images useful?



- Makes triangulation easy
- Makes the correspondence problem easier

Point triangulation



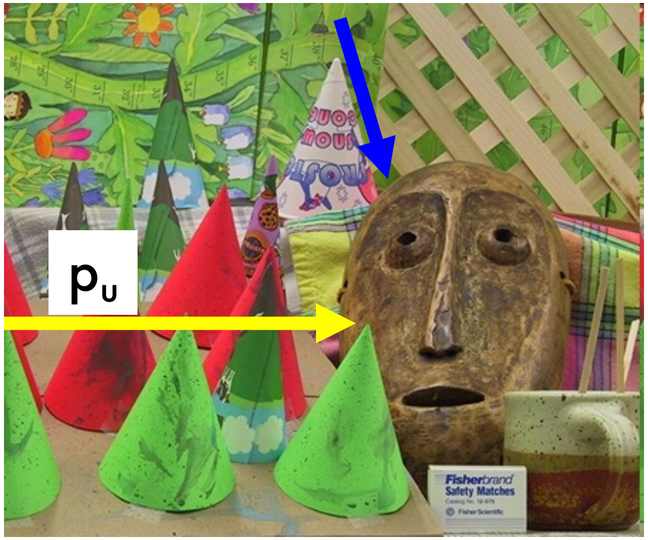
$$p = \begin{bmatrix} p_u \\ p_v \\ 1 \end{bmatrix} \quad p' = \begin{bmatrix} p'_u \\ p_v \\ 1 \end{bmatrix}$$

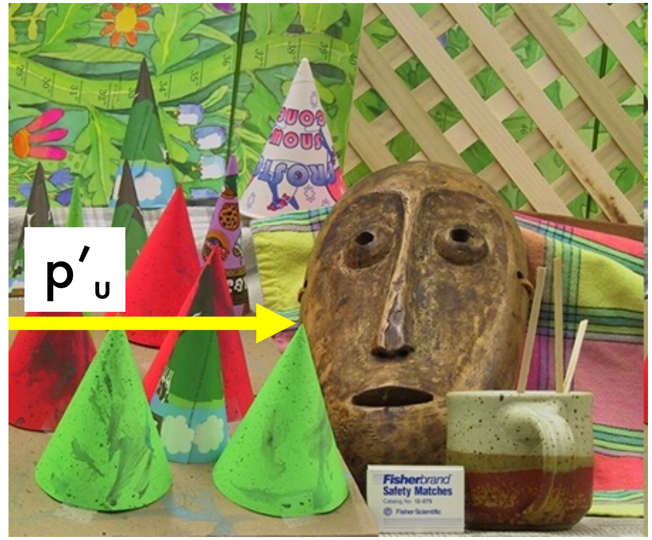
disparity = $p_u - p'_u \;\propto\; \dfrac{B \cdot f}{z}$ [Eq. 1]

Disparity is inversely proportional to depth z!

Disparity maps

http://vision.middlebury.edu/stereo/

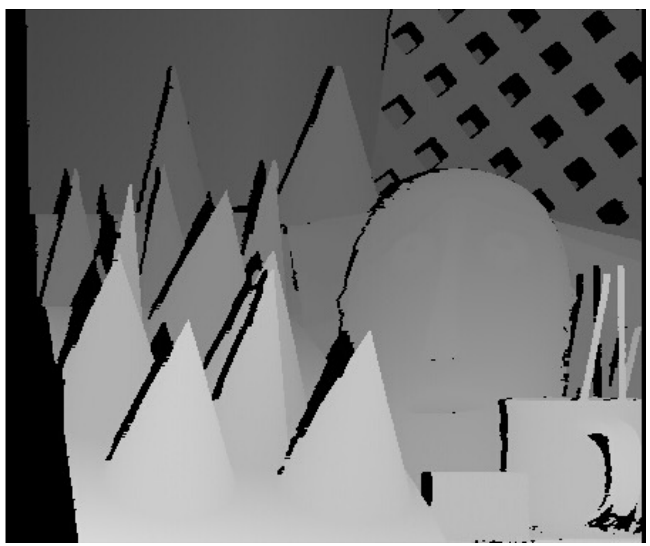




$$p_u - p'_u \propto \frac{B \cdot f}{z}$$

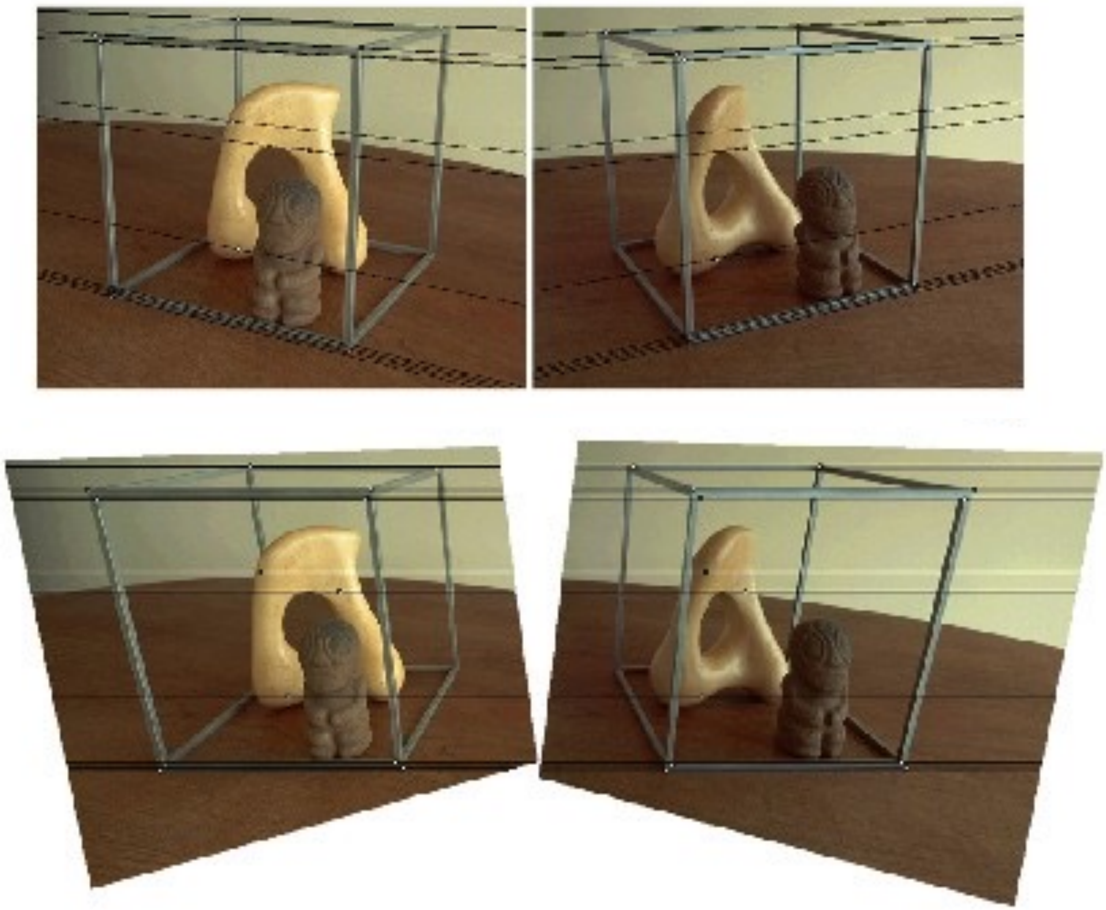
[Eq. 1]

