ASTM21: Project 3

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Introduction

Spectroscopy can be used in astronomy for revealing many properties of celestial objects, such as their chemical composition, luminosity and relative motion using Doppler shift measurements[1]. In this report we will use data from the Sloan Digital Sky Survey to determine the red shift of, in this case, quasar six. This is done by fitting a line profile to the Mg II emission line.

Theory

To model both the continuum and an emission line we use the model

$$f(\lambda|\vec{\theta}) = \theta_1 + \theta_2(\lambda - \lambda_{ref}) + \theta_3 L\left(\frac{\lambda - \theta_4}{\theta_5}\right) \tag{1}$$

here f is the theoretical flux, λ is the wavelength and λ_{ref} is a reference value chosen to be 6175 Å which is roughly at the center of the peak in question. $\vec{\theta}$ is a vector of shape parameters which are to be determine. The function L could be one of three different line profiles. Either a Gaussian

$$L(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-1\frac{1}{2}x^2\right) \tag{2}$$

or a Lorentzian

$$L(x) = \frac{1}{\pi}\frac{1}{1+x^2} \tag{3}$$

or lastly a Voigt profile

$$L(x) = \int_{-\infty}^{\infty} G(x')C(x-x')dx' \tag{4}$$

which is a convolution between the Gaussian, G and the Lorentzian, C, line profiles. The integral can be evaluated as

$$L(x) = \frac{\mathrm{Re}[W(z)]}{\sqrt{2\pi}} \tag{5}$$

where $\mathrm{Re}[W(z)]$ is the real part of the Faddeeva function evaluated for

$$z = \frac{x+i}{\sqrt{2}} \tag{6}$$

[2].
To get the parameters in equation 1, we use the non-linear least squares method which means that we try to optimize the χ^2-function

$$\chi^2(\vec{\theta}) = \sum_{i=1}^{n} \left(f_i - f(\lambda|\vec{\theta}) \right)^2 w_i \tag{7}$$

where f_i is the measured flux and w_i is the inverse variance of the flux.
The reduced χ^2-function

$$\chi^2_\nu = \frac{\chi^2}{\nu} \tag{8}$$

where ν equals the number of observations minus the number of fitted parameters, was also consider as a goodness of fit testing.
The red shift estimate is

$$\hat{z} = \frac{\hat{\theta}_4 - \lambda_{lab}}{\lambda_{lab}} \tag{9}$$

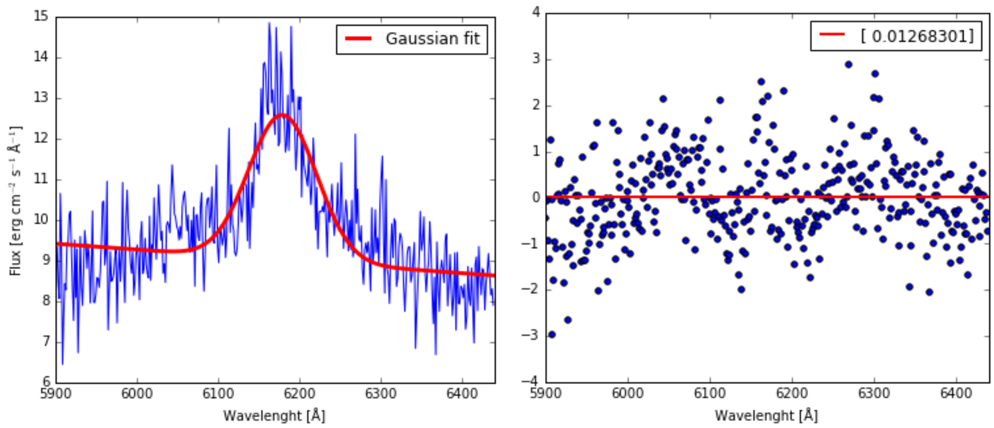
where $\hat{\theta}_4$ is the fourth estimated parameter from equation 1 and $\lambda_{lab} = 2800.3$ Å is the measured value for the Mg II emission line in a lab. To get the uncertainty in $\hat{z}$ we use a Monte Carlo simulation to generate synthetic data and calculate the standard deviation from this data. The synthetic data is generate according to

$$f^{synt} = f(\lambda|\vec{\theta}) + \sigma \times N(0,1) \tag{10}$$

where σ is the standard deviation, transform from w previously mentioned, and $N(0,1)$ is a random number taken from a standard distribution.

