Update for f11

November 16, 2014

$$S(U,V) = \prod_{ij} N(u_i^T v_j | m_{i,j}^{c,-11}, v_{i,j}^{c,-11}) \prod_i \prod_k N(u_{i,k} | m_{i,k}^{u,-11}, v_{i,k}^{u,-11} \prod_j \prod_k N(v_{j,k} | m_{j,k}^{v,-11}, v_{j,k}^{v,-11}) \tag{1}$$

$$Q_{U,V} = \prod_i \prod_k N(u_{i,k} | m_{i,k}^{u}, v_{i,k}^{u}) \prod_j \prod_k N(v_{j,k} | m_{j,k}^{v}, v_{j,k}^{v}) \tag{2}$$

We need to minimize the $KL(Q_{UV} \| S)$

$$\begin{aligned} KL(Q_{U,V} \| S) &= E_{Q_{U,V}} \ln \frac{Q_{U,V}}{S(U,V)} \\ &= E_{Q_{U,V}} \ln Q_{U,V} - E_{Q_{U,V}} \ln S(U,V) \\ &= E_U \ln Q_U + E_V \ln Q_V - E_{U,V} \ln S(U,V) \end{aligned} \tag{3}$$

$$\begin{aligned} & E_{u_i v_j} \ln N(\sum_k u_{ik} v_{jk} | m_{ij}^{c,-11}, v^{c,-11} ij) \\ &= -1/2 \ln(2\pi (v^{c,-11})^2) - 1/2 (v^{c,-11})^2 E(\sum_k u_{ik} v_{jk} - m_{ij}^{c,-11})^2 \end{aligned} \tag{4}$$

$$\begin{aligned} E(\sum_k u_{ik} v_{jk} - m_{ij}^{c,-11})^2 &= E(\sum_k u_{ik}^2 v_{jk}^2 + \sum_{k \neq t} u_{ik} v_{jk} u_{it} v_{jt} - 2 m_{ij}^{c,-11} \sum_k u_{ik} v_{jk} + (mij^{c,-11})^2) \\ &= \sum_k ((m_{ik}^u)^2 + (v_{jk}^u)^2)((m_{ik}^v)^2 + (v_{jk}^v)^2) + \sum_{k \neq t} m_{ik}^u m_{jk}^v m_{it}^u m_{jt}^v \\ &- 2 m_{ij}^{c,-11} \sum_k m_{ik}^u v_{jk}^u + (m_{ij}^{c,-11})^2 \\ &= (m_{ij}^{c,-11} - \sum_k m_{ik}^u m_{jk}^v)^2 + \sum_k ((v_{ik}^u)^2 (m_{jk}^v)^2 + (v_{jk}^v)^2 (m_{ik}^u)^2 + (v_{ik}^u)^2 (v_{jk}^v)^2) \end{aligned} \tag{5}$$

So this term is

$$E_{u_i v_j} \ln N(\sum_k u_{ik} v_{jk} | m_{ij}^{c,-11}, v_{ij}^{c,-11})$$
$$= -1/2 \ln(2\pi(v^{c,-11})^2) - \frac{(m_{ij}^{c,-11} - \sum_k m_{ik}^u m_{jk}^v)^2 + \sum_k ((v_{ik}^u)^2 (m_{jk}^v)^2 + (v_{jk}^v)^2 (m_{ik}^u)^2 + (v_{ik}^u)^2 (v_{jk}^v)^2)}{2(v_{ij}^{c,-11})^2} \tag{6}$$