

# The story of a proof: From paper to kernel.

Siddharth Bhat

IIIT Hyderabad

April 29, 2020

# The story of a proof: from paper to kernel

**Proposition 3.2.** Consider the composite system  $T = C \times_{\mathcal{I}} S$  working under the assumption that choice inputs arrive only at odd cycles. Then, the system correctly implements the starvation freedom constraint of the 1 Diner problem which states that the philosopher doesn't remain hungry forever. It defers to the philosopher's own choice (stay at the same state or switch to the next) when the philosopher is not hungry.

*Proof.* The result follows from the following propositions, which are simple consequences of the polled dynamics:

1. if  $a = h$ , then  $a' = e$ .
2. if  $a \neq h$ , then  $a' = f_P(a, b)$ .

□

# The story of a proof: from paper to kernel

**Proposition 3.2.** Consider the composite system  $T = C \times_I S$  working under the assumption that choice inputs arrive only at odd cycles. Then, the system correctly implements the starvation freedom constraint of the 1 Diner problem which states that the philosopher doesn't remain hungry forever. It defers to the philosopher's own choice (stay at the same state or switch to the next) when the philosopher is not hungry.

*Proof.* The result follows from the following propositions, which are simple consequences of the polled dynamics:

1. if  $a = h$ , then  $a' = e$ .
2. if  $a \neq h$ , then  $a' = f_P(a, b)$ .

□

```
Lemma system38_starvation_free:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SSSN: ValidTrace system38 ss ts (S(S(S n))))
    (BOTTOM_EVEN:
      forall (i: nat) (IEVEN: even i = true),
      snd (fst (ts i)) = nothing choice)
    (NOT_BOTTOM_ODD:
      forall (i: nat) (IODD: odd i = true),
      snd(fst (ts i)) <> nothing choice)
    (HUNGRY: fst (ss n) = h),
  exists (m: nat), m > n /\  fst (ss m) = e.
```

*Proof.*

*(\* 30 lines of proof,  
200 lines of supporting lemmas omitted \*)*

*Qed.*

```
Lemma system_38_phil_not_hungry_then_next_philo_choice:
  forall (n: nat) (ss: nat -> the * cmd) (c: choice)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SSSN: ValidTrace system38 ss ts (S(S(S n))))
    (NOTHUNGRY: fst (ss (S n)) <> h)
    (CHOICE: snd (fst (ts (S n))) = just choice c)
    (NEVEN: even n = true)
    (BOTTOM_EVEN:
      forall (i: nat) (IEVEN: even i = true),
      snd (fst (ts i)) = nothing choice),
  fst (ss (S(S(S n)))) = trans32fn (fst (ss (S n))) c.
```

*Proof.*

*(\* 20 lines of proof,  
200 lines of supporting lemmas omitted \*)*

*Qed.*

# Statistics

- ▶ 667 lines of coq code.

# Statistics

- ▶ 667 lines of coq code.
- ▶ All tables upto section 4 verified by computation.
- ▶ All theorems upto section 4 formally verified.

# The definitions: Systems

$\text{System} \equiv (X : \mathbf{Set}, U : \mathbf{Set}, X_0 \subseteq X, \rightarrow \subseteq X \times U \times X).$

# The definitions: Systems

$\text{System} \equiv (X : \mathbf{Set}, U : \mathbf{Set}, X_0 \subseteq X, \rightarrow \subseteq X \times U \times X).$

Recall:  $X_0 \subseteq X \iff \text{member}(X_0) : X \rightarrow \text{Bool}; \text{member}(X_0)(x) \equiv x \underset{?}{\in} X_0$

# The definitions: Systems

$\text{System} \equiv (X : \mathbf{Set}, U : \mathbf{Set}, X_0 \subseteq X, \rightarrow \subseteq X \times U \times X).$

Recall:  $X_0 \subseteq X \iff \text{member}(X_0) : X \rightarrow \text{Bool}; \text{member}(X_0)(x) \equiv x \underset{?}{\in} X_0$

*(\* 2.1: system specifcation \*)*

```
Record system (X: Set) (U: Set) :=  
  mkssystem { isx0: X -> Prop;  
              trans: X -> U -> X -> Prop }.
```

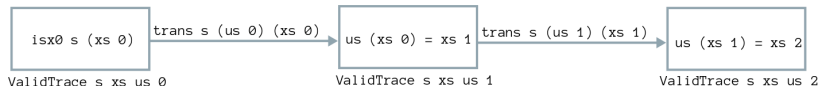


# Running a System: Valid Traces

```
Record system (X: Set) (U: Set) :=  
  mkssystem { isx0: X -> Prop;  
              trans: X -> U -> X -> Prop }.
```

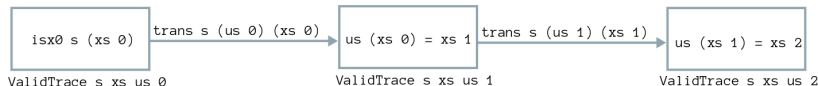
# Running a System: Valid Traces

```
Record system (X: Set) (U: Set) :=  
  mkssystem { isx0: X -> Prop;  
              trans: X -> U -> X -> Prop }.
```



