Sample title

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The story of a proof: from paper to kernel

Proposition 3.2. Consider the composite system $T = C \times_{\mathcal{I}} S$ working under the assumption that choice inputs arrive only at odd cycles. Then, the system correctly implements the starvation freedom constraint of the 1 Diner problem which states that the philosopher doesn't remain hungry forever. It defers to the philosopher's own choice (stay at the same state or switch to the next) when the philosopher is not hungry.

Proof. The result follows from the following propositions, which are simple consequences of the polled dynamics:

1. if $a = \mathsf{h}$, then $a' = \mathsf{e}$.
2. if $a \neq \mathsf{h}$, then $a' = f_P(a, b)$.

$\square$

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□

```
Lemma system38_starvation_free:
  forall (n: nat) (ss: nat -> the * cmd)
   (ts: nat -> cmd * maybe choice * the)
   (TRACE_SSSN: ValidTrace system38 ss ts (S(S(S n))))
   (BOTTOM_EVEN:
      forall (i: nat) (IEVEN: even i = true),
      snd (fst (ts i)) = nothing choice)
   (NOT_BOTTOM_ODD:
      forall (i: nat) (IODD: odd i = true),
      snd(fst (ts i)) <> nothing choice)
   (HUNGRY: fst (ss n) = h),
  exists (m: nat),  m > n /\  fst (ss m) = e.
Proof.
     (* 30 lines of proof,
        200 lines of supporting lemmas ommitted *)
Qed.

Lemma system_38_phil_not_hungry_then_next_philo_choice:
  forall (n: nat) (ss: nat -> the * cmd) (c: choice)
   (ts: nat -> cmd * maybe choice * the)
   (TRACE_SSSN: ValidTrace system38 ss ts (S(S(S n))))
   (NOTHUNGRY: fst (ss (S n)) <> h)
   (CHOICE: snd (fst (ts (S n))) = just choice c)
   (NEVEN: even n = true)
   (BOTTOM_EVEN:
     forall (i: nat) (IEVEN: even i = true),
     snd (fst (ts i)) = nothing choice),
    fst (ss (S(S(S n)))) = trans32fn (fst (ss (S n))) c.
Proof.
     (* 20 lines of proof,
        200 lines of supporting lemmas ommitted *)
Qed.
```

Statistics

- 667 lines of coq code.

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- All tables upto section 4 verified by computation.
- All theorems upto section 4 formally verified.

The definitions: Systems

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Recall: $X_0 \subseteq X \iff \texttt{member}(X_0) : X \to \texttt{Bool};\ \texttt{member}(X_0)(x) \equiv x \underset{?}{\in} X_0$

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```
(* 2.1: system specifcation *)
Record system (X: Set) (U: Set) :=
    mksystem { isx0: X -> Prop;
               trans: X -> U -> X -> Prop }.
```

The definitions: Tabuada Connection

The sets:

$$S \equiv (X : \mathbf{Set}, U_X : \mathbf{Set}, X_0 \subseteq X, \underset{X}{\rightarrow} \subseteq X \times U_X \times X).$$

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The composition:

$$S \times_{\mathcal{I}} T \equiv (Z \equiv X \times Y, U_Z \equiv U_X \times U_Y, X_0 \times Y_0, \underset{Z,\mathcal{I}}{\rightarrow} \subseteq Z \times U_Z \times Z).$$

$$(x, y) \xrightarrow[Z,\mathcal{I}]{u_X, u_Y} (x', y') \iff x \xrightarrow{u_X} x' \wedge y \xrightarrow{u_Y} y' \wedge (x, y, u_x, u_y) \in \mathcal{I}.$$

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```
(* 2.2: system composition *)
(* tabuada connection new system *)
Definition tabuada {X Y UX UY: Set}
    (sx: system X UX) (sy: system Y UY) (connect: X*Y->UX*UY->Prop): system (X*Y) (UX*UY) :=
  mksystem (X*Y) (UX*UY) (tabuada_start (isx0 X UX sx) (isx0 Y UY sy))
          (tabuada_trans connect (trans X UX sx) (trans Y UY sy)).

(* initial state for tabuada composition *)
Definition tabuada_start {X Y: Type} (isx0: X -> Prop) (isy0: Y -> Prop) (x: X * Y): Prop :=
  isx0 (fst  x) /\ isy0 (snd x).

(* transition fn for tabuada composition *)
Definition tabuada_trans {X Y: Type} {UX UY: Type}
    (connect: X*Y->UX*UY->Prop) (transx: X -> UX -> X -> Prop) (transy: Y -> UY -> Y -> Prop)
          (s: X*Y) (u: UX*UY) (s': X*Y): Prop :=
  transx (fst s) (fst u) (fst s') /\
  transy (snd s) (snd u) (snd s') /\
  (connect s u).
```

Pain point: Non determinism

- Coq has computational ability.
- Functions are computational; Relations are not.
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Inductive X: Set := X1 | X2 | X3.  Inductive Y: Set := Y1 | Y2 | Y3.

