Topics in Empirical Economics, Part III
Bayesian Inference & Latent Dirichlet Allocation

Stephen Hansen
Universitat Pompeu Fabra

Introduction

Recall we are interested in mixed-membership modeling, but that the pLSI model has a huge number of parameters to estimate.

One solution is to adopt a Bayesian approach; the pLSI model with a prior distribution on the document-specific mixing probabilities is called Latent Dirichlet Allocation (Blei, Ng, and Jordan 2003).

LDA is widely used within computer science and, increasingly, social sciences.

LDA forms the basis of many, more complicated mixed-membership models.

Outline

Bayesian models of discrete data

LDA model specification

Graphical models / Bayesian networks

Markov chain Monte Carlo estimation of LDA

Applications in economics

Review

Recall the simple unigram model of a document in which

$$\Pr\left[\mathbf{x}_d \mid \boldsymbol{\beta}\right] = \prod_v \beta_v^{x_{d,v}}.$$

The maximum likelihood estimate for the vth categorical probability is $\widehat{\beta}_v = \frac{x_{d,v}}{N_d}$

To maximize the probability of the observed data, we get parameters that exactly match the observed frequencies.

Black Swan Paradox

What would the model predict is the probability of seeing an unobserved term?

This is sometimes called the *black swan paradox*. Europeans assumed that the fact that they had never observed a black swan implied black swans could not exist.

Since the document-term matrix is sparse, the black swan paradox will be particularly relevant for text mining.

Bottom line is we need to incorporate some additional uncertainty in our inference procedure to not drive beliefs about unobserved events to zero.

Bayesian Inference

One solution is to adopt a Bayesian inference approach, which treats β as a random variable rather than a fixed parameter.

Recall that Bayes' rule states that

$$\Pr[\beta \mid \mathbf{x}_d] = \frac{\Pr[\mathbf{x}_d \mid \beta]\Pr[\beta]}{\Pr[\mathbf{x}_d]}$$

where

- $\Pr[\beta \mid \mathbf{x}_d]$ is the posterior distribution.
- $\Pr[\mathbf{x}_d \mid \beta]$ is the likelihood function.
- $\Pr[\beta]$ is the prior distribution on the parameter vector.
- $\Pr[\mathbf{x}_d]$ is a normalizing constant sometimes called the evidence.

The prior distribution introduces initial uncertainty about the value of the parameter vector.

Dirichlet Prior

One way of ensuring Bayesian inference is tractable is to select a prior distribution from a family that ensures the posterior will be in the same family given the likelihood function. This is called a *conjugate* prior.

The Dirichlet distribution is conjugate to the categorical likelihood function, and so is a popular choice for the prior in Bayesian models of discrete data.

The Dirichlet distribution is parametrized by $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_V)$; is defined on the $V-1$ simplex; and has probability density function

$$\text{Dir}(\boldsymbol{\beta} \mid \boldsymbol{\alpha}) \propto \prod_v \beta_v^{\alpha_v - 1}.$$

The normalization constant is $B(\boldsymbol{\alpha}) \equiv \prod_{v=1}^{V} \Gamma(\alpha_v) / \Gamma\left(\sum_{v=1}^{V} \alpha_v\right)$.

Interpreting the Dirichlet

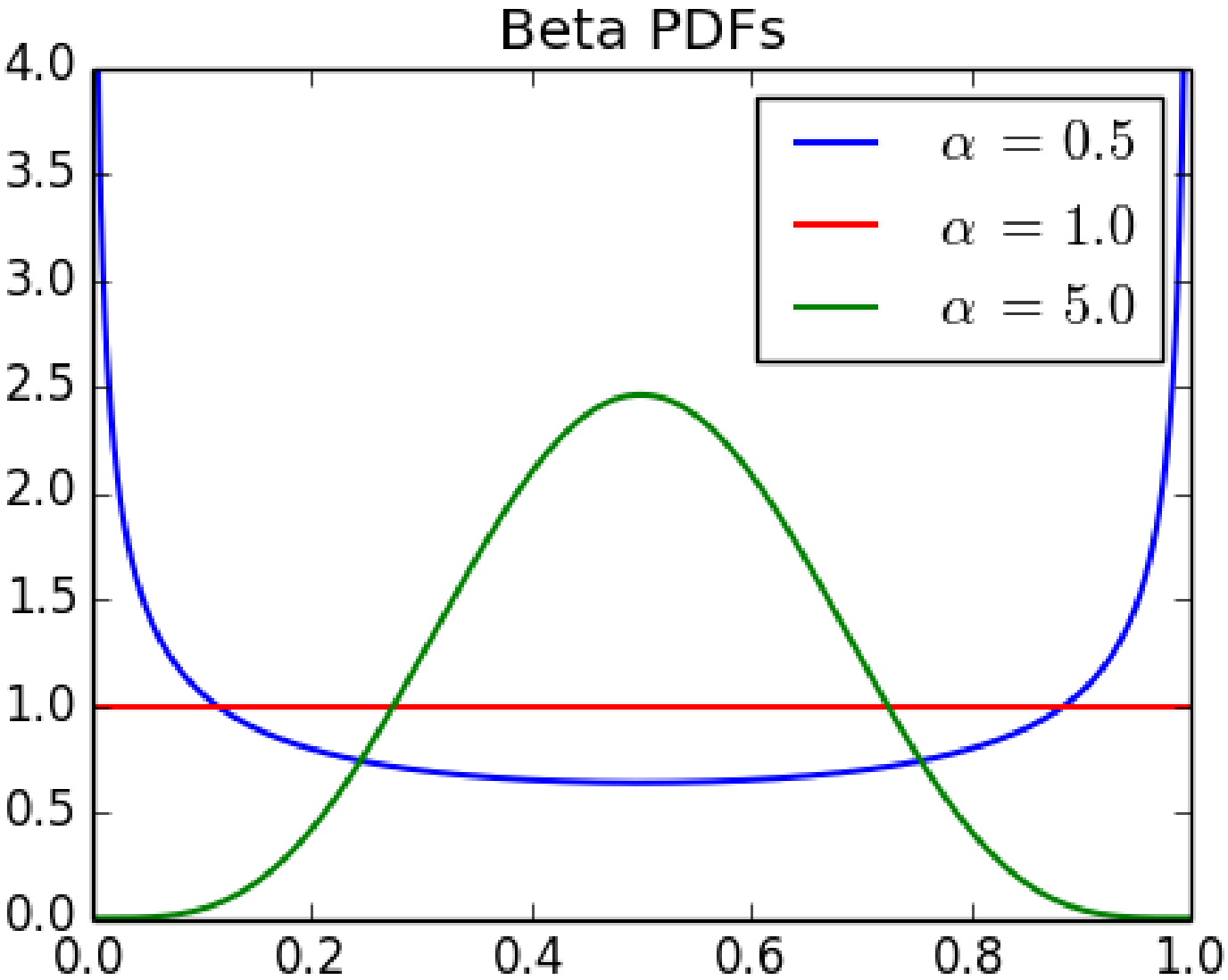
Consider a symmetric Dirichlet in which $\alpha_v = \alpha$ for all v. Agnostic about favoring one component over another.

Here the α parameter measures the concentration of distribution on the center of the simplex, where the mass on each term is more evenly spread out:

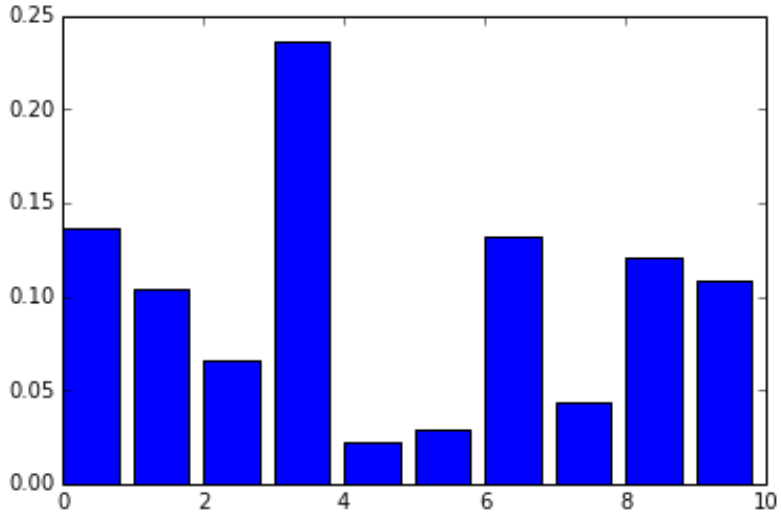
1. $\alpha = 1$ is a uniform distribution.
2. $\alpha > 1$ puts relatively more weight in center of simplex.
3. $\alpha < 1$ puts relatively more weight on corners of simplex.

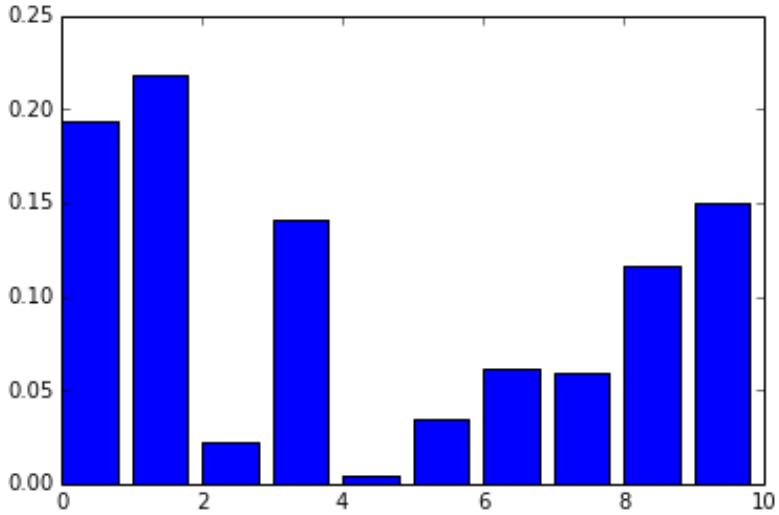
When $V = 2$, the Dirichlet is the beta distribution.