# Running a System: Valid Traces

```
Record system (X: Set) (U: Set) :=  
  mkssystem { isx0: X -> Prop;  
              trans: X -> U -> X -> Prop }.
```



```
(* ValidTrace s xs us n: trace suggested by xs, us is valid for n steps *)  
Inductive ValidTrace {X U: Set} (s: system X U) (xs: nat -> X) (us: nat -> U): nat -> Prop :=  
| Start: forall (VALID: (isx0 X U s) (xs 0)) , ValidTrace s xs us 0  
| Cons: forall (n: nat) (TILLN: ValidTrace s xs us n) (ATN: trans X U s (xs n) (us n) (xs (S n))),  
ValidTrace s xs us (S n).
```

# Composing Systems: Tabuada Connection

The sets:

$$S \equiv (X : \mathbf{Set}, U_X : \mathbf{Set}, X_0 \subseteq X, \overrightarrow{\quad}_X \subseteq X \times U_X \times X).$$

$$T \equiv (Y : \mathbf{Set}, U_Y : \mathbf{Set}, Y_0 \subseteq Y, \overrightarrow{\quad}_Y \subseteq Y \times U_Y \times Y).$$

# Composing Systems: Tabuada Connection

The sets:

$$S \equiv (X : \mathbf{Set}, U_X : \mathbf{Set}, X_0 \subseteq X, \xrightarrow{X} \subseteq X \times U_X \times X).$$

$$T \equiv (Y : \mathbf{Set}, U_Y : \mathbf{Set}, Y_0 \subseteq Y, \xrightarrow{Y} \subseteq Y \times U_Y \times Y).$$

The interconnect:

$$\mathcal{I} \subseteq (X \times Y) \times (U_X \times U_Y)$$

# Composing Systems: Tabuada Connection

The sets:

$$S \equiv (X : \mathbf{Set}, U_X : \mathbf{Set}, X_0 \subseteq X, \xrightarrow{X} \subseteq X \times U_X \times X).$$

$$T \equiv (Y : \mathbf{Set}, U_Y : \mathbf{Set}, Y_0 \subseteq Y, \xrightarrow{Y} \subseteq Y \times U_Y \times Y).$$

The interconnect:

$$\mathcal{I} \subseteq (X \times Y) \times (U_X \times U_Y)$$

The composition:

$$S \times_{\mathcal{I}} T \equiv (Z \equiv X \times Y, U_Z \equiv U_X \times U_Y, X_0 \times Y_0, \xrightarrow{Z, \mathcal{I}} \subseteq Z \times U_Z \times Z).$$

$$(x, y) \xrightarrow{Z, \mathcal{I}}^{u_x, u_y} (x', y') \iff x \xrightarrow{u_x} x' \wedge y \xrightarrow{u_y} y' \wedge (x, y, u_x, u_y) \in \mathcal{I}.$$

# Composing Systems: Tabuada Connection

The sets:

$$S \equiv (X : \mathbf{Set}, U_X : \mathbf{Set}, X_0 \subseteq X, \xrightarrow{X} \subseteq X \times U_X \times X).$$

$$T \equiv (Y : \mathbf{Set}, U_Y : \mathbf{Set}, Y_0 \subseteq Y, \xrightarrow{Y} \subseteq Y \times U_Y \times Y).$$

The interconnect:

$$\mathcal{I} \subseteq (X \times Y) \times (U_X \times U_Y)$$

The composition:

$$S \times_{\mathcal{I}} T \equiv (Z \equiv X \times Y, U_Z \equiv U_X \times U_Y, X_0 \times Y_0, \xrightarrow{Z, \mathcal{I}} \subseteq Z \times U_Z \times Z).$$

$$(x, y) \xrightarrow{U_X, U_Y}_{Z, \mathcal{I}} (x', y') \iff x \xrightarrow{U_X} x' \wedge y \xrightarrow{U_Y} y' \wedge (x, y, u_x, u_y) \in \mathcal{I}.$$

*(\* 2.2: system composition \*)*

*(\* tabuada connection new system \*)*

**Definition** tabuada {X Y UX UY: Set}

```
(sx: system X UX) (sy: system Y UY) (connect: X*Y->UX*UY->Prop): system (X*Y) (UX*UY) :=  
  mkssystem (X*Y) (UX*UY) (tabuada_start (isx0 X UX sx) (isy0 Y UY sy))  
  (tabuada_trans connect (trans X UX sx) (trans Y UY sy)).
```

*(\* initial state for tabuada composition \*)*

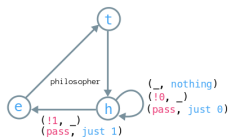
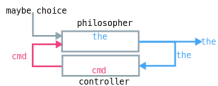
**Definition** tabuada\_start {X Y: Type} (isx0: X -> Prop) (isy0: Y -> Prop) (x: X \* Y): Prop :=  
 isx0 (fst x) /\ isy0 (snd x).

