

## Optimizing smallpt

Davean Scies, Siddharth Bhat

## Haskell Exchange

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## What is smallpt anyway?

## Optimizing smallpt

What is smallpt anyway?

2020-11-05

└ What is smallpt anyway?

1. **S**
2. 99 LoC C++: **small** path tracer.
3. Ported to many languages, including Haskell!
4. Haskell port was by Vo Minh Thu. Thanks a ton!
5. Start from noteed's original source; SHA the output image from the Haskell source for baseline.
6. Perfect for an optimization case study.
7. Plan: Quick walk through Haskell code, end up at C++ (clang++) performance.

# What is smallpt anyway?

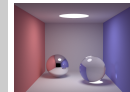


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## Optimizing smallpt

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### What is smallpt anyway?



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7. Plan: Quick walk through Haskell code, end up at C++ (clang++) performance.

# What is smallpt anyway?

```
struct Vec {
    double x, y, z; // position, also color (r,g,b)
    ... methods...
};

struct Ray { Vec o, d; Ray(Vec o_, Vec d_) : o(o_), d(d_) {} };

enum Refl_t { DIFF, SPEC, REFR }; // material types, used in radiance()

struct Sphere {
    double rad; // radius
    Vec p, e, c; // position, emission, color
    Refl_t refl; // reflection type (DIFFuse, SPECular, REFRactive)
    ... methods ...
    double intersect(const Ray &r) const // returns distance, 0 if nohit
};

Sphere spheres[] = { //Scene: radius, position, emission, color, material
    Sphere(1e5, Vec( 1e5+1,40.8,81.6), Vec(),Vec(.75,.25,.25),DIFF), //Left
    ... initialization ...
};
```

## Optimizing smallpt

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### What is smallpt anyway?

1. S
2. Has geometric primitives: vectors, spheres, materials
3. Entirely number-based, no real data structure

What is smallpt anyway?

```
struct Vec {
    double x, y, z; // position, also color (r,g,b)
    ... methods...
};

struct Ray { Vec o, d; Ray(Vec o_, Vec d_) : o(o_), d(d_) {} };

enum Refl_t { DIFF, SPEC, REFR }; // material types, used in radiance()

struct Sphere {
    double radi; // radius
    Vec p, e, c; // position, emission, color
    Refl_t refl; // reflection type (DIFFuse, SPECular, REFRactive)
    ... methods ...
    double intersect(const Ray &r) const // returns distance, 0 if nohit
};

Sphere spheres[] = { //Scene: radius, position, emission, color, material
    Sphere(1e5, Vec( 1e5+1,40.8,81.6), Vec(),Vec(.75,.25,.25),DIFF), //Left
    ... initialization ...
};
```

# What is smallpt anyway?

```
Vec radiance(const Ray &r, int depth, unsigned short *Xi){
```

```
}
```

## Optimizing smallpt

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What is smallpt anyway?

1. **S**
2. Most of the compute cost is spent in the function that traces rays.
3. is called radiance

# What is smallpt anyway?

```
Vec radiance(const Ray &r, int depth, unsigned short *Xi){
```

```
    radiance
```

```
    radiance
```

```
    radiance
```

```
    radiance  
    radiance
```

```
    radiance  
    radiance
```

```
}
```

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What is smallpt anyway?

1. **S**
2. Recursively calls itself a bunch of times



# What is smallpt anyway?

```
Vec radiance(const Ray &r, int depth, unsigned short *Xi){
```

```
    if (      ) if (      )      else
    if (      ){
```

```
        radiance
    } else if (      )
        radiance
```

```
    if (      )
        radiance
```

```
        radiance      radiance
        radiance      radiance
    }
```

## Optimizing smallpt

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What is smallpt anyway?

```
What is smallpt anyway?
Vec radiance(const Ray &r, int depth, unsigned short *Xi){

    if (      ) if (      )      else
    if (      ){

        radiance
    } else if (      )
        radiance

    if (      )
        radiance

    radiance      radiance
    radiance      radiance
}
```

1. S
2. Recursion is guarded by a lot of control flow

# What is smallpt anyway?

```
Vec radiance(const Ray &r, int depth, unsigned short *Xi){
```

```
Vec x=r.o+r.d*t, n=(x-obj.p).norm(), nl=n.dot(r.d)<0?n:n*-1, f=obj.c;
```

```
if ( ) if ( ) else  
if ( ){
```

```
radiance  
} else if ( )  
radiance
```

```
if ((cos2t=1-nnt*nnt*(1-ddn*ddn))<0)  
radiance
```

```
radiance radiance  
radiance radiance  
}
```

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What is smallpt anyway?

```
What is smallpt anyway?  
  
Vec radiance(const Ray &r, int depth, unsigned short *Xi){  
  
Vec x=r.o+r.d*t, n=(x-obj.p).norm(), nl=n.dot(r.d)<0?n:n*-1, f=obj.c;  
if ( ) if ( ) else  
if ( ){  
  
radiance  
radiance  
}  
else if ( )  
radiance  
radiance  
}  
if ((cos2t=1-nnt*nnt*(1-ddn*ddn))<0)  
radiance  
  
radiance radiance  
radiance radiance  
}
```

1. S
2. The control flow and computation is very numeric in nature



# What is smallpt anyway?

```
Vec radiance(const Ray &r, int depth, unsigned short *Xi){
```

```
Vec x=r.o+r.d*t, n=(x-obj.p).norm(), nl=n.dot(r.d)<0?n:n*-1, f=obj.c;
```

```
if ( ) if (erand48(Xi) ) else  
if ( ) {  
    erand48(Xi)    erand48(Xi)
```

```
    radiance  
} else if ( )  
    radiance
```

```
if ((cos2t=1-nnt*nnt*(1-ddn*ddn))<0)  
    radiance
```

```
    erand48(Xi)  
    radiance  
    radiance  
}
```

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What is smallpt anyway?

1. S
2. We use erand48 for randomness

What is smallpt anyway?

```
Vec radiance(const Ray &r, int depth, unsigned short *Xi){
```

```
Vec x=r.o+r.d*t, n=(x-obj.p).norm(), nl=n.dot(r.d)<0?n:n*-1, f=obj.c;  
if ( ) if (erand48(Xi) ) else  
if ( ) {  
    erand48(Xi)    erand48(Xi)  
    radiance  
} else if ( )  
    radiance  
if ((cos2t=1-nnt*nnt*(1-ddn*ddn))<0)  
    radiance  
    erand48(Xi)  
    radiance  
    radiance  
}
```

# What is smallpt anyway?

```
Vec radiance(const Ray &r, int depth, unsigned short *Xi){
    double t; // distance to intersection
    int id=0; // id of intersected object
    if (!intersect(r, t, id)) return Vec(); // if miss, return black
    const Sphere &obj = spheres[id]; // the hit object
    Vec x=r.o+r.d*t, n=(x-obj.p).norm(), nl=n.dot(r.d)<0?n:n*-1, f=obj.c;
    double p = f.x>f.y && f.x>f.z ? f.x : f.y>f.z ? f.y : f.z; // max refl
    if (++depth>5) if (erand48(Xi)<p) f=f*(1/p); else return obj.e; //R.R.
    if (obj.refl == DIFF){ // Ideal DIFFUSE reflection
        double r1=2*M_PI*erand48(Xi), r2=erand48(Xi), r2s=sqrt(r2);
        Vec w=nl, u=((fabs(w.x)>.1?Vec(0,1):Vec(1))%w).norm(), v=w%u;
        Vec d = (u*cos(r1)*r2s + v*sin(r1)*r2s + w*sqrt(1-r2)).norm();
        return obj.e + f.mult(radiance(Ray(x,d),depth,Xi));
    } else if (obj.refl == SPEC) // Ideal SPECULAR reflection
        return obj.e + f.mult(radiance(Ray(x,r.d-n*2*n.dot(r.d)),depth,Xi));
    Ray reflRay(x, r.d-n*2*n.dot(r.d)); // Ideal dielectric REFRACTION
    bool into = n.dot(nl)>0; // Ray from outside going in?
    double nc=1, nt=1.5, nnt=into?nc/nt:nt/nc, ddn=r.d.dot(nl), cos2t;
    if ((cos2t=1-nnt*nnt*(1-ddn*ddn))<0) // Total internal reflection
        return obj.e + f.mult(radiance(reflRay,depth,Xi));
    Vec tdir = (r.d*nnt - n*((into?-1):1)*(ddn*nnt+sqrt(cos2t))).norm();
    double a=nt-nc, b=nt+nc, R0=a*a/(b*b), c = 1-(into?-ddn:tdir.dot(n));
    double Re=R0+(1-R0)*c*c*c*c*c, Tr=1-Re, P=.25+.5*Re, RP=Re/P, TP=Tr/(1-P);
    return obj.e + f.mult(depth>2 ? (erand48(Xi)<P ? // Russian roulette
        radiance(reflRay,depth,Xi)*RP:radiance(Ray(x,tdir),depth,Xi)*TP) :
        radiance(reflRay,depth,Xi)*Re+radiance(Ray(x,tdir),depth,Xi)*Tr);
}
```

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What is smallpt anyway?

