the identity $\nabla \cdot (\nabla \times q(\mathbf{x})) = 0,$ (16)

To verify our implementation, we manufacture a solution that satisfies the continuity equation. Using

for any function $q(\mathbf{x})$, we can easily manufacture a solution by choosing some vector function $q(\mathbf{x})$,

projecting it onto the sphere via
$$P_x g(x)$$
 and applying the curl projection, Q_x :

$$Q_x = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.$$