Stereo pair



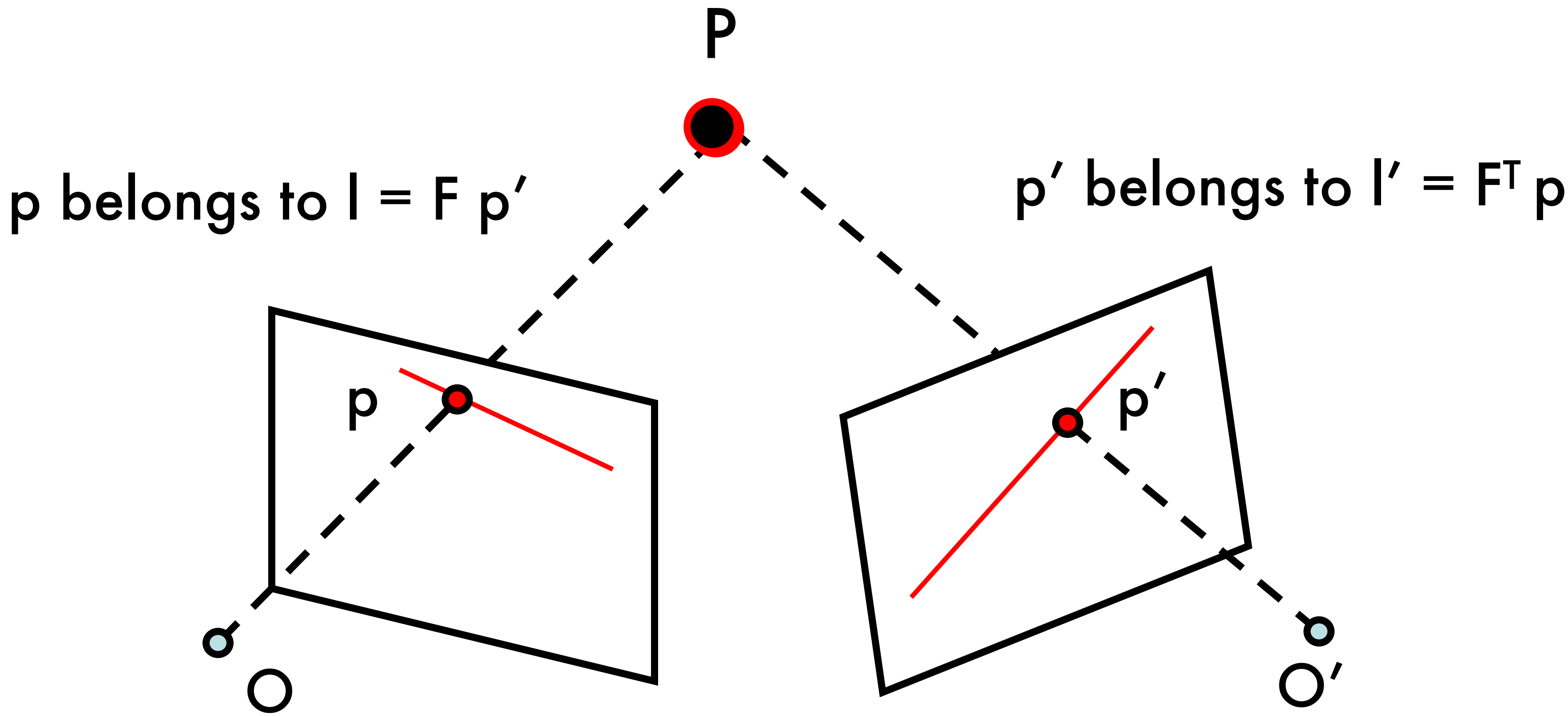
Disparity map / depth map

Why are parallel images useful?



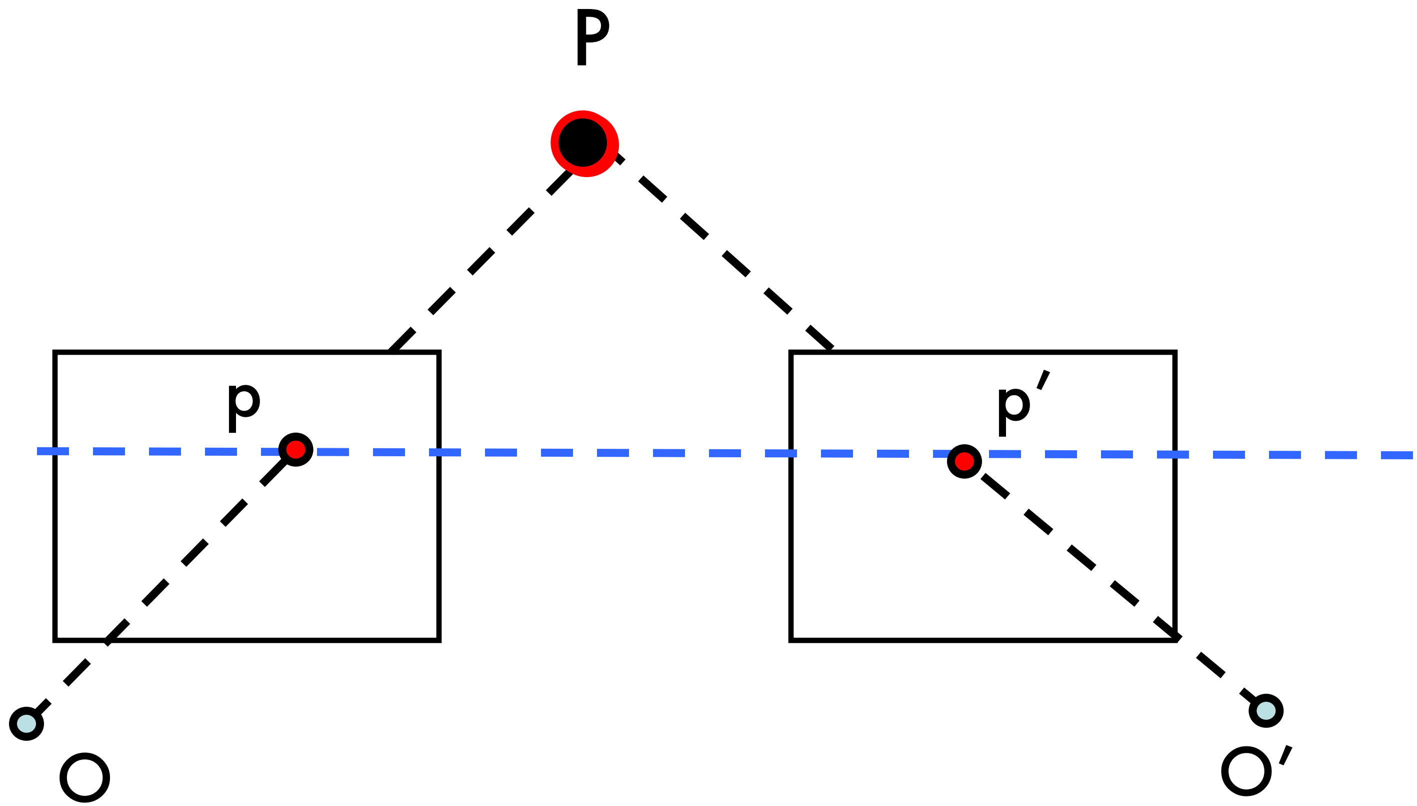
- Makes triangulation easy
- Makes the correspondence problem easier

Correspondence problem



Given a point in 3D, discover corresponding observations in left and right images [also called binocular fusion problem]

Correspondence problem



When images are rectified, this problem is much easier!

Correspondence problem

- A Cooperative Model (Marr and Poggio, 1976)
- Correlation Methods (1970–)
- Multi-Scale Edge Matching (Marr, Poggio and Grimson, 1979-81)

[FP] Chapters: 7

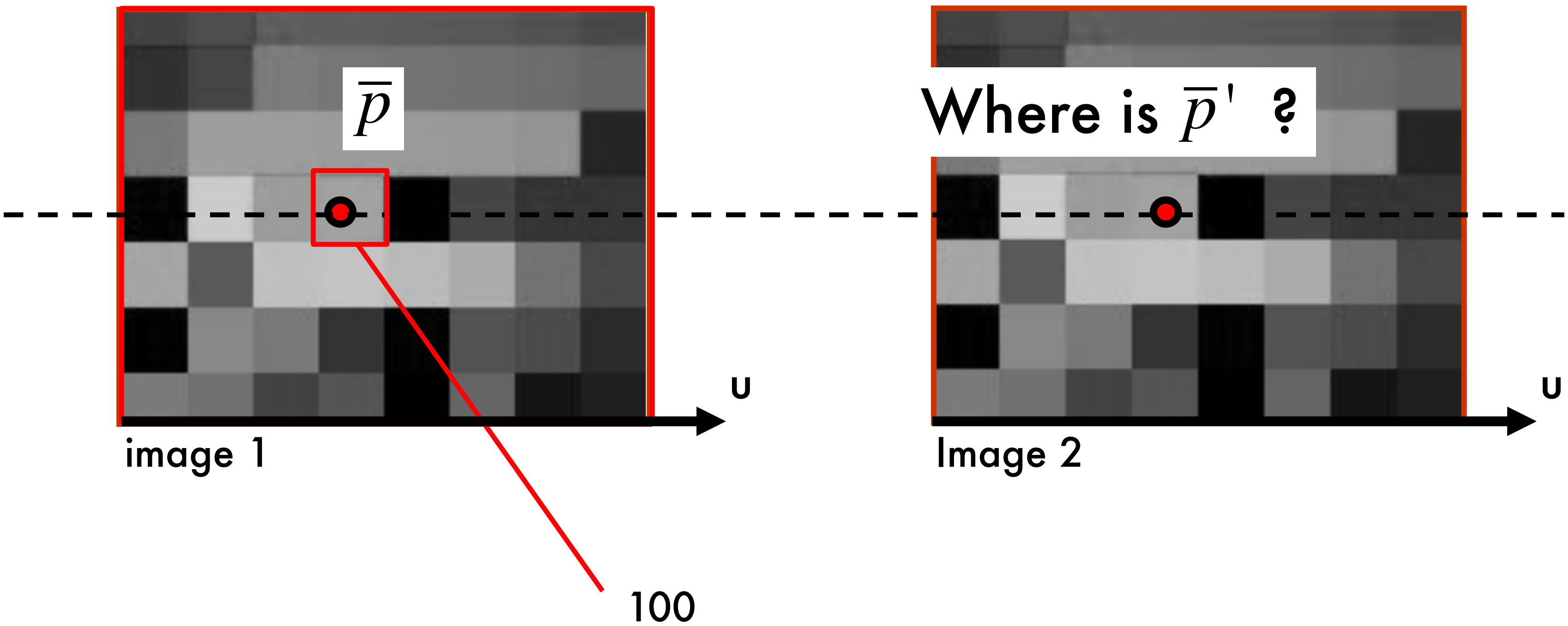
Correlation Methods (1970–)

