Synchronous single initiator spanning tree algorithm using flooding

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March 13, 2020

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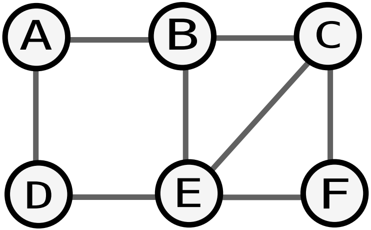
- We can distribute the sequential algorithm assuming *synchronous communication*.
- **Refresher:** Algorithm proceeds in rounds. All messages sent at round i are received at round i.
- **Flood:** everyone sends a message to their neighbours. Think flood fill. 

Example



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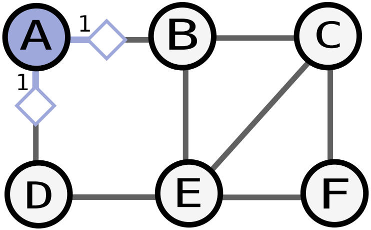

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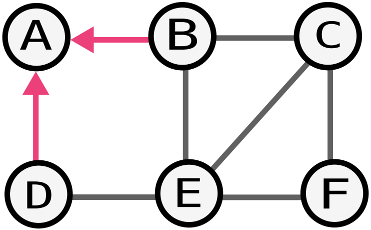

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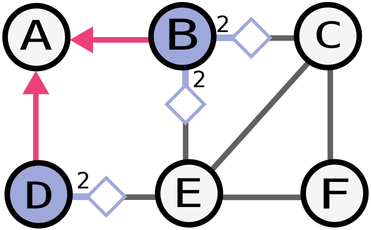

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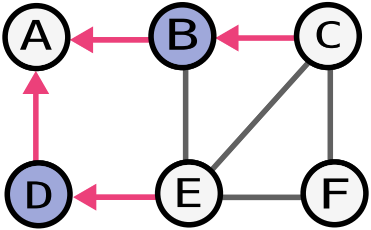

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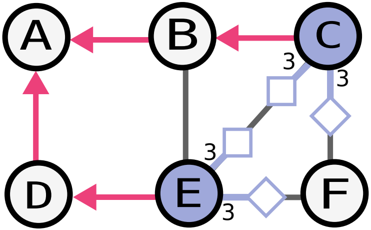

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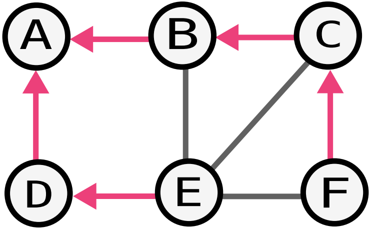

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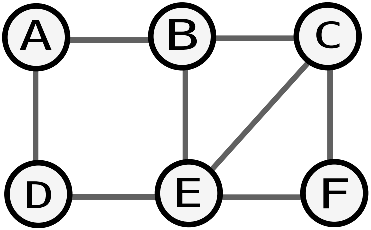

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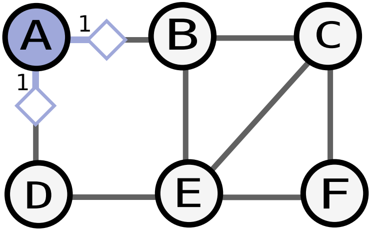

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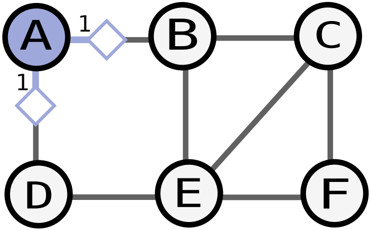

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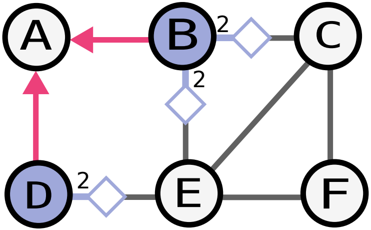

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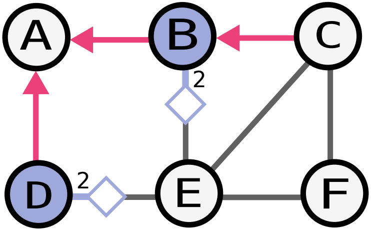

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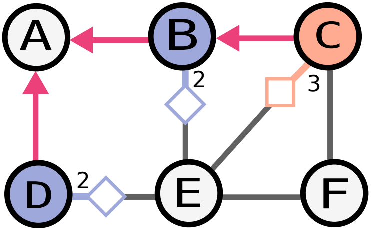

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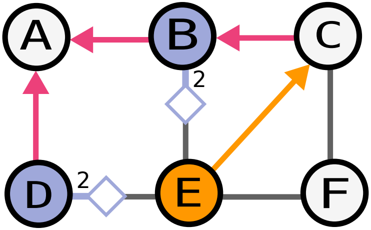

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      if self.queries: # if we have a query
        # randomly choose from queries
        parent = random.choice(self.query)
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        # synchronous
        for n in self.neighbours: n.send(self.id)
    self.queries = [];
```

Synchronous BFS (Ending earlier if visited)

```
def bfs_spanning_tree(self):

  if self.id == ROOT_ID:
    self.depth = 0;
    for n in self.neighbours: n.send(self.id)

    return # early-exit for root node

  for round in range(1, DIAMETER+1):

      if self.queries: # if we have a query
        # randomly choose from queries
        parent = random.choice(self.query)
        self.visited = True
        self.depth = round

        # synchronous
        for n in self.neighbours: n.send(self.id)

        return # early-exit for child
```

Synchronous BFS (Learning children)

- Assume root begins computation.
- Algorithm is synchronous.

```
def bfs_spanning_tree(self):
  if self.id == ROOT_ID:
    self.visited = True; self.depth = 0;
    for n in self.neighbours: n.send(self.id)
  for round in range(1, DIAMETER+1):
    if self.visited: # if visited, wait for children
      for q in self.queries: self.children.append(q)
    else: # if not visited, run code
      if self.queries: # if we have a query
        # randomly choose from queries
        parent = random.choice(self.query)
        self.visited = True
        self.depth = round
        # synchronous
        for n in self.neighbours: n.send(self.id)
        parent.send(self.id) # send to parent
    self.queries = [];
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- $|Diameter|$ rounds.
- 1 or 2 messages / edge. Message complexity $\leq 2|E|$.

Thank you!

Asynchronous Bounded Delay Network

- All processes have physical clocks: **need not be synchronized**.
- Message delivery time is bounded by constant $\mu \in \mathbb{R}$.

ABD Synchronizers: Bounded Delay $\rightarrow$ Synchronized

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Key idea

Chunk "real time" into units of μ. Each μ block of time behaves like a logical synchronized tick!

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- It then enters a state s' such that $(s, M) \vdash s'$.
- Note that $M\mathcal{G}$ is **deterministic**. All the nondeterminism is in the relation $(s, M) \vdash s'$.

Formal definition of synchronous system $\mathbb{P}$

- $\mathbb{P} \equiv (p_1, p_2, \ldots, p_n)$ where each $p_i \equiv (S_i, I_i, M\mathcal{G}_i, \vdash_i)$ is a synchronous process, is a synchronous system.

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- $\rightarrow \subseteq S \times S$ is the transition relation.

$$
\begin{aligned}
&(s_1, s_2, \ldots s_n) \rightarrow (s'_1, s'_2, \ldots, s'_n) \\
&\quad \text{iff} \quad \forall p_i \in \mathbb{P}; (s_i, \{\text{messages to } p_i \text{ from } p_{-i}\}) \vdash s'_i
\end{aligned}
$$