CS231A Midterm Review

2/11/2021

Agenda

- Exam logistics
- Preparation tips
- Core topics
- Review & practice question walkthrough

Midterm logistics

- Format:
  - Online exam, on Canvas
  - 15 True/False questions, 10 multiple choice questions, and 4 short answer questions
- 2 hour time limit (+ 5 mins to upload)
- Practice exam on Canvas
- Available for 24 hour window @ 2/14
- *Open notes & open book*

Preparing for the midterm

Resources:

- Lectures 1 - 10
- Lecture notes
- Homeworks

Again: open book & open notes!

- Focus on foundations & high-level understanding; you will have time to look up details.

Core topics (1/2)

- General background
  - Homogeneous coordinates
  - Transformations
  - Formulating & solving least squares problems (when do we use an SVD?)
- Camera models
  - Perspective & non-perspective
  - Degrees of freedom
  - Distortion
  - Calibration
- Single view metrology
  - Vanishing points, lines

(disclaimer: this guide is not meant to be comprehensive)

Core topics (2/2)

- Multiview geometry
  - Epipolar geometry; essential and fundamental matrices; 8-point algorithm
  - Structure from motion
  - Stereo
  - Perspective, affine, similarity ambiguities
- Active and volumetric stereo
  - Structured lighting
  - Voxel coloring
- Fitting and matching
  - Least squares
  - RANSAC
  - Hough transforms
- Representation learning

(disclaimer: this guide is not meant to be comprehensive)

Homogeneous Coordinates

Augmented space for writing coordinates:

2D:

$$\begin{bmatrix} x \\ y \end{bmatrix} \iff \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \iff \begin{bmatrix} wx \\ wy \\ w \end{bmatrix}$$

3D:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \iff \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \iff \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

2D Lines

Homogeneous coordinates give us a neat way of representing 2D lines as vectors/orthogonality constraints:

$$ax + by + c = 0$$

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x & y & 1 \end{bmatrix}^T = 0$$

=> symmetry between lines and points

=> cross products suddenly becomes very useful!

2D Lines

How can we get the line connecting two points?

Given:

$$\begin{bmatrix} x_1 & y_1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix}$$

Unknown: $\begin{bmatrix} a & b & c \end{bmatrix}$

Subject to:

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x_1 & y_1 & 1 \end{bmatrix}^T = 0$$

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix}^T = 0$$

Solution:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

2D Lines

How can we get the intersection of two lines?

Given:
$$\begin{bmatrix} a_1 & b_1 & c_1 \end{bmatrix}$$
$$\begin{bmatrix} a_2 & b_2 & c_2 \end{bmatrix}$$

Unknown: $\begin{bmatrix} x & y & 1 \end{bmatrix}$

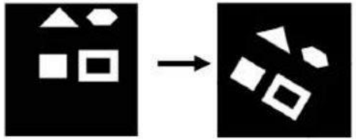
Subject to:
$$\begin{bmatrix} a_1 & b_1 & c_1 \end{bmatrix} \begin{bmatrix} x & y & 1 \end{bmatrix}^T = 0$$
$$\begin{bmatrix} a_2 & b_2 & c_2 \end{bmatrix} \begin{bmatrix} x & y & 1 \end{bmatrix}^T = 0$$

Solution:
$$\begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \times \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$$

Transformations

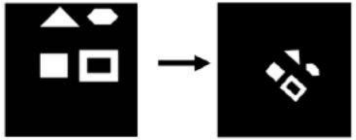
Isometric transformations:
Distances preserved

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



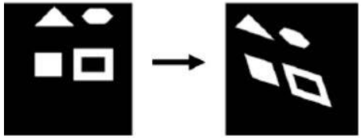
Similarity transformations:
Shapes preserved

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} SR & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad S = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$