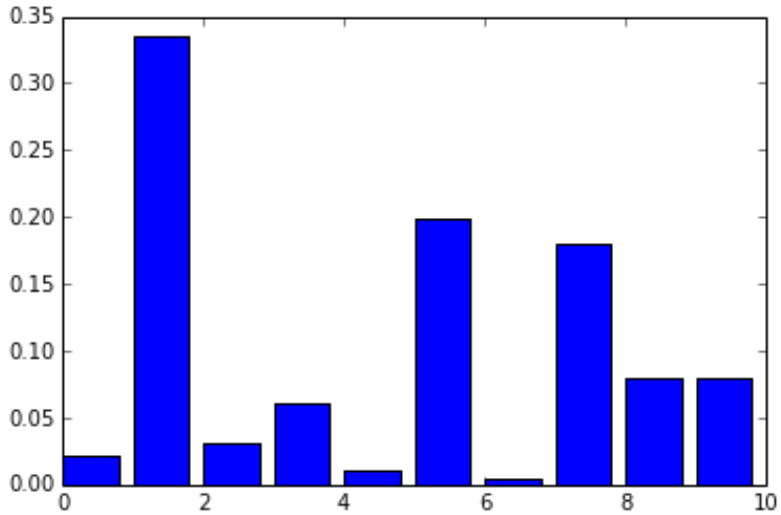
Beta with Different Parameters

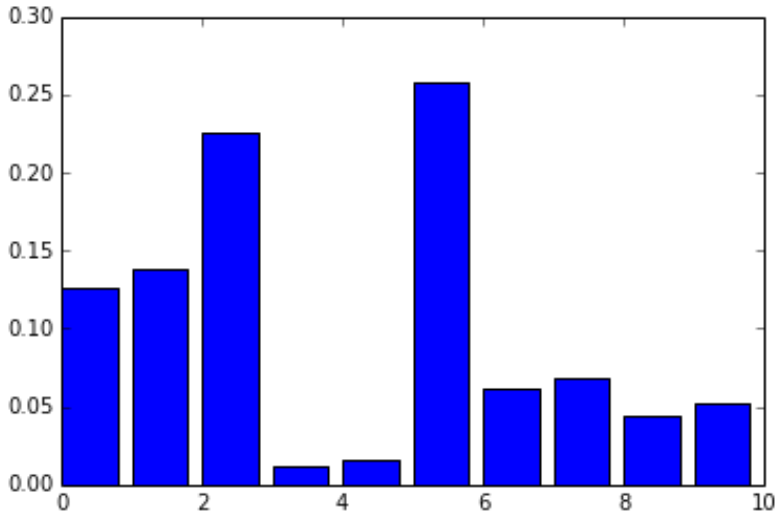


Draws from Dirichlet with $\alpha = 1$

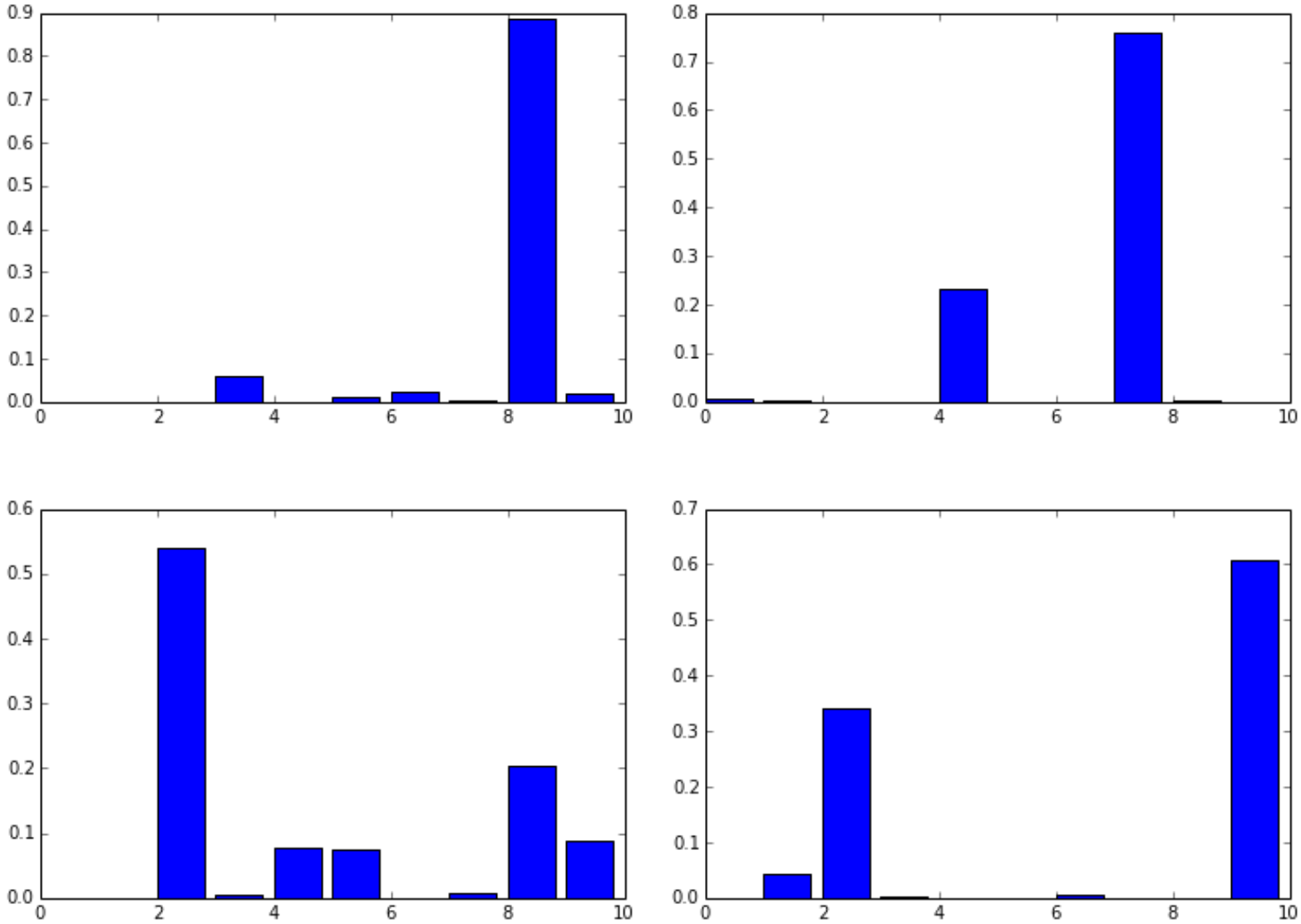




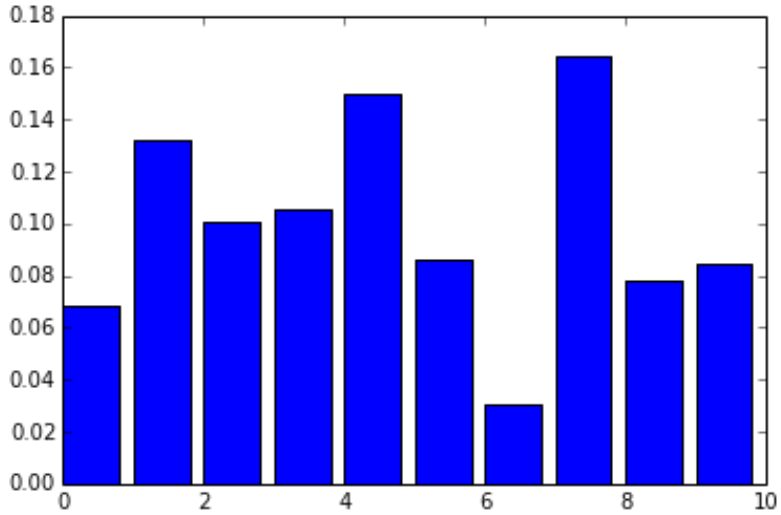


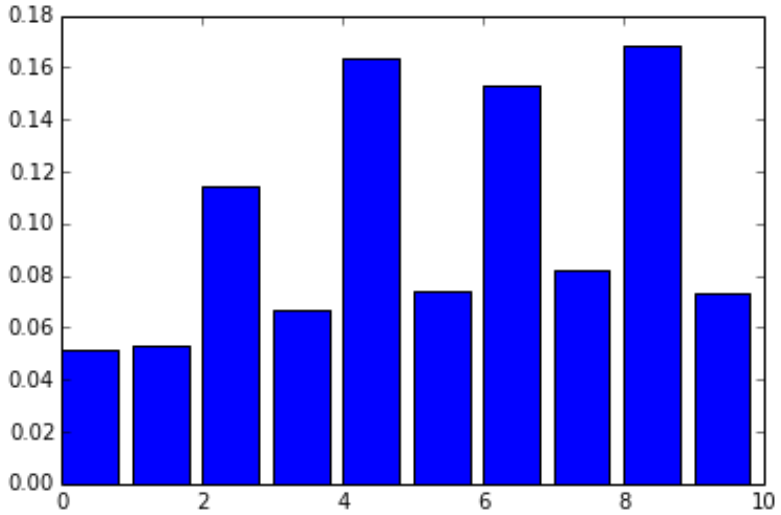


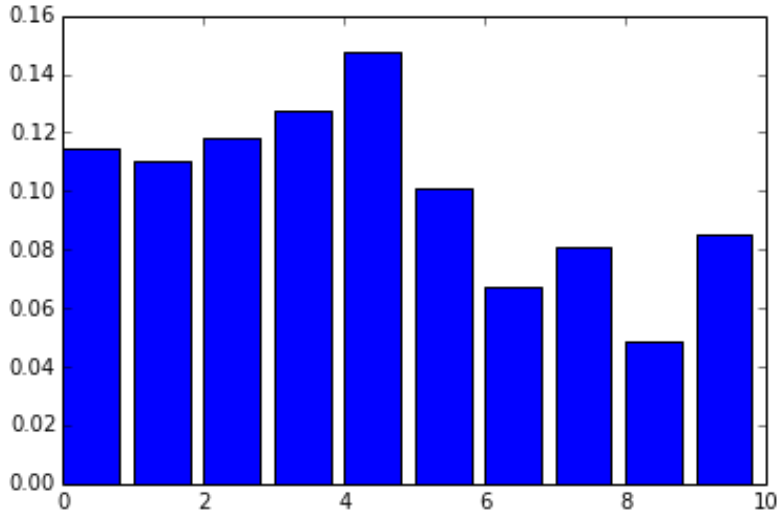
Draws from Dirichlet with $\alpha = 0.1$

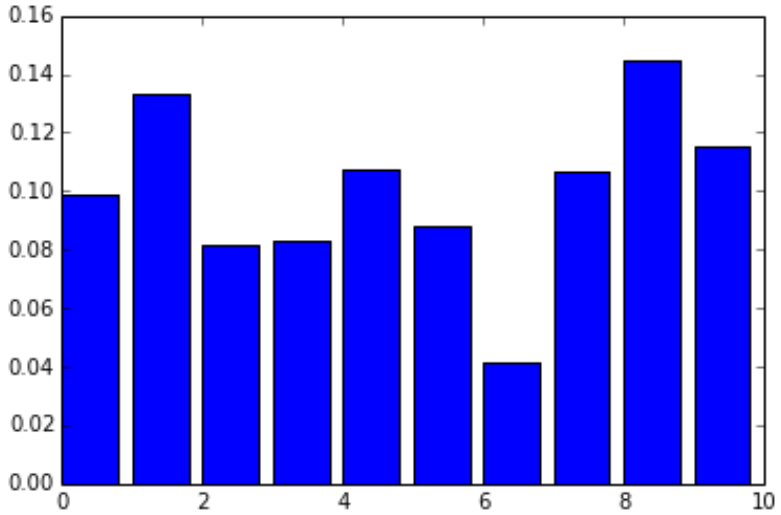


Draws from Dirichlet with $\alpha = 10$









Posterior Distribution

$$\Pr[\boldsymbol{\beta} \mid \mathbf{x}_d] \propto \Pr[\mathbf{x}_d \mid \boldsymbol{\beta}] \Pr[\boldsymbol{\beta}] \propto \prod_{v=1}^{V} \beta_v^{x_{d,v}} \prod_{v=1}^{V} \beta_v^{\alpha_v - 1} = \prod_{v=1}^{V} \beta_v^{x_{d,v} + \alpha_v - 1}.$$

Posterior is a Dirichlet with parameters $(\widehat{\alpha}_1, \ldots, \widehat{\alpha}_V)$ where $\widehat{\alpha}_v \equiv \alpha_v + x_{d,v}$.

Add term counts to the prior distribution's parameters to form posterior distribution.

The parameters in the prior distribution are sometimes called *pseudo-counts*, and can be viewed as observations made before $\mathbf{x}_d$.

Moments of the Posterior

One can show that a Dirichlet with parameter vector $\boldsymbol{\alpha}$ satisfies

$$\mathbb{E}[\,\beta_v\,] = \frac{\alpha_v}{\alpha} \text{ and } V[\,\beta_v\,] = \frac{\alpha_v(\alpha - \alpha_v)}{\alpha^2(\alpha+1)}, \text{ where } \alpha \equiv \sum_v \alpha_v.$$

If we apply these formulas to the posterior distribution we obtain

$$\mathbb{E}[\,\beta_v\,] = \frac{\alpha_v + x_{d,v}}{\alpha + N_d} \text{ and } V[\,\beta_v\,] = \frac{(\alpha_v + x_{d,v})(\alpha + N_d - \alpha_v - x_{d,v})}{(\alpha + N_d)^2(\alpha + N_d + 1)}.$$

One can also show that the mean corresponds to the predictive distribution for an additional word.

Data Overwhelming the Prior

Recall the MLE estimates for $\widehat{\beta}_v$ satisfies $N_d \widehat{\beta}_v = x_{d,v}$. We then have

$$\mathbb{E}[\,\beta_v\,] = \frac{\alpha_v + N_d \widehat{\beta}_v}{\alpha + N_d} \text{ and } V[\,\beta_v\,] = \frac{(\alpha_v + N_d \widehat{\beta}_v)(\alpha + N_d - \alpha_v - N_d \widehat{\beta}_v)}{(\alpha + N_d)^2(\alpha + N_d + 1)}.$$

If we take the limit as $N_d \to \infty$, we obtain a degenerate posterior distribution concentrated fully on the MLE parameter estimates.

Intuition: the more data we see, the less our priors should influence our beliefs.

Latent Dirichlet Allocation—Original

1. Draw $\boldsymbol{\theta}_d$ independently for $d = 1, \ldots, D$ from Dirichlet($\boldsymbol{\alpha}$).
2. Each word $w_{d,n}$ in document d is generated from a two-step process:
   - 2.1 Draw topic assignment $z_{d,n}$ from $\boldsymbol{\theta}_d$.
   - 2.2 Draw $w_{d,n}$ from $\boldsymbol{\beta}_{z_{d,n}}$.

Estimate hyperparameters $\boldsymbol{\alpha}$ and term probabilities $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K$.

