

Goals for Today

- Review Time-Varying LQR
- Motivating Vehicle Dynamics
- Coordinate System and Rigid Body Motion
- Longitudinal Vehicle Motion
- Lateral Vehicle Motion

Review Time-Varying LQR

1. Select a trajectory to be followed
2. Linearize about the trajectory
3. Apply Riccati Equation

Show sample code on ‘lanechange’

Goals for Today

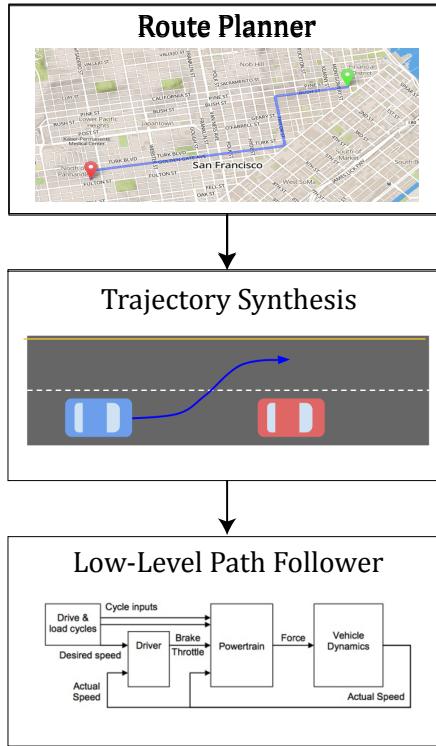
- Review Time-Varying LQR
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Review of Vehicle Dynamics

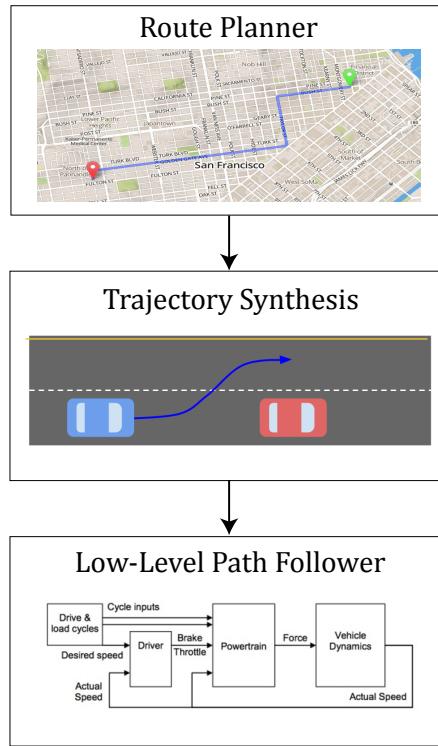
- Complete graduate level treatment is in ME542 (Vehicle Dynamics)
- Our goal:
 - Introduce basic concepts and terminology
 - Develop topics, and models, we will subsequently need for control system design
 - Longitudinal and Lateral Dynamics
 - Use textbook entitled Automotive Control Systems by Ulsoy, Peng, and Cakmakci



AV Control Currently



Strategies to Ignore Dynamics

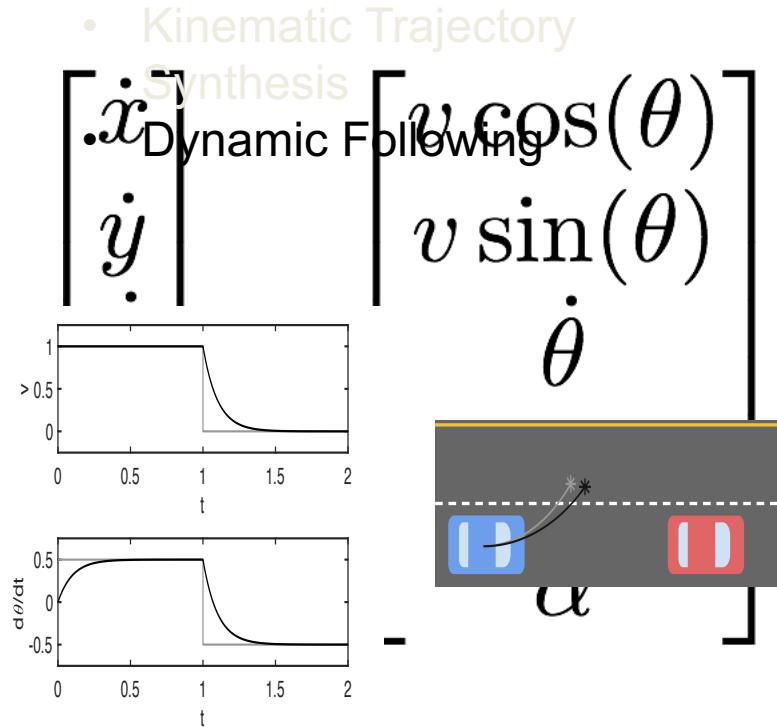
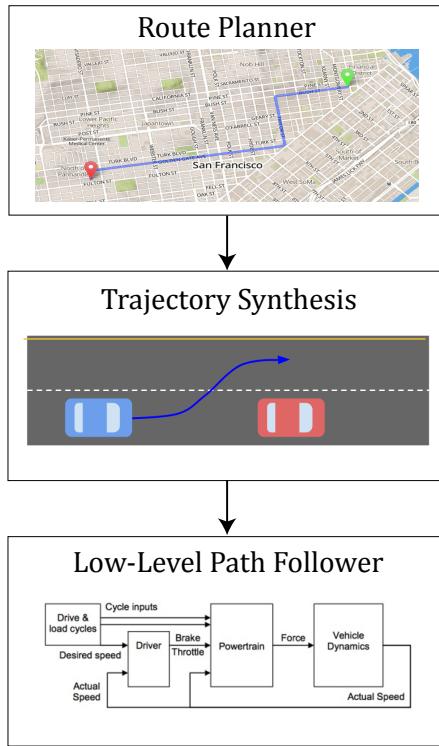


$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cos(\theta) \\ v \sin(\theta) \\ \dot{\theta} \end{bmatrix}$$

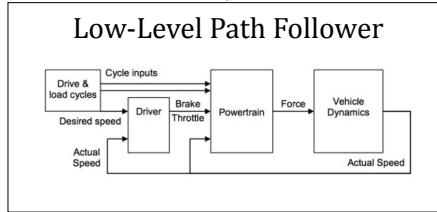
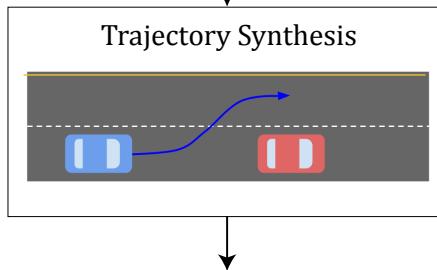
- Sample from control space to generate paths
- Sample from (x,y) space and then use pure pursuit



Real-World Has Dynamics



Don't Drive Aggressively or in Adverse Weather



Uber's autonomous cars drove 20,354 miles and had to be taken over at every mile, according to documents

A first look at Uber's progress.
By JHANA BHUVAN | @MBDOVYAH | MAR 16, 2012 6:44PM EST

A Google self-driving car caused a crash for the first time

A bad assumption led to a minor fender-bender
by Chris Ziegler | Feb 29, 2016, 1:50pm EST

Google self-driving car gets pulled over for driving too slowly

Automated vehicle caused traffic jam while travelling at just 24mph in a 35mph zone, causing police to pull the car over

Google's self-driving car in broadside collision after other car jumps red light

Autonomous Lexus SUV could not prevent accident that caved in front and rear passenger-side doors, setting off airbags and forcing it to be towed away



Is That Enough?

