slides

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Outline

Proof outline

- Algbra of polyhedra, $P(\mathbb{Q}^d)$
- $[\,] : \mathbb{Q}^d \to P(\mathbb{Q}^d)$
- Existence of $\mathcal{F} : P(\mathbb{Q}^d) \to \mathbb{C}(x)$, such that:
  - F is linear
  - P is a polyhedra, then $\mathcal{F}([P]) = \sum_{\vec{m} \in P \cap \mathbb{Z}^d} (x^{\vec{m}})$
  - $\mathcal{F}([\text{line}]) = 0$
- $\mathcal{F}(P)(1) =$ number of points in$P$
- reduction: F for cones gives full F
- reduction: F for simple cones gives F for cones

Caveats

- Do not understand subtleties of convergence arguments (how is evaluating at $\vec{1}$ correct?).
- No intuition for LLL, Lattice reduction.

Assuming $\mathcal{F}$ for cones, derive full $\mathcal{F}$: Part 1 (Polytopes)

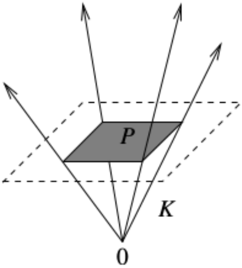


FIGURE 66. A polytope $P \subset \mathbb{R}^d$ and a cone $K \subset \mathbb{R}^{d+1}$ based on P.

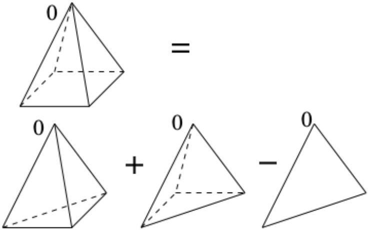
- Write polytope as intersection of hyperplane + cone.
- $\mathcal{F}(\text{polytope}) = (\frac{d}{dx}\mathcal{F}(\text{cone}))(1)$

Assuming $\mathcal{F}$ for cones, derive full $\mathcal{F}$: Part 2 (Lines)

- Line = cone + cone - point.
- Since line can be translated, $\forall \vec{x} \in L, L = \vec{x} + L$
  - $\forall x \in L, \mathcal{F}(L) = \mathcal{F}(L) + \mathcal{F}(\vec{x})$
  - $\mathcal{F}(L) = 0$

Assuming $\mathcal{F}$ for simple cone, derive for cone

- inclusion exclusion: decompose cone into simple cones.



References

- Lattice Points, Polyhedra, and Complexity: Alexander Barvinok
- Integer points in polyhedra: Alexander Barvinok