image 1

Image 2

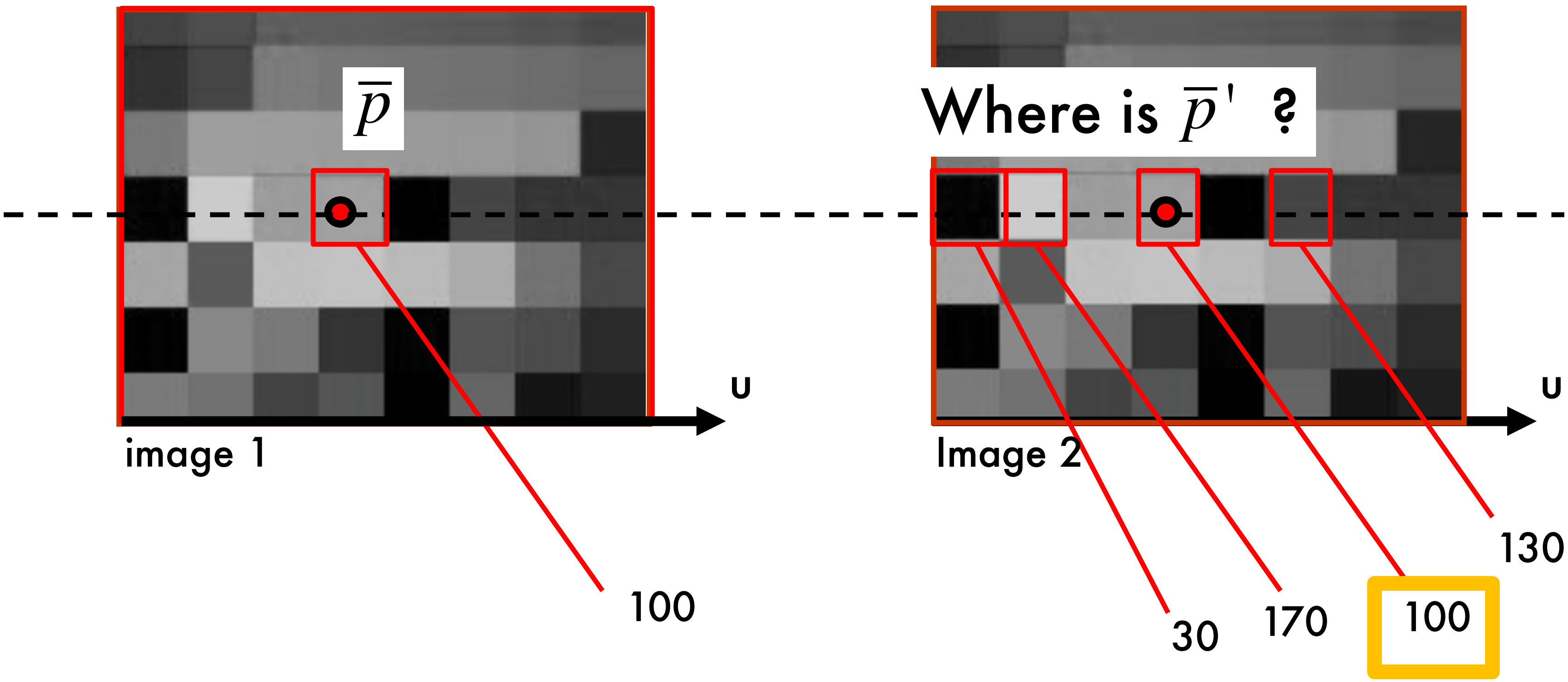
$$\bar{p} = \begin{bmatrix} \bar{u} \\ \bar{v} \\ 1 \end{bmatrix} \qquad \bar{p}' = \begin{bmatrix} \bar{u}' \\ \bar{v} \\ 1 \end{bmatrix}$$

Correlation Methods (1970–)



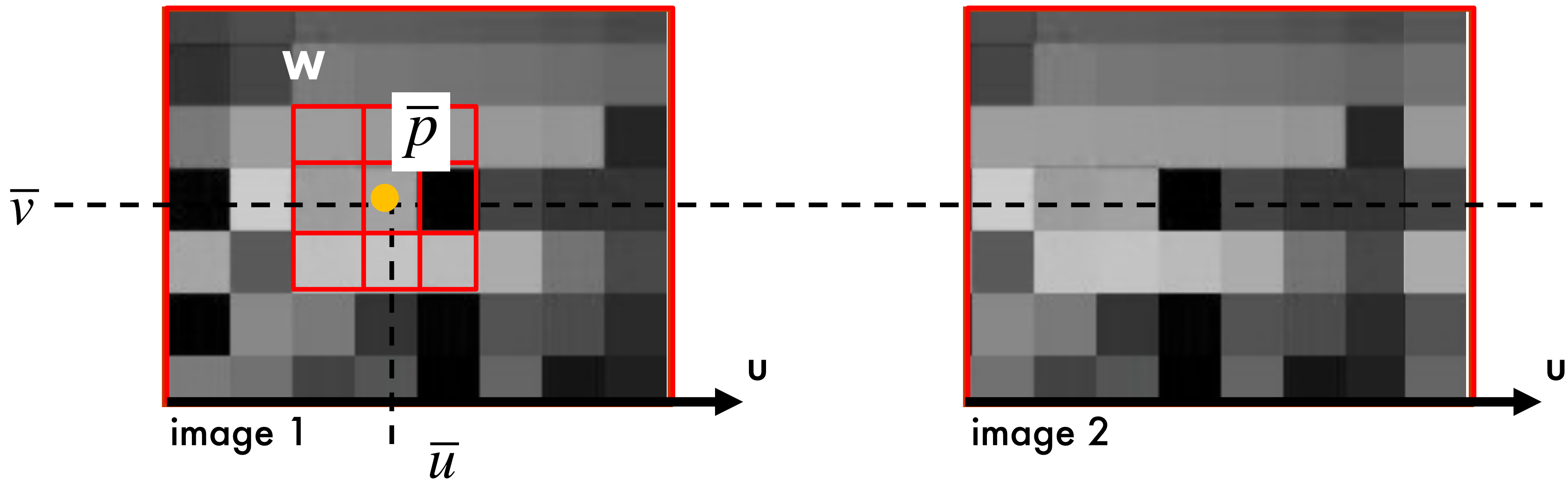
$$\overline{p} = \begin{bmatrix} \overline{u} \\ \overline{v} \\ 1 \end{bmatrix} \qquad \overline{p}\,' = \begin{bmatrix} \overline{u}\,' \\ \overline{v} \\ 1 \end{bmatrix}$$

Correlation Methods (1970–)



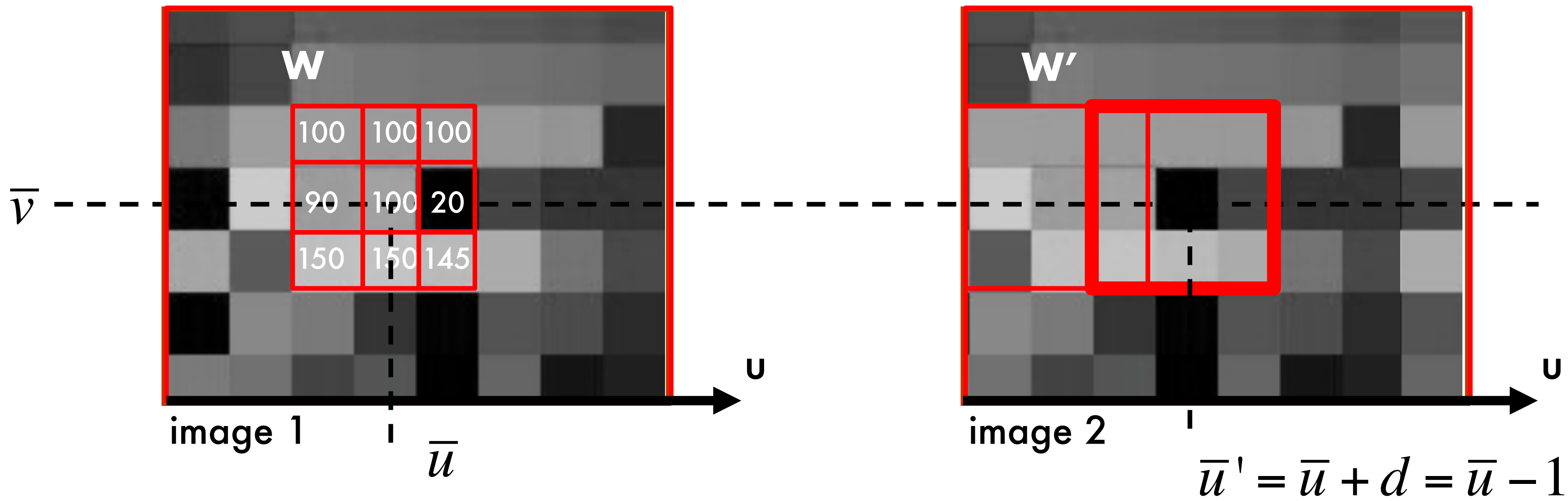
What's the problem with this?