*(\* transition fn for tabuada composition \*)*

**Definition** tabuada\_trans {X Y: Type} {UX UY: Type}

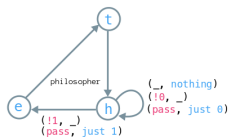
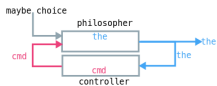
```
(connect: X*Y->UX*UY->Prop) (transx: X -> UX -> X -> Prop) (transy: Y -> UY -> Y -> Prop)  
  (s: X*Y) (u: UX*UY) (s': X*Y): Prop :=  
  transx (fst s) (fst u) (fst s') /\  
  transy (snd s) (snd u) (snd s') /\  
  (connect s u).
```

# One diner system





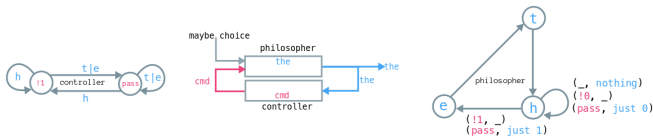
# One diner system



(\* 2.1: system specifcation \*)

**Record** system (X: Set) (U: Set) := mksystem { isx0: X -> Prop; trans: X -> U -> X -> Prop }.

# One diner system

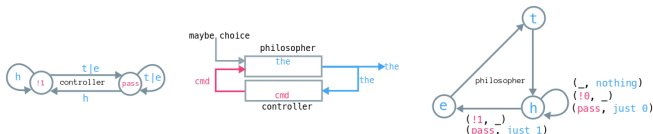


(\* 2.1: system specifcation \*)

**Record** system (X: Set) (U: Set) := mkssystem { isx0: X -> **Prop**; trans: X -> U -> X -> **Prop** }.

**Definition** system38 : system := tabuada phil37 controller34 connect38.

# One diner system



(\* 2.1: system specifcation \*)

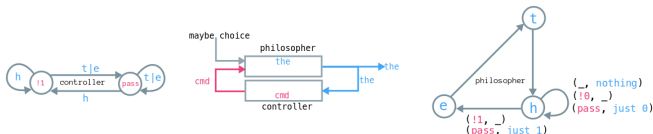
**Record** system (X: Set) (U: Set) := mkssystem { isx0: X -> Prop; trans: X -> U -> X -> Prop }.

**Definition** system38 : system := tabuada phil37 controller34 connect38.

**Definition** phil37 := mkssystem the (cmd \* maybe choice) isthinking (fun s u s' => trans37fn s u = s').

**Definition** controller34 := mkssystem cmd the ispass (fun s u s' => trans34fn s u = s').

# One diner system



(\* 2.1: system specification \*)

Record system (X: Set) (U: Set) := mkssystem { isx0: X -> Prop; trans: X -> U -> X -> Prop }.

Definition system38 : system := tabuada phil37 controller34 connect38.

Definition phil37 := mkssystem the (cmd \* maybe choice) isthinking (fun s u s' => trans37fn s u = s').

Definition controller34 := mkssystem cmd the ispass (fun s u s' => trans34fn s u = s').

(\* Philosopher spec \*)

Definition isthinking (s: the): Prop := s = t.

Inductive cmd := cmd\_pass | cmd\_bang0 | cmd\_bang1.

Definition trans37fnDEPR (s: the) (u: cmd \* maybe choice): the :=

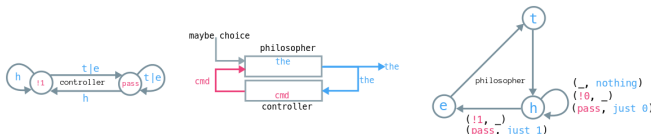
match u with

| (\_, nothing \_) => s (\* this looks fishy! This is order-sensitive \*)

| (cmd\_bang0, \_) => s | (cmd\_bang1, \_) => next s | (cmd\_pass, just \_ ch) => trans32fn s ch

end.

# One diner system



(\* 2.1: system specification \*)

**Record** system (X: Set) (U: Set) := mkssystem { isx0: X -> Prop; trans: X -> U -> X -> Prop }.

**Definition** system38 : system := tabuada phil37 controller34 connect38.

**Definition** phil37 := mkssystem the (cmd \* maybe choice) isthinking (fun s u s' => trans37fn s u = s').

**Definition** controller34 := mkssystem cmd the ispass (fun s u s' => trans34fn s u = s').

(\* Philosopher spec \*)

**Definition** isthinking (s: the): Prop := s = t.

**Inductive** cmd := cmd\_pass | cmd\_bang0 | cmd\_bang1.

**Definition** trans37fnDEPR (s: the) (u: cmd \* maybe choice): the :=

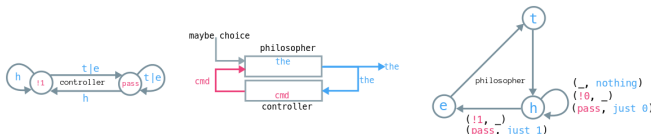
```
match u with
| (_, nothing _) => s (* this looks fishy! This is order-sensitive *)
| (cmd_bang0, _) => s | (cmd_bang1, _) => next s | (cmd_pass, just _ ch) => trans32fn s ch
end.
```

**Inductive** choice := choice\_0 | choice\_1.

**Definition** trans32fn (s: the) (c: choice): the :=

```
match c with
| choice_0 => s | choice_1 => next s end.
```

# One diner system



(\* 2.1: system specification \*)

Record system (X: Set) (U: Set) := mkssystem { isx0: X -> Prop; trans: X -> U -> X -> Prop }.