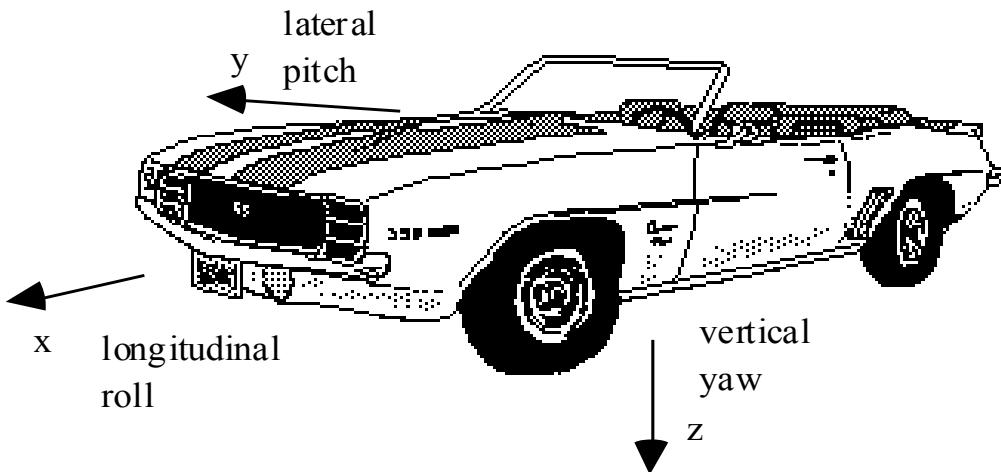
1. Is it feasible for AV to drive much slower than traffic on suburban/urban roads?
2. Is it feasible for AV to not operate in adverse weather conditions?

Outline

- Review Time-Varying LQR
- Motivating Vehicle Dynamics
- Coordinate System and Rigid Body Motion
 1. Vehicle coordinate system
 2. Rigid Body Dynamics
- Longitudinal Vehicle Motion
- Lateral Vehicle Motion

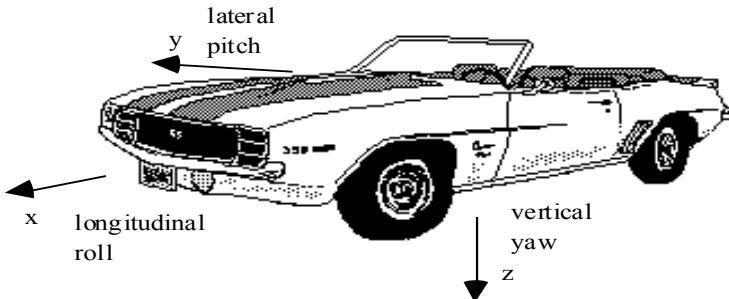
Vehicle Coordinate System

- The Society of Automotive Engineers (SAE) introduced standard coordinates and notations for describing vehicle dynamics
- Vehicle-fixed coordinate system:



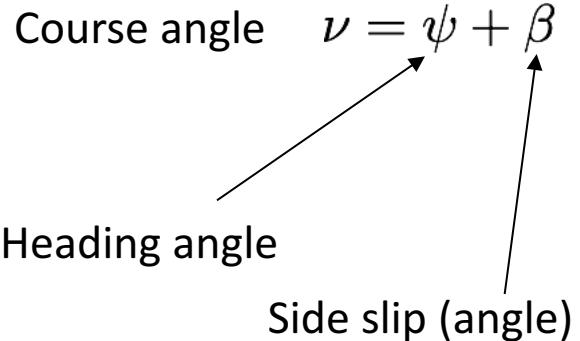
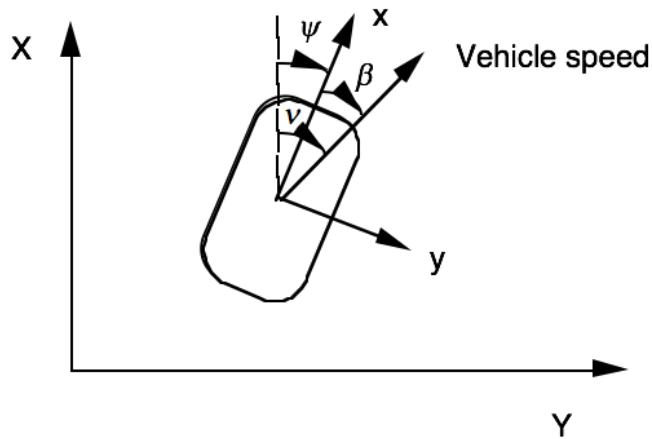
SAE Vehicle - Coordinate System

Axis	Translational Velocity	Angular Displacement	Angular Velocity	Force	Moment
x	u (forward)	ϕ	p or $\dot{\phi}$ (roll)	F_x	M_x
y	v (lateral)	θ	q or $\dot{\theta}$ (pitch)	F_y	M_y
z	w (vertical)	ψ	r or $\dot{\psi}$ (yaw)	F_z	M_z



Earth Fixed Coordinate + Side Slip

OXYZ fixed on Earth (does not turn with vehicle)



Tire Slip

- The tire coordinate system is not the same as XYZ or xyz.
- It is sometimes denoted by X', Y' and Z' to distinguish it.

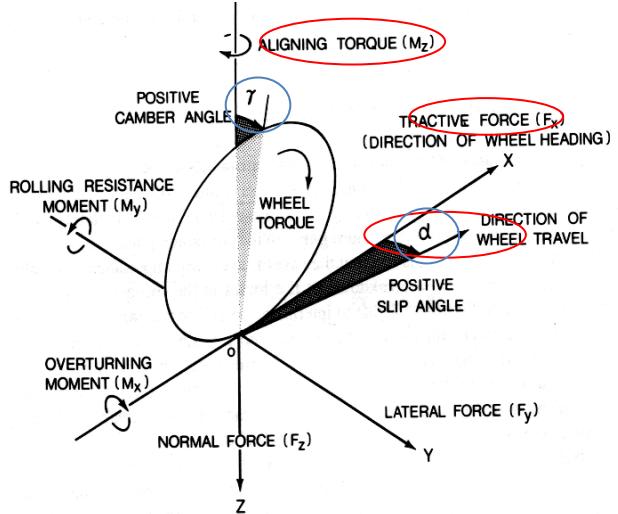


Fig. 1.2 Tire axis system.

Source: Wong page 7
SAE J760

o: center of tire contact patch

Z: perpendicular to the ground plane.

X and Y axes are on the ground plane.

Tire camber angle: angle between XZ and the wheel planes.