Window-based correlation



- Pick up a window **W** around $\overline{p} = (\overline{u}, \overline{v})$

Window-based correlation



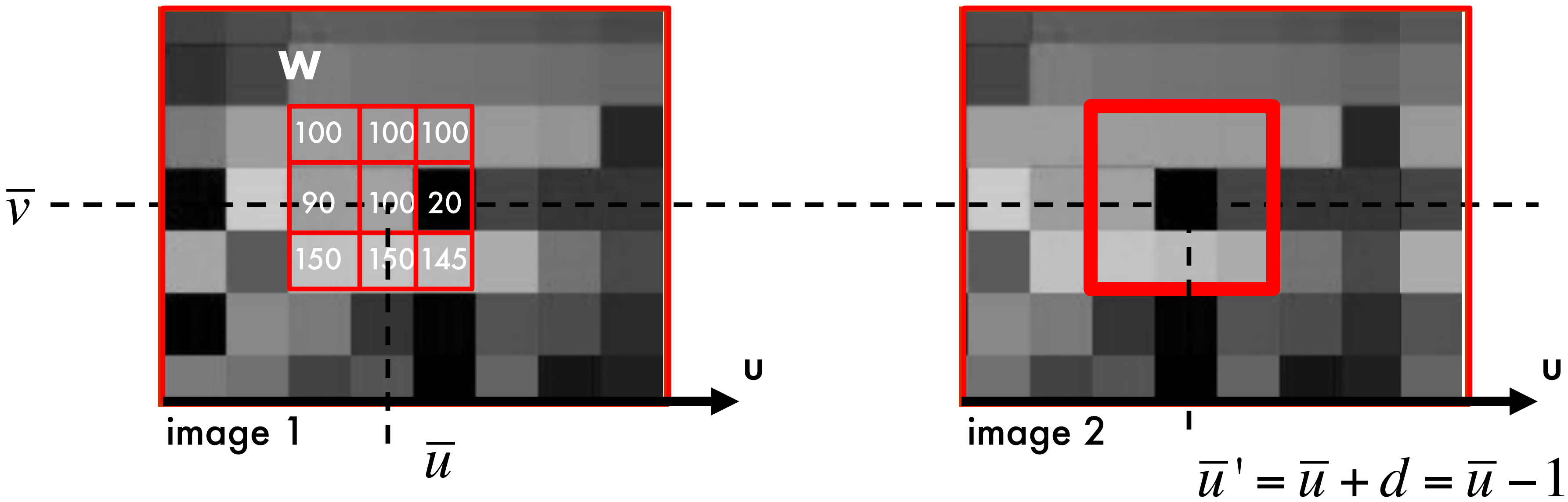
Example: **W** is a 3x3 window in red

w is a 9x1 vector

w = $[100, 100, 100, 90, 100, 20, 150, 150, 145]^T$

- Pick up a window **W** around $\bar{p} = (\bar{u}, \bar{v})$
- Build vector **w**
- Slide the window **W** along v = $\bar{v}$ in image 2 and compute **w'** (u) for each u
- Compute the dot product $\mathbf{w}^T\,\mathbf{w}'(u)$ for each u and retain the max value

Window-based correlation



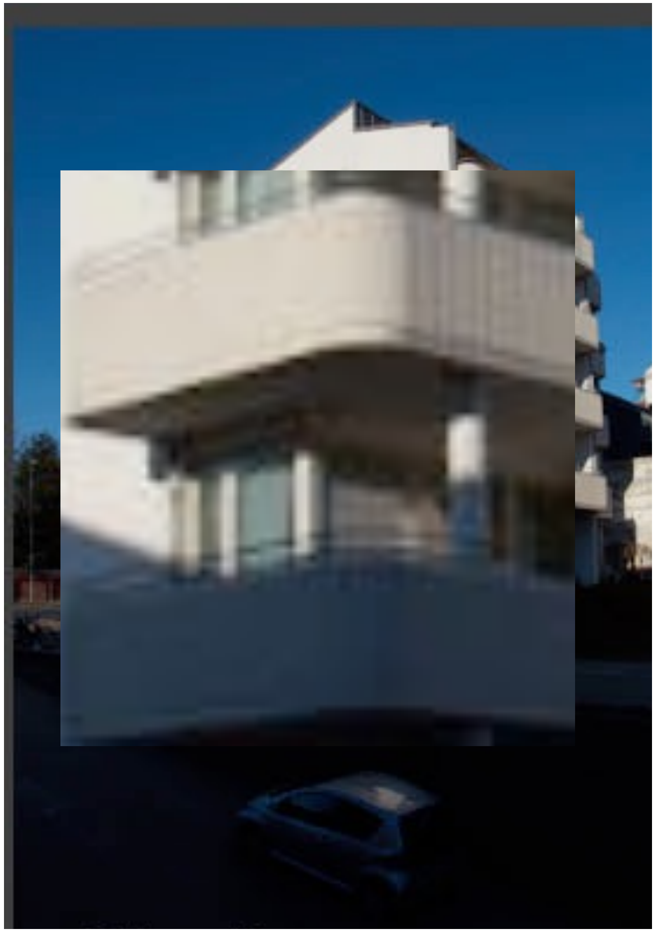
Example: **W** is a 3x3 window in red

w is a 9x1 vector

w = $[100, 100, 100, 90, 100, 20, 150, 150, 145]^T$

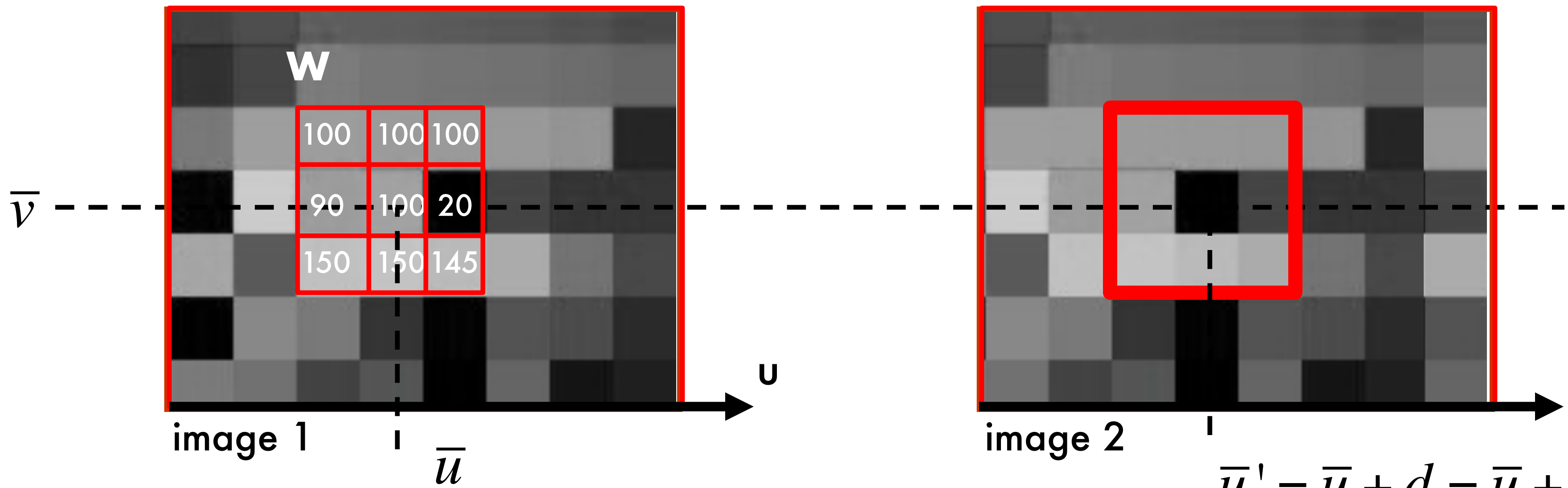
What's the problem with this?