Definition system38 : system := tabuada phil37 controller34 connect38.

Definition phil37 := mkssystem the (cmd \* maybe choice) isthinking (fun s u s' => trans37fn s u = s').

Definition controller34 := mkssystem cmd the ispass (fun s u s' => trans34fn s u = s').

(\* Philosopher spec \*)

Definition isthinking (s: the): Prop := s = t.

Inductive cmd := cmd\_pass | cmd\_bang0 | cmd\_bang1.

Definition trans37fnDEPR (s: the) (u: cmd \* maybe choice): the :=

```
match u with
| (_, nothing _) => s (* this looks fishy! This is order-sensitive *)
| (cmd_bang0, _) => s | (cmd_bang1, _) => next s | (cmd_pass, just _ ch) => trans32fn s ch
end.
```

Inductive choice := choice\_0 | choice\_1.

Definition trans32fn (s: the) (c: choice): the :=

```
match c with | choice_0 => s | choice_1 => next s end.
```

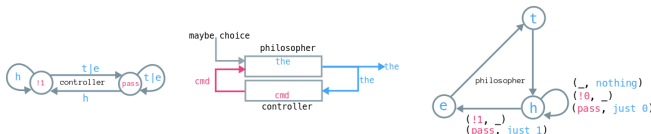
(\* Controller spec \*)

Definition ispass (c: cmd): Prop := c = cmd\_pass.

Definition trans34fn (s: cmd) (u: the): cmd :=

```
match u with | h => cmd_bang1 | e => cmd_pass | t => cmd_pass end.
```

# One diner system



(\* 2.1: system specification \*)

Record system (X: Set) (U: Set) := mkssystem { isx0: X -> Prop; trans: X -> U -> X -> Prop }.

Definition system38 : system := tabuada phil37 controller34 connect38.

Definition phil37 := mkssystem the (cmd \* maybe choice) isthinking (fun s u s' => trans37fn s u = s').

Definition controller34 := mkssystem cmd the ispass (fun s u s' => trans34fn s u = s').

(\* Philosopher spec \*)

Definition isthinking (s: the): Prop := s = t.

Inductive cmd := cmd\_pass | cmd\_bang0 | cmd\_bang1.

Definition trans37fnDEPR (s: the) (u: cmd \* maybe choice): the :=

```
match u with
| (_, nothing _) => s (* this looks fishy! This is order-sensitive *)
| (cmd_bang0, _) => s | (cmd_bang1, _) => next s | (cmd_pass, just _ ch) => trans32fn s ch
end.
```

Inductive choice := choice\_0 | choice\_1.

Definition trans32fn (s: the) (c: choice): the :=

```
match c with | choice_0 => s | choice_1 => next s end.
```

(\* Controller spec \*)

Definition ispass (c: cmd): Prop := c = cmd\_pass.

Definition trans34fn (s: cmd) (u: the): cmd :=

```
match u with | h => cmd_bang1 | e => cmd_pass | t => cmd_pass end.
```

(\* Connection Spec \*)

Definition connect38 (xy: the \* cmd)(ux\_uy: cmd \* (maybe choice) \* the): Prop :=

```
(fst xy) = (snd ux_uy) /\ (snd xy = fst (fst ux_uy)).
```

## Pain point: Non determinism

Functions are **computational**; Relations are not.



## Pain point: Non determinism

Functions are **computational**; Relations are not.

**Inductive** X: **Set** := X1 | X2 | X3.    **Inductive** Y: **Set** := Y1 | Y2 | Y3.

## Pain point: Non determinism

Functions are **computational**; Relations are not.

```
Inductive X: Set := X1 | X2 | X3. Inductive Y: Set := Y1 | Y2 | Y3.
```

```
Definition x2y_fn (x: X): Y :=  
  match x with | X1 => Y1 | X2 => Y2 | X3 => Y3 end.
```

## Pain point: Non determinism

Functions are **computational**; Relations are not.

```
Inductive X: Set := X1 | X2 | X3. Inductive Y: Set := Y1 | Y2 | Y3.
```

```
Definition x2y_fn (x: X): Y :=  
  match x with | X1 => Y1 | X2 => Y2 | X3 => Y3 end.
```

```
(* Works; computational *)
```

```
Lemma x1_to_y1_fn: x2y_fn X1 = Y1. Proof. reflexivity. Qed.
```