- 1. S
- 2. The full code continues to be more of the same

What is smallpt anyway?

```
Vec radiance(const Ray &r, int depth, unsigned short *Xi){
    double t; // distance to intersection
    int id=0; // id of intersected object
    if (!intersect(r, t, id)) return Vec(); // if miss, return black
    const Sphere &obj = spheres[id]; // the hit object
    Vec x=r.o+r.d*t, n=(x-obj.p).norm(), nl=n.dot(r.d)<0?n:n*-1, f=obj.c;
    double p = f.x>f.y && f.x>f.z ? f.x : f.y>f.z ? f.y : f.z; // max refl
    if (++depth>5) if (erand48(Xi)<p) f=f*(1/p); else return obj.e; //R.R.
    if (obj.refl == DIFF){ // Ideal DIFFUSE reflection
        double r1=2*M_PI*erand48(Xi), r2=erand48(Xi), r2s=sqrt(r2);
        Vec w=nl, u=((fabs(w.x)>.1?Vec(0,1):Vec(1))%w).norm(), v=w%u;
        Vec d = (u*cos(r1)*r2s + v*sin(r1)*r2s + w*sqrt(1-r2)).norm();
        return obj.e + f.mult(radiance(Ray(x,d),depth,Xi));
    } else if (obj.refl == SPEC) // Ideal SPECULAR reflection
        return obj.e + f.mult(radiance(Ray(x,r.d-n*2*n.dot(r.d)),depth,Xi));
    Ray reflRay(x, r.d-n*2*n.dot(r.d)); // Ideal dielectric REFRACTION
    bool into = n.dot(nl)>0; // Ray from outside going in?
    double nc=1, nt=1.5, nnt=into?nc/nt:nt/nc, ddn=r.d.dot(nl), cos2t;
    if ((cos2t=1-nnt*nnt*(1-ddn*ddn))<0) // Total internal reflection
        return obj.e + f.mult(radiance(reflRay,depth,Xi));
    Vec tdir = (r.d*nnt - n*((into?-1):1)*(ddn*nnt+sqrt(cos2t))).norm();
    double a=nt-nc, b=nt+nc, R0=a*a/(b*b), c = 1-(into?-ddn:tdir.dot(n));
    double Re=R0+(1-R0)*c*c*c*c*c, Tr=1-Re, P=.25+.5*Re, RP=Re/P, TP=Tr/(1-P);
    return obj.e + f.mult(depth>2 ? (erand48(Xi)<P ? // Russian roulette
        radiance(reflRay,depth,Xi)*RP:radiance(Ray(x,tdir),depth,Xi)*TP) :
        radiance(reflRay,depth,Xi)*Re+radiance(Ray(x,tdir),depth,Xi)*Tr);
}
```

# Initial Haskell Code: radiance (1×)

```
radiance :: Ray -> CInt -> Ptr CUSHort -> IO Vec
radiance ray@(Ray o d) depth xi = case intersects ray of
  (Nothing,_) -> return zerov
  (Just t,Sphere _r p e c refl) -> do
```

```
  continue f = case refl of - BRANCHING
    DIFF -> do
      r1 <- ((2*pi)*) `fmap` erand48 xi -- RNG
```

```
      radiance

SPEC -> do

  rad <- radiance -- RECURSION

REFR -> do
```

```
  if
    then do
      rad <- radiance
    ...
```

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### Initial Haskell Code: radiance (1×)

1. S
2. the original source has the same computation in haskell

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  (Just t,Sphere _r p e c refl) -> do
```

```
  continue f = case refl of - BRANCHING
    DIFF -> do
```

```
    radiance
    rad <- radiance -- RECURSION
    refr <- do
```

```
    if
      then do
        rad <- radiance
```

# Initial Haskell Code: Data structures

```
data Vec = Vec {-# UNPACK #-} !Double {-# UNPACK #-} !Double {-# UNPACK #-} !Double
```

```
cross :: Vec -> Vec -> Vec
(.*) :: Vec -> Double -> Vec
infixl 7 .*
len :: Vec -> Double
norm :: Vec -> Vec
norm v = v .* recip (len v)
dot :: Vec -> Vec -> Double
maxv :: Vec -> Double
```

```
data Ray = Ray Vec Vec -- origin, direction
data Refl = DIFF | SPEC | REFR -- material types, used in radiance
-- / radius, position, emission, color, reflection
data Sphere = Sphere Double Vec Vec Vec Refl
```

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### Initial Haskell Code: Data structures

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```
data Vec = Vec {-# UNPACK #-} !Double {-# UNPACK #-} !Double {-# UNPACK #-} !Double
cross :: Vec -> Vec -> Vec
(.*) :: Vec -> Double -> Vec
infixl 7 .*
len :: Vec -> Double
norm :: Vec -> Vec
norm v = v .* recip (len v)
dot :: Vec -> Vec -> Double
maxv :: Vec -> Double

data Ray = Ray Vec Vec -- origin, direction
data Refl = DIFF | SPEC | REFR -- material types, used in radiance
-- / radius, position, emission, color, reflection
data Sphere = Sphere Double Vec Vec Vec Refl
```

1. S
2. We implement the same data structures in Haskell
3. Note that Ray, Sphere not having unpack

# Initial Haskell Code: scene data

```
spheres :: [Sphere]
spheres =
  [ Sphere 1e5 (Vec (1e5+1) 40.8 81.6) 0 (Vec 0.75 0.25 0.25) DIFF --Left
  , Sphere 1e5 (Vec (99-1e5) 40.8 81.6) 0 (Vec 0.25 0.25 0.75) DIFF --Rght
  , Sphere 1e5 (Vec 50 40.8 1e5) 0 0.75 DIFF --Back
  , Sphere 1e5 (Vec 50 40.8 (170-1e5)) 0 0 DIFF --Frnt
  , Sphere 1e5 (Vec 50 1e5 81.6) 0 0.75 DIFF --Botm
  , Sphere 1e5 (Vec 50 (81.6-1e5) 81.6) 0 0.75 DIFF --Top
  , Sphere 16.5 (Vec 27 16.5 47) 0 0.999 SPEC --Mirr
  , Sphere 16.5 (Vec 73 16.5 78) 0 0.999 REFR --Glas
  , Sphere 600 (Vec 50 681.33 81.6) 12 0 DIFF] --Lite
```

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### Initial Haskell Code: scene data

```
Initial Haskell Code: scene data

spheres :: [Sphere]
spheres =
  [ Sphere 1e5 (Vec (1e5+1) 40.8 81.6) 0 (Vec 0.75 0.25 0.25) DIFF --Left
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  , Sphere 1e5 (Vec 50 40.8 1e5) 0 0.75 DIFF --Back
  , Sphere 1e5 (Vec 50 40.8 (170-1e5)) 0 0 DIFF --Frnt
  , Sphere 1e5 (Vec 50 1e5 81.6) 0 0.75 DIFF --Botm
  , Sphere 1e5 (Vec 50 (81.6-1e5) 81.6) 0 0.75 DIFF --Top
  , Sphere 16.5 (Vec 27 16.5 47) 0 0.999 SPEC --Mirr
  , Sphere 16.5 (Vec 73 16.5 78) 0 0.999 REFR --Glas
  , Sphere 600 (Vec 50 681.33 81.6) 12 0 DIFF] --Lite
```

1. S
2. this list will be walked many times, as it contains our scene information.

# Initial Haskell code: Sphere intersection

```
intersect :: Ray -> Sphere -> Maybe Double
intersect (Ray o d) (Sphere r p _e _c _refl) =
  if det<0 then Nothing else f (b-sdet) (b+sdet)
  where op = p - o -- Numeric
        eps = 1e-4
        b = dot op d
        det = b*b - dot op op + r*r -- Numeric
        sdet = sqrt det
        f a s = if a>eps then Just a else if s>eps then Just s else Nothing
intersects :: Ray -> (Maybe Double, Sphere)
intersects ray = (k, s)
  where (k,s) = foldl' f (Nothing,undefined) spheres -- Spheres iterated over
        f (k',sp) s' = case (k',intersect ray s') of
          (Nothing,Just x) -> (Just x,s')
          (Just y,Just x) | x < y -> (Just x,s')
          _ -> (k',sp)
```

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### Initial Haskell code: Sphere intersection