Changes of brightness/exposure



Changes in the mean and the variance of intensity values in corresponding windows!

Normalized cross-correlation



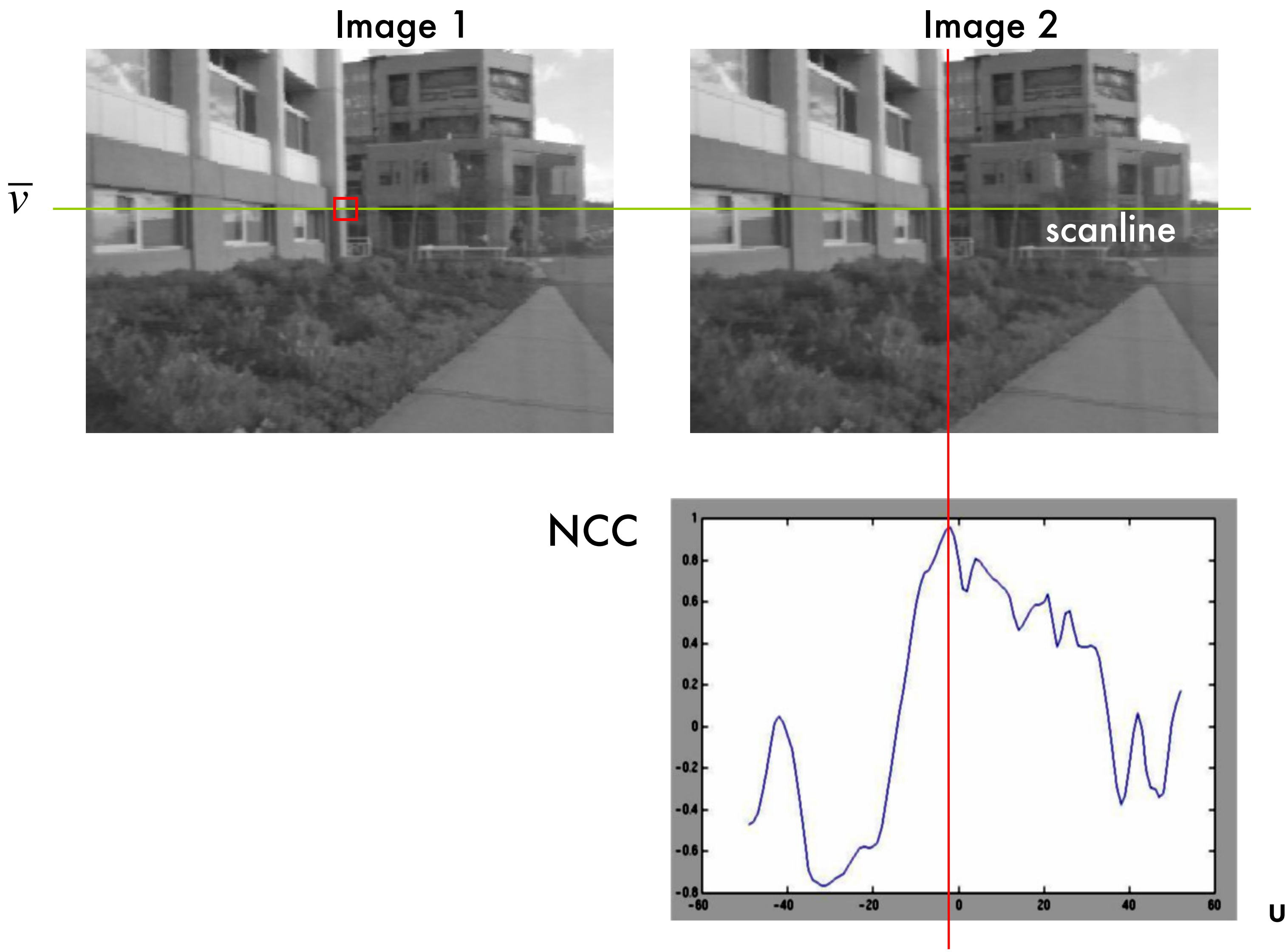
$$\bar{u}' = \bar{u} + d = \bar{u} + 1$$

Find u that maximizes: $$\frac{(w-\bar{w})^T (w'(u)-\bar{w}')}{\|(w-\bar{w})\| \, \|(w'(u)-\bar{w}')\|}$$ [Eq. 2]

$\bar{w} =$ mean value within **W** located at u^{bar} in image 1

$\bar{w}'(u) =$ mean value within **W** located at u in image 2

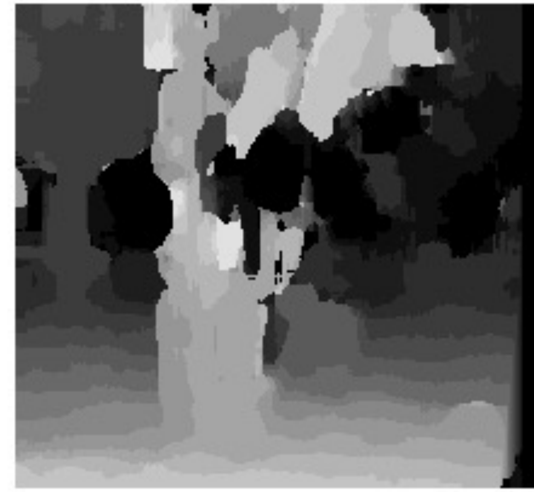
Example



Credit slide S. Lazebnik

Effect of the window's size

Window size = 3



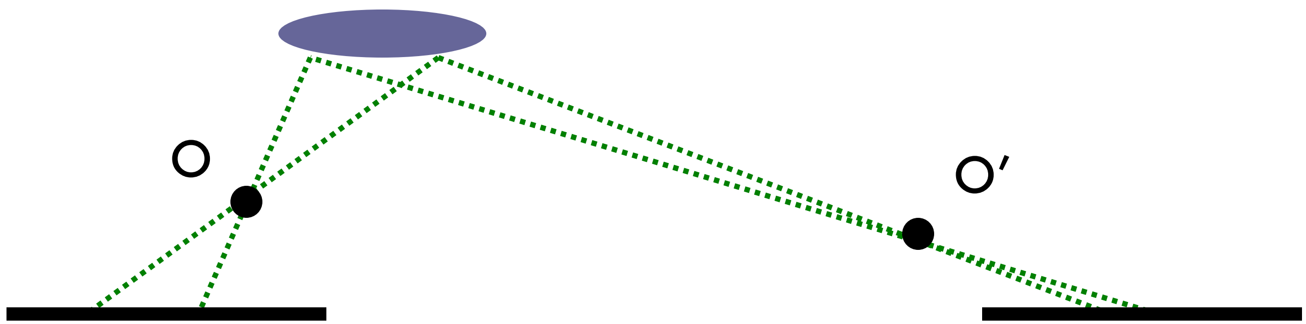
Window size = 20

- Smaller window
  - More detail
  - More noise
- Larger window
  - Smoother disparity maps
  - Less prone to noise

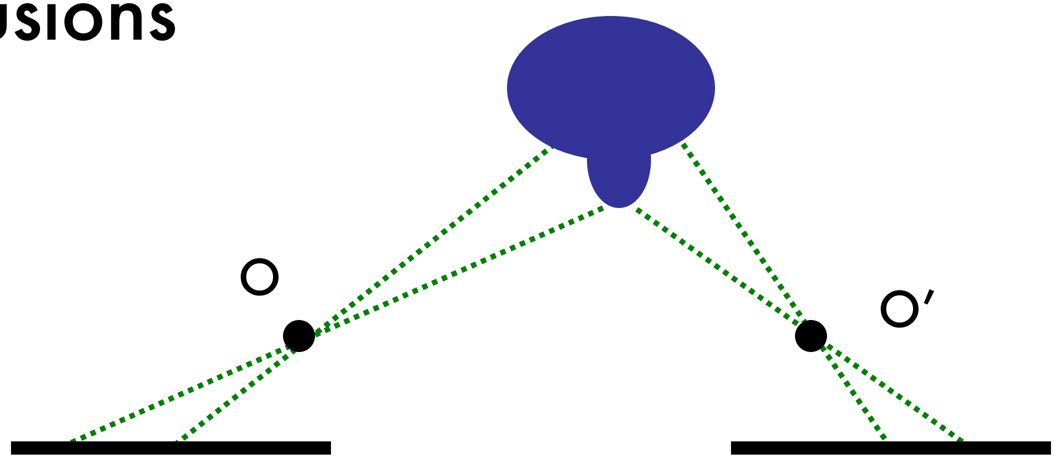
Credit slide S. Lazebnik

Issues

- Fore shortening effect



- Occlusions



Base line trade-off

- To reduce the effect of foreshortening and occlusions, it is desirable to have small B / z ratio!
- However, when B/z is small, small errors in measurements imply large error in estimating depth

