

TASK 8: Grover's algorithm for a 3-qubits database

Aim: To implement Grover's quantum search algorithm for a 3-qubit search space (8 items) using Cirq and demonstrate that the marked item (target state) can be found with high probability after the optimal number of iterations.

Algorithm - Grover's algorithm for a 3-qubits database

1. Initialize 3 qubits to $|0\rangle$.
2. Create uniform superposition by Hadamard gates $H^{\otimes 3}$.
3. Repeat $k = 2$ times: Apply oracle marking the target bit string with phase flip on the marked state. Apply diffusion operator (inversion about the mean).
4. Measure the qubits to obtain results peaked at the target state with high probability.

```
!pip install cirq

import cirq
import numpy as np
import matplotlib.pyplot as plt

def grover_3_qubit(target_binary):
    qubits = [cirq.GridQubit(0, i) for i in range(3)]
    circuit = cirq.Circuit()

    # Initialize superposition
    circuit.append(cirq.H.on_each(*qubits))

    # Number of Grover iterations
    N = 2 ** 3
    iterations = int(np.floor(np.pi / 4 * np.sqrt(N)))

    for _ in range(iterations):
        # Oracle
        apply_oracle(circuit, qubits, target_binary)

        # Diffusion
        apply_diffusion(circuit, qubits)

    # Measurement (only once at the end)
    circuit.append(cirq.measure(*qubits, key='result'))

    return circuit, qubits
```

```

def apply_oracle(circuit, qubits, target_binary):
    # Apply X gates where target bit is 0
    for i, bit in enumerate(target_binary):
        if bit == '0':
            circuit.append(cirq.X(qubits[i]))

    # Multi-controlled Z via H + CCX
    circuit.append(cirq.H(qubits[-1]))
    circuit.append(cirq.CCX(qubits[0], qubits[1], qubits[2]))
    circuit.append(cirq.H(qubits[-1]))

    # Undo X gates
    for i, bit in enumerate(target_binary):
        if bit == '0':
            circuit.append(cirq.X(qubits[i]))


def apply_diffusion(circuit, qubits):
    circuit.append(cirq.H.on_each(*qubits))
    circuit.append(cirq.X.on_each(*qubits))

    circuit.append(cirq.H(qubits[-1]))
    circuit.append(cirq.CCX(qubits[0], qubits[1], qubits[2]))
    circuit.append(cirq.H(qubits[-1]))

    circuit.append(cirq.X.on_each(*qubits))
    circuit.append(cirq.H.on_each(*qubits))


def analyze_results(counts, target):
    total = sum(counts.values())
    success = counts.get(int(target), 0)
    success_rate = success / total * 100

    print(f"Measurement results for target |{target}>:")
    for state in range(8):
        bitstr = format(state, '03b')
        count = counts.get(state, 0)
        pct = count / total * 100
        marker = "<-- Target" if bitstr == target else ""
        print(f"State |{bitstr}>: {count} times ({pct:.2f}%) {marker}")

    print(f"\nSuccess rate: {success_rate:.2f}% (optimal ~94% after 2 iterations)")

    states = [format(i, '03b') for i in range(8)]
    values = [counts.get(i, 0) for i in range(8)]
    colors = ['red' if s == target else 'blue' for s in states]

    plt.bar(states, values, color=colors)
    plt.title(f"Grover's Algorithm Results (Target: |{target}>)")
    plt.xlabel("States")
    plt.ylabel("Counts")
    plt.show()

```

```
if __name__ == "__main__":
    target = "101"
    circuit, qubits = grover_3_qubit(target)

    print("Circuit diagram:")
    print(circuit)

    simulator = cirq.Simulator()
    result = simulator.run(circuit, repetitions=1000)

    counts = result.histogram(key='result')
    analyze_results(counts, target)
```



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Collecting cirq
  Downloading cirq-1.6.1-py3-none-any.whl.metadata (16 kB)
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