

RBF-FD Weights (for one n -node stencil centered at x_j)

$$\begin{pmatrix} \phi(\epsilon ||\mathbf{x}_1 - \mathbf{x}_1||) & \phi(\epsilon ||\mathbf{x}_1 - \mathbf{x}_2||) & \cdots & \phi(\epsilon ||\mathbf{x}_1 - \mathbf{x}_n||) & 1 \\ \phi(\epsilon ||\mathbf{x}_2 - \mathbf{x}_1||) & \phi(\epsilon ||\mathbf{x}_2 - \mathbf{x}_2||) & \cdots & \phi(\epsilon ||\mathbf{x}_2 - \mathbf{x}_n||) & 1 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \phi(\epsilon ||\mathbf{x}_n - \mathbf{x}_1||) & \phi(\epsilon ||\mathbf{x}_n - \mathbf{x}_2||) & \cdots & \phi(\epsilon ||\mathbf{x}_n - \mathbf{x}_n||) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \\ c_{n+1} \end{bmatrix} = \begin{bmatrix} \mathcal{L}\phi(\epsilon ||\mathbf{x} - \mathbf{x}_1||)|_{\mathbf{x}=\mathbf{x}_j} \\ \mathcal{L}\phi(\epsilon ||\mathbf{x} - \mathbf{x}_2||)|_{\mathbf{x}=\mathbf{x}_j} \\ \vdots \\ \mathcal{L}\phi(\epsilon ||\mathbf{x} - \mathbf{x}_n||)|_{\mathbf{x}=\mathbf{x}_j} \\ 0 \end{bmatrix}$$

- ▶ ϕ is Gaussian RBF centered at x_k , $k = 1, \dots, n$
- ▶ \mathcal{L} is some differential operator (i.e., $\frac{\partial}{\partial \lambda}, \frac{\partial}{\partial \theta}, \nabla^k$, etc.); form multiple RHS system for efficiency
- ▶ Repeat this $n \times n$ system solve for all N stencils.