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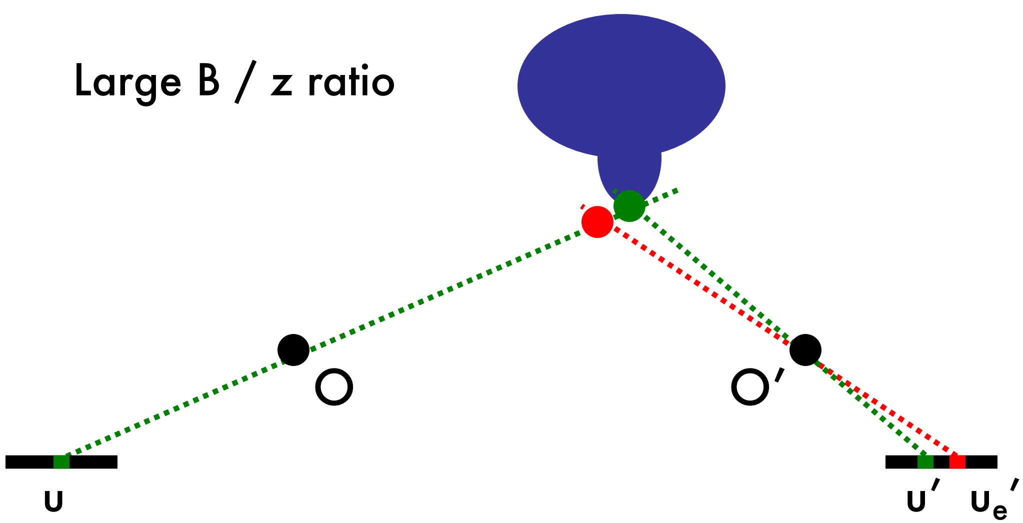
Large B / z ratio



Base line trade-off

- To reduce the effect of foreshortening and occlusions, it is desirable to have small B / z ratio!
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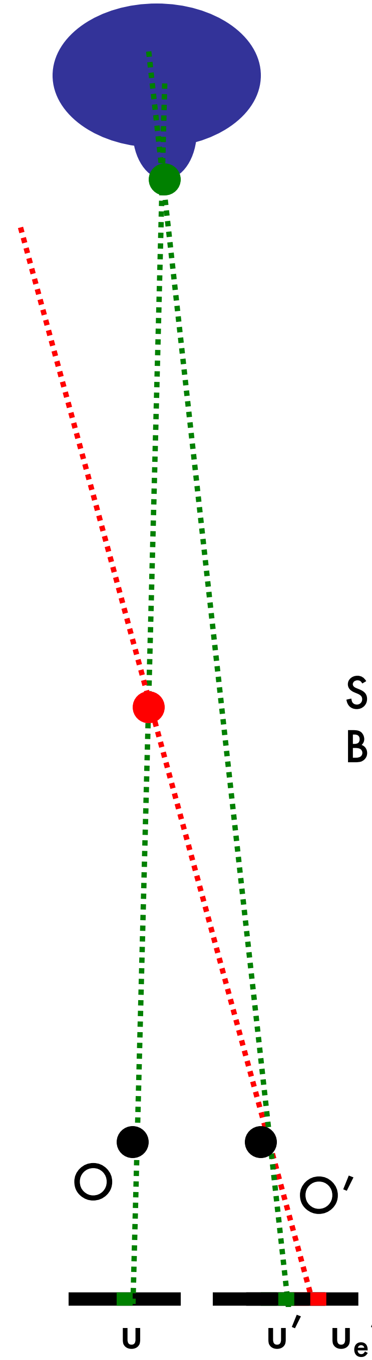
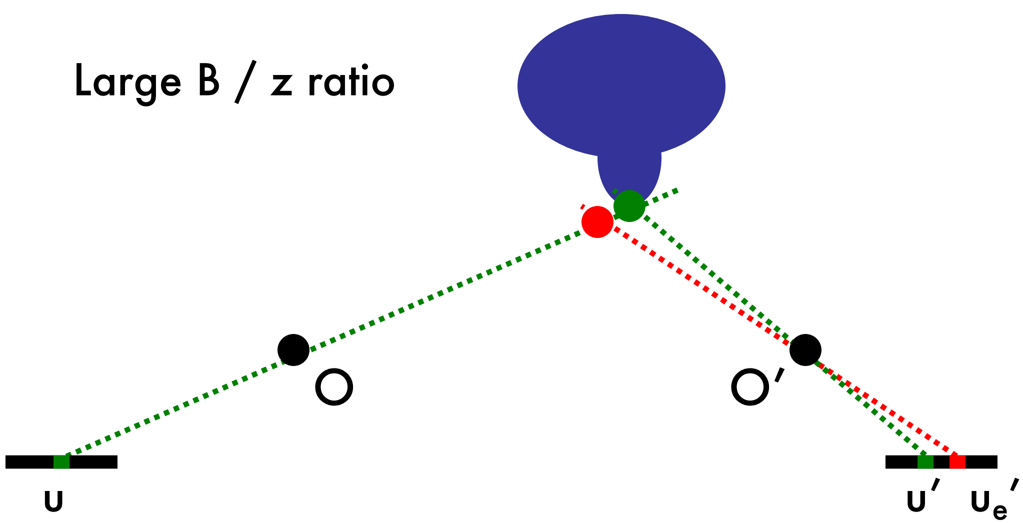


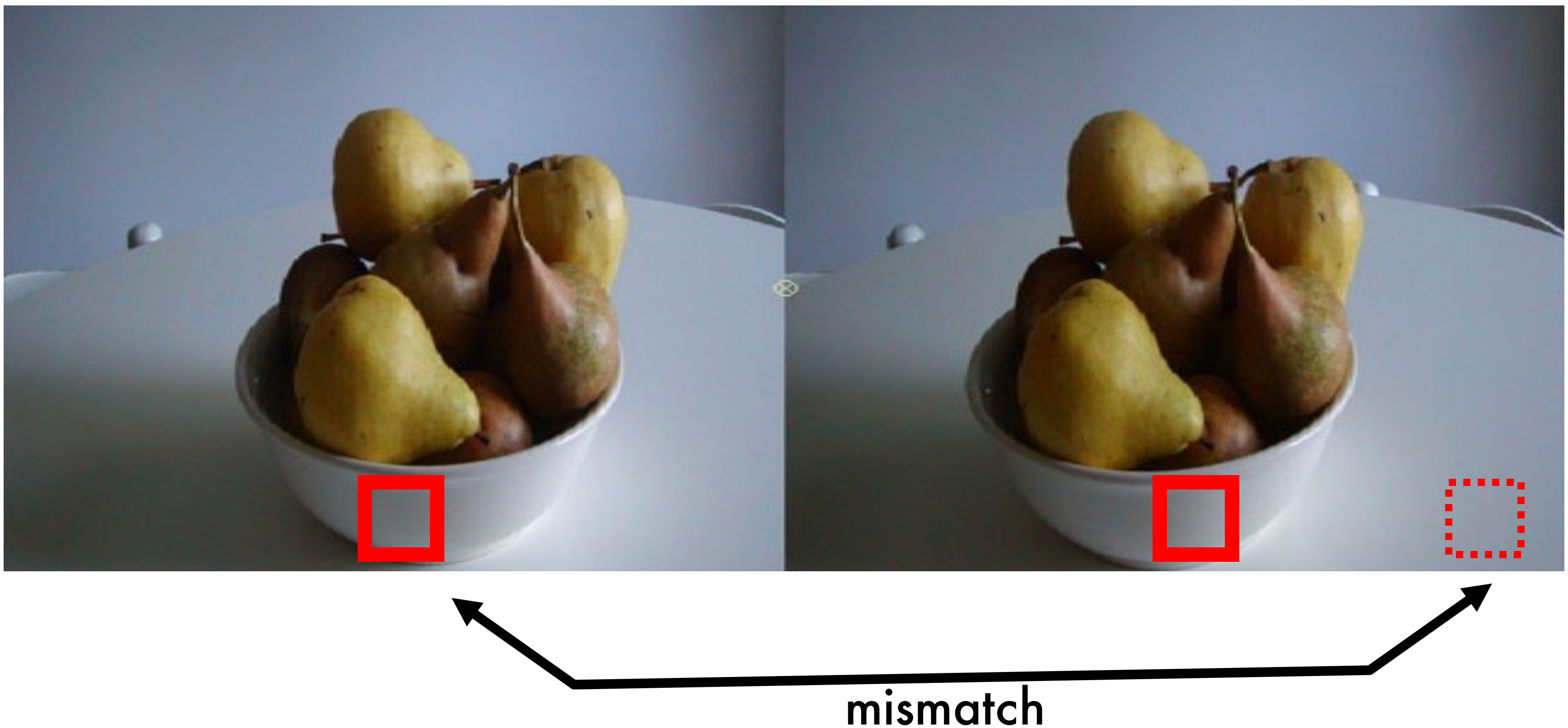
Small B / z ratio

Large B / z ratio



More issues!

