Algorithm 1 Left-preconditioned GMRES(k) with Given's Rotations

1: ε (tolerance for the residual norm r), x_0 (initial guess), and set $convergence = false$
2: MPI_Alltoallv(x_0)
3: **while** $convergence == false$ **do**
4: $r_0 = M^{-1}(b - Ax_0)$
5: MPI_Alltoallv(r_0)
6: $\beta = ||r_0||_2$ ▷ MPI_Allreduce($< r_0, r_0 >$)
7: $v_1 = r_0/\beta$
8: **for** $j = 1$ to k **do**
9: $w_j = M^{-1}Av_j$ ▷ MPI_Alltoallv(w_j)
10: **for** $i = 1$ to j **do**
11: $h_{i,j} =< w_j, v_i >$ ▷ MPI_Allreduce
12: $w_j = w_j - h_{i,j}v_i$
13: **end for**
14: $h_{j+1,j} = ||w_j||_2$ ▷ MPI_Allreduce
15: $v_{j+1} = w_j/h_{j+1,j}$
16: **end for**
17: Set $V_k = [v_1, \cdots, v_k]$ and $\bar{H}_k = (h_{i,j})$ an upper Hesssenberg matrix of order $(m + 1) \times m$
18: Solve a least-square problem of size m: $\min_{y \in \mathbb{R}^k} ||\beta e_1 - \bar{H}_k y||_2$
19: $x_k = x_0 + V_k y_k$
20: **if** $||M^{-1}(b - Ax_k)||_2 < \varepsilon$ **then**
21: $convergence = true$
22: **end if**
23: $x_0 = x_k$ ▷ MPI_Alltoallv(x˙0)
24: **end while**