Latent Dirichlet Allocation—Modified

1. Draw $\boldsymbol{\beta}_k$ independently for $k = 1, \ldots, K$ from Dirichlet(η).
2. Draw $\boldsymbol{\theta}_d$ independently for $d = 1, \ldots, D$ from Dirichlet(α).
3. Each word $w_{d,n}$ in document d is generated from a two-step process:
   - 3.1 Draw topic assignment $z_{d,n}$ from θ_d.
   - 3.2 Draw $w_{d,n}$ from $\boldsymbol{\beta}_{z_{d,n}}$.

Fix scalar values for η and α.

Graphical Models

Consider a probabilistic model with joint distribution $f(\mathbf{x})$ over the random variables $\mathbf{x} = (x_1, \ldots, x_N)$.

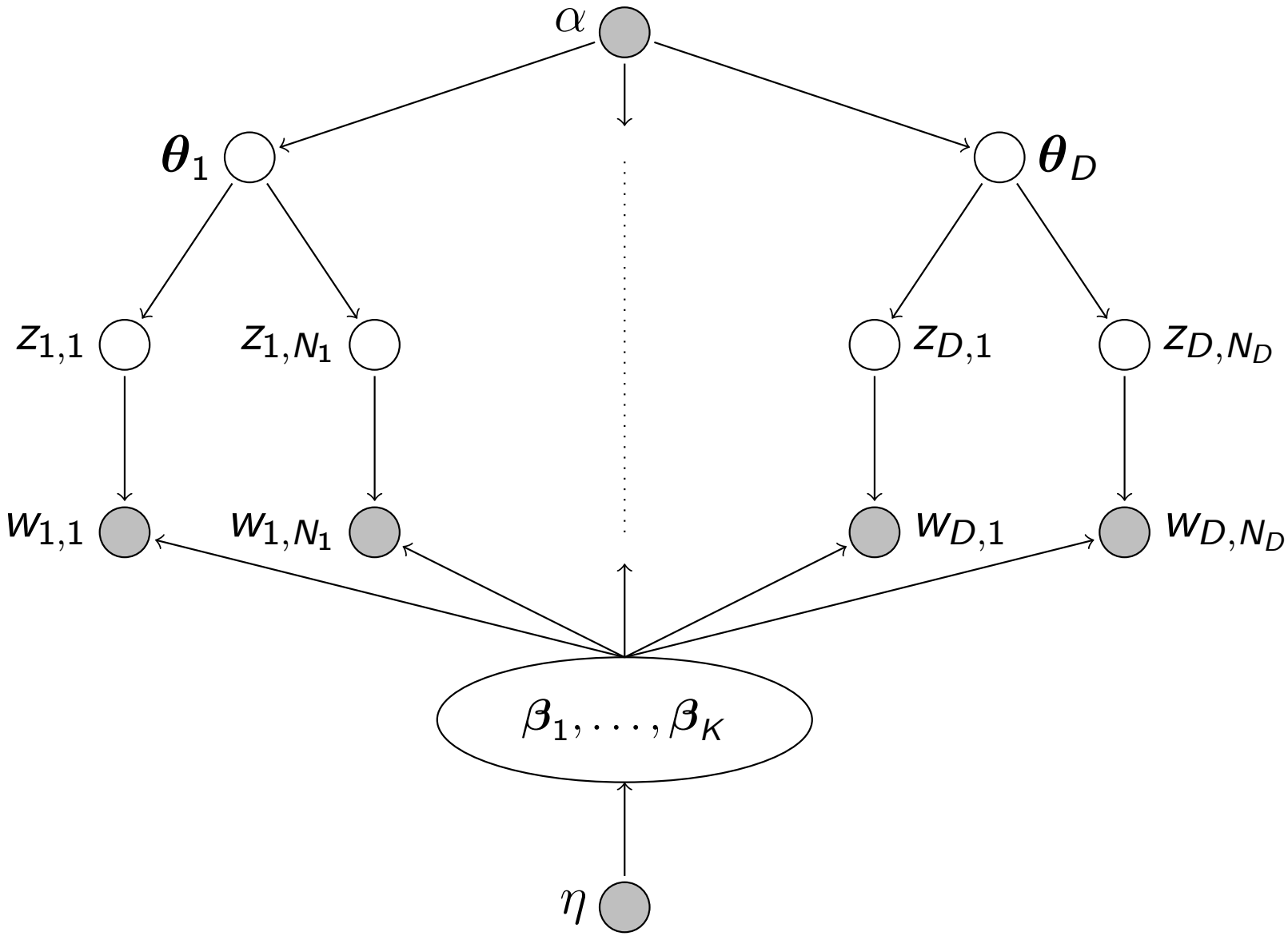
In high-dimensional models, it is useful to summarize relationships among random variables with directed graphs in which nodes represent random variables and links between nodes represent dependencies.

A node's *parents* are the set of nodes that link to it; a node's *children* are the the set of nodes that it links to.

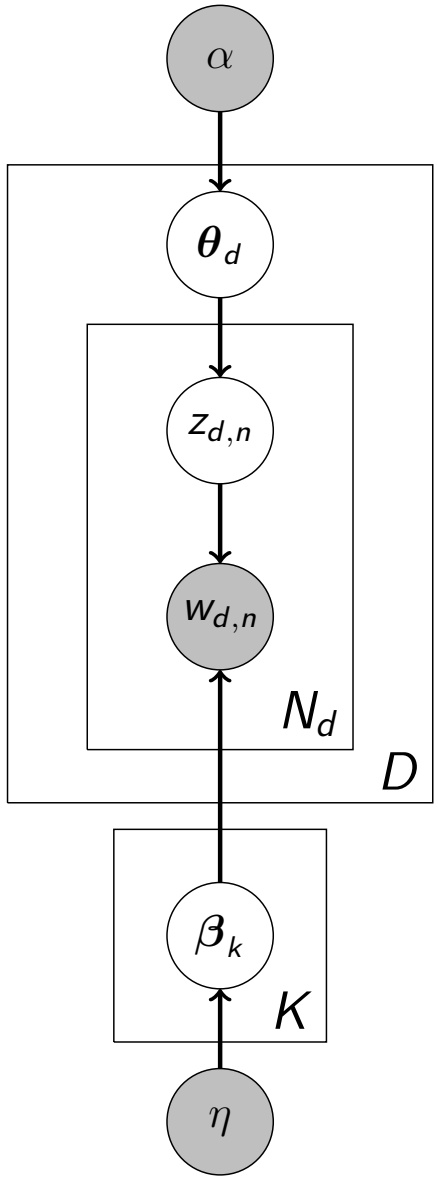
A *Bayesian network* is a probabilistic model whose joint distribution can be represented by a directed acyclic graph (DAG).

The nodes in a DAG can be ordered so that parents precede children.

LDA as a Bayesian Network



LDA Plate Diagram



Graph Properties

Let $\mathbf{B} = (\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K)$ and $\mathbf{T} = (\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_D)$.

Object	Parents	Children
α	$\emptyset$	$\mathbf{T}$
$\boldsymbol{\theta}_d$	α	$\mathbf{z}_d$
$z_{d,n}$	$\boldsymbol{\theta}_d$	$w_{d,n}$
$w_{d,n}$	$z_{d,n}, \mathbf{B}$	$\emptyset$
$\boldsymbol{\beta}_k$	η	$\mathbf{w}_1, \ldots, \mathbf{w}_D$
η	$\emptyset$	$\mathbf{B}$

Conditional Independence Property I

In a Bayesian network, nodes are independent of their ancestors conditional on their parents.

This means we can write $f(\mathbf{x}) = \prod_{i=1}^{N} f(x_i \mid \text{parents}(x_i))$, which can greatly simplify joint distributions.

Applying this formula to LDA yields

$$\left(\prod_d \prod_n \Pr[w_{d,n} \mid z_{d,n}, \mathbf{B}]\right)\left(\prod_d \prod_n \Pr[z_{d,n} \mid \boldsymbol{\theta}_d]\right) \times \left(\prod_d \Pr[\boldsymbol{\theta}_d \mid \alpha]\right)\left(\prod_k \Pr[\boldsymbol{\beta}_k \mid \eta]\right)$$

Conditional Independence Property II

The *Markov blanket* $\mathrm{MB}(x_i)$ of a node x_i in a Bayesian network is the set of nodes consisting of x_i's parents, children, and children's parents.

Conditional on its Markov blanket, the node x_i is independent of all nodes outside its Markov blanket.

So $f(x_i \mid \mathbf{x}_{-i})$ has the same distribution as $f(x_i \mid \mathrm{MB}(x_i))$.

Posterior Distribution

The inference problem in LDA is to compute the posterior distribution over $\mathbf{z}$, $\mathbf{T}$, and $\mathbf{B}$ given the data $\mathbf{w}$ and Dirichlet hyperparameters.

Let's consider the simpler problem of inferring the latent variables taking the parameters as given. Posterior distribution is

$$\Pr[\mathbf{z} = z' \mid \mathbf{w}, \mathbf{T}, \mathbf{B}] = \frac{\Pr[\mathbf{w} \mid \mathbf{z} = z', \mathbf{T}, \mathbf{B}] \Pr[\mathbf{z} = z' \mid \mathbf{T}, \mathbf{B}]}{\sum_{z'} \Pr[\mathbf{w} \mid \mathbf{z} = z', \mathbf{T}, \mathbf{B}] \Pr[\mathbf{z} = z' \mid \mathbf{T}, \mathbf{B}]}.$$

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We can compute the numerator easily, and each element of denominator.

But $z' \in \{1, \ldots, K\}^N \Rightarrow$ there are K^N terms in the sum $\Rightarrow$ intractable problem.

For example, a 100 word corpus with 50 topics has $\approx$ 7.88e169 terms.

Approximate Inference

Instead of obtaining a closed-form solution for the posterior distribution, we must approximate it.

Markov chain Monte Carlo methods provide a stochastic approximation to the true posterior.

The general idea is to define a Markov chain whose stationary distribution is equivalent to the posterior distribution, which we then draw samples from.

There are several MCMC methods, but we will consider Gibbs sampling.

Gibbs Sampling

We want to draw samples from some joint distribution over $\mathbf{x} = (x_1, \ldots, x_N)$ given by $f(\mathbf{x})$ (e.g. a posterior distribution).

Suppose we can compute the conditional distribution $f_i \equiv f(x_i \mid \mathbf{x}_{-i})$.

Then we can use the following algorithm:

1. Randomly allocate an initial value for $\mathbf{x}$, say $\mathbf{x}^0$
2. Let S be the number of iterations to run chain. For each $s \in \{1, \ldots, S\}$, draw x_i^s according to

$$x_i^s \sim f(x_i \mid x_1^s, \ldots, x_{i-1}^s, x_{i+1}^{s-1}, \ldots, x_N^{s-1}).$$

3. Discard initial iterations (burn in), and collect samples from every mth (thinning interval) iteration thereafter.
4. Use collected samples to approximate joint distribution, or related distributions and moments.

Sampling Equations for θ_d

The Markov blanket of $\boldsymbol{\theta}_d$ is:

- The parent α.
- The children $\mathbf{z}_d$.

So we need to draw samples from $\Pr[\boldsymbol{\theta}_d \mid \alpha, \mathbf{z}_d]$. This is the posterior distribution for $\boldsymbol{\theta}_d$ given a fixed value for the vector of allocation variables $\mathbf{z}_d$.

We computed this posterior above. Let $n_{d,k}$ be the number of words in document d that have topic allocation k.

Then $\Pr[\boldsymbol{\theta}_d \mid \alpha, \mathbf{z}_d] = \text{Dir}(\alpha + n_{d,1}, \ldots, \alpha + n_{d,K})$.

Sampling Equations for β_k

The Markov blanket of β_k is:

- The parent η.
- The children $\mathbf{w} = (\mathbf{w}_1, \ldots, \mathbf{w}_D)$.
- The children's parents $\mathbf{z} = (\mathbf{z}_1, \ldots, \mathbf{z}_D)$.

Consider $\Pr[\beta_k \mid \eta, \mathbf{w}, \mathbf{z}]$. Only the allocation variables assigned to k—and their associated words—are informative about β_k.

Let $m_{k,v}$ be the number of times topic k allocation variables generate term v.

Then $\Pr[\beta_k \mid \eta, \mathbf{w}, \mathbf{z}] = \text{Dir}(\eta + m_{k,1}, \ldots, \eta + m_{k,V})$.