# Pain point: Non determinism

Functions are **computational**; Relations are not.

```
Inductive X: Set := X1 | X2 | X3. Inductive Y: Set := Y1 | Y2 | Y3.
```

```
Definition x2y_fn (x: X): Y :=  
  match x with | X1 => Y1 | X2 => Y2 | X3 => Y3 end.
```

```
(* Works; computational *)
```

```
Lemma x1_to_y1_fn: x2y_fn X1 = Y1. Proof. reflexivity. Qed.
```

```
Definition x2y_rel (x: X) (y: Y): Prop :=  
  (x = X1 /\ y = Y1) \/ (x = X2 /\ y = Y2) \/ (x = X3 /\ y = Y3).
```

```
Lemma x1_to_y2_rel: x2y_rel X1 Y1.
```

```
Proof.
```

```
  try reflexivity. (* Does not work! Not computational *)
```

```
  unfold x2y_rel. left. auto.
```

```
Qed.
```

# Pain point: Non determinism

Functions are **computational**; Relations are not.

```
Inductive X: Set := X1 | X2 | X3. Inductive Y: Set := Y1 | Y2 | Y3.
```

```
Definition x2y_fn (x: X): Y :=  
  match x with | X1 => Y1 | X2 => Y2 | X3 => Y3 end.
```

```
(* Works; computational *)
```

```
Lemma x1_to_y1_fn: x2y_fn X1 = Y1. Proof. reflexivity. Qed.
```

```
Definition x2y_rel (x: X) (y: Y): Prop :=  
  (x = X1 /\ y = Y1) \/ (x = X2 /\ y = Y2) \/ (x = X3 /\ y = Y3).
```

```
Lemma x1_to_y2_rel: x2y_rel X1 Y1.
```

```
Proof.
```

```
  try reflexivity. (* Does not work! Not computational *)
```

```
  unfold x2y_rel. left. auto.
```

```
Qed.
```

```
(* 2.1: system specification *)
```

```
Record system (X: Set) (U: Set) :=
```

```
  mkssystem { isx0: X -> Prop; trans: X -> U -> X -> Prop }.
```

## Pain point: Modularity

`ValidTrace`  $\mapsto$  `system38`  $\mapsto$  `tabuada`  $\mapsto$  `phil37`  $\mapsto$  `trans37`  $\mapsto$  `trans32`

# Pain point: Modularity

`ValidTrace`  $\mapsto$  `system38`  $\mapsto$  `tabuada`  $\mapsto$  `phil37`  $\mapsto$  `trans37`  $\mapsto$  `trans32`

```
1  Lemma system38_s_the_to_t_the:
2    forall (n: nat) (ss: nat -> the * cmd) (ts: nat -> cmd * maybe choice * the)
3      (TRACE: ValidTrace system38 ss ts (S n)), snd (ts n) = fst (ss n).
4  Proof.
5    intros.
6    inversion TRACE as [TRACE1 | npred TRACE1 AT1]. subst. (* 1 *)
7    inversion AT1 as [AT11 [AT12 AT13]]. (* 2 *)
8    set (s1 := ss 1) in *.
9    destruct s1 as [s1_the s1_cmd].
10   set (t1 := ts 1) in *.
11   destruct t1 as [[t1_cmd t1_mchoice] t1_the]. simpl in *.
12   inversion AT13; simpl in *. (* 3; useful info *)
13   auto.
14  Qed.
```

# Pain point: Modularity

ValidTrace  $\mapsto$  system38  $\mapsto$  tabuada  $\mapsto$  phil37  $\mapsto$  trans37  $\mapsto$  trans32

```
1  Lemma system38_s_the_to_t_the:
2    forall (n: nat) (ss: nat -> the * cmd) (ts: nat -> cmd * maybe choice * the)
3      (TRACE: ValidTrace system38 ss ts (S n)), snd (ts n) = fst (ss n).
4  Proof.
5    intros.
6    inversion TRACE as [TRACE1 | npred TRACE1 AT1]. subst. (* 1 *)
7    inversion AT1 as [AT11 [AT12 AT13]]. (* 2 *)
8    set (s1 := ss 1) in *.
9    destruct s1 as [s1_the s1_cmd].
10   set (t1 := ts 1) in *.
11   destruct t1 as [[t1_cmd t1_mchoice] t1_the]. simpl in *.
12   inversion AT13; simpl in *. (* 3; useful info *)
13   auto.
14  Qed.

n : nat
ss : nat -> the * cmd
ts : nat -> cmd * maybe choice * the
TRACE : ValidTrace system38 ss ts (S n)
TRACE1 : ValidTrace system38 ss ts n
AT1 : tabuada.trans connect38
      (fun (s : the) (u : cmd * maybe choice) (s' : the) => trans37fn s u = s')
      (fun (s : cmd) (u : the) (s' : cmd) => trans34fn s u = s') (ss n)
      (ts n) (ss (S n))
AT11 : trans37fn (fst (ss n)) (fst (ts n)) = fst (ss (S n))
AT12 : trans34fn (snd (ss n)) (snd (ts n)) = snd (ss (S n))
AT13 : connect38 (ss n) (ts n)
s1_the : the
s1_cmd, t1_cmd : cmd
t1_mchoice : maybe choice
t1_the : the
H : fst (ss n) = snd (ts n)
H0 : snd (ss n) = fst (fst (ts n))
=====
snd (ts n) = fst (ss n)
```