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```
intersect :: Ray -> Sphere -> Maybe Double
intersect (Ray o d) (Sphere r p _e _c _refl) =
  if det<0 then Nothing else f (b-sdet) (b+sdet)
  where op = p - o -- Numeric
        eps = 1e-4
        b = dot op d
        det = b*b - dot op op + r*r -- Numeric
        sdet = sqrt det
        f a s = if a>eps then Just a else if s>eps then Just s else Nothing
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intersects ray = (k, s)
  where (k,s) = foldl' f (Nothing,undefined) spheres -- Spheres iterated over
        f (k',sp) s' = case (k',intersect ray s') of
          (Nothing,Just x) -> (Just x,s')
          (Just y,Just x) | x < y -> (Just x,s')
          _ -> (k',sp)
```

1. S
2. Responsible for figuring out what the ray hits.
3. We iterate over the list of spheres.
4. Once again, numeric heavy.
5. Use a Maybe to indicate whether we've found an answer or not.

# Initial Haskell Code: radiance (1×)

```
radiance :: Ray -> CInt -> Ptr CUSHort -> IO Vec
radiance ray@(Ray o d) depth xi = case intersects ray of
  (Nothing,_) -> return zeroV
  (Just t,Sphere _r p e c refl) -> do
    let x = o `addv` (d `mulvs` t)
        n = norm $ x `subv` p
        nl = if n `dot` d < 0 then n else n `mulvs` (-1)
        pr = maxv c
        depth' = depth + 1
        continue f = case refl of
          DIFF -> do
            r1 <- ((2*pi)* `fmap` erand48 xi)
            r2 <- erand48 xi
            let r2s = sqrt r2
                w@(Vec wx _ _) = nl
                u = norm $ (if abs wx > 0.1 then (Vec 0 1 0) else (Vec 1 0 0)) `cross` w
                v = w `cross` u
                d' = norm $ (u`mulvs`(cos r1*r2s)) `addv` (v`mulvs`(sin r1*r2s)) `addv` (w`mulvs`sqrt (1-r2))
            rad <- radiance (Ray x d') depth' xi
            return $ e `addv` (f `mulv` rad)
          SPEC -> do
            let d' = d `subv` (n `mulvs` (2 * (n`dot`d)))
            rad <- radiance (Ray x d') depth' xi
            return $ e `addv` (f `mulv` rad)
          REFR -> do
            let reflRay = Ray x (d `subv` (n `mulvs` (2* n`dot`d)))
                into = n`dot`nl > 0
                nc = 1
                nt = 1.5
                nnt = if into then nc/nt else nt/nc
                ddn = d`dot`nl
                cos2t = 1-nnt*nnt*(1-ddn*ddn)
            if cos2t<0
            then do
              rad <- radiance reflRay depth' xi
            ...
```

## Optimizing smallpt

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### Initial Haskell Code: radiance (1×)

1. S
2. Branch heavy
3. Recursive
4. Uses an RNG

```
radiance :: Ray -> CInt -> Ptr CUSHort -> IO Vec
radiance ray@(Ray o d) depth xi = case intersects ray of
  (Nothing,_) -> return zeroV
  (Just t,Sphere _r p e c refl) -> do
    let x = o `addv` (d `mulvs` t)
        n = norm $ x `subv` p
        nl = if n `dot` d < 0 then n else n `mulvs` (-1)
        pr = maxv c
        depth' = depth + 1
        continue f = case refl of
          DIFF -> do
            r1 <- ((2*pi)* `fmap` erand48 xi)
            r2 <- erand48 xi
            let r2s = sqrt r2
                w@(Vec wx _ _) = nl
                u = norm $ (if abs wx > 0.1 then (Vec 0 1 0) else (Vec 1 0 0)) `cross` w
                v = w `cross` u
                d' = norm $ (u`mulvs`(cos r1*r2s)) `addv` (v`mulvs`(sin r1*r2s)) `addv` (w`mulvs`sqrt (1-r2))
            rad <- radiance (Ray x d') depth' xi
            return $ e `addv` (f `mulv` rad)
          SPEC -> do
            let d' = d `subv` (n `mulvs` (2 * (n`dot`d)))
            rad <- radiance (Ray x d') depth' xi
            return $ e `addv` (f `mulv` rad)
          REFR -> do
            let reflRay = Ray x (d `subv` (n `mulvs` (2* n`dot`d)))
                into = n`dot`nl > 0
                nc = 1
                nt = 1.5
                nnt = if into then nc/nt else nt/nc
                ddn = d`dot`nl
                cos2t = 1-nnt*nnt*(1-ddn*ddn)
            if cos2t<0
            then do
              rad <- radiance reflRay depth' xi
            ...
```

# Initial Haskell Code: Entry point (1×)

`smallpt :: Int -> Int -> Int -> IO ()`

`smallpt w h nsamps = do`

```
...
c <- VM.replicate (w * h) 0
allocaArray 3 \xi -> -- Create mutable memory
  flip mapM_ [0..h-1] $ \y -> do -- Loop
    writeXi xi y
    for_ [0..w-1] \x -> do -- Loop
      let i = (h-y-1) * w + x
      for_ [0..1] \sy -> do -- Loop
        for_ [0..1] \sx -> do -- Loop
          r <- newIORef 0 -- Create mutable memory
          for_ [0..samps-1] \s -> do -- Loops, Loops
            r1 <- (2*) <$> erand48 xi
            ...
            rad <- radiance (Ray (org+d.*140) (norm d)) 0 xi -- Crunch
            ...
            modifyIORef r (+ rad .* recip (fromIntegral samps)) -- Write
ci <- VM.unsafeRead c i
Vec rr rg rb <- readIORef r
VM.unsafeWrite c i $
  ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25 -- Write
...
```

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### Initial Haskell Code: Entry point (1×)

```
Initial Haskell Code: Entry point (1×)
smallpt :: Int -> Int -> Int -> IO ()
smallpt w h nsamps = do
  ...
  d <- VM.replicate (w * h) 0
  allocaArray 3 \xi -> -- Create mutable memory
  flip mapM_ [0..h-1] $ \y -> do -- Loop
    writeXi xi y
    for_ [0..w-1] \x -> do -- Loop
      let i = (h-y-1) * w + x
      for_ [0..1] \sy -> do -- Loop
        for_ [0..1] \sx -> do -- Loop
          r <- newIORef 0 -- Create mutable memory
          for_ [0..samps-1] \s -> do -- Loops, Loops
            r1 <- (2*) <$> erand48 xi
            ...
            rad <- radiance (Ray (org+d.*140) (norm d)) 0 xi -- Crunch
            ...
            modifyIORef r (+ rad .* recip (fromIntegral samps)) -- Write
ci <- VM.unsafeRead c i
Vec rr rg rb <- readIORef r
VM.unsafeWrite c i $
  ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25 -- Write
...
```

1. S
2. Uses mutability
3. Performs number crunchy loops
4. Finally, writes results out



## Initial Haskell Code: File I/O (1×)

```
withFile "image.ppm" WriteMode $ \hdl -> do
  hPrintf hdl "P3\n%d %d\n%d\n" w h (255::Int)
  flip mapM_ [0..w*h-1] \i -> do
    Vec r g b <- VM.unsafeRead c i
    hPrintf hdl "%d %d %d " (toInt r) (toInt g) (toInt b)
```

## Optimizing smallpt

2020-11-05

### Initial Haskell Code: File I/O (1×)

```
Initial Haskell Code: File I/O (1×)

withFile "image.ppm" WriteMode $ \hdl -> do
  hPrintf hdl "P3\n%d %d\n%d\n" w h (255::Int)
  flip mapM_ [0..w*h-1] \i -> do
    Vec r g b <- VM.unsafeRead c i
    hPrintf hdl "%d %d %d " (toInt r) (toInt g) (toInt b)
```

## Initial Haskell Code: RNG (1×)

```
foreign import ccall unsafe "erand48"  
  erand48 :: Ptr CUShort -> IO Double
```

## Optimizing smallpt

2020-11-05

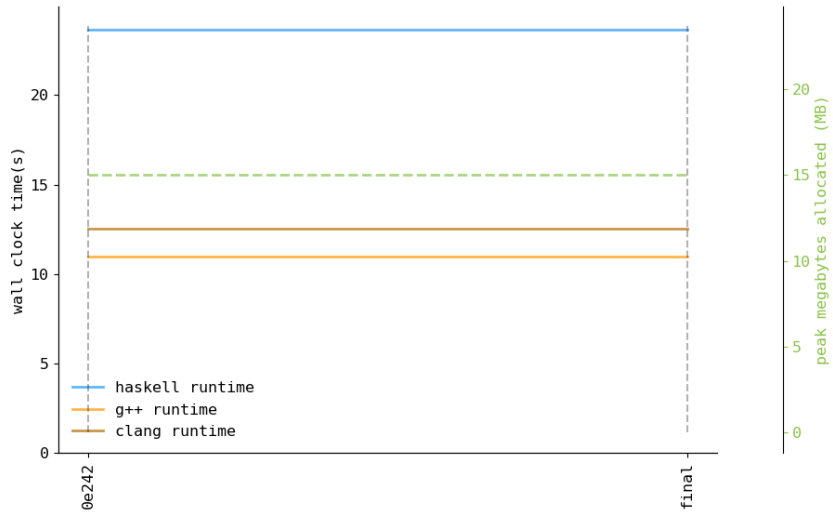
Initial Haskell Code: RNG (1×)

Initial Haskell Code: RNG (1×)