- Homogeneous regions



More issues!

- Repetitive patterns

Correspondence problem is difficult!

- Occlusions
- Fore shortening
- Baseline trade-off
- Homogeneous regions
- Repetitive patterns

Apply non-local constraints to help enforce the correspondences

Non-local constraints

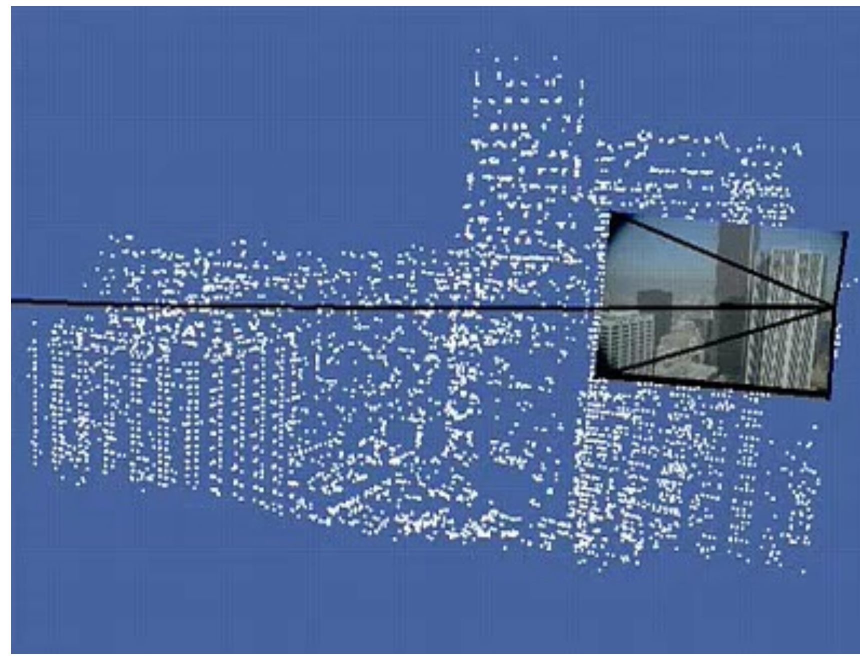
- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views
- Smoothness
  - Disparity is typically a smooth function of x (except in occluding boundaries)

Lecture 6

Stereo Systems
Multi-view geometry

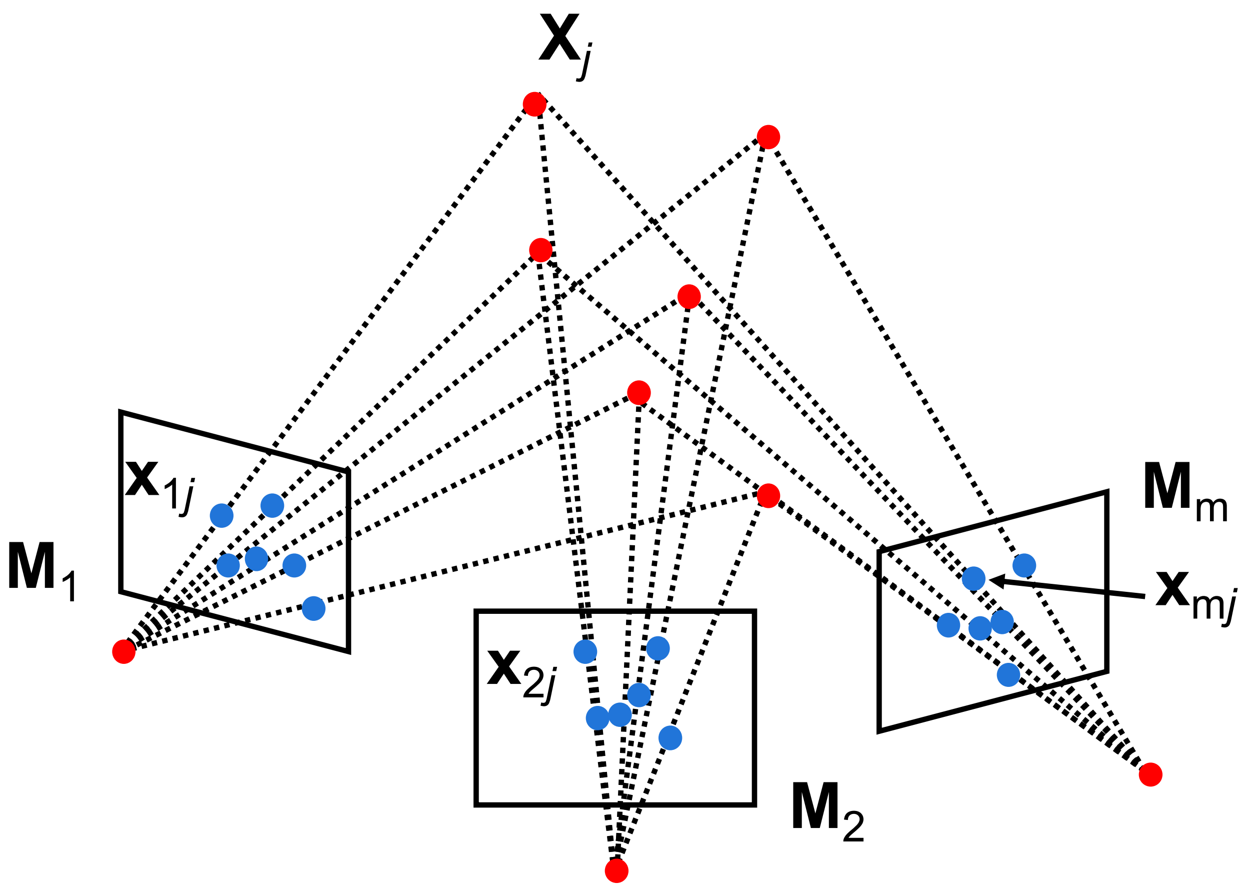
- Stereo systems
  - Rectification
  - Correspondence problem
- Multi-view geometry
  - The SFM problem
  - Affine SFM

Structure from motion problem



Courtesy of Oxford **Visual Geometry Group**

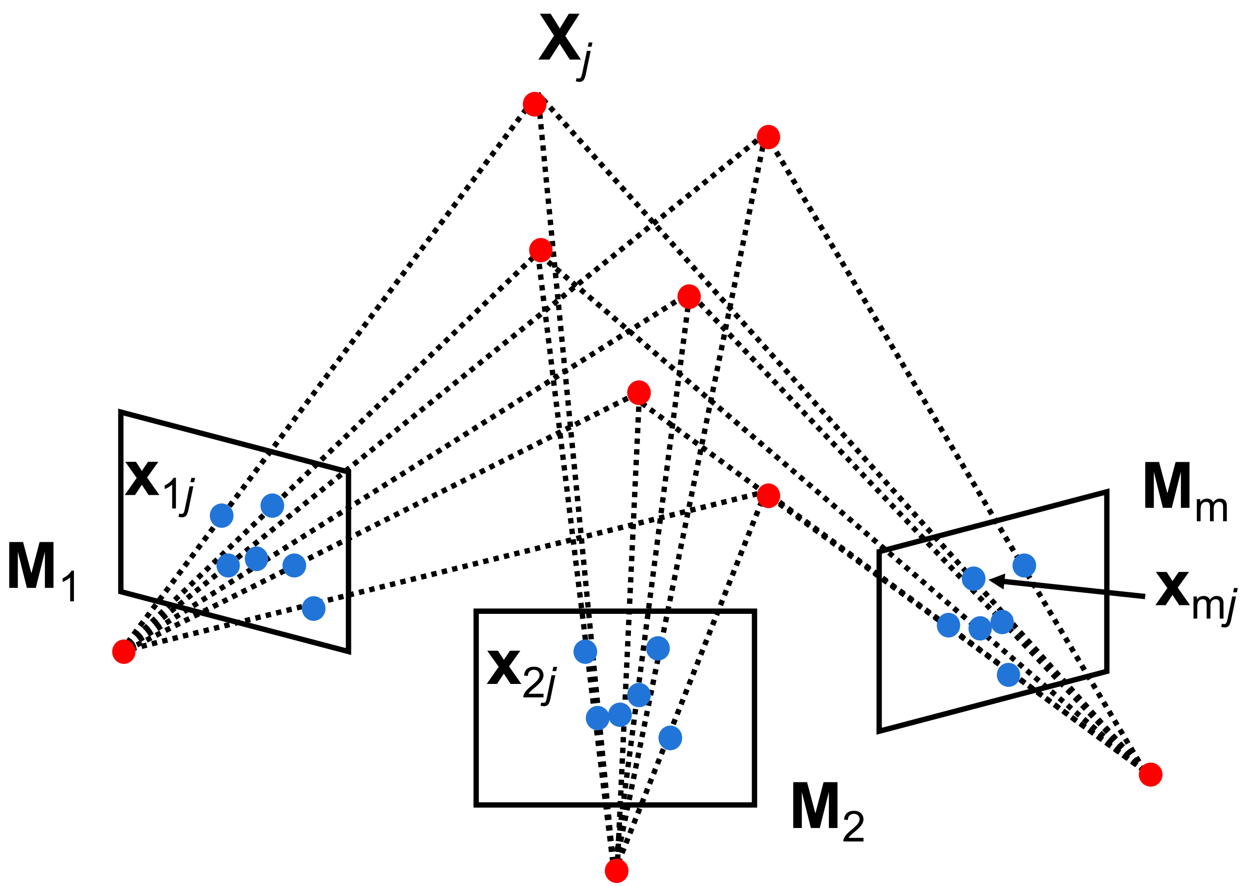
Structure from motion problem



Given *m* images of *n* fixed 3D points

- $\mathbf{x}_{ij} = \mathbf{M}_i \mathbf{X}_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$