Sampling Equations for Allocations

The Markov blanket of $z_{d,n}$ is:

- The parent $\boldsymbol{\theta}_d$.
- The child $w_{d,n}$.
- The child's parents $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K$.

$$\Pr\left[z_{d,n} = k \mid w_{d,n} = v, B, \boldsymbol{\theta}_d \right] =$$

$$\frac{\Pr\left[w_{d,n} = v \mid z_{d,n} = k, B, \boldsymbol{\theta}_d \right] \Pr\left[z_{d,n} = k \mid B, \boldsymbol{\theta}_d \right]}{\sum_k \Pr\left[w_{d,n} = v \mid z_{d,n} = k, B, \boldsymbol{\theta}_d \right] \Pr\left[z_{d,n} = k \mid B, \boldsymbol{\theta}_d \right]} = \frac{\theta_d^k \beta_k^v}{\sum_k \theta_d^k \beta_k^v}.$$

Summary

To complete one iteration of Gibbs sampling, we need to:

1. Sample from a multinomial distribution N times for the topic allocation variables.
2. Sample from a Dirichlet D times for the document-specific mixing probabilities.
3. Sample from a Dirichlet K times for the topic-specific term probabilities.

Sampling from these distributions is standard, and implemented in many programming languages.

Collapsed Sampling

Collapsed sampling refers to analytically integrating out some variables in the joint likelihood and sampling the remainder.

This tends to be more efficient because we reduce the dimensionality of the space we sample from.

Griffiths and Steyvers (2004) proposed a collapsed sampler for LDA that integrates out the $\mathbf{T}$ and $\mathbf{B}$ terms and samples only $\mathbf{z}$.

For details see Heinrich (2009) and technical appendix of Hansen, McMahon, and Prat (2015).

Collapsed Sampling Equation for LDA

The sampling equation for the nth allocation variable in document d is:

$$\Pr\left[z_{d,n} = k \mid \mathbf{z}_{-(d,n)}, \mathbf{w}, \alpha, \eta \right] \propto \frac{m^{-}_{k,v_{d,n}} + \eta}{\sum_{v} m^{-}_{k,v} + \eta V} \left(n^{-}_{d,k} + \alpha \right)$$

where the $-$ superscript denotes counts excluding (d, n) term.

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where the $-$ superscript denotes counts excluding (d, n) term.

Probability term n in document d is assigned to topic k is increasing in:

1. The number of other terms in document d that are currently assigned to k.
2. The number of other occurrences of the term $v_{d,n}$ in the entire corpus that are currently assigned to k.

Both mean that terms that regularly co-occur in documents will be grouped together to form topics.

Property 1 means that terms within a document will tend to be grouped together into few topics rather than spread across many separate topics.

Predictive Distributions

Collapsed sampling gives the distribution of the allocation variables, but we also care about variables we integrated out.

Their predictive distributions are easy to form given topic assignments:

$$\widehat{\beta}_{k,v} = \frac{m_{k,v} + \eta}{\sum_{v=1}^{V} (m_{k,v} + \eta)} \quad \text{and} \quad \widehat{\theta}_{d,k} = \frac{n_{d,k} + \alpha}{\sum_{k=1}^{K} (n_{d,k} + \alpha)}.$$

Example: Seed

	term1		term2		term3		term4		
Doc	T0	T1	T0	T1	T0	T1	T0	T1	$\widehat{\theta}_i$
A	0	0	1	3	0	2	82	80	0.506
B	9	4	1	0	5	4	12	19	0.5
C	35	43	0	0	38	30	0	0	0.5
	0.24	0.254	0.011	0.017	0.235	0.195	0.513	0.535	
	$\widehat{\beta}_0^1$	$\widehat{\beta}_1^1$	$\widehat{\beta}_0^2$	$\widehat{\beta}_1^2$	$\widehat{\beta}_0^3$	$\widehat{\beta}_1^3$	$\widehat{\beta}_0^4$	$\widehat{\beta}_1^4$	

Example: Iteration 2

	term1		term2		term3		term4		
Doc	T0	T1	T0	T1	T0	T1	T0	T1	$\widehat{\theta}_i$
A	0	0	4	0	2	0	35	127	0.753
B	10	3	1	0	5	4	4	27	0.625
C	73	5	0	0	63	5	0	0	0.074
	0.421	0.047	0.026	0.001	0.355	0.053	0.198	0.899	
	$\widehat{\beta}_0^1$	$\widehat{\beta}_1^1$	$\widehat{\beta}_0^2$	$\widehat{\beta}_1^2$	$\widehat{\beta}_0^3$	$\widehat{\beta}_1^3$	$\widehat{\beta}_0^4$	$\widehat{\beta}_1^4$	

Example: Iteration 5

	term1		term2		term3		term4		
Doc	T0	T1	T0	T1	T0	T1	T0	T1	$\widehat{\theta}_i$
A	0	0	0	4	2	0	0	162	0.982
B	13	0	0	1	9	0	0	31	0.589
C	78	0	0	0	68	0	0	0	0.007
	0.535	0.001	0.001	0.026	0.464	0.001	0.001	0.973	
	$\widehat{\beta}_0^1$	$\widehat{\beta}_1^1$	$\widehat{\beta}_0^2$	$\widehat{\beta}_1^2$	$\widehat{\beta}_0^3$	$\widehat{\beta}_1^3$	$\widehat{\beta}_0^4$	$\widehat{\beta}_1^4$	

Example statement: Yellen, March 2006, #51

We have noticed a change in the relationship between the core CPI and the chained core CPI, which suggested to us that maybe something is going on relating to substitution bias at the upper level of the index. You focused on the nonmarket component of the PCE, and I wondered if something unusual might be happening with the core CPI relative to other measures.

Example statement: Yellen, March 2006, #51

noticed change relationship between core CPI chained core CPI suggested maybe something going relating substitution bias upper level index focused nonmarket component PCE wondered something unusual happening core CPI relative measures

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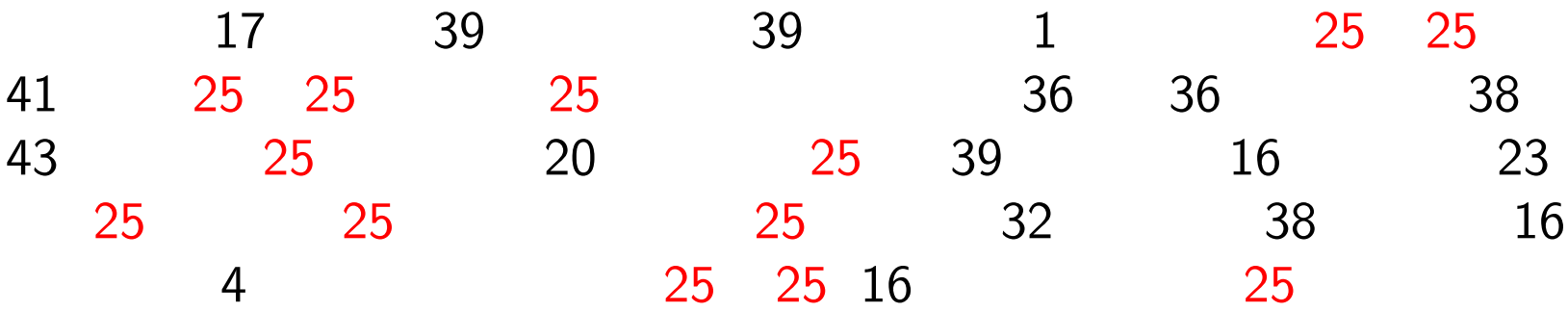
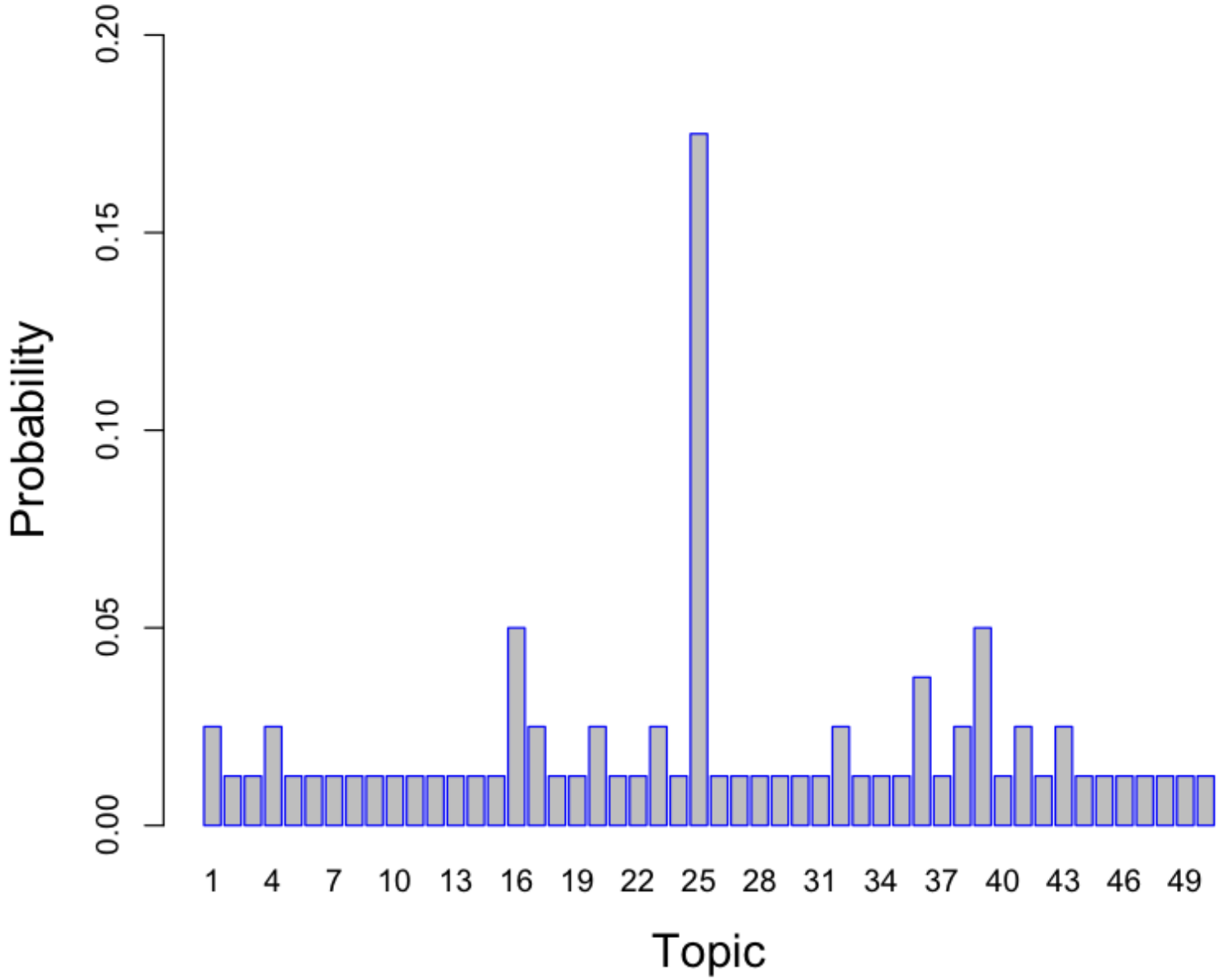
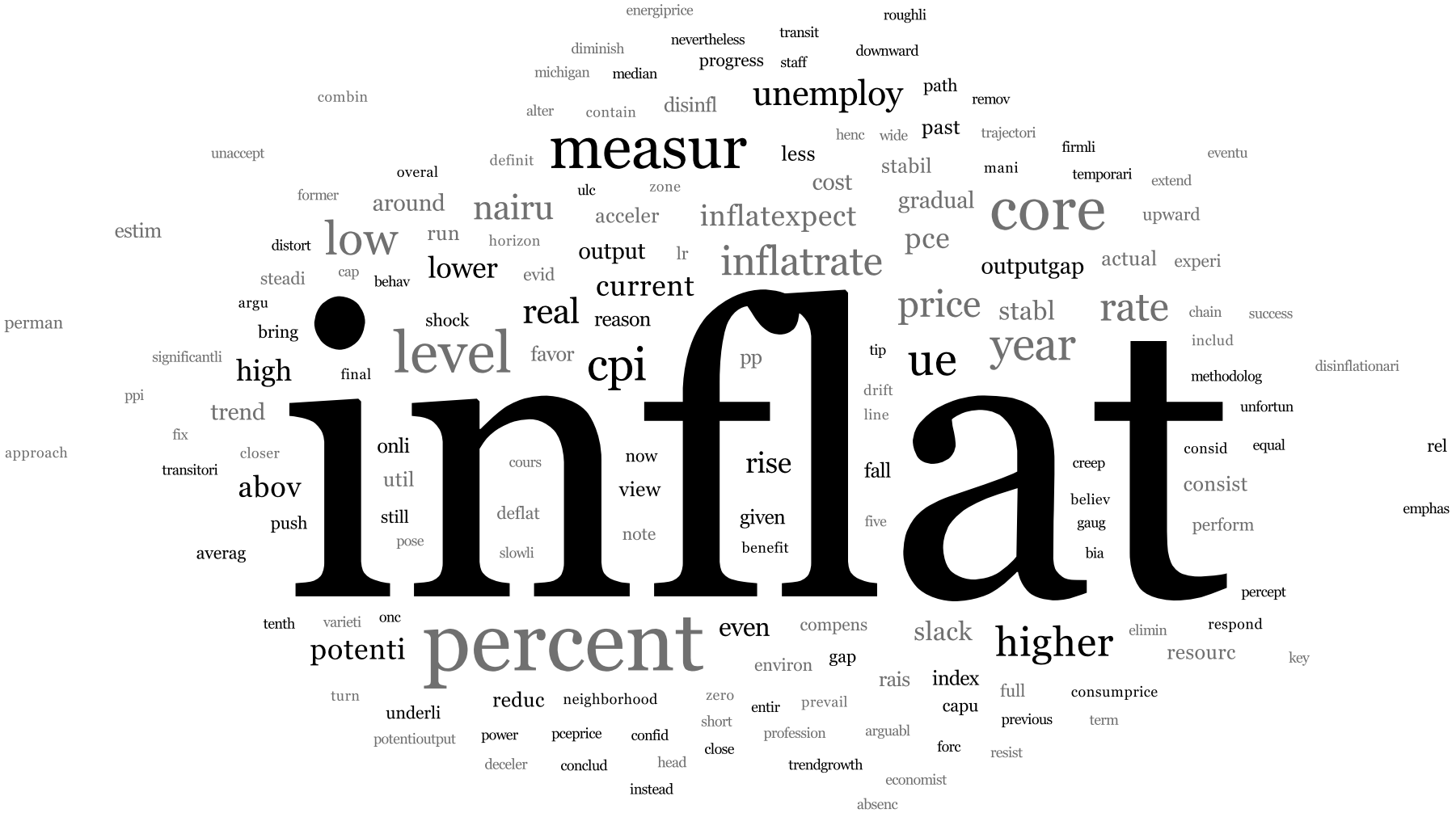