# What about tactics?!

# What about tactics?!

Tactics work for  $LHS = RHS$ :

# What about tactics?!

Tactics work for  $LHS = RHS$ :

$t$	$b_{\perp}$	desired	$a$	$c$
0	$\perp$	t	t	pass
1	1	t	t	pass
2	$\perp$	h	h	pass
3	0	h	h	!1
4	$\perp$	e	e	!1
5	0	e	e	pass
6	$\perp$	e	e	pass
7	0	e	e	pass
8	$\perp$	e	e	pass
9	1	e	e	pass
10	$\perp$	t	t	pass

```
(* Verify table 3 *)
Definition states_table_3
(n: nat): the * cmd :=
match n with
| 0 => (t, cmd_pass)
| 1 => (t, cmd_pass)
| 2 => (h, cmd_pass)
| 3 => (h, cmd_bang1)
| 4 => (e, cmd_bang1)
| 5 => (e, cmd_pass)
| 6 => (e, cmd_pass)
| 7 => (e, cmd_pass)
| 8 => (e, cmd_pass)
| 9 => (e, cmd_pass)
| 10 => (t, cmd_pass)
| _ => (t, cmd_pass)
end.
```

```
(* Verify table 3 *)
Definition trans_table_3
(n: nat): cmd * maybe choice * the :=
match n with
| 0 => (cmd_pass, nothing _, t)
| 1 => (cmd_pass, just _ choice_1, t)
| 2 => (cmd_pass, nothing _, h)
| 3 => (cmd_bang1, just _ choice_0, h)
| 4 => (cmd_bang1, nothing _, e)
| 5 => (cmd_pass, just _ choice_0, e)
| 6 => (cmd_pass, nothing _, e)
| 7 => (cmd_pass, just _ choice_0, e)
| 8 => (cmd_pass, nothing _, e)
| 9 => (cmd_pass, just _ choice_1, e)
| 10 => (cmd_pass, nothing _, t)
| _ => (cmd_pass, nothing _, t)
end.
```

Example valid\_trace\_table3\_step10: ValidTrace system38 states\_table\_3 trans\_table\_3 10.

Proof. repeat (try constructor; simpl; try apply valid\_trace\_system35\_step1; try apply tabuada\_start). Qed.

# What about tactics?!

Tactics work for  $LHS = RHS$ :

$t$	$b_{\perp}$	desired	$a$	$c$
0	$\perp$	t	t	pass
1	1	t	t	pass
2	$\perp$	h	h	pass
3	0	h	h	!1
4	$\perp$	e	e	!1
5	0	e	e	pass
6	$\perp$	e	e	pass
7	0	e	e	pass
8	$\perp$	e	e	pass
9	1	e	e	pass
10	$\perp$	t	t	pass

```
(* Verify table 3 *)
Definition states_table_3
(n: nat): the * cmd :=
match n with
| 0 => (t, cmd_pass)
| 1 => (t, cmd_pass)
| 2 => (h, cmd_pass)
| 3 => (h, cmd_bang1)
| 4 => (e, cmd_bang1)
| 5 => (e, cmd_pass)
| 6 => (e, cmd_pass)
| 7 => (e, cmd_pass)
| 8 => (e, cmd_pass)
| 9 => (e, cmd_pass)
| 10 => (t, cmd_pass)
| _ => (t, cmd_pass)
end.
```

```
(* Verify table 3 *)
Definition trans_table_3
(n: nat): cmd * maybe choice * the :=
match n with
| 0 => (cmd_pass, nothing _, t)
| 1 => (cmd_pass, just _ choice_1, t)
| 2 => (cmd_pass, nothing _, h)
| 3 => (cmd_bang1, just _ choice_0, h)
| 4 => (cmd_bang1, nothing _, e)
| 5 => (cmd_pass, just _ choice_0, e)
| 6 => (cmd_pass, nothing _, e)
| 7 => (cmd_pass, just _ choice_0, e)
| 8 => (cmd_pass, nothing _, e)
| 9 => (cmd_pass, just _ choice_1, e)
| 10 => (cmd_pass, nothing _, t)
| _ => (cmd_pass, nothing _, t)
end.
```

Example valid\_trace\_table3\_step10: ValidTrace system38 states\_table\_3 trans\_table\_3 10.

Proof. repeat (try constructor; simpl; try apply valid\_trace\_system35\_step1; try apply tabuada\_start). Qed.