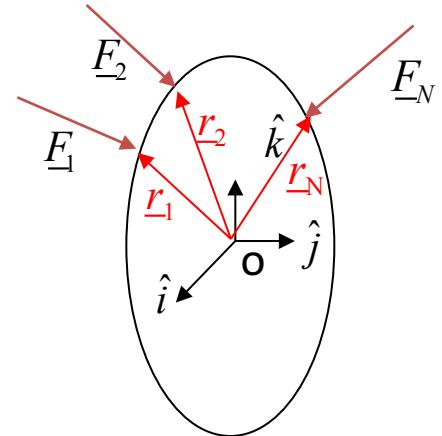
Newton/Euler Equations

Translation $\sum_i^N \underline{F}_i = \underline{F} = m \frac{dV}{dt}$

Rotation $\sum_i^N \underline{r}_i \times \underline{F}_i = \underline{M}_o = \frac{dH}{dt}$

Where H is the *angular momentum* of the rigid body:

$$\underline{H} = \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{yx} & I_{zx} \\ I_{xy} & I_{yy} & I_{zy} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$



$$I_{xx} = \int_V (y^2 + z^2) \rho \cdot dV$$

$$I_{xy} = \int_V (xy) \rho \cdot dV$$

Vehicle Rigid Body Motion

Translational motion: $\sum F = ma$

$$\begin{aligned}\sum F &= ma = m \frac{d}{dt} (u\hat{i} + v\hat{j} + w\hat{k}) \\&= m \cdot (\dot{u}\hat{i} + \dot{v}\hat{j} + \dot{w}\hat{k} + u\dot{\hat{i}} + v\dot{\hat{j}} + w\dot{\hat{k}}) \\&= m \left(\frac{d}{dt} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} \right)\end{aligned}$$

$$\sum F_x = m(\dot{u} + qw - rv)$$

$$\sum F_y = m(\dot{v} + ru - pw)$$

$$\sum F_z = m(\dot{w} + pv - qu)$$

Vehicle Rigid Body Motion

Rotational motion: $M = \dot{H}$

$$\sum M = \frac{d}{dt} \begin{bmatrix} I_{xx}p - I_{xy}q - I_{xz}r \\ -I_{xy}p + I_{yy}q - I_{yz}r \\ -I_{xz}p - I_{yz}q + I_{zz}r \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} I_{xx}p - I_{xy}q - I_{xz}r \\ I_{xy}p + I_{yy}q - I_{yz}r \\ -I_{xz}p - I_{yz}q + I_{zz}r \end{bmatrix}$$

$$\sum M_x = I_{xx}\dot{p} - I_{xy}\dot{q} - I_{xz}\dot{r} - I_{xz}pq - I_{yz}q^2 + I_{zz}rq + I_{xy}pr - I_{yy}qr + I_{yz}r^2$$

$$\sum M_y = -I_{xy}\dot{p} + I_{yy}\dot{q} - I_{yz}\dot{r} + I_{xx}pr - I_{xy}qr - I_{xz}r^2 + I_{xz}p^2 + I_{yz}qp - I_{zz}rp$$

$$\sum M_z = -I_{xz}\dot{p} - I_{yz}\dot{q} + I_{zz}\dot{r} - I_{xy}p^2 + I_{yy}qp - I_{yz}rp - I_{xx}pq + I_{xy}q^2 + I_{xz}rq$$

Simplified Vehicle Rigid Body Motion

- Assume
 - vehicle is symmetric in the xz plane ($I_{xy} = I_{yz} = 0$)
 - p, q, r, v , and w are small, i.e., their products are negligible.
 - $u = u_o + u'$, and u' is small compared with u_o .

$$\sum F_x = m\dot{u}$$

$$\sum F_y = m(\dot{v} + ru_0)$$

$$\sum F_z = m(\dot{w} - qu_0)$$

$$\sum M_x = I_{xx}\dot{p} - I_{xz}\dot{r}$$

$$\sum M_y = I_{yy}\dot{q}$$

$$\sum M_z = -I_{xz}\dot{p} + I_{zz}\dot{r}$$

Linear!

Naturally groups into 3 sets:

1. Longitudinal

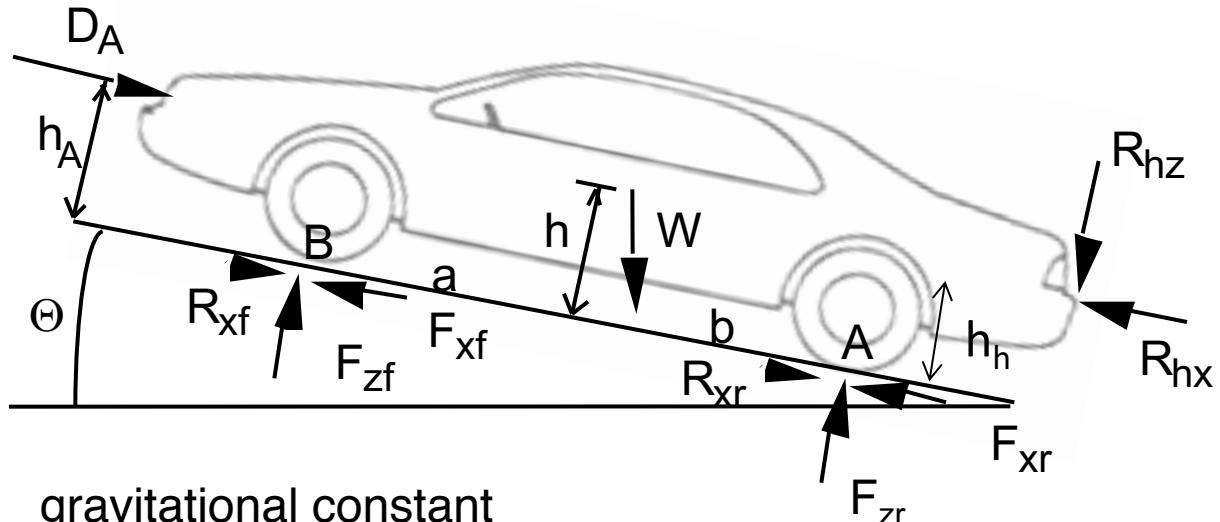
2. Lateral/yaw/roll

3. Vertical/pitch

Outline

- Review Time-Varying LQR
- Motivating Vehicle Dynamics
- Coordinate System and Rigid Body Motion
- Longitudinal Vehicle Motion
 1. Force Balance
 2. Longitudinal Slip and Tire Friction
- Lateral Vehicle Motion
- Vertical Vehicle Motion

Longitudinal Vehicle Motion



g : gravitational constant

D_A : aerodynamic drag force

R_h : drawbar force

W : weight of the vehicle ($= mg$)