Affine transformations:
Parallelism preserved

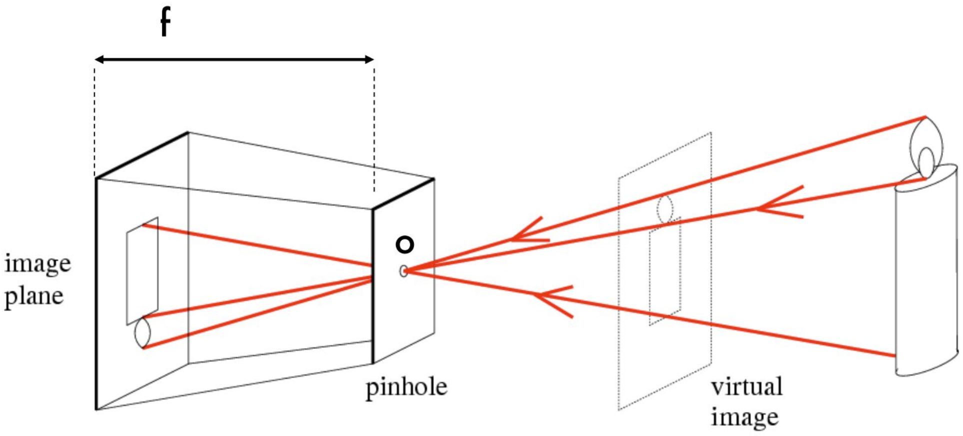
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



Projective transformations:
Lines preserved

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Pinhole Cameras

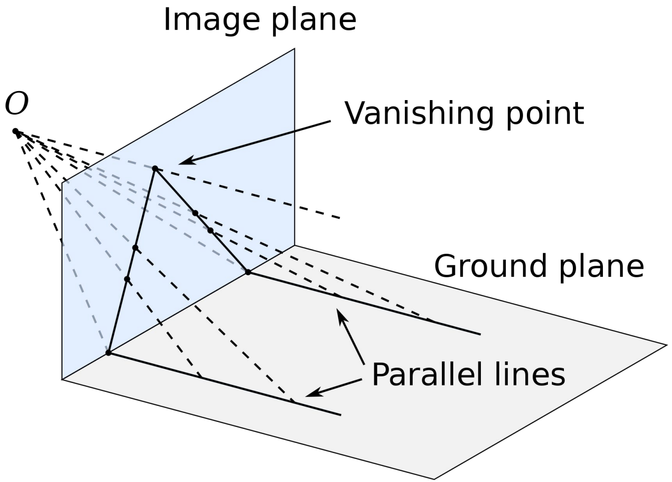


$$P' = K \begin{bmatrix} R & T \end{bmatrix} P_w$$

$$K = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

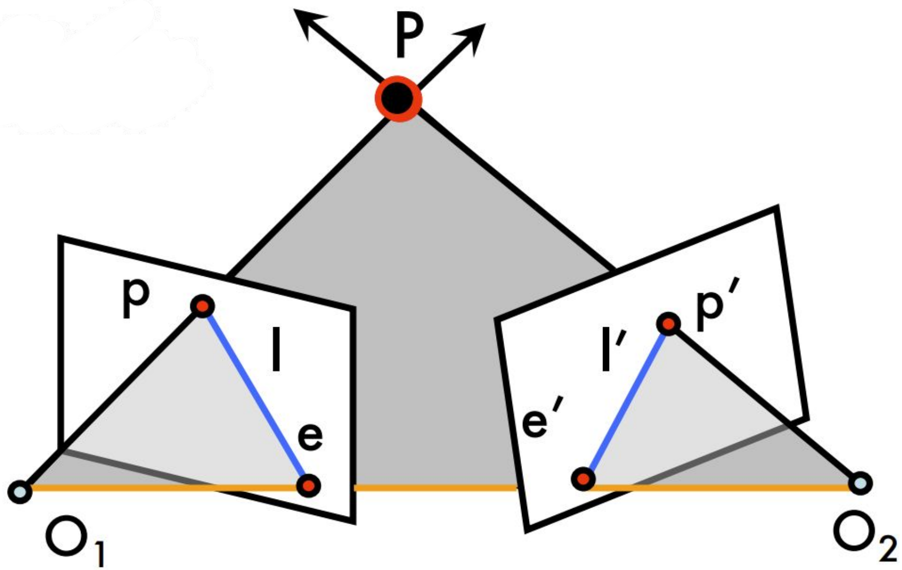
Single View Metrology

Under projective transformation, parallel lines converge to a vanishing point:



We used this for camera calibration in your homework!