Distribution of Attention



Topic 25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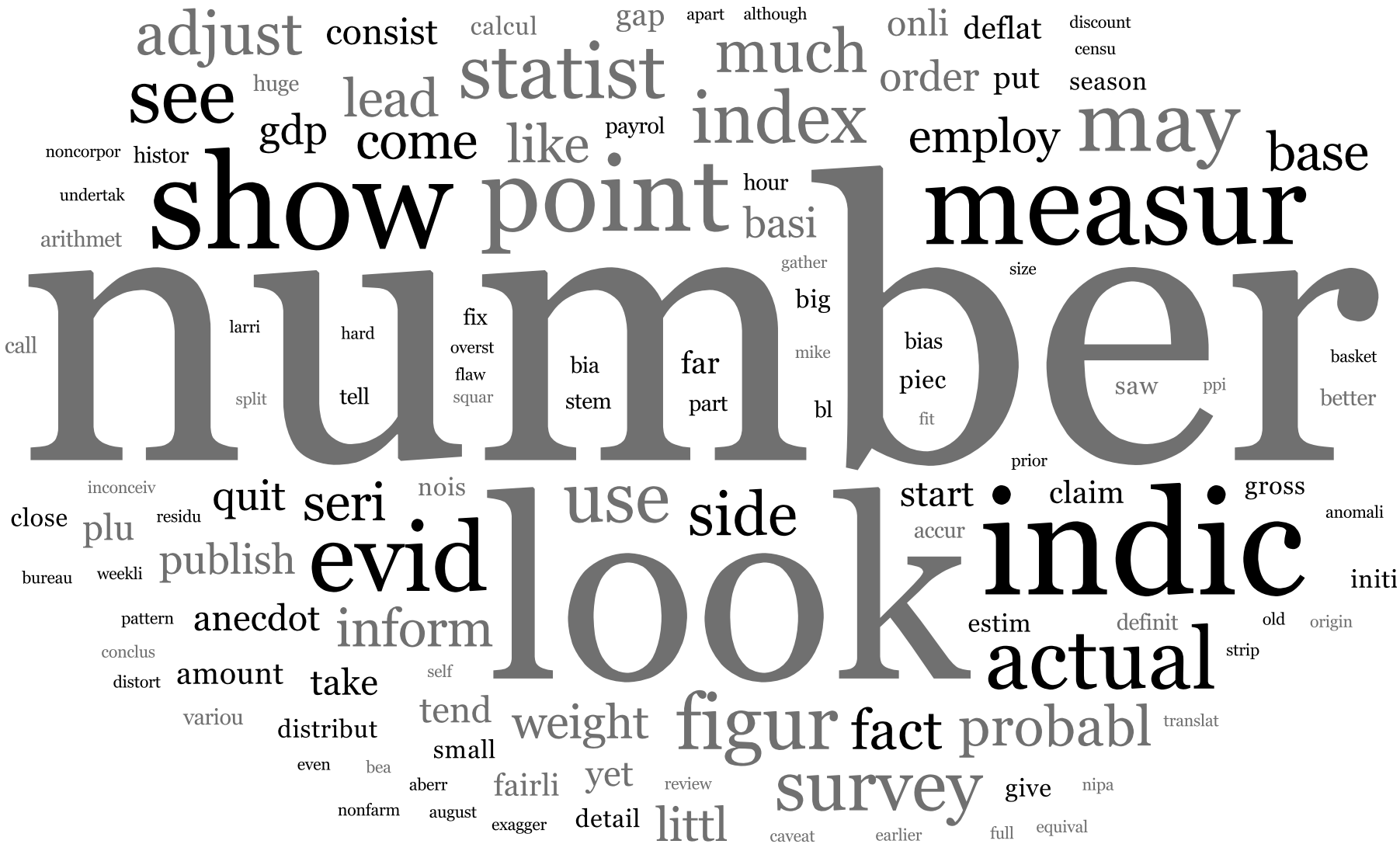
Advantage of Flexibility

'measur' has probability 0.026 in topic 25, and probability 0.021 in topic 11

Topic 11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Advantage of Flexibility

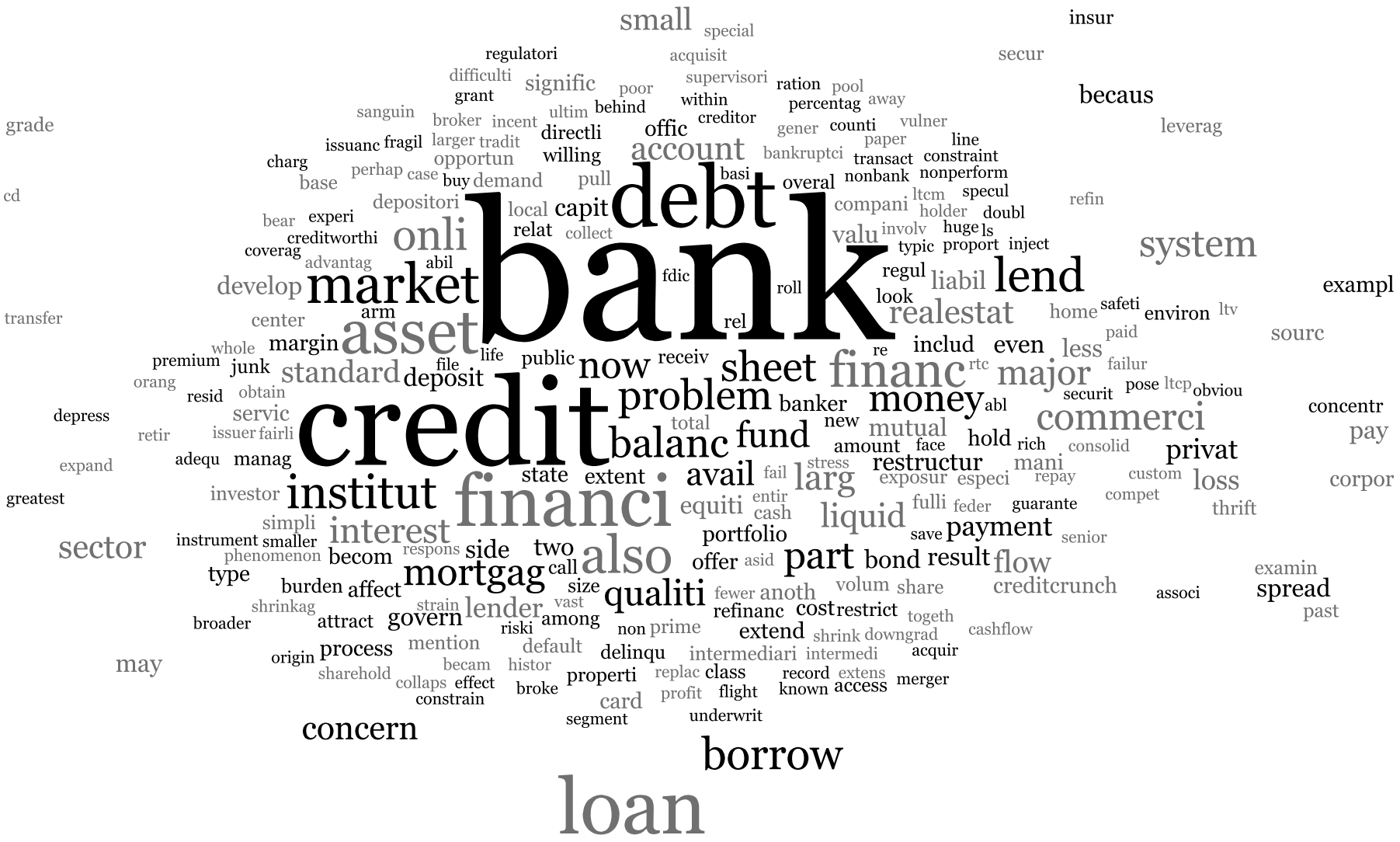
'measur' has probability 0.026 in topic 25, and probability 0.021 in topic 11.

It gets assigned to 25 in this statement consistently due to the presence of other topic 25 words.

In statements containing words on evidence and numbers, it consistently gets assigned to 11.

Sampling algorithm can help place words in their appropriate context.

Topic 38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Topic 23


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Topic 29


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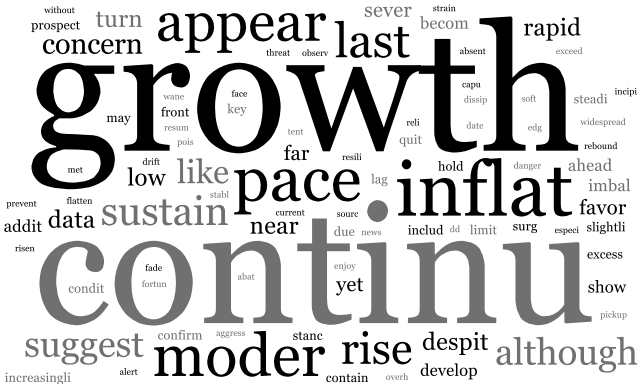
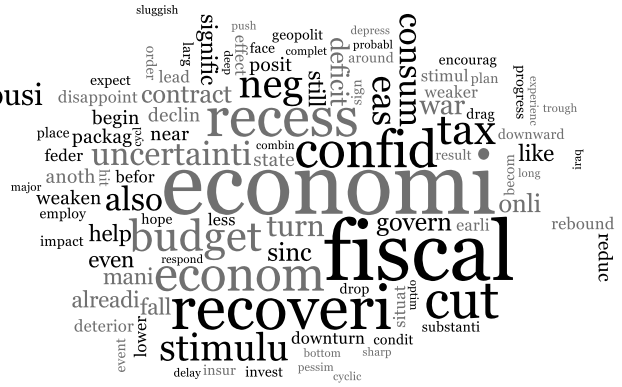
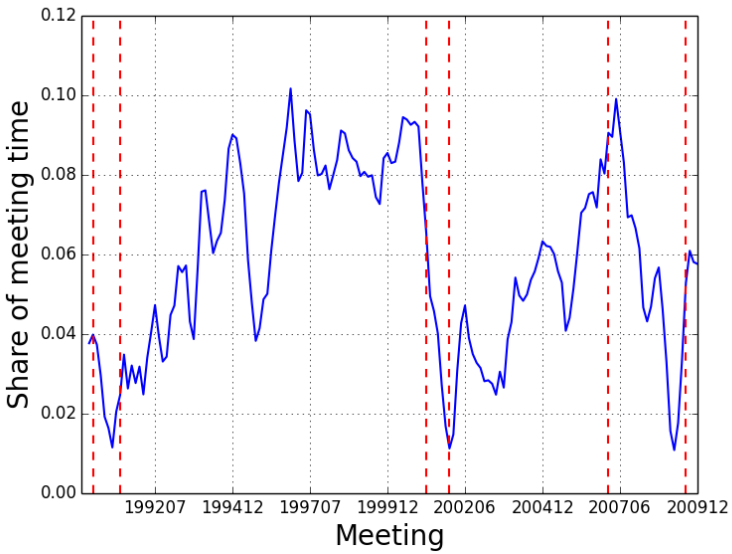
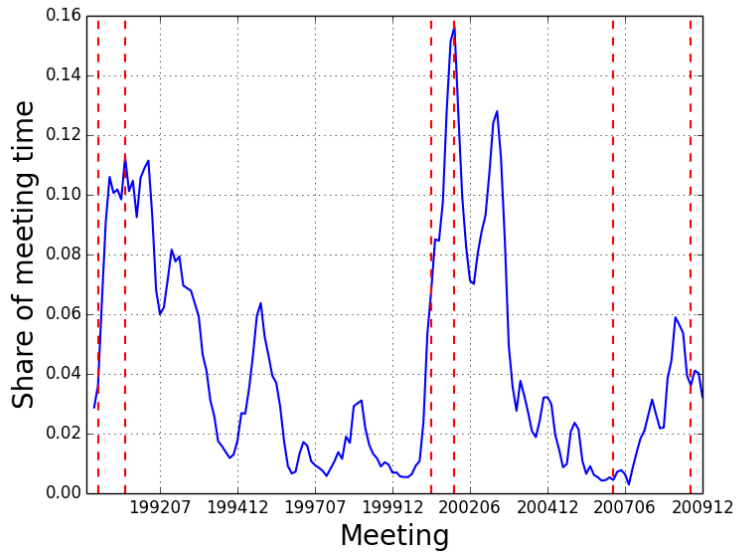
External Validation