```
foreign import ccall unsafe "erand48"  
  erand48 :: Ptr CUShort -> IO Double
```

1. **S**
2. We use the RNG to decide randomly in which direction to send rays
3. Point out the use of foreign CCall.

# Performance: Initial Haskell Code



2020-11-05

## Optimizing smallpt

Performance: Initial Haskell Code



Restrict export list to main (1.13×)

```
-module Main where
+module Main (main) where
```

## Optimizing smallpt

2020-11-05

- └ Restrict export list to main (1.13x)

Restrict export list to main (1.13x)

```
-module Main where
+module Main (main) where
```

1. **S**
2. The very first thing to do is to let the compiler actually optimize.
3. Compiler can't know how exported functions are used.
4. Export lists not just about encapsulation.

Restrict export list to main (1.13x)

```
-module Main where
+module Main (main) where
```

- ▶ Exported functions could be used by something unknown.
- ▶ Original versions must be available.

## Optimizing smallpt

2020-11-05

- Restrict export list to main ( $1.13\times$ )

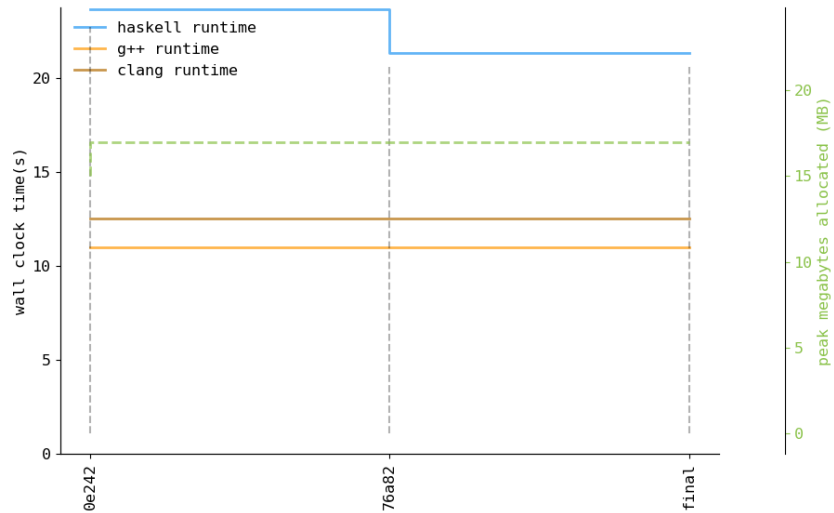
Restrict export list to main (1.13x)

```
-module Main where
+module Main (main) where
```

- ▶ Exported functions could be used by something unknown
- ▶ Original versions must be available.

1. **S**
2. The very first thing to do is to let the compiler actually optimize.
3. Compiler can't know how exported functions are used.
4. Export lists not just about encapsulation.

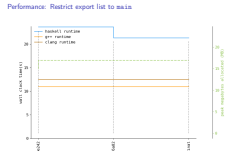
## Performance: Restrict export list to main



## Optimizing smallpt

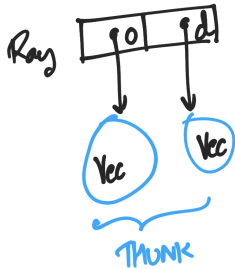
2020-11-05

Performance: Restrict export list to main



## Mark entries of Ray and Sphere as UNPACK and Strict (1.07×)

```
data Ray = Ray Vec Vec
```



- ▶ By default, all fields are *thunks* to rest of computation
- ▶ Pure, allow equational reasoning.

## Optimizing smallpt

2020-11-05

└ Mark entries of Ray and Sphere as UNPACK and Strict (1.07×)

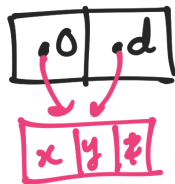
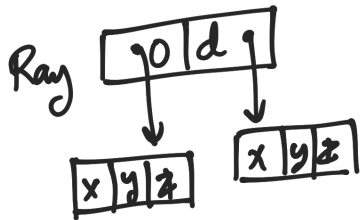
data Ray = Ray Vec Vec



- ▶ By default, all fields are *thunks* to rest of computation
- ▶ Pure, allow equational reasoning.

## Mark entries of Ray and Sphere as UNPACK and Strict (1.07x)

```
data Ray = Ray !Vec !Vec
```

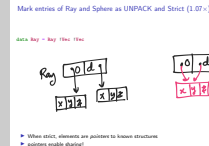


- ▶ When strict, elements are *pointers* to known structures
- ▶ pointers enable sharing!

## Optimizing smallpt

2020-11-05

Mark entries of Ray and Sphere as UNPACK and Strict (1.07x)





## Mark entries of Ray and Sphere as UNPACK and Strict (1.07×)

```
data Ray = Ray {-# UNPACK #-} !Vec {-# UNPACK #-} !Vec
```



- ▶ When unpacked, elements are *members* of the parent.
- ▶ Larger, but eliminate pointer chasing.

## Optimizing smallpt

2020-11-05

└ Mark entries of Ray and Sphere as UNPACK and Strict (1.07×)

```
data Ray = Ray {-# UNPACK #-} !Vec {-# UNPACK #-} !Vec
```



- ▶ When unpacked, elements are members of the parent.
- ▶ Larger, but eliminate pointer chasing.

# Mark entries of Ray and Sphere as UNPACK and Strict (1.07x)

```
data Vec = Vec {-# UNPACK #-} !Double
             {-# UNPACK #-} !Double
             {-# UNPACK #-} !Double
```

```
-data Ray = Ray Vec Vec -- origin, direction
+data Ray = Ray !Vec !Vec -- origin, direction
```

```
data Refl = DIFF | SPEC | REFR -- material types, used in radiance
```

```
-- radius, position, emission, color, reflection
-data Sphere = Sphere Double Vec Vec Vec !Refl
+data Sphere = Sphere {-# UNPACK #-} !Double
+                    {-# UNPACK #-} !Vec
+                    {-# UNPACK #-} !Vec
+                    {-# UNPACK #-} !Vec !Refl
```

```
struct Vec { double x, y, z; }
struct Ray { std::function<Vec()> v; std::function<Vec()> w; };
struct RayUnpack { double xv, yv, int zv;
                  double xw, yw, zw; };
```

## Optimizing smallpt

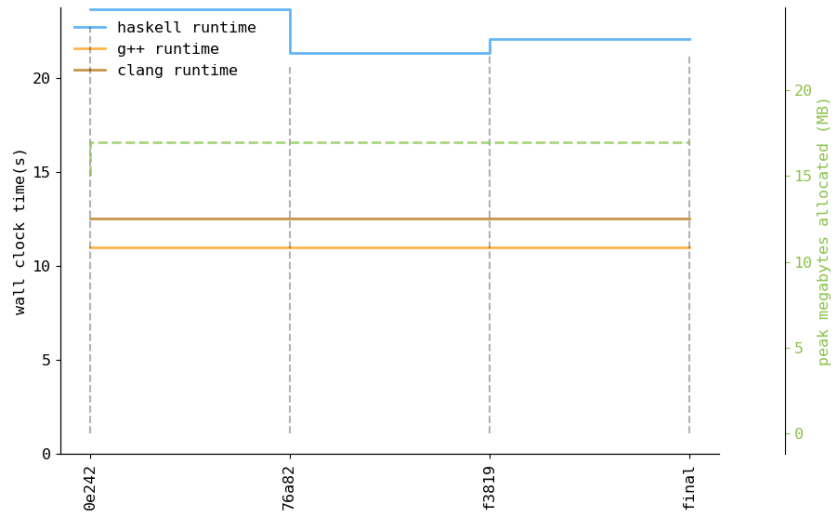
2020-11-05

### Mark entries of Ray and Sphere as UNPACK and Strict (1.07x)