... Not to derive a proof:

# What about tactics?

Tactics work for  $LHS = RHS$ :

$t$	$b_{\perp}$	desired	$a$	$c$
0	$\perp$	t	t	pass
1	1	t	t	pass
2	$\perp$	h	h	pass
3	0	h	h	!t
4	$\perp$	e	e	!t
5	0	e	e	pass
6	$\perp$	e	e	pass
7	0	e	e	pass
8	$\perp$	e	e	pass
9	1	e	e	pass
10	$\perp$	t	t	pass

```
(* Verify table 3 *)
Definition states_table_3
(n: nat): the * cmd :=
match n with
| 0 => (t, cmd_pass)
| 1 => (t, cmd_pass)
| 2 => (h, cmd_pass)
| 3 => (h, cmd_bang1)
| 4 => (e, cmd_bang1)
| 5 => (e, cmd_pass)
| 6 => (e, cmd_pass)
| 7 => (e, cmd_pass)
| 8 => (e, cmd_pass)
| 9 => (e, cmd_pass)
| 10 => (t, cmd_pass)
| _ => (t, cmd_pass)
end.
```

```
(* Verify table 3 *)
Definition trans_table_3
(n: nat): cmd * maybe choice * the :=
match n with
| 0 => (cmd_pass, nothing _, t)
| 1 => (cmd_pass, just _ choice_1, t)
| 2 => (cmd_pass, nothing _, h)
| 3 => (cmd_bang1, just _ choice_0, h)
| 4 => (cmd_bang1, nothing _, e)
| 5 => (cmd_pass, just _ choice_0, e)
| 6 => (cmd_pass, nothing _, e)
| 7 => (cmd_pass, just _ choice_0, e)
| 8 => (cmd_pass, nothing _, e)
| 9 => (cmd_pass, just _ choice_1, e)
| 10 => (cmd_pass, nothing _, t)
| _ => (cmd_pass, nothing _, t)
end.
```

Example valid\_trace\_table3\_step10: ValidTrace system38 states\_table\_3 trans\_table\_3 10.

Proof. repeat (try constructor; simpl; try apply valid\_trace\_system35\_step1; try apply tabuada\_start). Qed.

... Not to derive a proof:

```
Lemma system38_s_the_to_t_the_tactics:
forall (n: nat)
(ss: nat -> the * cmd)
(ts: nat -> cmd * maybe choice * the)
(TRACE: ValidTrace system38 ss ts (S n)),
snd (ts n) = fst (ss n).
Proof.
intros.
repeat (try constructor;
simpl;
try apply valid_trace_system35_step1;
try apply tabuada_start).
Abort.
```

```
Lemma system38_s_the_to_t_the:
forall (n: nat)
(ss: nat -> the * cmd)
(ts: nat -> cmd * maybe choice * the)
(TRACE: ValidTrace system38 ss ts (S n)),
snd (ts n) = fst (ss n).
Proof.
intros.
inversion TRACE as [TRACE1 | npred TRACE1 AT1]. subst. (* 1 *)
inversion AT1 as [AT11 [AT12 AT13]]. (* 2 *)
set (s1 := ss 1) in *.
destruct s1 as [s1_the s1_cmd].
set (t1 := ts 1) in *.
destruct t1 as [[t1_cmd t1_mchoice] t1_the]. simpl in *.
inversion AT13; simpl in *. (* 3; useful info *)
auto.
Qed.
```

# Hand-write Lemmas to drive reasoning

```
(* Helper: reason about even/odd *)
Lemma even_n_odd_Sn: forall (n: nat),
  (even n = true) <-> (odd (S n) = true).
(* Helper: reason about even/odd *)
Lemma odd_n_even_Sn: forall (n: nat),
  (odd n = true) <-> (even (S n) = true).
(* Rewrite ts in terms of ss *)
Lemma s_cmd_to_t_cmd:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE: ValidTrace system38 ss ts (S n)),
    fst (fst (ts n)) = snd(ss n).
(* Rewrite ss in terms of ts *)
Lemma s_the_to_t_the:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE: ValidTrace system38 ss ts (S n)),
    snd (ts n) = fst (ss n).
(* 'h' leads to '!1' command next. *)
Lemma phil_hungry_then_next_controller_bang1:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts (S n))
    (HUNGRY: fst (ss n) = h),
    snd (ss (S n)) = cmd_bang1.
```

```
(* not 'h' in leads to pass next *)
Lemma phil_not_hungry_then_next_controller_pass:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts (S n))
    (NOTHUNGRY: fst (ss n) <> h),
    snd (ss (S n)) = cmd_pass.
(* Phil. state in the next-odd-state = cur-state *)
Lemma phil_even_state_next_state:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts ((S n)))
    (BOTTOM_EVEN: ...)
    (NEVEN: even n = true),
    fst (ss (S n)) = fst (ss n).
(* Phil. state in the cur-odd-state = prev-state *)
Lemma phil_odd_state_prev_state:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts ((S n)))
    (BOTTOM_EVEN: ...)
    (SNODD: odd (S n) = true),
    fst (ss (S n)) = fst (ss n).
(* state after odd cycle has taken transition *)
Lemma odd_state_next_phil_state:
  forall (n: nat) (ss: nat -> the * cmd)
    (ts: nat -> cmd * maybe choice * the)
    (TRACE_SN: ValidTrace system38 ss ts ((S n)))
    (BOTTOM_EVEN: ...)
    (SNODD: odd n = true),
    fst (ss (S n)) = trans37fn (fst (ss n)) (fst (ts n)).
```

# Takeaways

- ▶ Coq is no silver bullet.