LDA on Survey Data

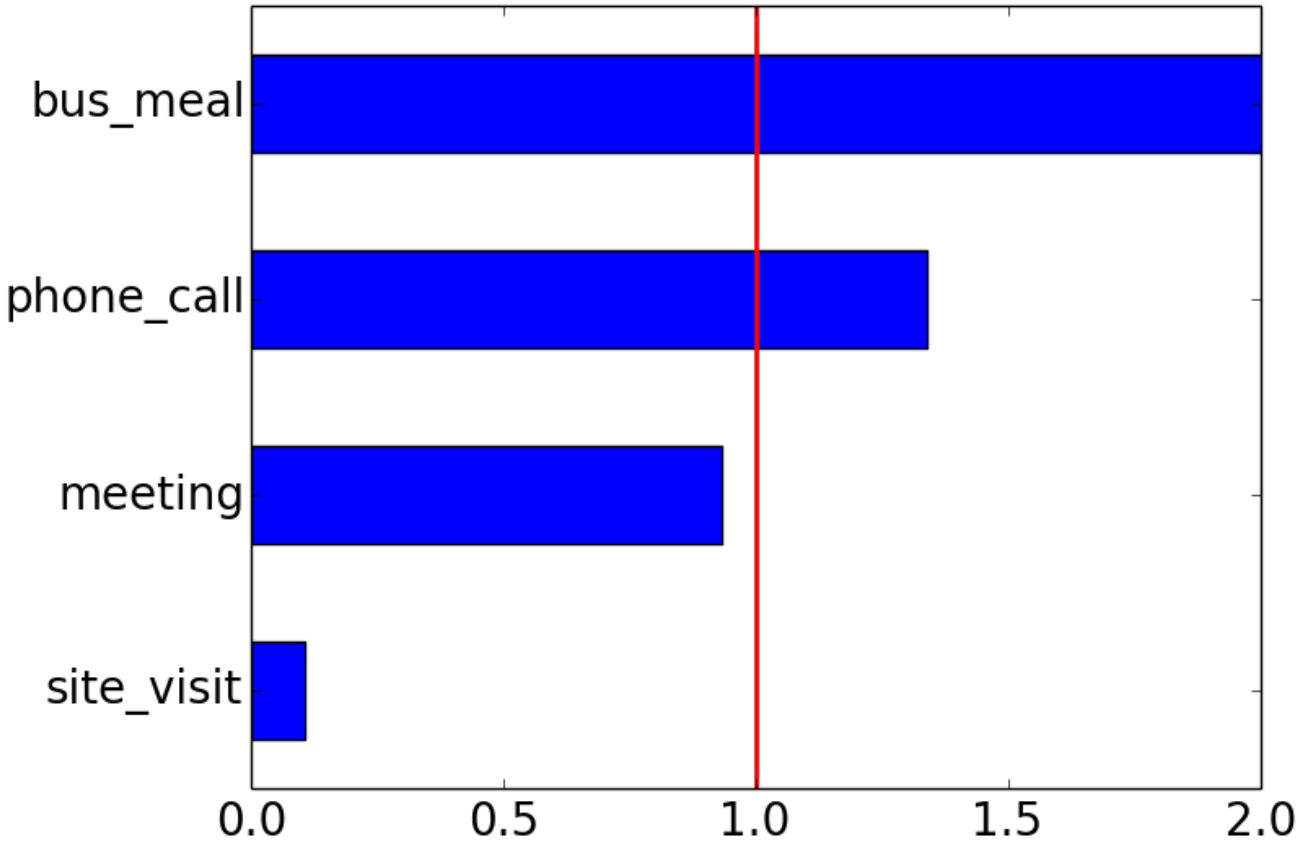
Recall from the first lecture that text data is one instance of count data.

Although typically applied to natural language, LDA is in principle applicable to *any* count data.

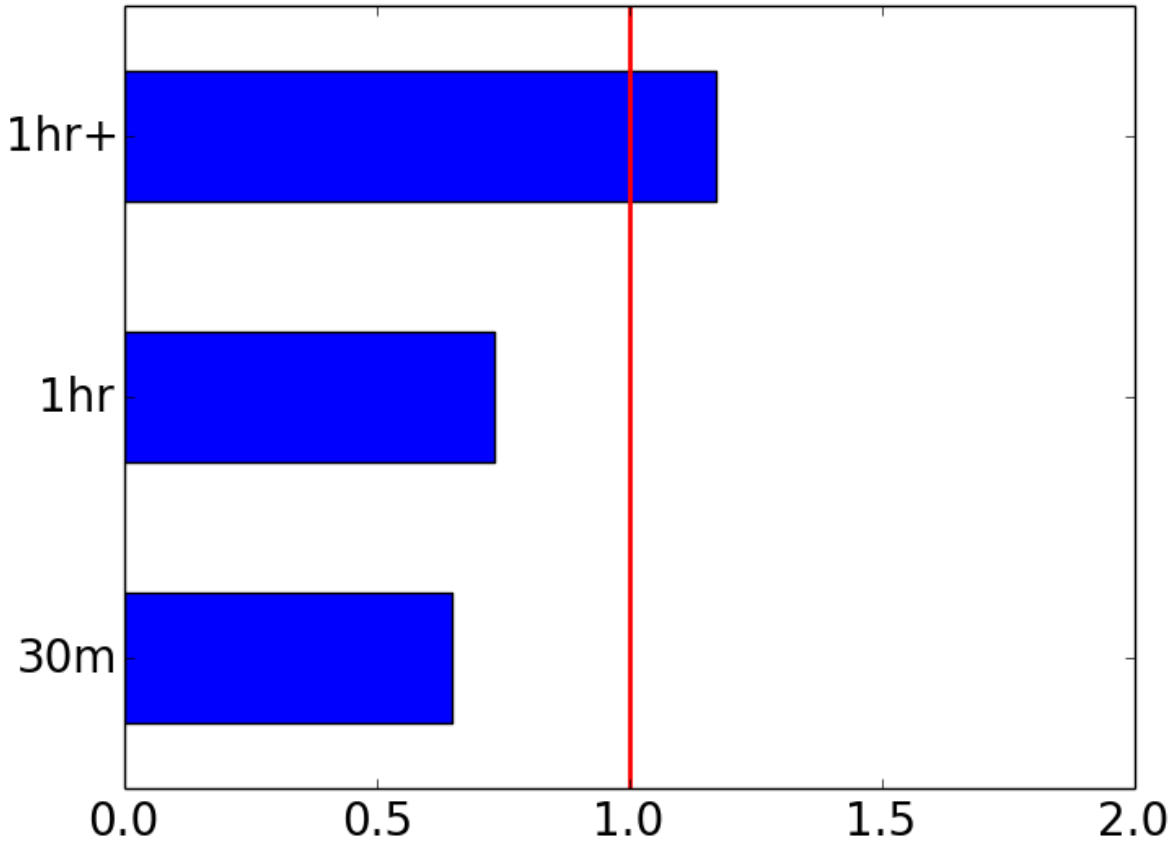
We recently applied it to CEO survey data to estimate management "behaviors" with $K = 2$.

Can visualize results in terms of likelihood ratios of marginals over specific data features.