```
data Vec = Vec {-# UNPACK #-} !Double
             {-# UNPACK #-} !Double
             {-# UNPACK #-} !Double
data Ray = Ray Vec Vec -- origin, direction
data Refl = DIFF | SPEC | REFR -- material types, used in radiance
-- radius, position, emission, color, reflection
data Sphere = Sphere Double Vec Vec Vec !Refl
data Sphere = Sphere {-# UNPACK #-} !Double
+                  {-# UNPACK #-} !Vec
+                  {-# UNPACK #-} !Vec
+                  {-# UNPACK #-} !Vec !Refl
struct Vec { double x, y, z; }
struct Ray { std::function<Vec()> v; std::function<Vec()> w; };
struct RayUnpack { double xv, yv, int zv;
                  double xw, yw, zw; };
```

1. D
2. Strictness in the arguments means that they're evaluated when instantiated, not when demanded.
3. Where as Unpacking removes indirection from doing a memory lookup for components.
4. Means we have to copy everything into the data structure that it is unpacked into.
5. We don't unpack ray (Lots of calculations on its components, want those to fuse)
6. Unpack Sphere - its static from compile time
7. Don't unpack Ray, because each Vec undergoes a lot of computation.

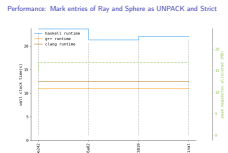
# Performance: Mark entries of Ray and Sphere as UNPACK and Strict



## Optimizing smallpt

2020-11-05

Performance: Mark entries of Ray and Sphere as UNPACK and Strict



Use a pattern synonym to unpack Refl in Sphere ( $1.07\times$ )

```
+{-# LANGUAGE PatternSynonyms #-}
```

```
-data Refl = DIFF | SPEC | REFR -- material types, used in radiance
+newtype Refl = Refl Int -- material types, used in radiance
+pattern DIFF,SPEC,REFR :: Refl
+pattern DIFF = Refl 0
+pattern SPEC = Refl 1
+pattern REFR = Refl 2
+{-# COMPLETE DIFF, SPEC, REFR #-}
```

```
-- radius, position, emission, color, reflection
data Sphere = Sphere {-# UNPACK #-} !Double
                    {-# UNPACK #-} !Vec
                    {-# UNPACK #-} !Vec
-                    {-# UNPACK #-} !Vec !Refl
+                    {-# UNPACK #-} !Vec {-# UNPACK #-} !Refl
```

## Optimizing smallpt

2020-11-05

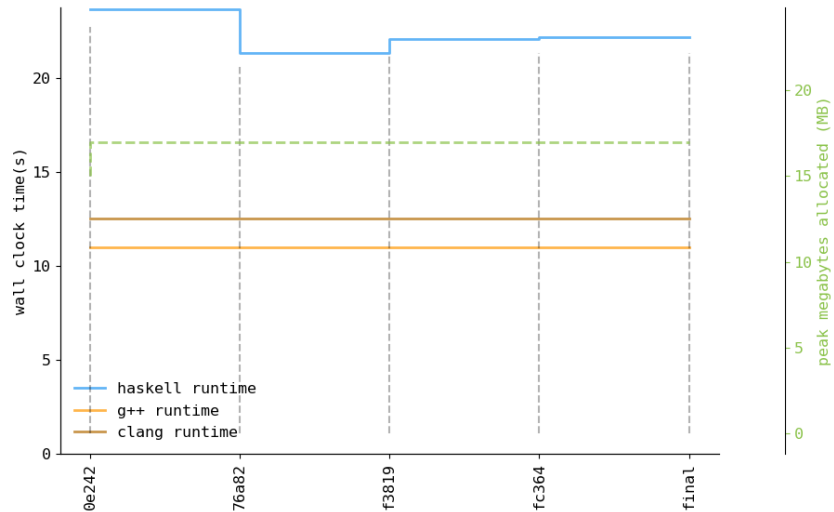
- Use a pattern synonym to unpack Refl in Sphere (1.07x)

### Use a pattern synonym to unpack Refl in Sphere (1.07)

[illegible]

1. **D**
2. Was unable to unpack Refl
3. UnboxedSums are recent
4. UnboxedSums are very unpleasant
5. We're using an older trick to fake the unboxing here instead.
6. In this case it isn't much of a win, but it illustrates the technique.

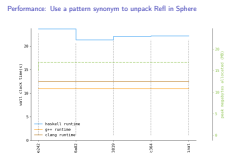
## Performance: Use a pattern synonym to unpack Refl in Sphere



## Optimizing smallpt

2020-11-05

Performance: Use a pattern synonym to unpack Refl in Sphere



## Change from maximum on a list to max (1.08×)

```
-maxv (Vec a b c) = maximum [a,b,c]
+maxv (Vec a b c) = max a (max b c)

    let x = o `addv` (d `mulvs` t)
        n = norm $ x `subv` p
        nl = if n `dot` d < 0 then n else n `mulvs` (-1)
-    pr = maxv c
    depth' = depth + 1
    continue f = case refl of
        DIFF -> do
...
    if depth'>5
    then do
        er <- erand48 xi
+    let !pr = maxv c
```

## Optimizing smallpt

2020-11-05

Change from maximum on a list to max (1.08×)

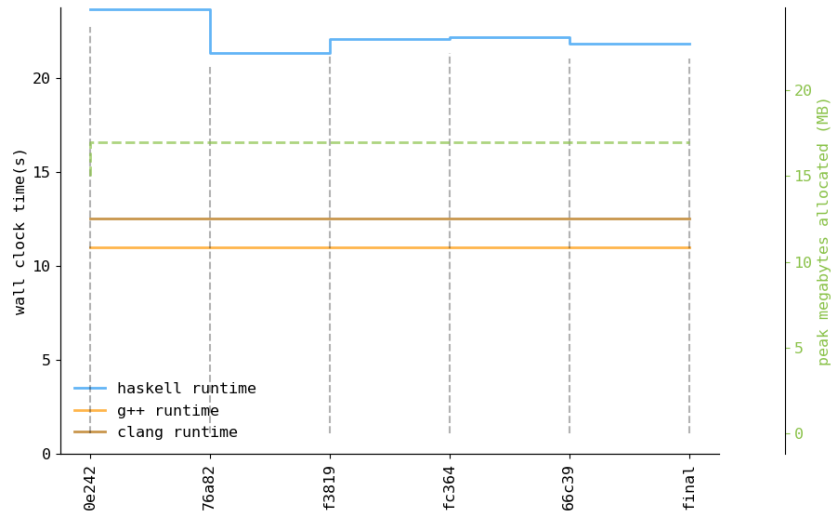
```
Change from maximum on a list to max (1.08×)

-maxv (Vec a b c) = maximum [a,b,c]
+maxv (Vec a b c) = max a (max b c)

    let x = o `addv` (d `mulvs` t)
        n = norm $ x `subv` p
        nl = if n `dot` d < 0 then n else n `mulvs` (-1)
-    pr = maxv c
    depth' = depth + 1
    continue f = case refl of
        DIFF -> do
...
    if depth'>5
    then do
        er <- erand48 xi
+    let !pr = maxv c
```

1. D
2. Prebuild comparison
3. Don't go via list
4. GHC does not evaluate at compile time, only has RULES
5. Doesn't really help much in this case

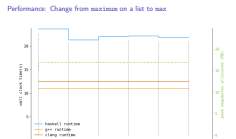
## Performance: Change from maximum on a list to max



## Optimizing smallpt

2020-11-05

Performance: Change from maximum on a list to max



# Convert erand48 to pure Haskell (1.09x)

```
-foreign import ccall unsafe "erand48"
- erand48 :: Ptr CUShort -> IO Double
```

```
+erand48 :: IORef Word64 -> IO Double
+erand48 !t = do -- | Some number crunchy thing.
+  r <- readIORef t
+  let x' = 0x5deece66d * r + 0xb
+      d_word = 0x3fff000000000000 .|. ((x' .&. 0xffffffff) `unsafeShiftL` 4)
+      d = castWord64ToDouble d_word - 1.0
+  writeIORef t x'
+  pure d
+...
+--radiance :: Ray -> CInt -> Ptr CUShort -> IO Vec
+radiance :: Ray -> Int -> IORef Word64 -> IO Vec -- IORef with state
+radiance ray@(Ray o d) depth xi = case intersects ray of
+  ...
+  c <- VM.replicate (w * h) zeroV
+-  allocaArray 3 $ \xi -> -- Old RNG state
+-    flip mapM_ [0..h-1] $ \y -> do
++  xi <- newIORef 0 -- New RNG state
++  flip mapM_ [0..h-1] $ \y -> do
+    writeXi xi y
```

## Optimizing smallpt

2020-11-05

### Convert erand48 to pure Haskell (1.09x)

```
Convert erand48 to pure Haskell (1.09x)