Epipolar Geometry



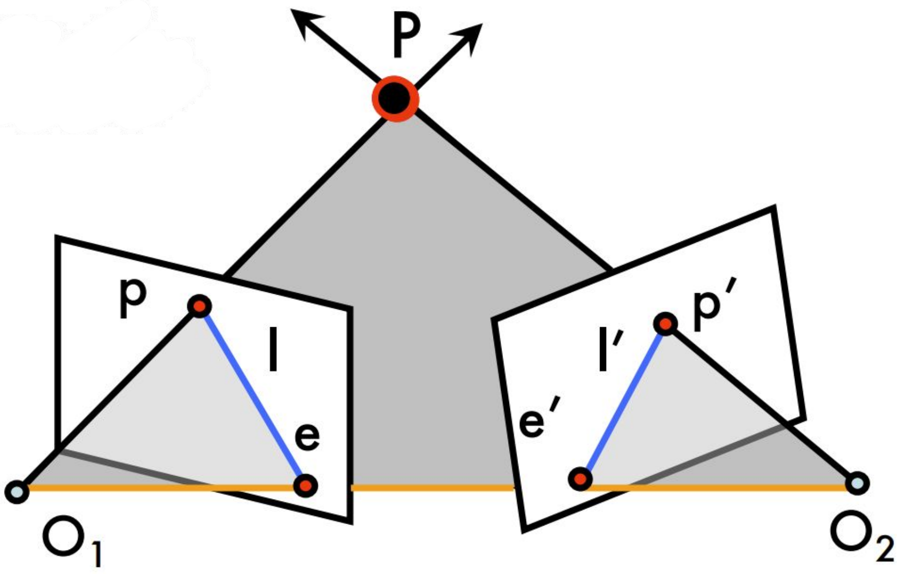
Essential matrix:
A point → epipolar line mapping for canonical cameras (K = I)

$$l' = E^T p$$

$$l = Ep'$$

$$p^T E p' = 0$$

Epipolar Geometry



Given a 2D point in one camera, a correspondence in the other must lie on an epipolar line

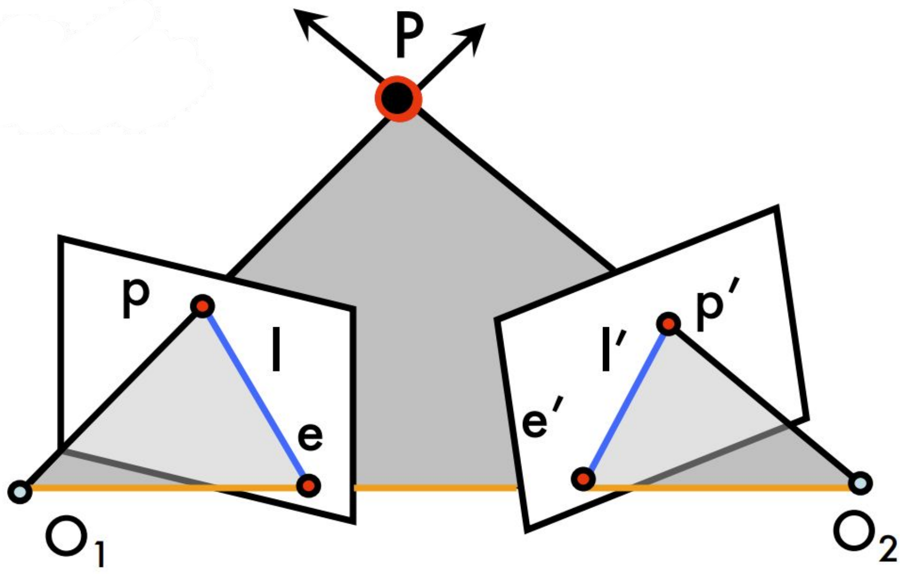
If ***p'*** is known, we can compute ***l*** and search for ***p*** using:

$$l^T p = 0$$

If ***p*** is known, we can compute ***l'*** and search for ***p'*** using:

$$l'^T p' = 0$$

Epipolar Geometry



Fundamental matrix:
A point → epipolar line mapping for general projective cameras

$$l' = F^T p$$

$$l = F p'$$

$$p^T F p' = 0$$

Epipolar Geometry

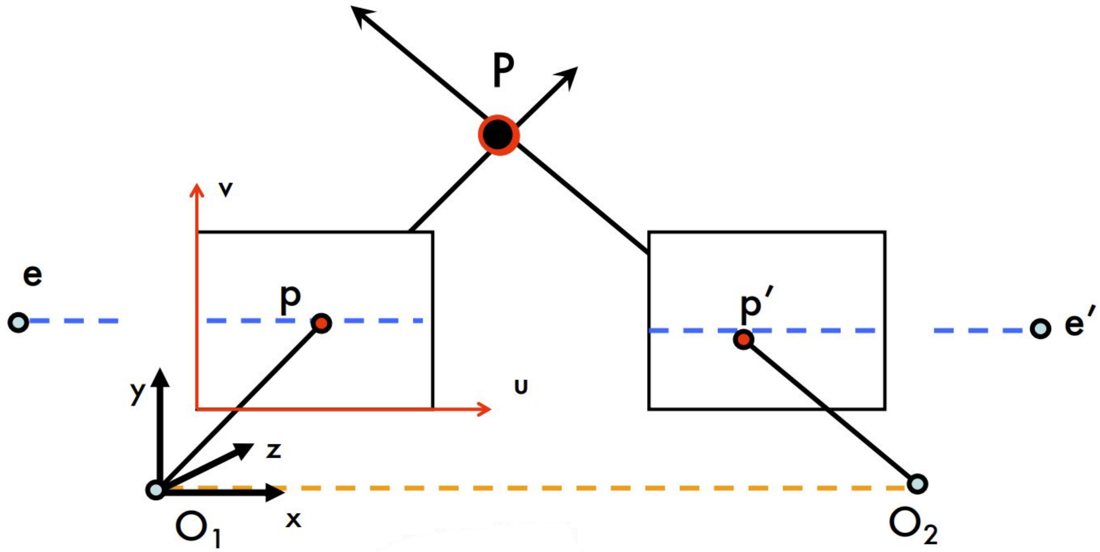
Computing the fundamental matrix with the 8-point algorithm:

$$\mathrm{p}^{\mathrm{T}}\,\mathrm{F}\,\mathrm{p}' = 0$$

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ u_3u_3' & v_3u_3' & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 & 1 \\ u_4u_4' & v_4u_4' & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 & 1 \\ u_5u_5' & v_5u_5' & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 & 1 \\ u_6u_6' & v_6u_6' & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 & 1 \\ u_7u_7' & v_7u_7' & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 & 1 \\ u_8u_8' & v_8u_8' & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

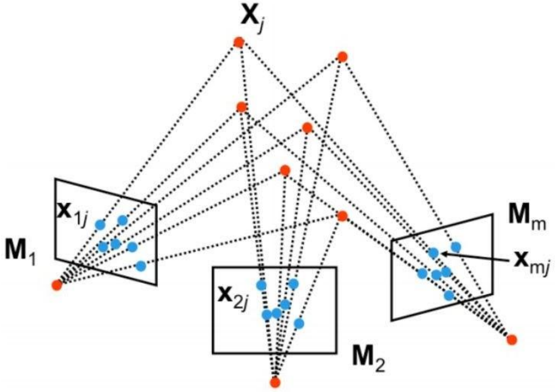
=> Solve with SVD,
then project to rank 2

Epipolar Geometry



Parallel images planes or rectification:
simplifies correspondence problem, moves epipoles to infinity

Structure from Motion



Determining structure *and* motion

You've implemented a few algorithms for this!