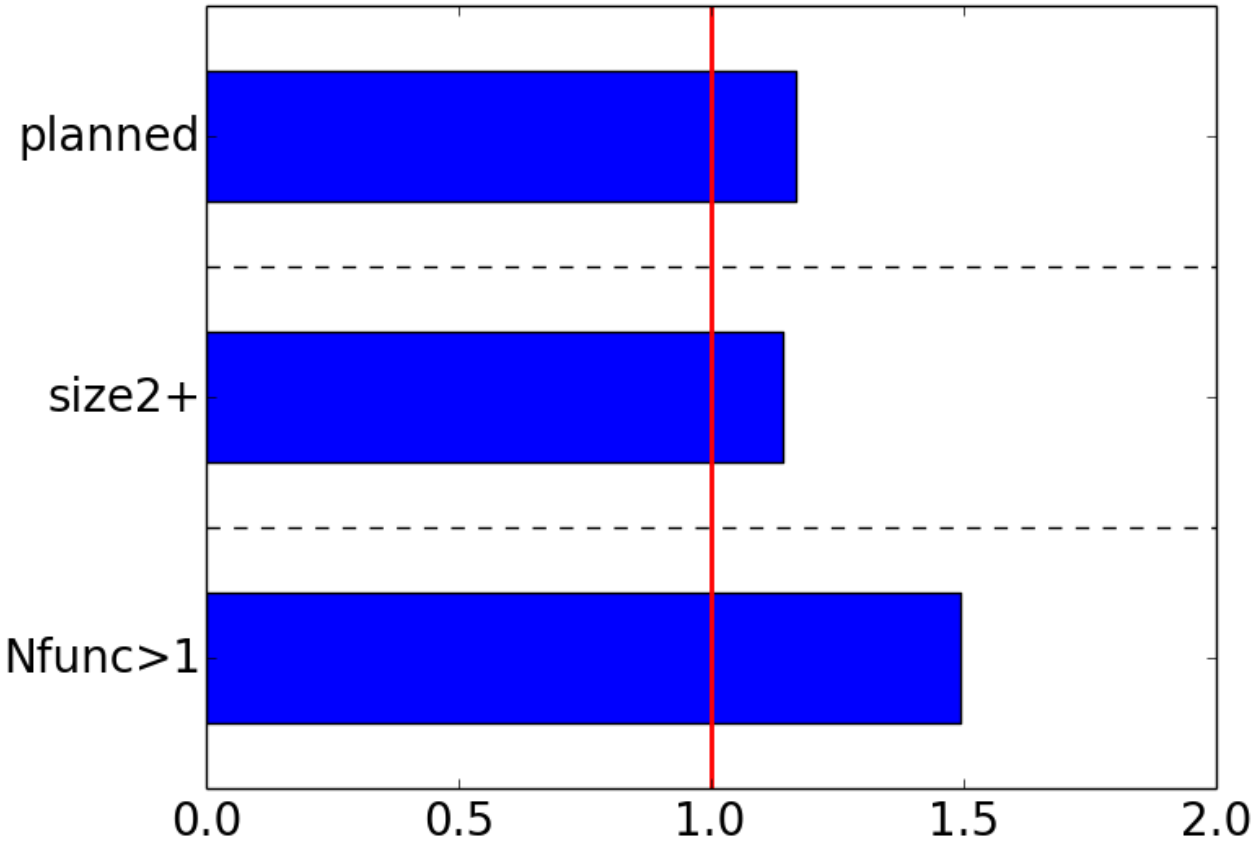
Activity Type



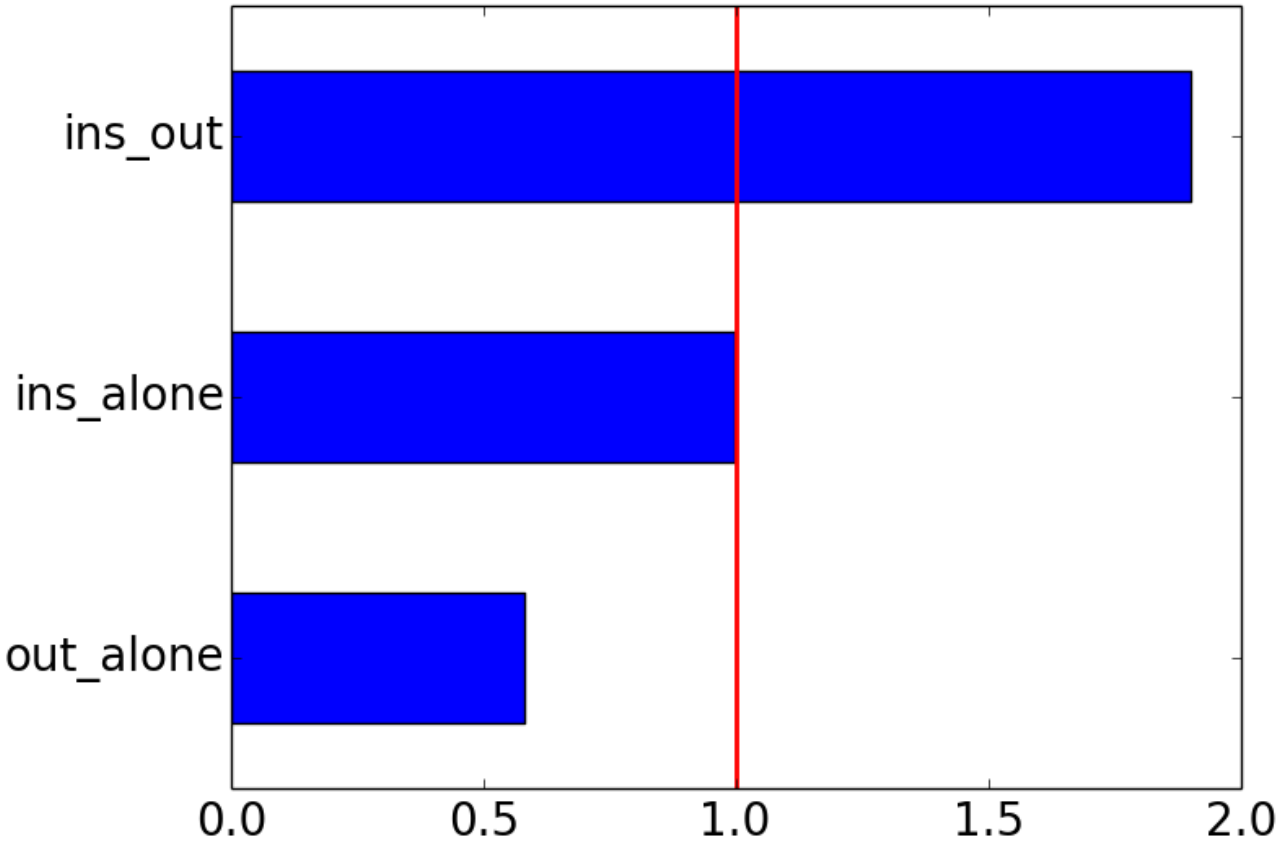
Activity Duration



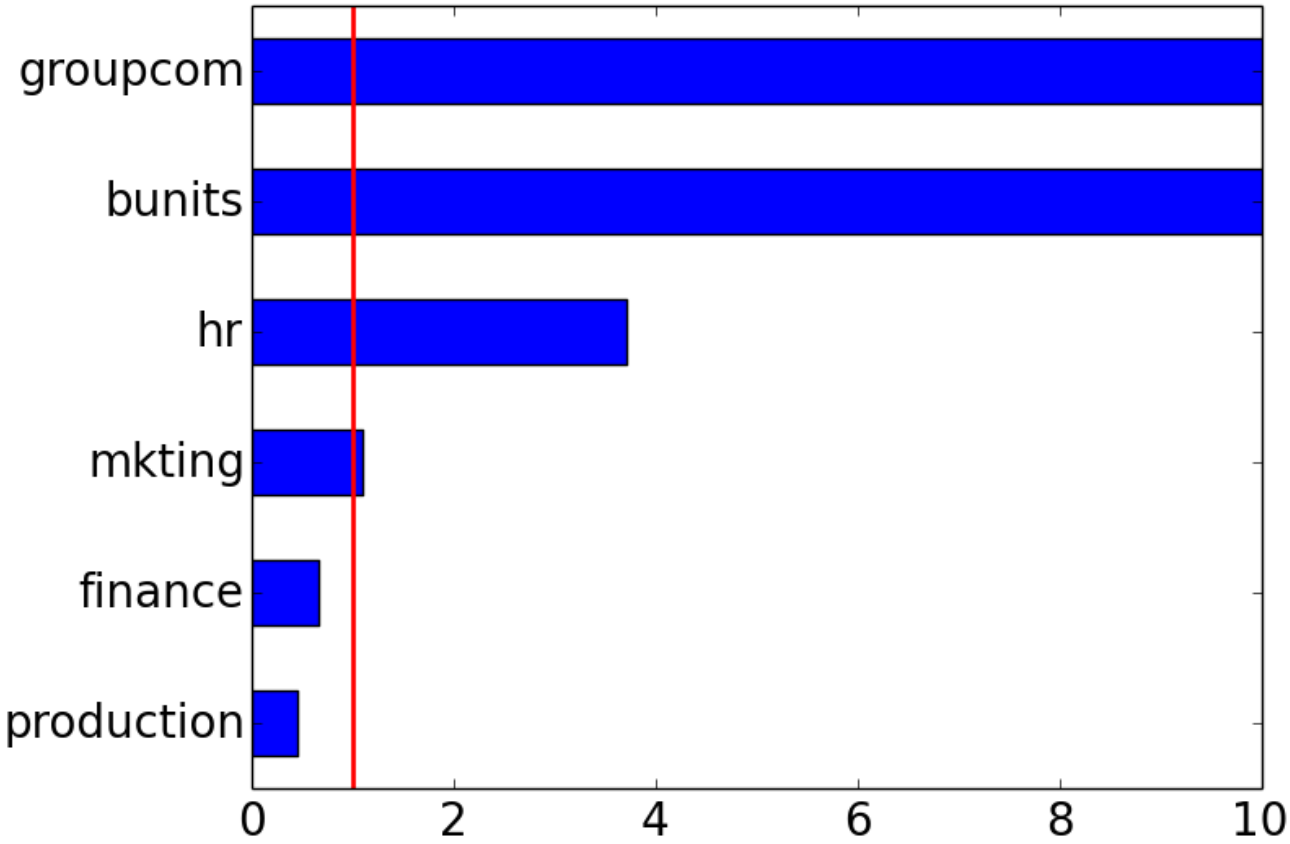
Planning; Size; Number of Functions



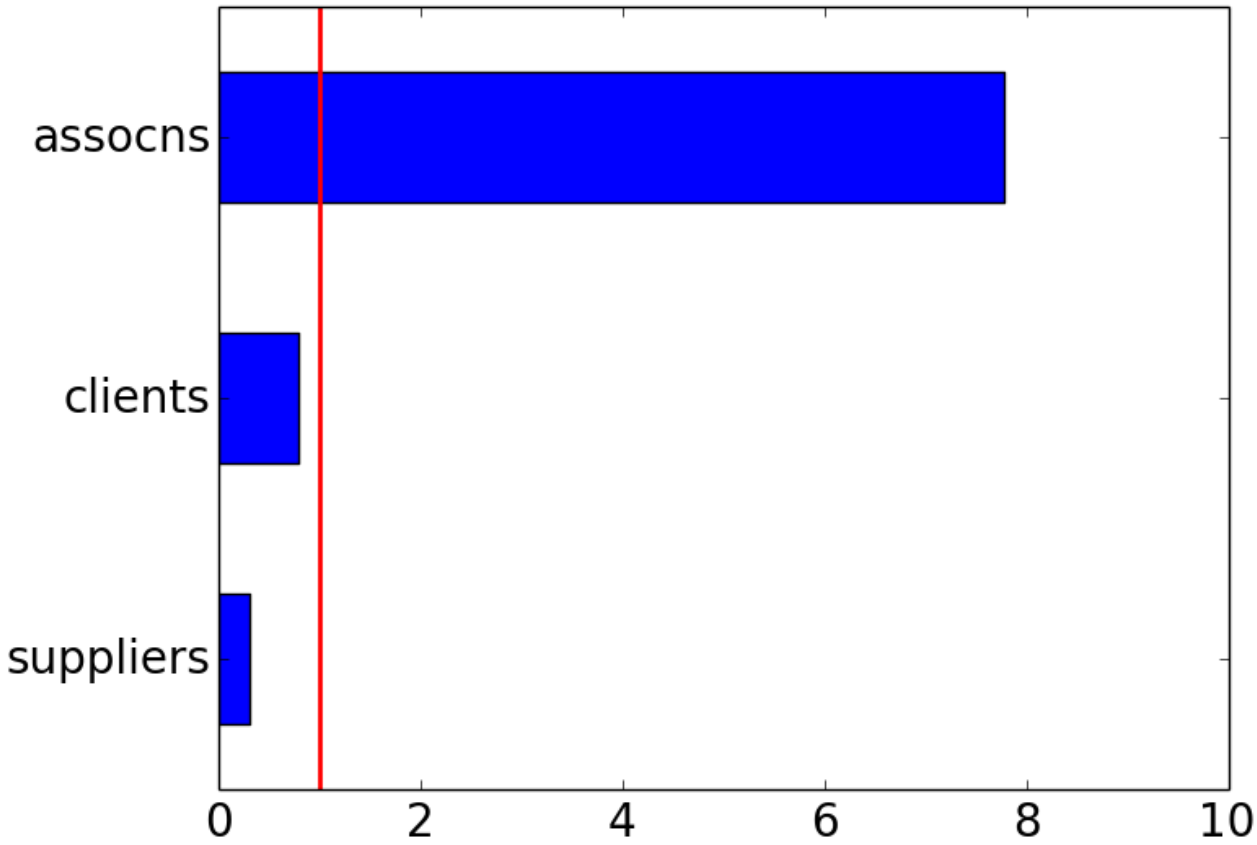
Insiders vs Outsiders



Inside Functions



Outside Functions



Model Selection

There are three parameters to set to run the Gibbs sampling algorithm: number of topics K and hyperparameters α, η.

Priors don't receive too much attention in literature. Griffiths and Steyvers recommend $\eta = 200/V$ and $\alpha = 50/K$. Smaller values will tend to generate more concentrated distributions. (See also Wallach et. al. 2009).

K is less clear. Two potential goals:

1. Predict text well. Statistical criteria to select K.
2. Interpretability. General versus specific.

Formalizing Interpretablility

Chang et. al. (2009) propose an objective way of determining whether topics are indeed interpretable.

Two tests:

1. *Word intrusion*. Form set of words consisting of top five words from topic k + word with low probability in topic k. Ask subjects to identify inserted word.
2. *Topic intrusion*. Show subjects a snippet of a document + top three topics associated to it + randomly drawn other topic. Ask to identify inserted topic.

Estimate LDA and other topic models on NYT and Wikipedia articles for $K = 50, 100, 150$.