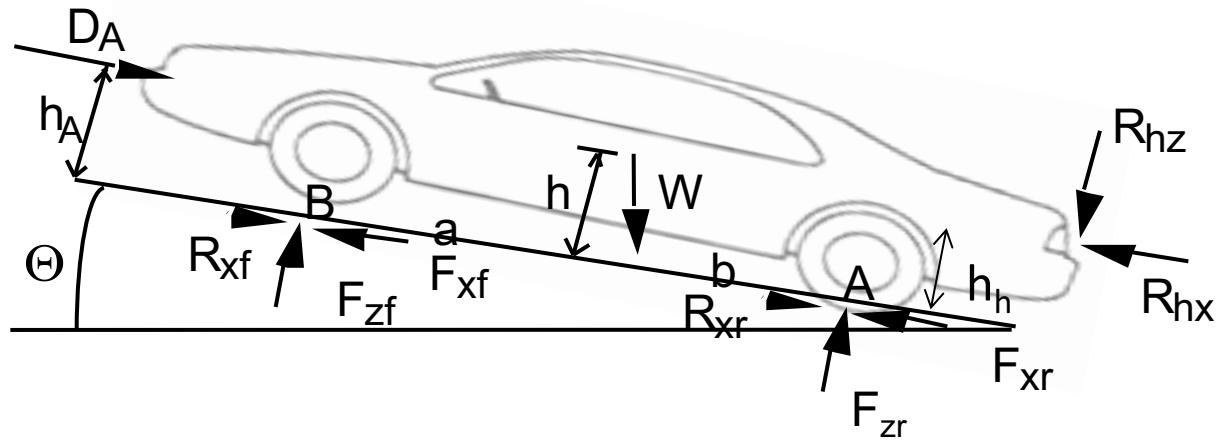
F_x : tractive force

F_z : tire normal force

R_x : rolling resistance force

ma_x : inertial force

Input Classification



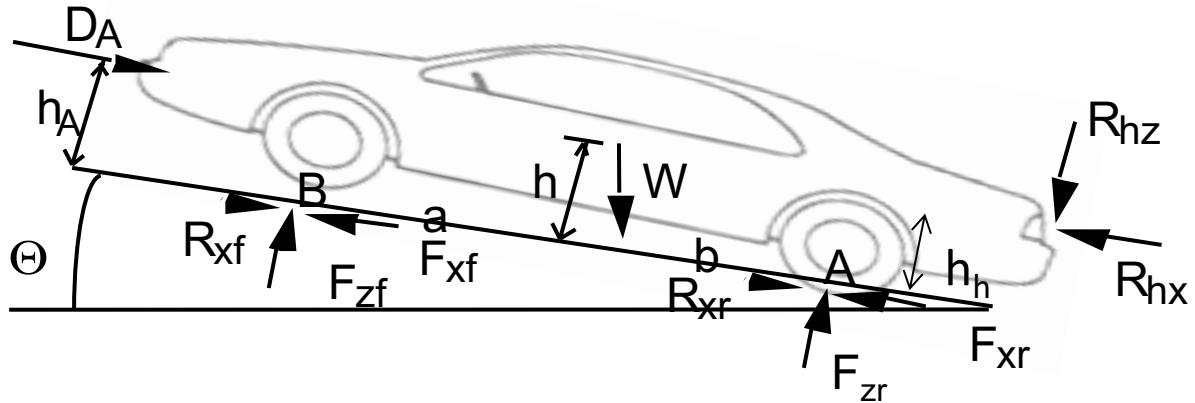
Controlled Inputs

Tractive forces (2)

Uncontrolled Inputs

Aerodynamic drag(1)
Rolling resistance(2)
Hitch forces(2)
Normal forces(2)
Gravity forces(1)

Force Balance (constraints)



Force balance in the vertical (z) direction:

$$0 = W \cos \Theta - F_{zf} - F_{zr} + R_{hz}$$

Force balance in the longitudinal (x) direction:

$$ma_x = \frac{W}{g}a_x = F_{xr} + F_{xf} - W \sin(\Theta) - R_{xr} - R_{xf} - D_A + R_{hx}$$

Moment balance (about point A or point B)

Example 4.1

Aerodynamic Drag and Rolling Resistance

- Assumptions: no gradient, no hitch force, neutral gear (i.e., no tractive forces)
- Perform two coast-down tests:

	High speed test	Low speed test
Initial speed	V_{i1}	V_{i2}
Final speed	V_{f1}	V_{f2}
Time duration	t_1	t_2
Average speed	$V_1 = \frac{V_{i1} + V_{f1}}{2}$	$V_2 = \frac{V_{i2} + V_{f2}}{2}$
Average deceleration	$a_1 = \frac{V_{i1} - V_{f1}}{t_1}$	$a_2 = \frac{V_{i2} - V_{f2}}{t_2}$

Ex. 4.1 (cont.)

$$ma_x = \frac{W}{g}a_x = F_{xr} + F_{xf} - W \sin(\Theta) - R_{xr} - R_{xf} - D_A + R_{hx}$$

No traction Flat road No hitch force

$$D_{A1} + R_x = 0.5\rho C_d A V_1^2 + fmg = ma_1$$
$$D_{A2} + R_x = 0.5\rho C_d A V_2^2 + fmg = ma_2$$

$C_d = \frac{m(a_1 - a_2)}{0.5\rho A(V_1^2 - V_2^2)}$

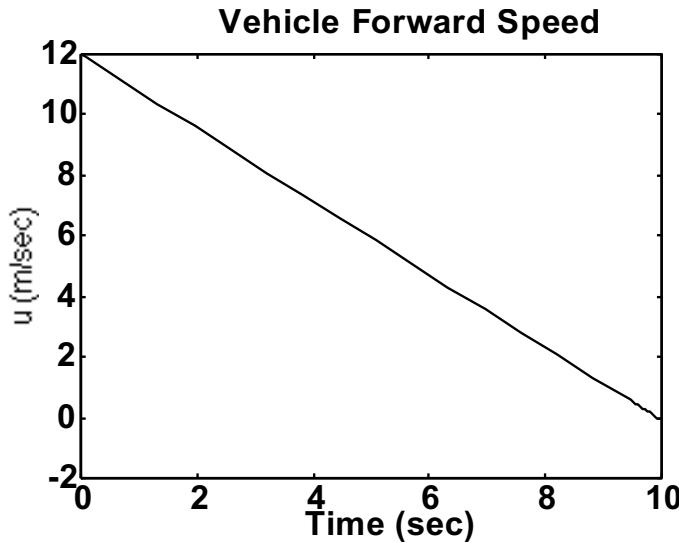
$f = \frac{a_1 V_2^2 - a_2 V_1^2}{g(V_2^2 - V_1^2)}$

Example 4.3

Vehicle Longitudinal Dynamics Simulation (No Tire Dynamics)

$$ma_x = \frac{W}{g}a_x = F_{xr} + F_{xf} - W \sin(\Theta) - R_{xr} - R_{xf} - D_A + R_{hx}$$

```
% Ex4_3.m
% Init. time, final time
% values of the variables
ti=0.0; tf=10.0; ui =
% Tol and trace are not used by the integration
tol = 1.0E-4; trace
% Perform integration
% the results in x
[t,u] = ode23('Ex4_3', [ti, tf], ui);
% Plot the results
plot(t,u,'r')
title('Vehicle Forward Speed')
xlabel('Time (sec)')
ylabel('u (m/sec)'); grid;
```

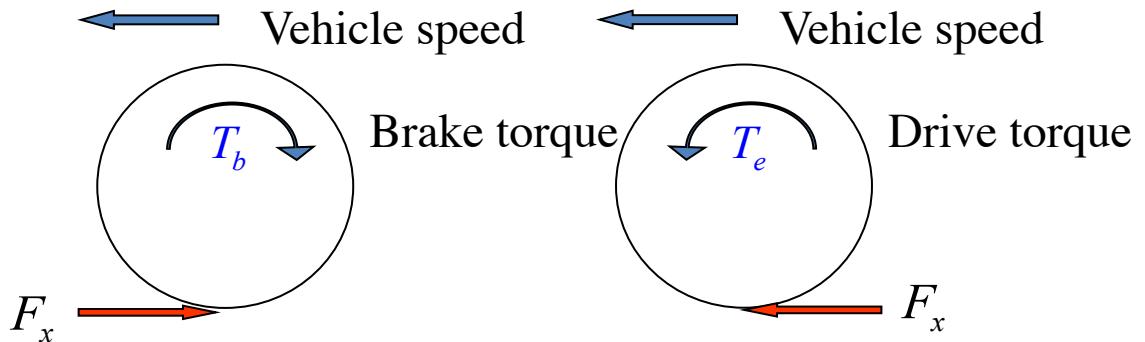


$a(t,u);$
 $\beta; =0.02;$
 $\gamma; uw=0.0;$
 $\sin(\Theta) ...$
 $..$
 $lw)^2];$