- Factorization, triangulation

Factorization Method

[Eq. 10]

$$\underbrace{\mathbf{D} = \begin{bmatrix} \hat{\mathbf{x}}_{11} & \hat{\mathbf{x}}_{12} & \cdots & \hat{\mathbf{x}}_{1n} \\ \hat{\mathbf{x}}_{21} & \hat{\mathbf{x}}_{22} & \cdots & \hat{\mathbf{x}}_{2n} \\ & & \ddots & \\ \hat{\mathbf{x}}_{m1} & \hat{\mathbf{x}}_{m2} & \cdots & \hat{\mathbf{x}}_{mn} \end{bmatrix}}_{(2m \times n)} = \underbrace{\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_m \end{bmatrix}}_{\substack{\text{cameras} \\ (2m \times 3)}} \underbrace{\begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix}}_{\text{points } (3 \times n)}$$

M

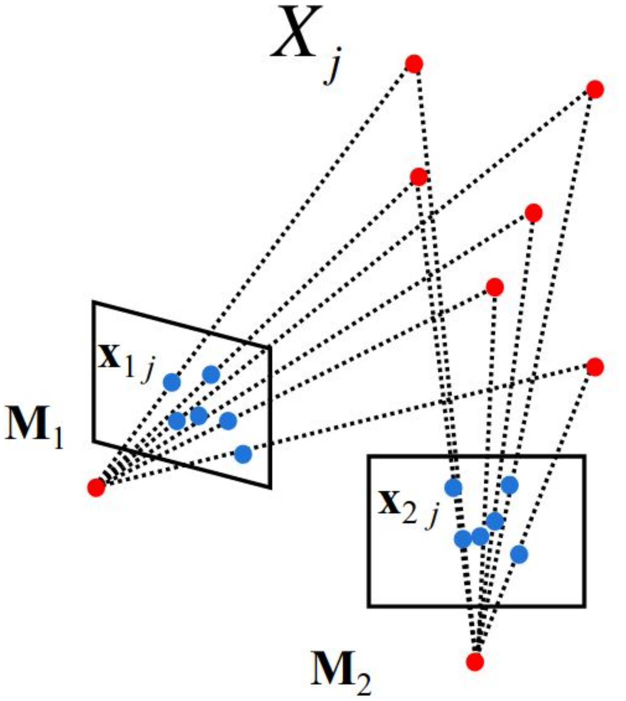
S

Assume all points are visible

Algebraic approach

- Compute fundamental matrix F
- Use F to estimate projective cameras
- Use these cameras to triangulate and estimate points in 3D