Results

Gaussian fit



(a) Using the Gaussian line profile to fit to the data.

(b) The residuals between the data and the fitted values, that is, $f - f(\lambda|\vec{\theta})$. The red line is a fitted constant value to the data points.

The left part of the figure show the data plotted between wavelengths 5900 Å and 6450 Å, where the peak is the Mg II emission line should be. The red line is a the fitted function from equation 1 using the Gaussian model.

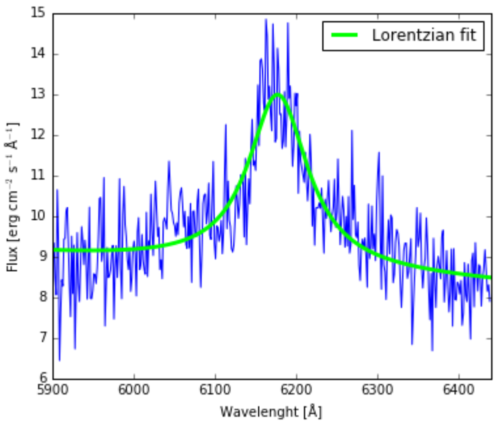
The right part of the figure shows the residues between the fitted function and the actual data in the same wavelength interval. The red line is a fitted constant value that all of the data points is centered around to get an estimation on how the good the fit was in the right part of the figure.

	Value	Standard deviation
$\hat{\theta}_1$	9.015	0.78
$\hat{\theta}_2$	-0.0014	0.00070
$\hat{\theta}_3$	8.95	0.20
$\hat{\theta}_4$	6179.53	0.77
$\hat{\theta}_5$	41.82	1.33
χ^2_ν	1.39	-
$\hat{z}$	1.21	0.00027

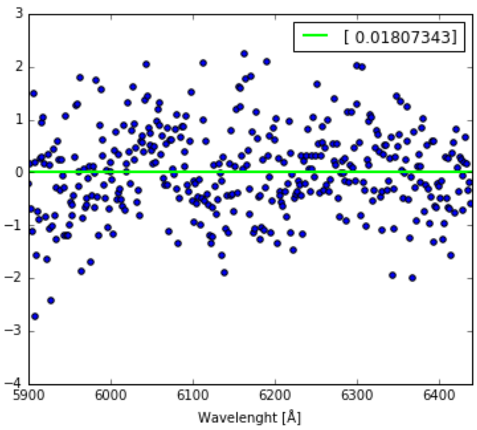
Table 1: Table over the estimated parameters, reduced χ^2 and the red shift, with their corresponding standard deviations. The standard deviations are calculated from 100 synthetics data sets.

Table 1 shows the estimated parameters in equation 1, reduced χ^2 calculated using equation 8 and the estimated red shift, calculated using equation 9, and their corresponding standard deviations.

Lorentzian fit



(a) Using the Lorentzian line profile to fit to the data.



(b) The residuals between the data and the fitted values, that is, $f - f(\lambda|\vec{\theta})$. The green line is a fitted constant value to the data points.