# Takeaways

- ▶ Coq is no silver bullet.
- ▶ Non-determinism + Modularity  $\implies$  Coq's metatheory is useless.



# Takeaways

- ▶ Coq is no silver bullet.
- ▶ Non-determinism + Modularity  $\implies$  Coq's metatheory is useless.
- ▶ N-diner will be *extremely* painful, is my estimate.

# Thank you

All code available at:

<https://github.com/bollu/IIIT-H-code/tree/master/softwarefoundations/project>

# Proof structure: By-hand

$t$	$b_{\perp}$	desired	$a$	$c$
0	$\perp$	t	t	pass
1	1	t	t	pass
2	$\perp$	h	h	pass
3	0	h	h	!1
4	$\perp$	e	e	!1
5	0	e	e	pass
6	$\perp$	e	e	pass
7	0	e	e	pass
8	$\perp$	e	e	pass
9	1	e	e	pass
10	$\perp$	t	t	pass

```
PAST EVEN: n    (s| phil: h);  
| [phil sends pass]  
CURRENT ODD: n + 1 (s| phil: h, cont: !1); [cont receives h]  
| [cont sends !1]  
NEXT EVEN: n + 2 (s| phil: e)  
| [phil sends pass]  
NEXT NEXT ODD: n + 3 (s| phil: e)
```

# Proof structure: Coq

```
1  Lemma system38_polled_if_hungry_odd_state_then_eat:
2    forall (n: nat)
3      (NODD: odd (S n) = true)
4      (ss: nat -> the * cmd)
5      (ts: nat -> cmd * maybe choice * the)
6      (TRACE_SSSN: ValidTrace system38 ss ts (S (S (S n))))
7      (BOTTOM_EVEN: forall (i: nat) (IEVEN: even i = true), snd (fst (ts i)) = nothing choice)
8      (NOT_BOTTOM_ODD: forall (i: nat) (IODD: odd i = true), snd (fst (ts i)) <> nothing choice)
9      (HUNGRY: fst (ss (S n)) = h),
10     fst (ss (S (S (S n)))) = e.
11 Proof.
12   intros.
13   assert (PREV_STATE_H: fst (ss n) = h). ... (* prev state same as cur state *)
14   assert (CUR_CMD_BANG1: snd (ss (S n)) = cmd_bang1).
15   eapply system38_phil_hungry_then_next_controller_bang1; ...
16
17   assert (NEXT_STATE: fst (ss (S (S n))) = trans37fn (fst (ss (S n))) (fst (ts (S n)))).
18   eapply system38_odd_state_next_phil_state; ...
19
20   assert (NEXT_NEXT_STATE_EQ_NEXT_STATE: fst (ss (S (S (S n)))) = fst (ss (S (S n))) ).
21   eapply system38_phil_even_state_next_state; ...
22
23   assert (CHOICE_NEXT: snd (fst (ts (S n))) <> nothing choice). ...
24
25   rewrite NEXT_NEXT_STATE_EQ_NEXT_STATE.
26   rewrite NEXT_STATE.
27   rewrite CUR_CMD_BANG1.
28   rewrite HUNGRY.
29   (* prove what the value of the transition will be using case analysis. *)
30   destruct (snd (fst (ts (S n)))) ; auto ; try contradiction.
31   inversion TRACE_SSSN ; auto.
32 Qed.
```

# Phrasing of the Tabuada connection

Phrased as:

$$\mathcal{I} \subseteq S_X \times S_Y \times U_X \times U_Y$$

# Phrasing of the Tabuada connection

Phrased as:

$$\mathcal{I} \subseteq S_X \times S_Y \times U_X \times U_Y$$

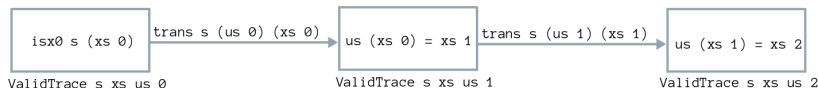
This definition has no sense of the *causality* inherent to the notion of ValidTrace.

# Phrasing of the Tabuada connection

Phrased as:

$$\mathcal{I} \subseteq S_X \times S_Y \times U_X \times U_Y$$

This definition has no sense of the *causality* inherent to the notion of ValidTrace.

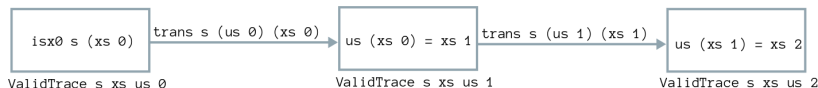


# Phrasing of the Tabuada connection

Phrased as:

$$\mathcal{I} \subseteq S_X \times S_Y \times U_X \times U_Y$$

This definition has no sense of the *causality* inherent to the notion of ValidTrace.



Better definition:

$$\mathcal{I} : S_X \times S_Y \rightarrow 2^{U_X \times U_Y}$$