Works with 2 views



Bundle Adjustment

Non-linear method for refining structure and motion

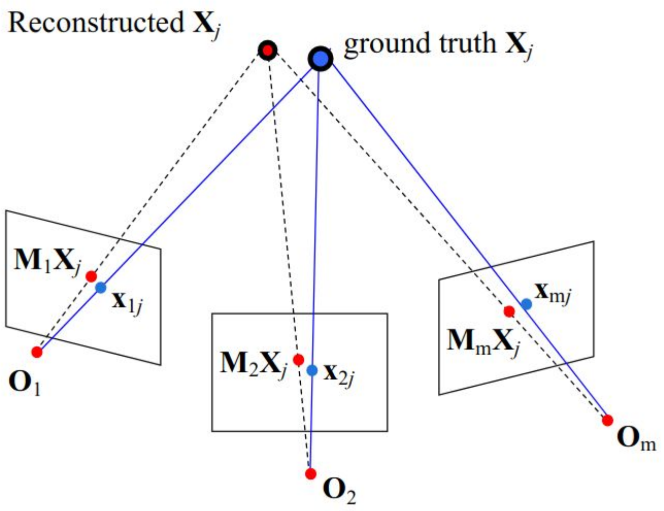
Goal: minimize reprojection error

Advantages

- Handle large number of views
- Handle missing data

Limitations

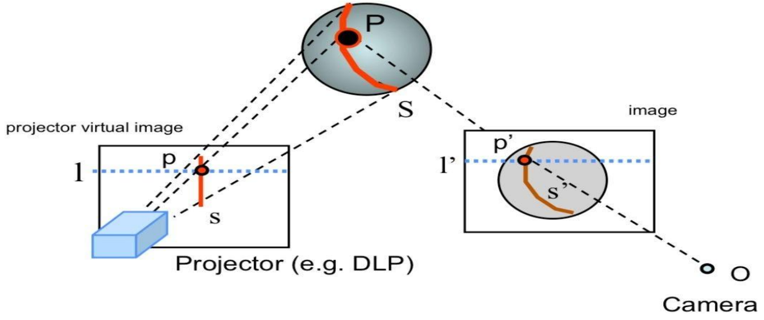
- Large minimization problem
- Require good initialization

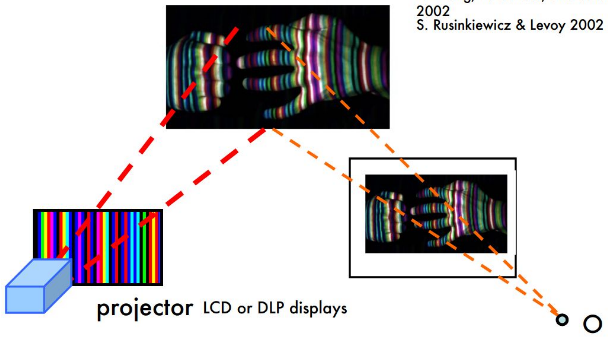


Active Stereo

Active Stereo

- Replaces one camera with a projector
- Solves matching problem



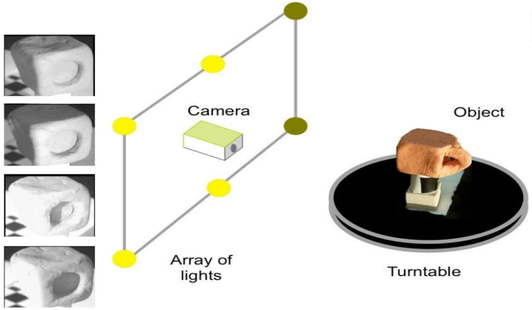


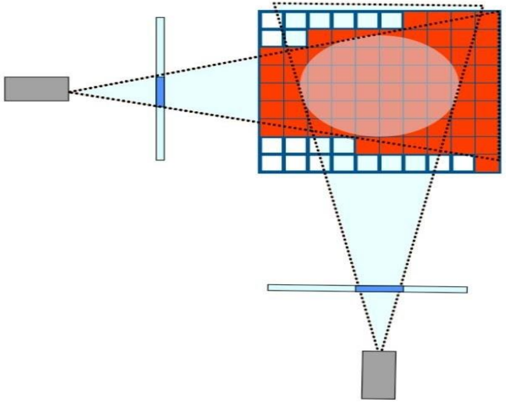
L. Zhang, B. Curless, and S. M. Seitz 2002
S. Rusinkiewicz & Levoy 2002