Definition x2y_fn (x: X): Y :=
  match x with | X1 => Y1 | X2 => Y2 | X3 => Y3 end.
(* Works; computational *)
Lemma x1_to_y1_fn: x2y_fn X1 = Y1. Proof. reflexivity. Qed.
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(* Works; computational *)
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Definition x2y_rel (x: X) (y: Y): Prop :=
  (x = X1 /\ y = Y1) \/ (x = X2 /\ y = Y2) \/  (x = X3 /\ y = Y3).
Lemma x1_to_y2_rel: x2y_rel X1 Y1.
Proof.
  try reflexivity. (* Does not work! Not computational *)
  unfold x2y_rel. left. auto.
Qed.
```

Pain point: Modularity

```
(* Helper: rewrite ss in terms of ts for the *)
Lemma system38_s_the_to_t_the:
  forall (n: nat) (ss: nat -> the * cmd) (ts: nat -> cmd * maybe choice * the)
         (TRACE: ValidTrace system38 ss ts (S n)),
    snd (ts n) = fst (ss n).
Proof.
  intros.
  inversion TRACE as [TRACE1 | npred TRACE1 AT1].
  subst.
  inversion AT1 as [AT11 [AT12 AT13]].
  set (s1 := ss 1) in *.
  destruct s1 as [s1_the s1_cmd].
  set (t1 := ts 1) in *.
  destruct t1 as [[t1_cmd t1_mchoice] t1_the].
  simpl in *.
  inversion AT13; simpl in *. (* step where we can see useful info *)
```

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  destruct t1 as [[t1_cmd t1_mchoice] t1_the].
  simpl in *.
  inversion AT13; simpl in *. (* step where we can see useful info *)
```

```
n : nat
ss : nat -> the * cmd
ts : nat -> cmd * maybe choice * the
TRACE : ValidTrace system38 ss ts (S n)
TRACE1 : ValidTrace system38 ss ts n
AT1 : trans (the * cmd) (cmd * maybe choice * the) system38 (ss n) (ts n) (ss (S n))
AT11 : trans the (cmd * maybe choice) phil37 (fst (ss n)) (fst (ts n)) (fst (ss (S n)))
AT12 : trans cmd the controller34 (snd (ss n)) (snd (ts n)) (snd (ss (S n)))
AT13 : connect38 (ss n) (ts n)
s1_the : the
s1_cmd, t1_cmd : cmd
t1_mchoice : maybe choice
t1_the : the
============================
snd (ts n) = fst (ss n)
```

Lemmas to drive reasoning

```
(* Helper: reason about even/odd *)
Lemma even_n_odd_Sn: forall (n: nat),
    (even n = true) <-> (odd (S n) = true).
(* Helper: reason about even/odd *)
Lemma odd_n_even_Sn: forall (n: nat),
    (odd n = true) <-> (even (S n) = true).
(* Rewrite ts in terms of ss *)
Lemma s_cmd_to_t_cmd:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE: ValidTrace system38 ss ts (S n)),
    fst (fst (ts n)) = snd(ss n).
(* Rewrite ss in terms of ts *)
Lemma s_the_to_t_the:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE: ValidTrace system38 ss ts (S n)),
    snd (ts n) = fst (ss n).
(* 'h' leads to '!1' command next. *)
Lemma phil_hungry_then_next_controller_bang1:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts (S n))
    (HUNGRY: fst (ss n) = h),
    snd (ss (S n)) = cmd_bang1.
```

```
(* not 'h' in leads to pass next *)
Lemma phil_not_hungry_then_next_controller_pass:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts (S n))
    (NOTHUNGRY: fst (ss n) <> h),
    snd (ss (S n)) = cmd_pass.
(* Phil. state in the next-odd-state = cur-state *)
Lemma phil_even_state_next_state:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts ((S n)))
    (BOTTOM_EVEN: ...)
    (NEVEN: even n = true),
    fst (ss (S n)) = fst (ss n).
(* Phil. state in the cur-odd-state = prev-state *)
Lemma phil_odd_state_prev_state:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts ((S n)))
    (BOTTOM_EVEN: ...)
    (SNODD: odd (S n) = true),
    fst (ss (S n)) = fst (ss n).
(* state after odd cycle has taken transition *)
Lemma odd_state_next_phil_state:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts ((S n)))
    (BOTTOM_EVEN: ...)
    (SNODD: odd n = true),
    fst (ss (S n)) = trans37fn  (fst (ss n)) (fst (ts n)).
```

Thank you

All code available at:

https://github.com/bollu/IIIT-H-code/tree/master/softwarefoundations/project