D. Inverses

Def: Let $A \in \mathbb{R}^{n\times n}$, Suppose $\exists B \in \mathbb{R}^{n\times n}$ s.t. $AB = I$ then B is called the inverse of A denoted A^{-1}.

Ex: (a) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ then $A^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ does not have an inverse.

Theorem: Let $A \in \mathbb{R}^{n\times n}$

a) If A^{-1} exists it is unique

b) $\det(A) \neq 0$ iff A^{-1} exists

c) If $\det(A) \neq 0$ then $\det(A^{-1}) = (\det(A))^{-1}$

Note: There are numerical implementations to compute A^{-1} using MATLAB.

E. Singular Value Decomposition.

Theorem: Suppose $M \in \mathbb{R}^{m\times n}$ then $\exists U \in \mathbb{R}^{m\times m}$, $\Sigma \in \mathbb{R}^{m\times n}$, and $V \in \mathbb{R}^{n\times n}$ s.t.

(a) $M = U \Sigma V^T$

(b) the columns of U are the set of orthonormal eigenvectors of MM^T s.t. $U^TU = I$

(c) the columns of V are the set of orthonormal eigenvectors of M^TM s.t. $VV^T = I$

(d) Σ is a diagonal matrix whose diagonal elements are the square root of nonzero eigenvalues of M^TM and MM^T

<u>Def</u>: The diagonal entries σ_i of Σ are called the <u>singular values</u> of M.

<u>Remark</u>: 1) Matrices with $U^TU = I$ are called unitary matrices and describe transformations between coordinate systems (note they are invertible)

2) As a result U and V can be viewed as transformations between different coordinate systems.

3) Σ just scales the coordinate axes.

4)