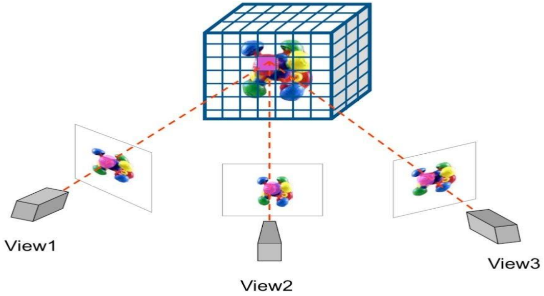
Volumetric Stereo

Volumetric Stereo

- Space carving
  - Cannot carve concavity
- Shadow carving
- Voxel coloring



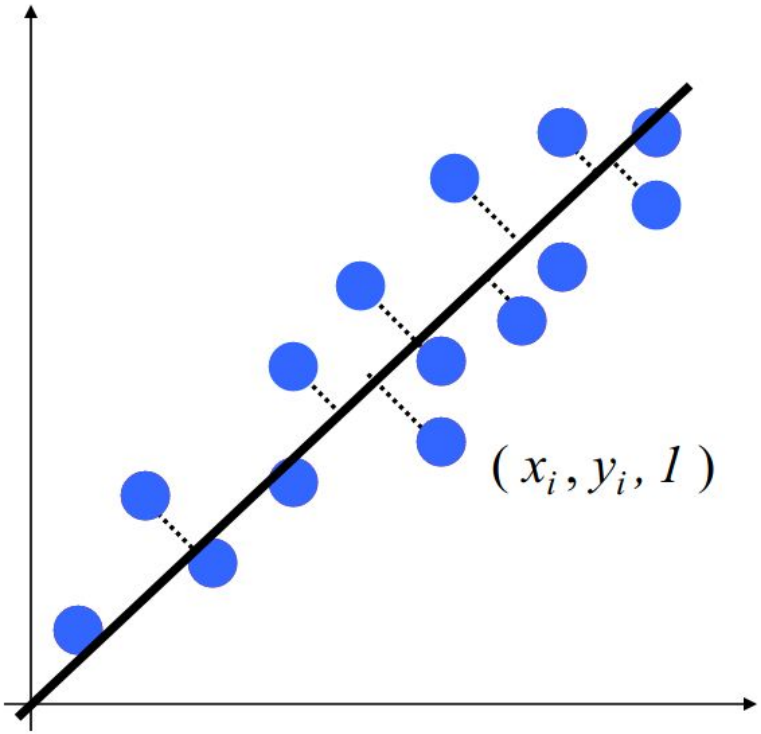




Least square

Find a line to minimize the sum of squared distance to the points

$$E = \sum_{i=1}^{n} (ax_i + by_i + d)^2$$



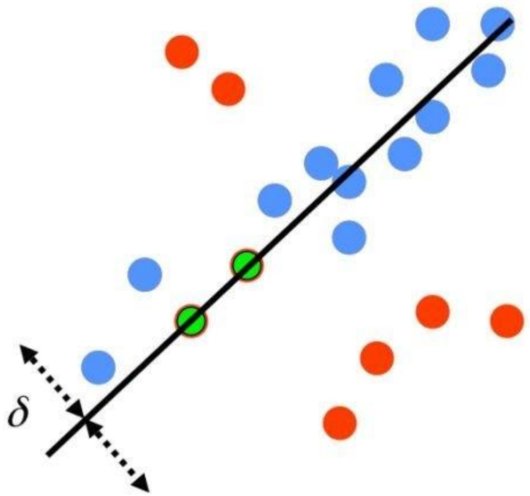
RANSAC

Ran*dom* ***sa****mple* ***c****onsensus*

For fitting a model to noisy data!

Iterative approach:

- Sample a subset of points
- Fit our model
- Count the total # of inliers that match this model
- Repeat

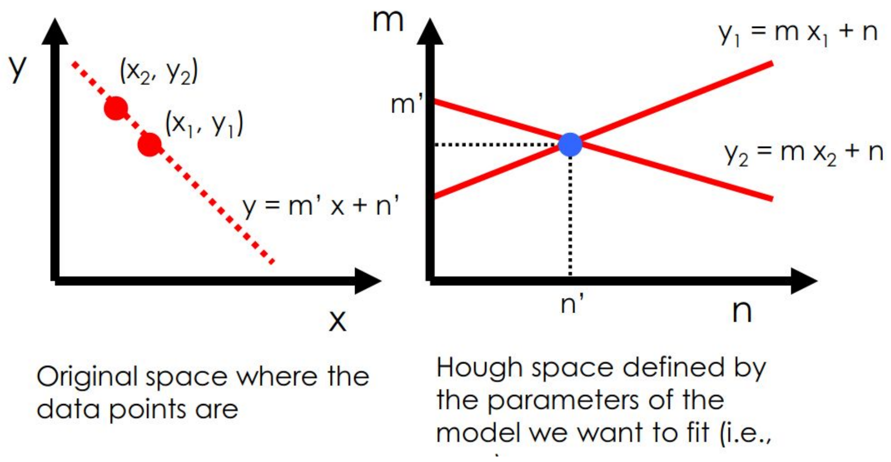


Hough Transforms

Key idea for line fitting:

- Map points in (x,y) to a line in our Hough space
- Each point in our Hough space represents a line in our (x,y) space
- Intersection of lines in hough space = line

\+ Polar line representation
\+ Discretization and voting

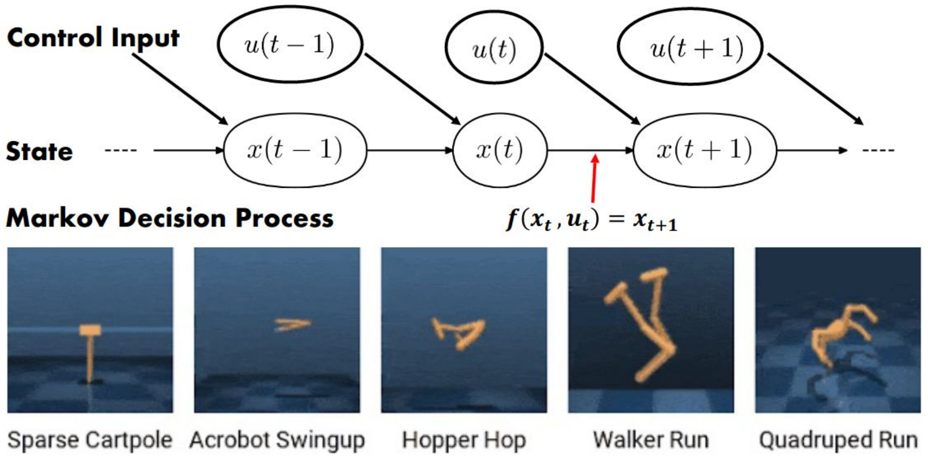


Representation Learning

State: Quantity that describes the most important aspect of a dynamical system at time t

Representation: data format of input or output including a low-dimensional representation of sensor data

- Compact
- Explanatory
- Disentangled
- Hierarchical
- Makes subsequent problem easier

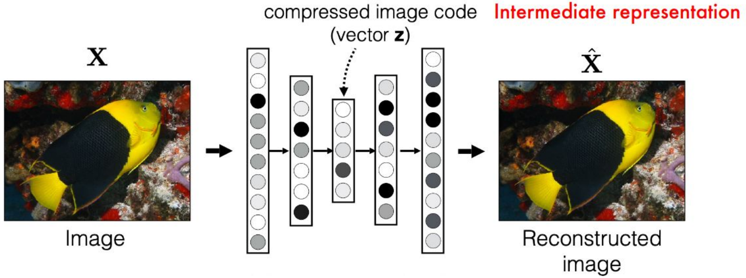


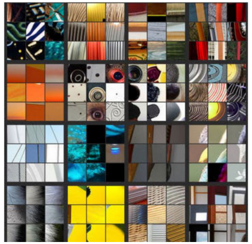
Representation

Learned representation

- Supervised
- Unsupervised
- Self-supervised

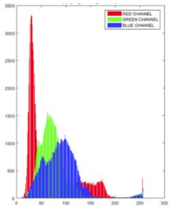
Visualizing learned representation

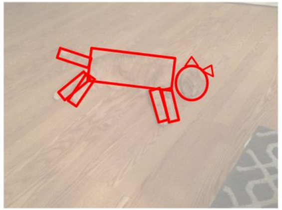




Interpretable representation

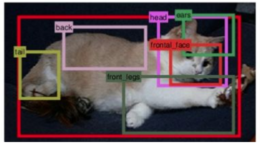
Color Histograms

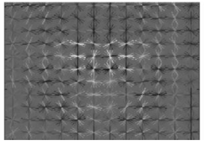




Model based Shapes

Deformable Part based Models (DPM)





Histogram of Gradients (HOG)