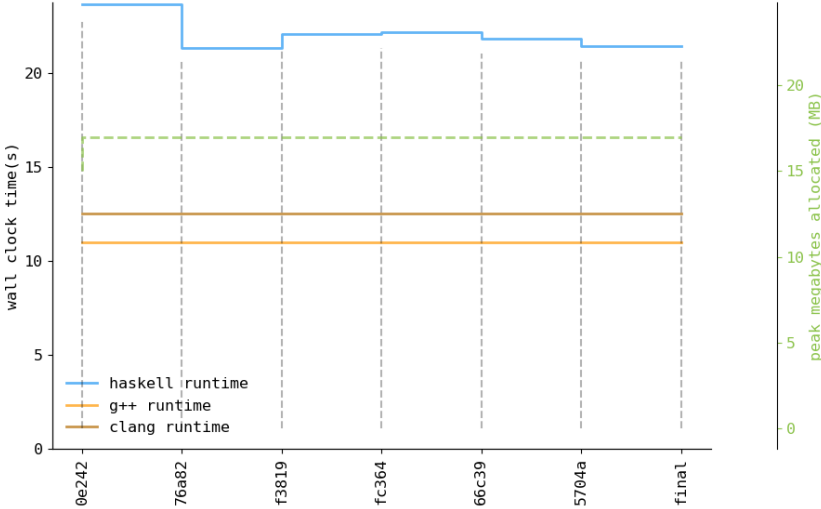
--foreign import ccall unsafe "erand48"
-- erand48 :: Ptr CUShort -> IO Double

--erand48 :: IORef Word64 -> IO Double
--erand48 !t = do -- | Some number crunchy thing.
--  r <- readIORef t
--  let x' = 0x5deece66d * r + 0xb
--      d_word = 0x3fff000000000000 .|. ((x' .&. 0xffffffff) `unsafeShiftL` 4)
--      d = castWord64ToDouble d_word - 1.0
--  writeIORef t x'
--  pure d
--...
--radiance :: Ray -> CInt -> Ptr CUShort -> IO Vec
--radiance :: Ray -> Int -> IORef Word64 -> IO Vec -- IORef with state
--radiance ray@(Ray o d) depth xi = case intersects ray of
--  ...
--  c <- VM.replicate (w * h) zeroV
--  allocaArray 3 $ \xi -> -- Old RNG state
--    flip mapM_ [0..h-1] $ \y -> do
--  xi <- newIORef 0 -- New RNG state
--  flip mapM_ [0..h-1] $ \y -> do
--    writeXi xi y
```

1. S
2. The entire premise of this talk is that Haskell can be as fast as C.
3. We're opening the black box of what erand48 does to GHC
4. Further any impedance mismatch, such as FFI almost universally has to have, carries some bookkeeping overhead.
5. If our Haskell code was as fast as the C code moving the code into Haskell would be a win, if it was slightly slower it could still be a win.
6. Often considering your Haskell code's performance is a better option and easier than reimplementing something in C.
7. As is the way with optimizations, this is not universally true.



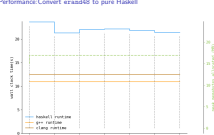
# Performance:Convert erand48 to pure Haskell



## Optimizing smallpt

2020-11-05

Performance:Convert erand48 to pure Haskell



# Remove mutability: Erand48 Monad

```
-erand48 :: IORef Word64 -> IO Double
-erand48 !t = do
-  r <- readIORef t
+data ET a = ET !Word64 !a deriving Functor
+newtype Erand48 a = Erand48 { runErand48' :: Word64 -> ET a } deriving Functor
+instance Applicative Erand48 where
+instance Monad Erand48 where
+runWithErand48 :: Int -> Erand48 a -> a
+erand48 :: Erand48 Double
+...
-radiance :: Ray -> Int -> IORef Word64 -> IO Vec
-radiance ray@(Ray o d) depth xi = case intersects ray of
+radiance :: Ray -> Int -> Erand48 Vec
+radiance ray@(Ray o d) depth = case intersects ray of
+...
-      r1 <- (2*pi*) <$> erand48 xi
-      r2 <- erand48 xi
+      r1 <- (2*pi*) <$> erand48
+      r2 <- erand48
+...
-      then (* rp) <$> radiance reflRay depth' xi
-      else (* tp) <$> radiance (Ray x tdir) depth' xi
+      then (* rp) <$> radiance reflRay depth'
+      else (* tp) <$> radiance (Ray x tdir) depth'
```

## Optimizing smallpt

2020-11-05

### Remove mutability: Erand48 Monad

```
erand48 :: IORef Word64 -> IO Double
erand48 !t = do
  r <- readIORef t
  return ET a = ET !Word64 !a deriving Functor
newtype Erand48 a = Erand48 { runErand48' :: Word64 -> ET a } deriving Functor
instance Applicative Erand48 where
instance Monad Erand48 where
runWithErand48 :: Int -> Erand48 a -> a
erand48 :: Erand48 Double
...
radiance :: Ray -> Int -> IORef Word64 -> IO Vec
radiance ray@(Ray o d) depth xi = case intersects ray of
radiance :: Ray -> Int -> Erand48 Vec
radiance ray@(Ray o d) depth = case intersects ray of
...
  r1 <- (2*pi*) <$> erand48 xi
  r2 <- erand48 xi
  r3 <- (2*pi*) <$> erand48
  r4 <- erand48
  ...
  when (* rp) <$> radiance reflRay depth' xi
  when (* tp) <$> radiance (Ray x tdir) depth' xi
  when (* rp) <$> radiance reflRay depth'
  when (* tp) <$> radiance (Ray x tdir) depth'
```

1. D
2. All these mutability locations throw in extra RTS code, extra sequencing that blocks the compiler's optimization, and dependency chains.
3. Sometimes we need mutability for performance.
4. SSA is normal to compilers though.
5. We almost start at SSA as a functional language.
6. don't break it when you don't have a good reason.

# Removing mutation: eliminate IORef and Data.Vector.Mutable

```
- c <- VM.replicate (w * h) 0
- xi <- newIORef 0
- flip mapM_ [0..h-1] $ \y -> do
-   writeXi xi y
-   for_ [0..w-1] \x -> do
-     let i = (h-y-1) * w + x
-     for_ [0..1] \sy -> do
-       for_ [0..1] \sx -> do
-         r <- newIORef 0
-         for_ [0..samps-1] \_s -> do
-           r1 <- (2*) <$> erand48 xi
+ img = (`concatMap` [(h-1),(h-2)..0]) $ \y -> runWithErand48 y do
+   for [0..w-1] \x -> do
+     (\pf -> foldlM pf 0 [(sy, sx) | sy <- [0,1], sx <- [0,1]]) \ci (sy, sx) -> do
+       Vec rr rg rb <- (\f -> foldlM f 0 [0..samps-1]) \ !r _s -> do
+         r1 <- (2*) <$> erand48
+
+ ...
-       modifyIORef r (+ rad .* recip (fromIntegral samps))
-       ci <- VM.unsafeRead c i
-       Vec rr rg rb <- readIORef r
-       VM.unsafeWrite c i $ ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25
+       pure (r + rad .* recip (fromIntegral samps))
+       pure (ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25)
+
+ ...
```

## Optimizing smallpt

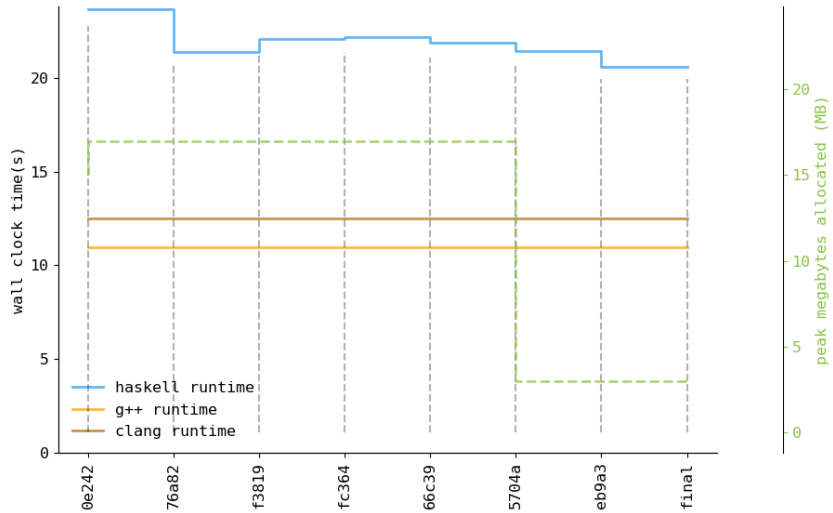
2020-11-05

### Removing mutation: eliminate IORef and Data.Vector.Mutable