Longitudinal Dynamics

- Mainly influenced by power generation (engine) and transfer (transmission, tires) devices
- When road friction is high, tire is mainly linear and can be ignored (part of vehicle inertia)
- When road friction is low, tire motion may be decoupled from vehicle motion, and a tire force model becomes important.
- Radius of wheel: r_w
- Tire rotational speed: ω

Longitudinal Dynamics



$$\dot{\omega} = \frac{T_e - T_b - r_w F_x - f_w F_z - b_w \omega}{J}$$

Longitudinal Dynamics

$$\dot{\omega} = \frac{T_e - T_b - r_w F_x - f_w F_z - b_w \omega}{J}$$

ω	wheel rotational velocity	f_w	dry friction at wheel
T_e	engine torque	b_w	viscous friction at wheel
T_b	braking torque	J	wheel inertia

Longitudinal slip



M

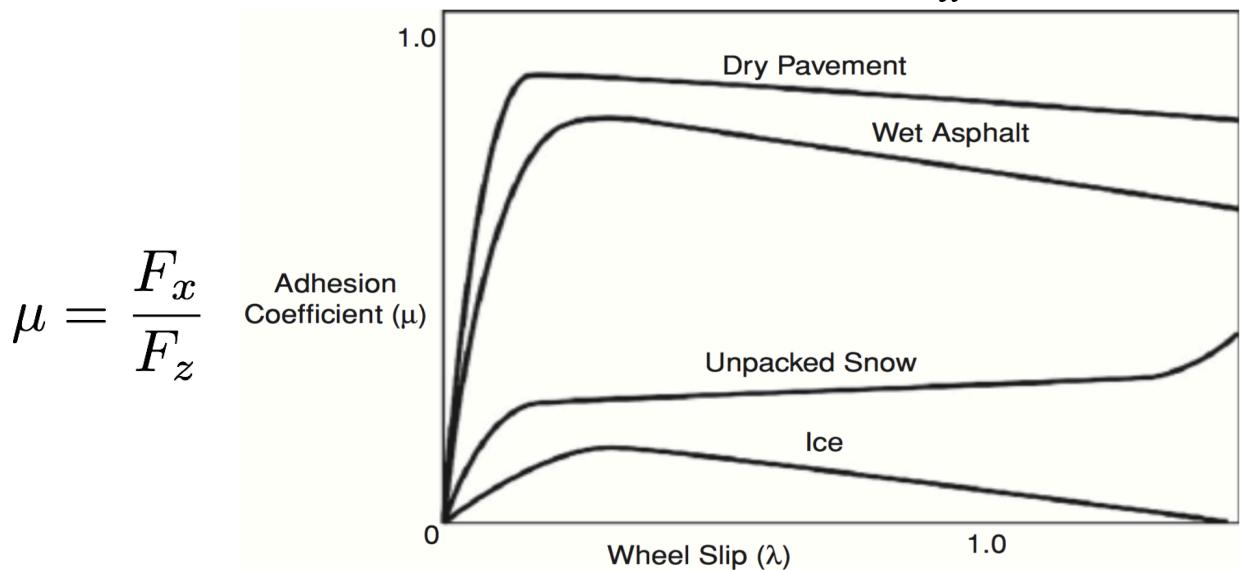
Longitudinal slip

Driven (accelerating) case

$$\lambda \equiv \frac{\omega r_w - u}{\omega r_w}$$

Braking case

$$\lambda \equiv \frac{\omega r_w - u}{u}$$



$$\mu = \frac{F_x}{F_z}$$

Complete Longitudinal Model

$$\dot{u} = \frac{-0.5\rho C_d A(u + u_w)^2 - fmg \cos \Theta - mg \sin \Theta + N_w F_x}{m}$$

$$\dot{\omega} = \frac{T_e - T_b - r_w F_x - f_w F_z - b_w \omega}{J}$$

$$\dot{u} = \frac{-0.5\rho C_d A(u + u_w)^2 - fmg \cos \Theta - mg \sin \Theta + N_w F_z \mu(\lambda)}{m}$$

$$\dot{\omega} = \frac{T_e - T_b - r_w F_z \mu(\lambda) - f_w F_z - b_w \omega}{J}$$

Modeling Tire Friction

- Tire forces and moments are highly nonlinear and difficult to model.
- Thus, models such as the Brush or Pacejka “Magic Formula” models, have been developed for use in simulation studies.
- We will discuss one of these soon!

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 - 1. 2 DOF Transient Model
 - 2. Nonlinear Tire Model

Vehicle Lateral Models

Lateral	$\sum F_x = m\dot{u}$
	$\sum F_y = m(\dot{v} + ru_0)$
	$\sum F_z = m(\dot{w} - qu_0)$
Roll	$\sum M_x = I_{xx}\dot{p} - I_{xz}\dot{r}$
	$\sum M_y = I_{yy}\dot{q}$
Yaw	$\sum M_z = -I_{xz}\dot{p} + I_{zz}\dot{r}$

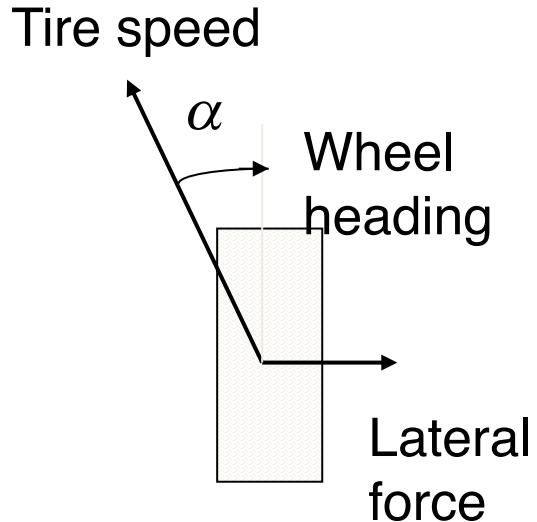
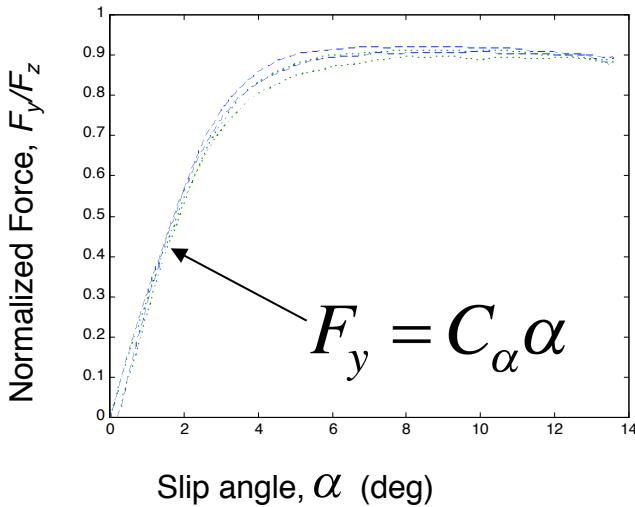
Two popular types of models:

- | | |
|------------------|------------------------------|
| Lateral/Yaw | (2DOF Handling, Lateral/Yaw) |
| Lateral/Yaw/Roll | (3DOF Lateral/Yaw/Roll) |