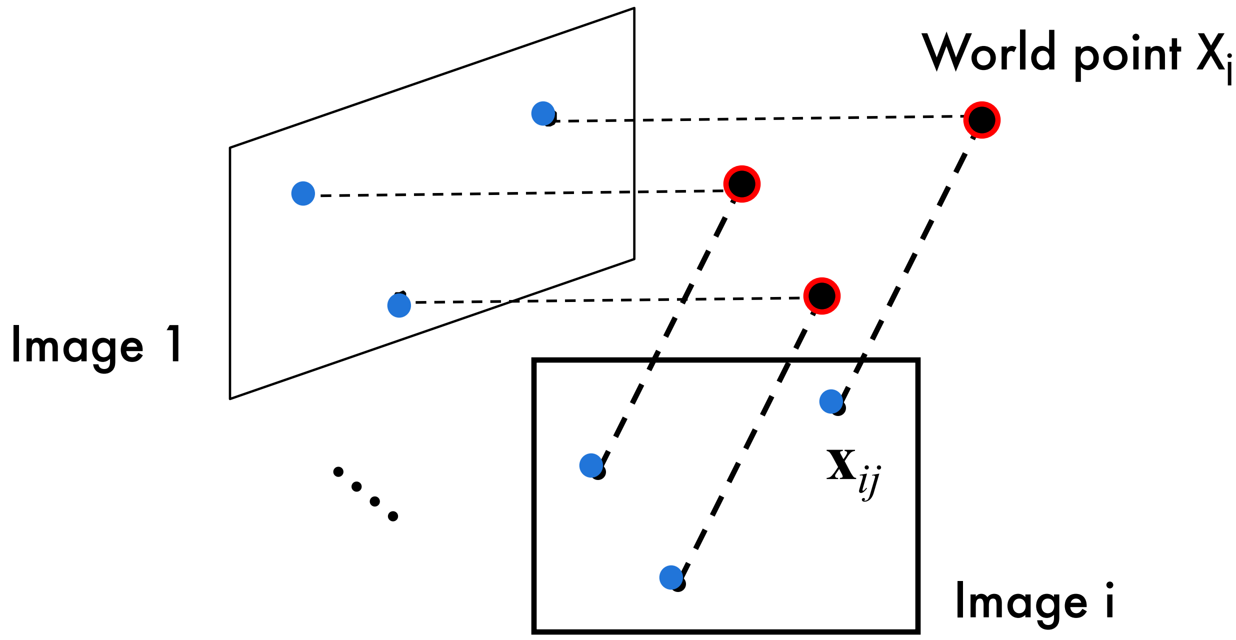
Structure from motion problem



From the m×n observations $\mathbf{x}_{ij}$, estimate:

- m projection matrices $\mathbf{M}_i$ — motion
- n 3D points $\mathbf{X}_j$ — structure

Affine structure from motion
(simpler problem)



From the m×n observations $\mathbf{x}_{ij}$, estimate:

- *m* projection matrices $\mathbf{M}_i$ (affine cameras)
- *n* 3D points $\mathbf{X}_j$

Perspective

$$\mathbf{x} = M\ \mathbf{X} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathbf{m}_1\mathbf{X} \\ \mathbf{m}_2\mathbf{X} \\ \mathbf{m}_3\mathbf{X} \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$\mathbf{x}^E = \left(\frac{\mathbf{m}_1\mathbf{X}}{\mathbf{m}_3\mathbf{X}}, \frac{\mathbf{m}_2\mathbf{X}}{\mathbf{m}_3\mathbf{X}}\right)^T$$

Affine

$$\mathbf{x} = M\ \mathbf{X} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathbf{m}_1\mathbf{X} \\ \mathbf{m}_2\mathbf{X} \\ 1 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{2x3} & \mathbf{b}_{2x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix} = \begin{bmatrix} & \mathbf{m}_1 & & \\ & \mathbf{m}_2 & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

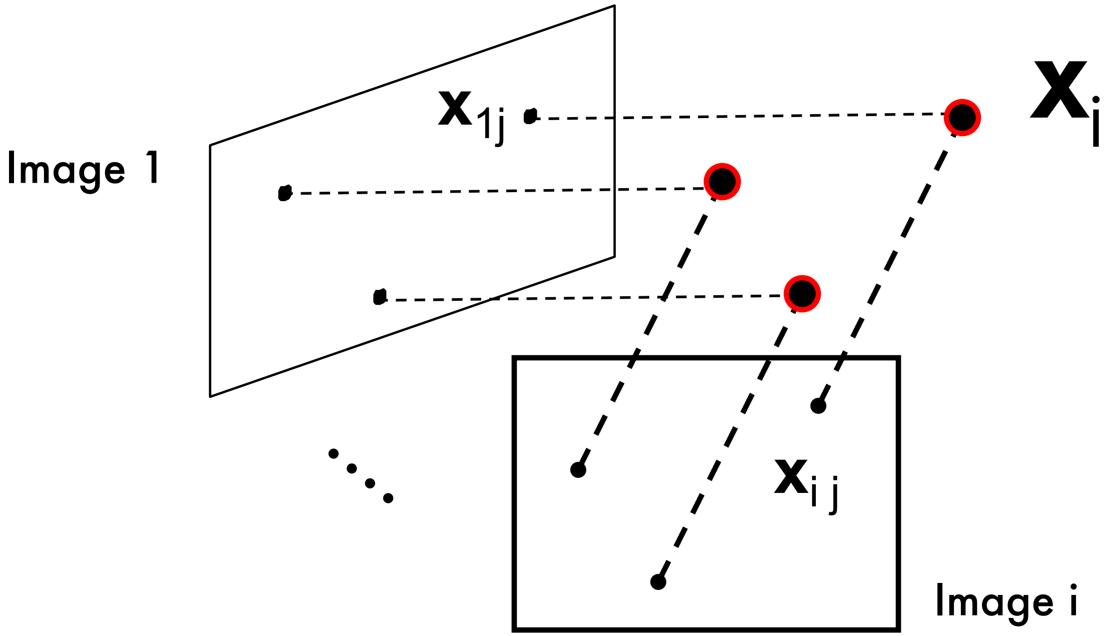
$$\mathbf{x}^E = (\underset{\uparrow}{\mathbf{m}_1}\mathbf{X}, \underset{\uparrow}{\mathbf{m}_2}\mathbf{X})^T = \begin{bmatrix} \mathbf{A} & \mathbf{b} \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \mathbf{A}\mathbf{X}^E + \mathbf{b}$$

magnification

[Eq. 3]

$$\mathbf{X}^E = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Affine cameras



For the affine case (in Euclidean space)

$$\underset{2\times1}{\mathbf{x}_{ij}} = \underset{2\times3}{\mathbf{A}_i}\,\underset{3\times1}{\mathbf{X}_j} + \underset{2\times1}{\mathbf{b}_i} \qquad \text{[Eq. 4]}$$

The Affine Structure-from-Motion Problem

Given *m* images of *n* fixed points $\mathbf{X}_j$ we can write

$$\mathbf{x}_{ij} = \mathbf{A}_i \mathbf{X}_j + \mathbf{b}_i \qquad \text{for } i = 1, \ldots, \boxed{m} \text{ and } j = 1, \ldots, \boxed{n}$$

N. of cameras N. of points

Problem: estimate m matrices A_i, m matrices b_i and the n positions $\mathbf{X}_j$ from the m×n observations $\mathbf{x}_{ij}$.

The Affine Structure-from-Motion Problem

Two approaches:

- Algebraic approach (affine epipolar geometry; estimate F; cameras; points)

- Factorization method

Next lecture

Multiple view geometry:
Affine and Perspective structure from Motion