```
- w <- VM.replicate (w * h) 0
- xi <- newIORef 0
- flip mapM_ [0..h-1] $ \y -> do
-   writeXi xi y
-   for_ [0..w-1] \x -> do
-     let i = (h-y-1) * w + x
-     for_ [0..1] \sy -> do
-       for_ [0..1] \sx -> do
-         r <- newIORef 0
-         for_ [0..samps-1] \_s -> do
-           r1 <- (2*) <$> erand48 xi
+ img = `concatMap` [(h-1),(h-2)..0] $ \y -> multiWithIORef y do
+   for [0..w-1] \x -> do
+     (\pf -> foldlM pf 0 [(sy, sx) | sy <- [0,1], sx <- [0,1]]) \ci (sy, sx) -> do
+       Vec rr rg rb <- (\f -> foldlM f 0 [0..samps-1]) \ !r _s -> do
+         r1 <- (2*) <$> erand48
+
+ ...
-       modifyIORef r (+ rad .* recip (fromIntegral samps))
-       ci <- VM.unsafeRead c i
-       Vec rr rg rb <- readIORef r
-       VM.unsafeWrite c i $ ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25
+       pure (r + rad .* recip (fromIntegral samps))
+       pure (ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25)
+
+ ...
```

## 1. D

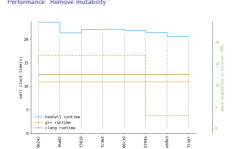
# Performance: Remove mutability



## Optimizing smallpt

2020-11-05

Performance: Remove mutability



## Set everything in smallpt to be strict (1.17x)

```
smallpt :: Int -> Int -> Int -> IO ()
smallpt w h nsamps = do
-   let samps = nsamps `div` 4
-   org = Vec 50 52 295.6
-   dir = norm $ Vec 0 (-0.042612) (-1)
-   cx = Vec (fromIntegral w * 0.5135 / fromIntegral h) 0 0
-   cy = norm (cx `cross` dir) `mulvs` 0.5135
+   let !samps = nsamps `div` 4
+   !org = Vec 50 52 295.6
+   !dir = norm $ Vec 0 (-0.042612) (-1)
+   !cx = Vec (fromIntegral w * 0.5135 / fromIntegral h) 0 0
+   !cy = norm (cx `cross` dir) `mulvs` 0.5135
...
-   r1 <- (2*) `fmap` erand48 xi
-   let dx = if r1<1 then sqrt r1-1 else 1-sqrt(2-r1)
-   r2 <- (2*) `fmap` erand48 xi
-   let dy = if r2<1 then sqrt r2-1 else 1-sqrt(2-r2)
-   d = ...
-   rad <- radiance (Ray (org`addv`(d`mulvs`140)) (norm d)) 0 xi
+   !r1 <- (2*) `fmap` erand48 xi
+   let !dx = if r1<1 then sqrt r1-1 else 1-sqrt(2-r1)
+   !r2 <- (2*) `fmap` erand48 xi
+   let !dy = if r2<1 then sqrt r2-1 else 1-sqrt(2-r2)
+   !d = ...
...
+   pure $! r + rad .* recip (fromIntegral samps)
+   pure $! ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25
```

## Optimizing smallpt

2020-11-05

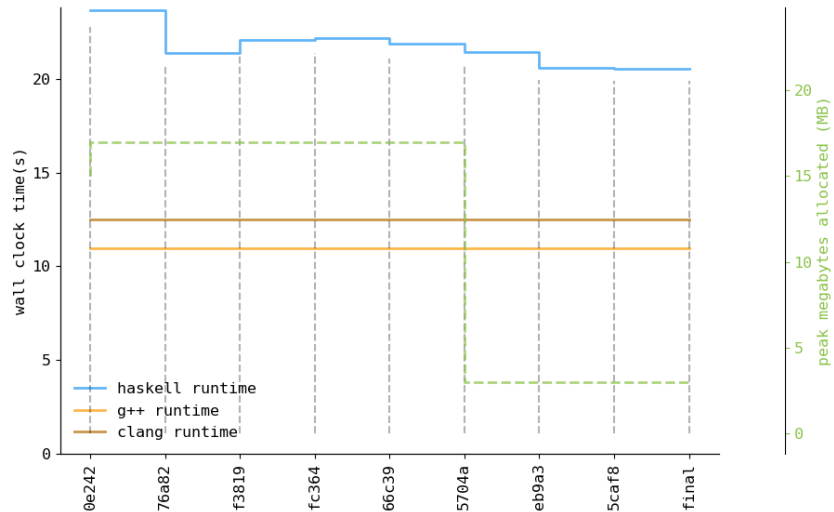
Set everything in smallpt to be strict (1.17x)

```
Set everything in smallpt to be strict (1.17x)
smallpt :: Int -> Int -> Int -> IO ()
smallpt w h nsamps = do
-   let samps = nsamps `div` 4
-   org = Vec 50 52 295.6
-   dir = norm $ Vec 0 (-0.042612) (-1)
-   cx = Vec (fromIntegral w * 0.5135 / fromIntegral h) 0 0
-   cy = norm (cx `cross` dir) `mulvs` 0.5135
+   let !samps = nsamps `div` 4
+   !org = Vec 50 52 295.6
+   !dir = norm $ Vec 0 (-0.042612) (-1)
+   !cx = Vec (fromIntegral w * 0.5135 / fromIntegral h) 0 0
+   !cy = norm (cx `cross` dir) `mulvs` 0.5135
...
-   r1 <- (2*) `fmap` erand48 xi
-   let dx = if r1<1 then sqrt r1-1 else 1-sqrt(2-r1)
-   r2 <- (2*) `fmap` erand48 xi
-   let dy = if r2<1 then sqrt r2-1 else 1-sqrt(2-r2)
-   d = ...
-   rad <- radiance (Ray (org`addv`(d`mulvs`140)) (norm d)) 0 xi
+   !r1 <- (2*) `fmap` erand48 xi
+   !dx = if r1<1 then sqrt r1-1 else 1-sqrt(2-r1)
+   !r2 <- (2*) `fmap` erand48 xi
+   !dy = if r2<1 then sqrt r2-1 else 1-sqrt(2-r2)
+   !d = ...
...
+   pure $! r + rad .* recip (fromIntegral samps)
+   pure $! ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25
```

Don't senselessly bang everything in sight.

1. D
2. This is not a recommendation, this is a warning.
3. We get a speedup here but it can also regress performance. Some of these bangs are regressions that are hidden.

## Performance: Set everything in smallpt to be strict



## Optimizing smallpt

2020-11-05

Performance: Set everything in smallpt to be strict



# Why strictness may be bad

```
let foo = let x = error "ERR" in \y -> y
let fooOpt = \y -> y
```

## Optimizing smallpt

2020-11-05

### Why strictness may be bad

Why strictness may be bad

```
let foo = let x = error "ERR" in \y -> y
let fooOpt = \y -> y
```

1. **S**
2. Consider the function foo and fooOpt. These are equivalent
3. The fact that x is not used allows us to eliminate computing x
4. Consider the next version
5. Illegal, we need to have x, because it doesn't produce ERR
6. we can't equationally reason about the program anymore.
7. Makes it harder for GHC. GHC is conservative about bangs
8. Inhibits compiler from optimizing

# Why strictness may be bad

```
let foo = let x = error "ERR" in \y -> y
let fooOpt = \y -> y

let foo' = let !x = error "ERR" in \y -> y
```

## Optimizing smallpt

2020-11-05

### Why strictness may be bad

Why strictness may be bad

```
let foo = let x = error "ERR" in \y -> y
let fooOpt = \y -> y
let foo' = let !x = error "ERR" in \y -> y
```

1. **S**
2. Consider the function foo and fooOpt. These are equivalent
3. The fact that x is not used allows us to eliminate computing x
4. Consider the next version
5. Illegal, we need to have x, because it doesn't produce ERR
6. we can't equationally reason about the program anymore.
7. Makes it harder for GHC. GHC is conservative about bangs
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# Why strictness may be bad

```
let foo = let x = error "ERR" in \y -> y
let fooOpt = \y -> y

let foo' = let !x = error "ERR" in \y -> y
let foo'Opt = \y -> y  -- ERROR! forcing foo'=foo'Opt should give "ERR"
```

## Optimizing smallpt

2020-11-05

### Why strictness may be bad

Why strictness may be bad

```
let foo = let x = error "ERR" in \y -> y
let fooOpt = \y -> y
let foo' = let !x = error "ERR" in \y -> y
let foo'Opt = \y -> y  -- ERROR! forcing foo'=foo'Opt should give "ERR"
```

1. S
2. Consider the function foo and fooOpt. These are equivalent
3. The fact that x is not used allows us to eliminate computing x
4. Consider the next version
5. Illegal, we need to have x, because it doesn't produce ERR
6. we can't equationally reason about the program anymore.
7. Makes it harder for GHC. GHC is conservative about bangs
8. Inhibits compiler from optimizing

## Reduce to only useful strictnesses in smallpt(1.17x)