Vehicle Lateral Dynamics

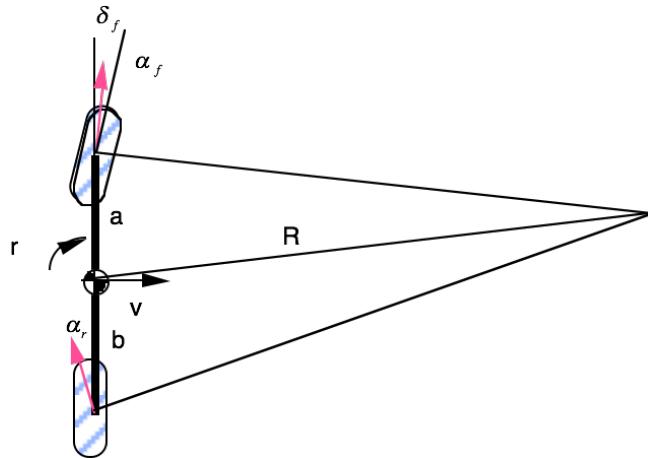
Lateral and yaw (i.e., need to find equations for \dot{v} and \dot{r})

Critical step is to link input (steering) with tire lateral forces
— which are the real inputs to lateral dynamics.



The 2 DOF “Bicycle” Model

Relationship between (front axle) steering angle and slip angles:



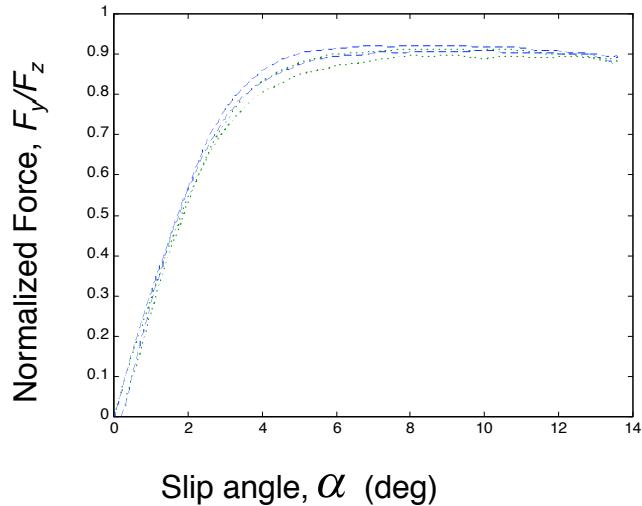
$$\alpha_f = \delta_f - \tan^{-1} \left(\frac{v + ar}{u} \right) \approx \delta_f - \frac{v + ar}{u}$$

$$\alpha_r = \delta_r - \tan^{-1} \left(\frac{v - br}{u} \right) \approx -\frac{v - br}{u}$$

$$\delta_f = \frac{L}{R} + \alpha_f - \alpha_r$$

Ackermann angle

Tire Lateral Forces



Tire lateral forces:

- depend on normal force
- are a nonlinear function of tire slip, and
- depend on tire design (e.g., radial, bias-ply) and inflation pressure.

Small slip angle:

$$F_y = C_\alpha \alpha$$

2DOF Vehicle Transient Model

$$\sum F_x = m\dot{u}$$

$$\sum F_y = m(\dot{v} + ru_o)$$

$$\sum F_z = m(\dot{w} - qu_o)$$

$$\sum M_x = I_{xx}\dot{p} - I_{xz}\dot{r}$$

$$\sum M_y = I_{yy}\dot{q}$$

$$\sum M_z = -I_{xz}\dot{p} + I_{zz}\dot{r}$$

Figure out the tire forces and corresponding yaw moment

Keep the model linear

- small angle assumption
- linear tire approximation

2DOF Vehicle Transient Model

Slip angles:

$$\alpha_f = \delta_f - \tan^{-1}\left(\frac{v + ar}{u}\right) \approx \delta_f - \frac{v + ar}{u}$$

$$\alpha_r = \delta_r - \tan^{-1}\left(\frac{v - br}{u}\right) \approx -\frac{v - br}{u}$$

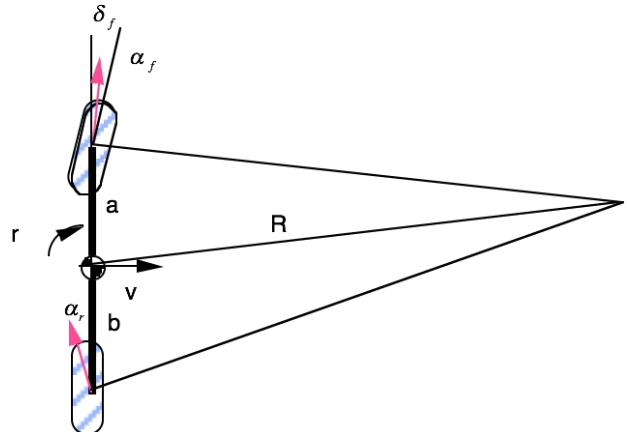
Lateral tire force: $F_y = C_\alpha \alpha$

$$\sum F_y = m(\dot{v} + u_0 r) :$$

$$C_{\alpha f} \left[\delta_f - \frac{v + ar}{u_0} \right] + C_{\alpha r} \left[-\frac{v - br}{u_0} \right] = m(\dot{v} + u_0 r)$$

$$\sum M_z = I_z \dot{r} :$$

$$C_{\alpha f} \left[\delta_f - \frac{v + ar}{u_0} \right] a + C_{\alpha r} \left[-\frac{v - br}{u_0} \right] b = I_z \dot{r}$$



State Space Form of the 2DOF Model

$$\frac{d}{dt} \begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} -(C_{af} + C_{ar}) & bC_{ar} - aC_{af} - u_o \\ mu_o & -(a^2C_{af} + b^2C_{ar}) \\ \hline bC_{ar} - aC_{af} & I_z u_o \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_{af}}{m} \\ \frac{aC_{af}}{I_z} \end{bmatrix} \delta_f$$

Or equivalently, if β is the sideslip angle at the CG:

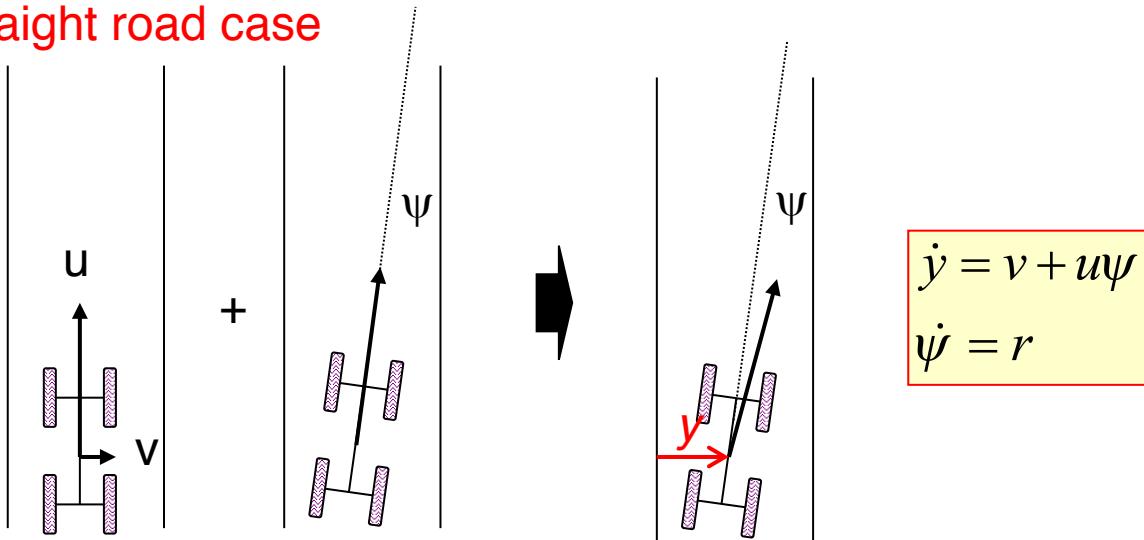
$$\tan \beta = \frac{v}{u_0} \quad \text{or} \quad v \approx \beta u_0$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\left(\frac{C_{af} + C_{ar}}{mu_o}\right) & -\left(\frac{aC_{af} - bC_{ar}}{mu_o^2}\right) - 1 \\ -\left(\frac{aC_{af} - bC_{ar}}{I_z}\right) & -\left(\frac{C_{af}a^2 + C_{ar}b^2}{I_z u_o}\right) \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_{af}}{mu_o} \\ \frac{aC_{af}}{I_z} \end{bmatrix} \delta_f$$

