

Assuming η is a constant (i.e., $\nabla\eta = 0$), our system simplifies to

$$\begin{pmatrix} -\eta\nabla^2 & 0 & 0 & \frac{\partial}{\partial x_1} \\ 0 & -\eta\nabla^2 & 0 & \frac{\partial}{\partial x_2} \\ 0 & 0 & -\eta\nabla^2 & \frac{\partial}{\partial x_3} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ p \end{pmatrix} = \frac{RaT}{\sqrt{x_1^2 + x_2^2 + x_3^2}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{pmatrix}. \quad (7)$$

where the ∇^2 operator in spherical polar coordinates for \mathbb{R}^3 is:

$$\nabla^2 = \underbrace{\frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} \left(\hat{r}^2 \frac{\partial}{\partial \hat{r}} \right)}_{\text{radial}} + \underbrace{\frac{1}{\hat{r}^2} \Delta_S}_{\text{angular}}. \quad (8)$$