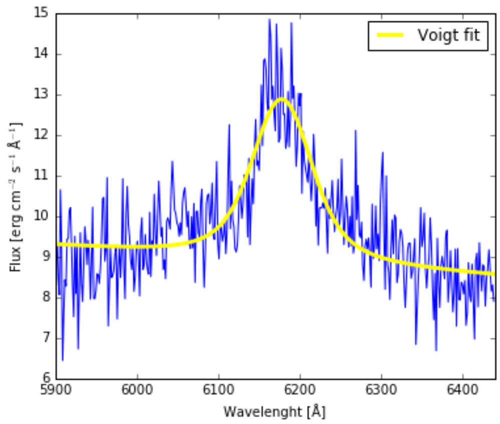
The left part of the figure show the data plotted between wavelengths 5900 Å and 6450 Å, where the peak is the Mg II emission line should be. The green line is a the fitted function from equation 1 using the Lorentzian model.
The right part of the figure shows the residues between the fitted function and the actual data in the same wavelength interval. The green line is a fitted constant value that all of the data points is centered around to get an estimation on how the good the fit was in the right part of the figure.

	Value	Standard deviation
$\hat{\theta}_1$	8.71	0.75
$\hat{\theta}_2$	-0.0013	0.00066
$\hat{\theta}_3$	13.47	0.27
$\hat{\theta}_4$	6177.66	0.62
$\hat{\theta}_5$	44.60	1.97
χ^2_ν	1.22	-
$\hat{z}$	1.21	0.00022

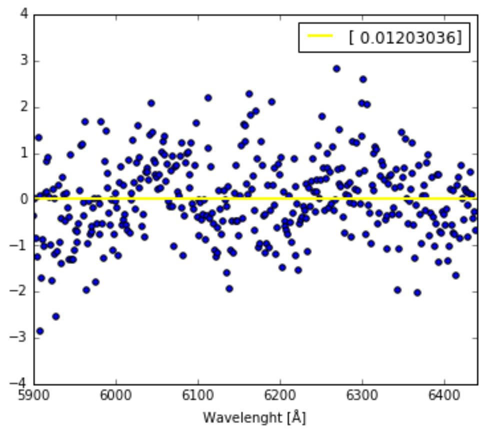
Table 2: Table over the estimated parameters, reduced χ^2 and the red shift, with their corresponding standard deviations. The standard deviations are calculated from 100 synthetics data sets.

Table 2 shows the estimated parameters in equation 1, reduced χ^2 calculated using equation 8 and the estimated red shift, calculated using equation 9, and their corresponding standard deviations.

Voigt profile fit



(a) Using the Voigt line profile to fit to the data.



(b) The residuals between the data and the fitted values, that is, $f - f(\lambda|\vec{\theta})$. The yellow line is a fitted constant value to the data points.

The left part of the figure show the data plotted between wavelengths 5900 Å and 6450 Å, where the peak is the Mg II emission line should be. The yellow line is a the fitted function from equation 1 using the Voigt model.
The right part of the figure shows the residues between the fitted function and the actual data in the same wavelength interval. The yellow line is a fitted constant value that all of the data points is centered around to get an estimation on how the good the fit was in the right part of the figure.

	Value	Standard deviation
$\hat{\theta}_1$	8.88	0.73
$\hat{\theta}_2$	-0.0014	0.00065
$\hat{\theta}_3$	19.17	0.41
$\hat{\theta}_4$	6178.36	0.62
$\hat{\theta}_5$	24.72	0.86
χ^2_ν	1.29	-
$\hat{z}$	1.21	0.00022

Table 3: Table over the estimated parameters, reduced χ^2 and the red shift, with their corresponding standard deviations. The standard deviations are calculated from 100 synthetics data sets.

Table 3 shows the estimated parameters in equation 1, reduced χ^2 calculated using equation 8 and the estimated red shift, calculated using equation 9, and their corresponding standard deviations.

Discussion

Visually all three models fitted the data quite well. The wings might be a bit off, which might be fixed if we use a larger range for the wavelength data.
The residuals looks similarly spread for all three profiles and the fitted value is close to zero which one expects if the optimization of χ^2 was preformed well. The Lorentzian line profile sticks out in the reduced-χ^2 as the best and the Gaussian line profile the worst. For all three line profiles the red shit estimate is the same in order of two significant values.

References

[1] https://en.wikipedia.org/wiki/Astronomical_spectroscopy

[2] https://en.wikipedia.org/wiki/Voigt_profile