Bicycle Model State Equation (Road Following)

Augment the handling model with the dynamics of the “displacement variables” (both lateral displacement and yaw angle)

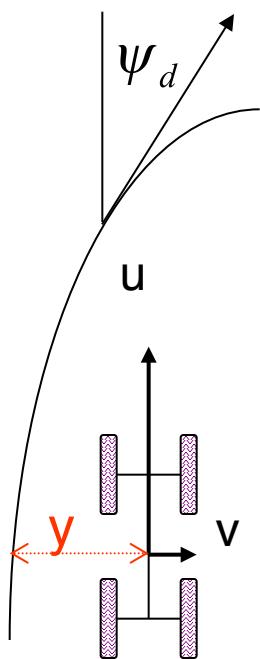
Straight road case



$$\begin{aligned}\dot{y} &= v + u\psi \\ \dot{\psi} &= r\end{aligned}$$

Road Input to Vehicle Dynamics

Curved road case: need to define a road “disturbance” input



Road radius of curvature = R

Desired yaw rate $r_d = \frac{u}{R}$ $\int r_d = \psi_d$

$$\begin{aligned}\dot{y} &= v + u\psi \\ \dot{\psi} &= r\end{aligned}$$



$$\begin{aligned}\dot{y} &= v + u(\psi - \psi_d) \\ \frac{d}{dt}(\psi - \psi_d) &= r - r_d\end{aligned}$$

Road Following Model

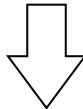
- Standard “bicycle model” only has 2 states (v and r).
- When we need to simulate vehicle **road following** behavior, 2 extra states (y, ψ) are included:

$$\frac{d}{dt} \begin{bmatrix} y \\ \dot{y} \\ \psi - \psi_d \\ r \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\left(\frac{C_{af} + C_{ar}}{mu_o}\right) & \frac{C_{af} + C_{ar}}{m} & \frac{-aC_{af} + bC_{ar}}{mu_o} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-aC_{af} + bC_{ar}}{I_z u_o} & \frac{aC_{af} - bC_{ar}}{I_z} & -\left(\frac{C_{af}a^2 + C_{ar}b^2}{I_z u_o}\right) \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \psi - \psi_d \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_{af}}{m} \\ 0 \\ \frac{aC_{af}}{I_z} \end{bmatrix} \delta_f + \begin{bmatrix} 0 \\ -u_o \\ -1 \\ 0 \end{bmatrix} r_d$$

- When the road is curved, the orientation (yaw angle) of the road is denoted as ψ_d , and its rate of change as r_d
- For a straight road, set $\psi_d = r_d = 0$.

Bicycle 2DoF Model Equation

$$\frac{d}{dt} \begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} \frac{-(C_{\alpha f} + C_{\alpha r})}{mu} & \frac{bC_{\alpha r} - aC_{\alpha f}}{mu} - u \\ \frac{bC_{\alpha r} - aC_{\alpha f}}{I_z u} & \frac{-(a^2 C_{\alpha f} + b^2 C_{\alpha r})}{I_z u} \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_{\alpha f}}{m} \\ \frac{aC_{\alpha f}}{I_z} \end{bmatrix} \delta_f$$



$$\frac{d}{dt} \begin{bmatrix} y \\ v \\ \psi - \psi_d \\ r \end{bmatrix} = \begin{bmatrix} 0 & 1 & u & 0 \\ 0 & -\frac{C_{\alpha f} + C_{\alpha r}}{mu} & 0 & \frac{bC_{\alpha r} - aC_{\alpha f}}{mu} - u \\ 0 & 0 & 0 & 1 \\ 0 & \frac{bC_{\alpha r} - aC_{\alpha f}}{I_z u} & 0 & -\frac{a^2 C_{\alpha f} + b^2 C_{\alpha r}}{I_z u} \end{bmatrix} \begin{bmatrix} y \\ v \\ \psi - \psi_d \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_{\alpha f}}{m} \\ 0 \\ \frac{aC_{\alpha f}}{I_z} \end{bmatrix} \delta_f + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} r_d$$

Ex. 4.6 Vehicle Handling Simulations

```
% Ex4_6a.m
% 2DOF model vehicle parameters
a = 1.14; % distance c.g. to front axle (m)
L = 2.54; % wheel base (m)
m = 1500; % mass (kg)
Iz = 2420.0; % yaw moment of inertia (kg-m^2)
Caf = 44000*2; % cornering stiffness--front axle (N/rad)
Car = 47000*2; % cornering stiffness-- rear axle (N/rad)
b=L-a; g=9.81;
Kus = m*b/(L*Caf) - m*a/(L*Car); % (rad/(m/sec^2))
u=20.0; % forward speed in m/sec
A=[-(Caf+Car)/(m*u), (b*Car-a*Caf)/(m*u)-u
   (b*Car-a*Caf)/(Iz*u), -(a^2*Caf+b^2*Car)/(Iz*u)];
B=[Caf/m; a*Caf/Iz];
C_lat = [1 0]; D_lat = 0; % Lateral speed
C_yaw = [0 1]; D_yaw = 0; % Yaw rate
C_acc=A(1,:)+u*[0,1];
D_acc = B(1); % Lateral acceleration
C = [C_lat; C_yaw; C_acc];
D = [D_lat; D_yaw; D_acc];

t=[0:0.01:6];
U=0.5*pi/180*sin(1/3*2*pi*t);
```

```
Y=lsim(A,B,C,D,U,t);
```

```
Subplot, subplot(221)
plot(t,Y(:,1),'r'); grid
xlabel('time (sec)')
ylabel('Lateral speed (m/sec)')
```