Results

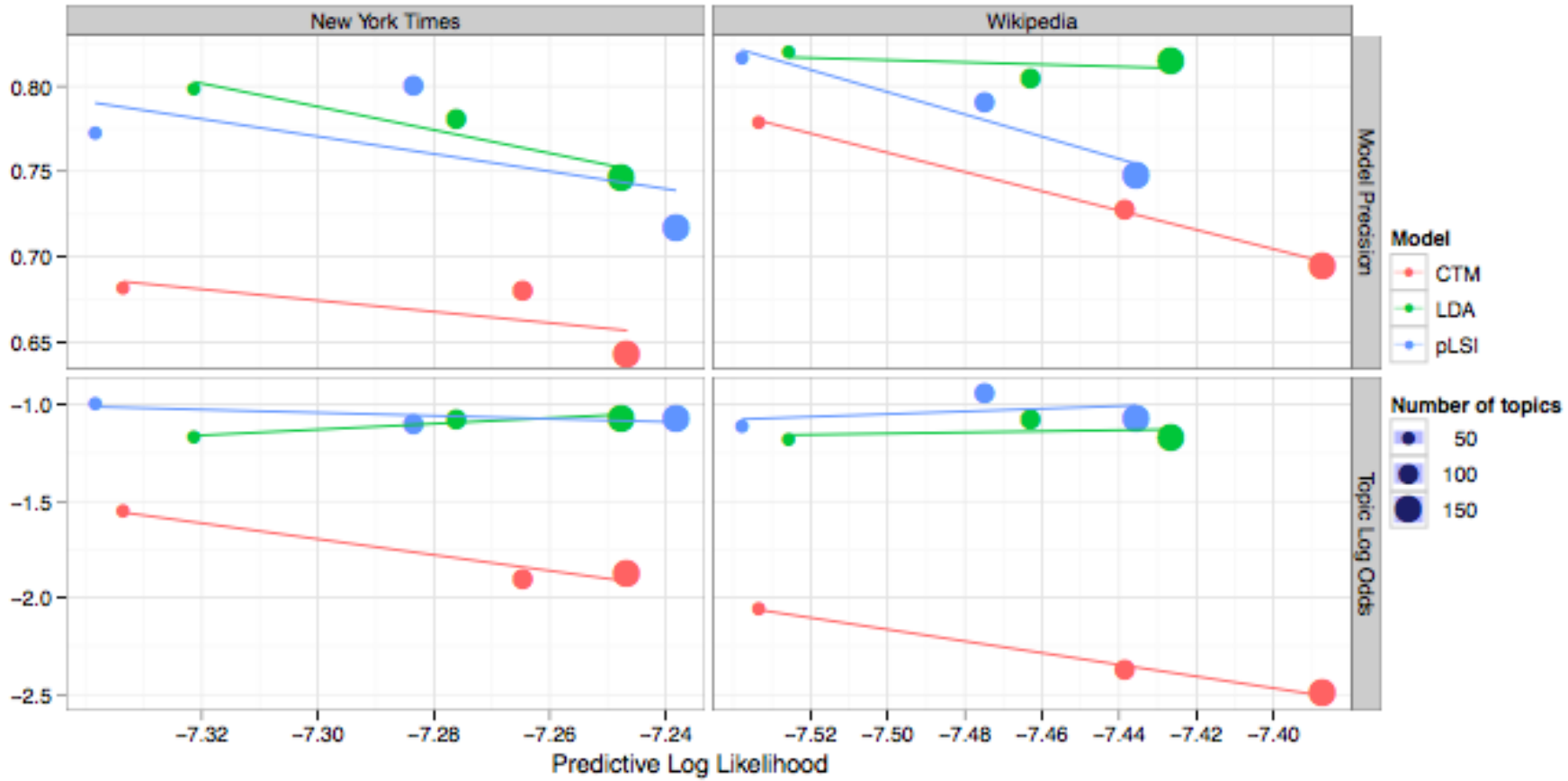


Figure 5: A scatter plot of model precision (top row) and topic log odds (bottom row) vs. predictive log likelihood. Each point is colored by model and sized according to the number of topics used to fit the model. Each model is accompanied by a regression line. Increasing likelihood does not increase the agreement between human subjects and the model for either task (as shown by the downward-sloping regression lines).

Implications

Topics seem objectively interpretable in many contexts.

Tradeoff between goodness-of-fit and interpretablility, which is generally more important in social science.

Potential development of statistical models in future to explicitly maximize interpretablility.

Chain Convergence and Selection

Determining when a chain has converged can be tricky.

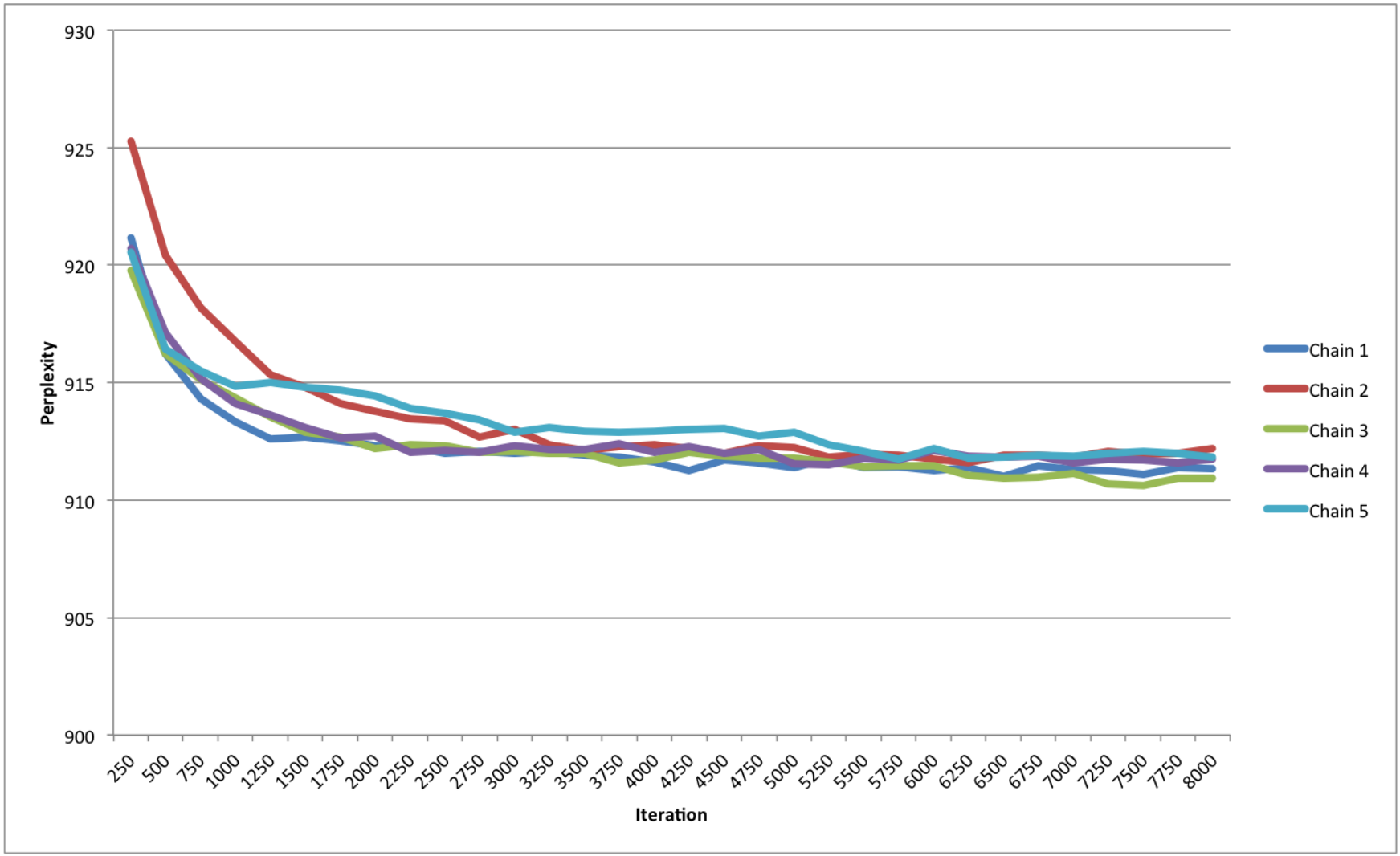
One approach is to measure how well different states of the chain predict the data, and determine convergence in terms of its stability.

Standard practice is run chains from different starting values, after which you can select the best-fit chain for analysis.

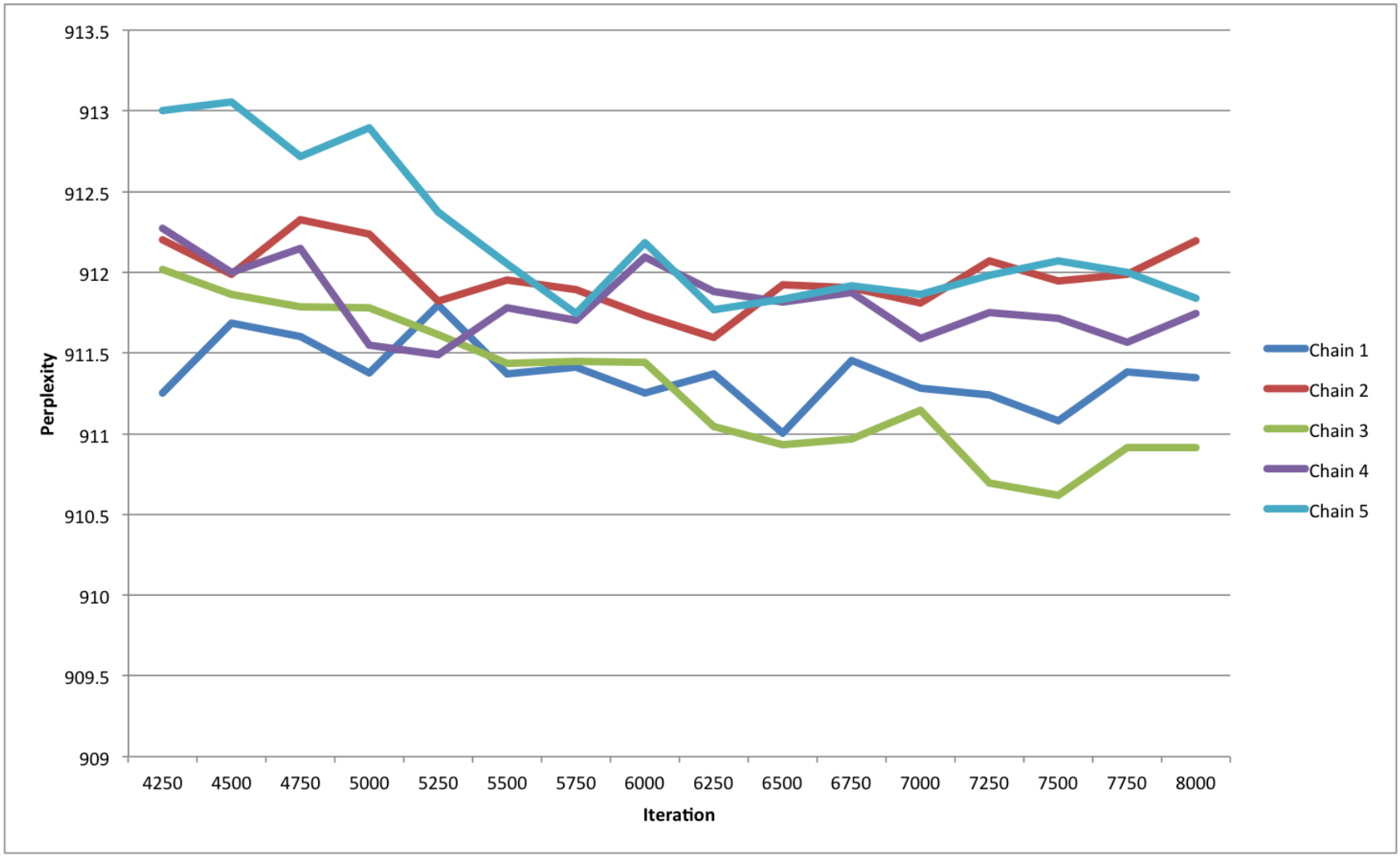
For LDA, a typical goodness-of-fit measure is *perplexity*, given by

$$\exp\left[-\frac{\sum_{d=1}^{D}\sum_{v=1}^{V} x_{d,v}\log\left(\sum_{k=1}^{K}\widehat{\theta}_{d,k}\widehat{\beta}_{k,v}\right)}{\sum_{d=1}^{D} N_d}\right].$$

Perplexity 1



Perplexity 2



Variational Inference

Blei, Ng, and Jordan (2003) use variational inference rather than MCMC to approximate the posterior.

Approximate the true posterior distribution with a simpler functional form that depends on a set of variational parameters.

Then optimize the approximate posterior with respect to the variational parameters so that it lies "close to" the true posterior.

The inference problem becomes an optimization problem.

But note that the family of distributions used to approximate the posterior typically does not include the true posterior.

Out-of-Sample Documents

We are sometimes interested in obtaining the document-topic distribution for out-of-sample documents.

We can perform Gibbs sampling treating estimated topics as fixed

$$\Pr\left[z_{d,n} = k \;\middle|\; \mathbf{z}_{-(d,n)}, \mathbf{w}_d, \alpha, \eta \right] \propto \widehat{\beta}_{k,v_{d,n}} \left[n^{-}_{d,k} + \alpha \right]$$

for each out-of-sample document d.

Only 10-20 iterations necessary since topics already estimated.

Gibbs Sampling / Variational Inference

Advantages of sampling:

1. Typically easier to derive sampling algorithms
2. More accurate, especially for approximating features of posterior distribution beyond the mode

Advantages of variational inference:

1. Faster, especially when optimized
2. Deterministic
3. Convergence easy to assess

Conclusion

Key ideas from this lecture:

1. Latent Dirichlet Allocation. Influential in computer science, many potential applications in economics.
2. Graphical models to simplify and visualize complex joint likelihoods.
3. Gibbs sampling to stochastically approximate posterior.
4. Interpretable output in several domains.

Topic for future: incorporate economic structure into LDA.