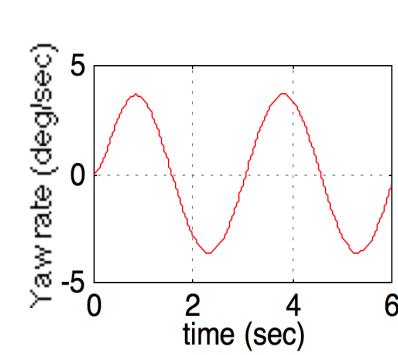
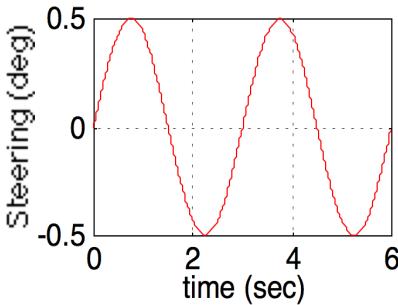
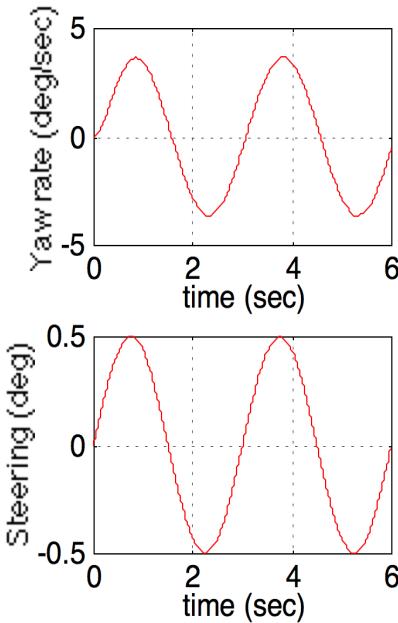
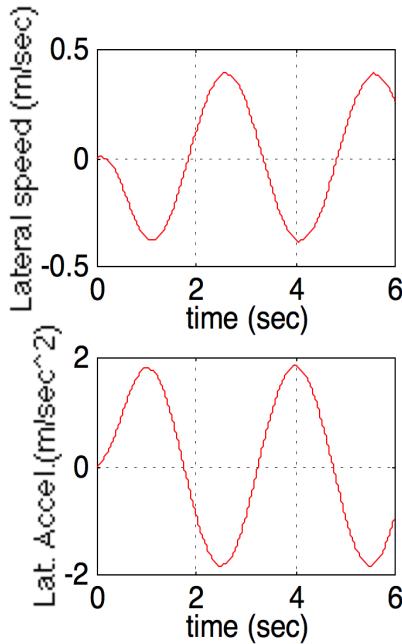
```
subplot(222)
plot(t,Y(:,2)*180/pi,'r'); grid
xlabel('time (sec)')
ylabel('Yaw rate (deg/sec)')
```

```
subplot(223)
plot(t,Y(:,3),'r'); grid
xlabel('time (sec)')
ylabel('Lat. Accel.(m/sec^2)')
```

```
subplot(224)
plot(t,U*180/pi,'r'); grid
xlabel('time (sec)')
ylabel('Steering (deg)')
```



Ex. 4.6 Vehicle Handling Simulations (cont.)



```
Subplot, subplot(221)  
plot(t,Y(:,1),'r'); grid  
xlabel('time (sec)')  
ylabel('Lateral speed (m/sec)')
```

```
subplot(222)  
plot(t,Y(:,2)*180/pi,'r'); grid  
xlabel('time (sec)')  
ylabel('Yaw rate (deg/sec)')
```

```
subplot(223)  
plot(t,Y(:,3),'r'); grid  
xlabel('time (sec)')  
ylabel('Lat. Accel.(m/sec^2)')
```

```
subplot(224)  
plot(t,U*180/pi,'r'); grid  
xlabel('time (sec)')  
ylabel('Steering (deg)')
```



Quality of Approximations: Magic Formula

$$F_x = D_x \sin(C_x \tan^{-1}(B_x \phi_x)) + S_{vx}$$

$$F_y = D_y \sin(C_y \tan^{-1}(B_y \phi_y)) + S_{vy}$$

$$\phi_x = (1 - E_x)(\lambda + S_{hx}) + \frac{E_x}{B_x} \tan^{-1}(B_x(\lambda + S_{hx}))$$

$$\phi_y = (1 - E_y)(\alpha + S_{hy}) + \frac{E_y}{B_y} \tan^{-1}(B_y(\alpha + S_{hy}))$$

B=stiffness factor (B_x for **longitudinal**, B_y for **lateral**)

C=shape factor

D=Peak factor (determines peak magnitude)

E=curvature factor

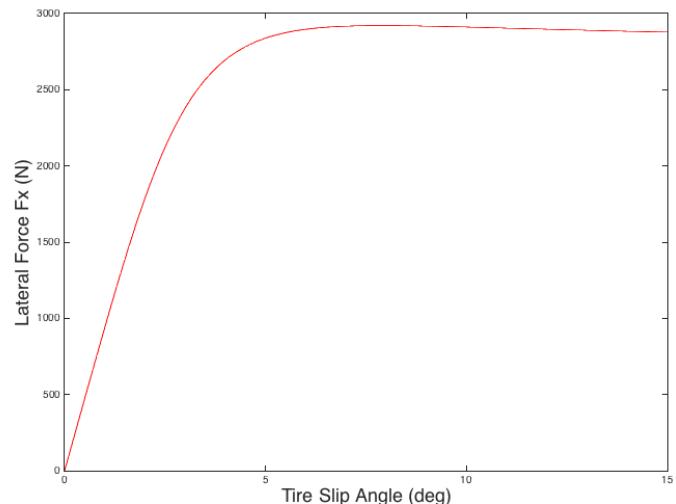
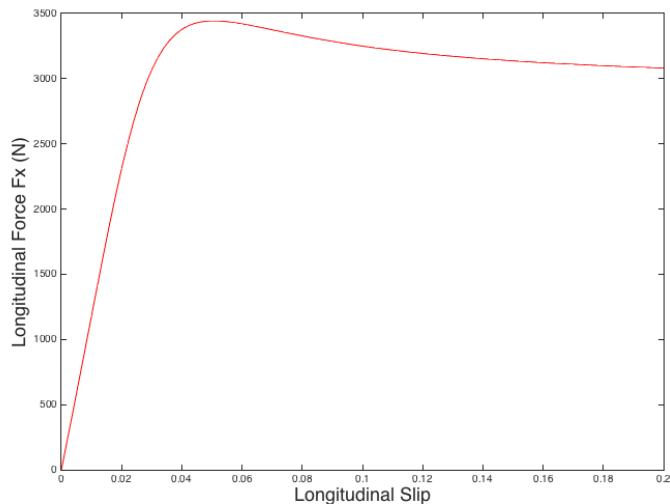
S_h =horizontal offset

S_v =vertical offset

MAKE SURE SLIP ANGLE, α ,IS GIVEN IN DEGREES!



Longitudinal and Lateral Forces



check [magicformulaslip.m](#)



Combined Longitudinal/Lateral/Yaw Nonlinear Model

$$\dot{x} = u \cos \psi - v \sin \psi$$

$$\dot{u} = \frac{fW + N_w F_x}{m}$$

$$\dot{\omega} = \frac{T - r_w F_x - f_w F_z - b_w \omega}{J}$$

$$\dot{y} = u \sin \psi + v \cos \psi$$

$$\dot{v} = \frac{F_{yf} + F_{yr}}{m} - ur$$

$$\dot{\psi} = r$$

$$\dot{r} = \frac{F_{yf}a + F_{yr}b}{I_z}$$

- Inputs are: T and δ_f
- Use Magic Formula for F_x , F_{yf} , F_{yr}
- $\alpha_f = \delta_f - \tan^{-1}\left(\frac{v+ar}{u}\right)$
- $\alpha_r = \tan^{-1}\left(\frac{v-br}{u}\right)$



Simulate Nonlinear Model

dynamicbicyclecontroltest.m

Summary

- Coordinate System and Rigid Body Motion
- Longitudinal Vehicle Motion
- Lateral Vehicle Motion.