```
- let !samps = nsamps `div` 4
- !org = Vec 50 52 295.6
- !dir = norm $ Vec 0 (-0.042612) (-1)
- !cx = Vec (fromIntegral w * 0.5135 / fromIntegral h) 0 0
- !cy = norm (cx `cross` dir) .* 0.5135
+ let samps = nsamps `div` 4
+ org = Vec 50 52 295.6
+ dir = norm $ Vec 0 (-0.042612) (-1)
+ cx = Vec (fromIntegral w * 0.5135 / fromIntegral h) 0 0
+ cy = norm (cx `cross` dir) .* 0.5135
...
- !r1 <- (2*) <$> erand48
+ r1 <- (2*) <$> erand48
...
- !r2 <- (2*) <$> erand48
+ r2 <- (2*) <$> erand48
...
- !rad <- radiance (Ray (org+d.*140) (norm d)) 0
+ rad <- radiance (Ray (org+d.*140) (norm d)) 0
...
- pure $! r + rad .* recip (fromIntegral samps)
- pure $! ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25
+ pure (r + rad .* recip (fromIntegral samps))
+ pure (ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25)
```

## Optimizing smallpt

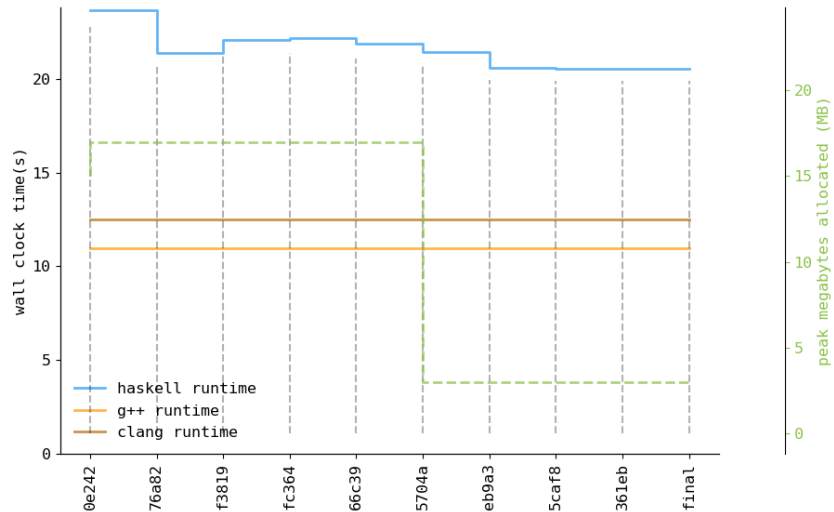
2020-11-05

Reduce to only useful strictnesses in smallpt(1.17x)

```
Reduce to only useful strictnesses in smallpt(1.17x)
- let tsamps = nsamps `div` 4
- !org = Vec 50 52 295.6
- !dir = norm $ Vec 0 (-0.042612) (-1)
- !cx = Vec (fromIntegral w * 0.5135 / fromIntegral h) 0 0
- !cy = norm (cx `cross` dir) .* 0.5135
+ let samps = nsamps `div` 4
+ org = Vec 50 52 295.6
+ dir = norm $ Vec 0 (-0.042612) (-1)
+ cx = Vec (fromIntegral w * 0.5135 / fromIntegral h) 0 0
+ cy = norm (cx `cross` dir) .* 0.5135
...
- !r1 <- (2*) <$> erand48
- !r2 <- (2*) <$> erand48
+ r1 <- (2*) <$> erand48
+ r2 <- (2*) <$> erand48
...
- !rad <- radiance (Ray (org+d.*140) (norm d)) 0
- !rad <- radiance (Ray (org+d.*140) (norm d)) 0
+ rad <- radiance (Ray (org+d.*140) (norm d)) 0
...
- pure $! r + rad .* recip (fromIntegral samps)
- pure $! ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25
+ pure (r + rad .* recip (fromIntegral samps))
+ pure (ci + Vec (clamp rr) (clamp rg) (clamp rb) .* 0.25)
```

1. D
2. Thus the compiler can no longer move the computation around or simplify it.
3. Force useless work.
4. A little thinking about how the variables are used or looking at core allows us to select which ones we bang selectively.

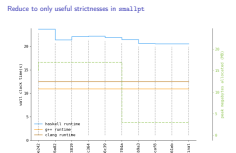
## Reduce to only useful strictnesses in smallpt



## Optimizing smallpt

2020-11-05

Reduce to only useful strictnesses in smallpt



# Use strictness strategically in entire project

```
...
- if det<0 then Nothing else f (b-sdet) (b+sdet)
- where op = p - o
-       eps = 1e-4
-       b = dot op d
-       det = b*b - dot op op + r*r
-       sdet = sqrt det
-       f a s = if a>eps then Just a else if s>eps then Just s else Nothing
+ if det<0
+ then Nothing
+ else
+   let !eps = 1e-4
+       !sdet = sqrt det
+       !a = b-sdet
+       !s = b+sdet
+   in if a>eps then Just a else if s>eps then Just s else Nothing
...
```

## Optimizing smallpt

2020-11-05

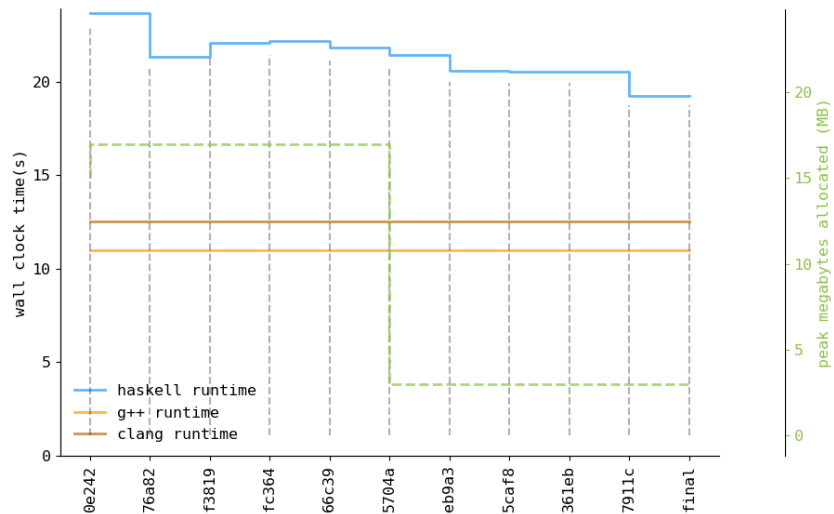
### Use strictness strategically in entire project

```
Use strictness strategically in entire project

...
- if det<0 then Nothing else f (b-sdet) (b+sdet)
- where op = p - o
-       eps = 1e-4
-       b = dot op d
-       det = b*b - dot op op + r*r
-       sdet = sqrt det
-       f a s = if a>eps then Just a else if s>eps then Just s else Nothing
+ if det<0
+ then Nothing
+ else
+   let !eps = 1e-4
+       !sdet = sqrt det
+       !a = b-sdet
+       !s = b+sdet
+   in if a>eps then Just a else if s>eps then Just s else Nothing
...
```

1. **D**
2. Sometimes (point out 'intersect') we have to rearrange the code though when we use bangs.
3. Bangs tell the compiler to make more efficient code, but take away the compiler's options in how to do so.
4. Only take away the compiler's liberties when it's using them poorly.
5. Becomes intuitive.

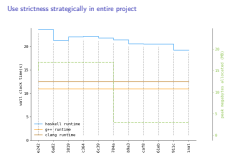
## Use strictness strategically in entire project



## Optimizing smallpt

2020-11-05

Use strictness strategically in entire project



## Remove Maybe from intersect(s) (1.32x)

| Old: Use Maybe Double to represent (was-hit?:bool, hit-distance: Double)

| New: use (1/0) to represent not (was-hit?)

-intersect :: Ray -> Sphere -> Maybe Double

+intersect :: Ray -> Sphere -> Double

intersect (Ray o d) (Sphere r p \_e \_c \_refl) =

- if det<0 then Nothing else f (b-sdet) (b+sdet)

+ if det<0 then (1/0.0) else f (b-sdet) (b+sdet)

where op = p `subv` o

...

- f a s = if a>eps then Just a else if s>eps then Just s else Nothing

+ f a s = if a>eps then a else if s>eps then s else (1/0.0)

-intersects :: Ray -> (Maybe Double, Sphere)

+intersects :: Ray -> (Double, Sphere)

intersects ray = (k, s)

- where (k,s) = foldl' f (Nothing,undefined) spheres

- f (k',sp) s' = case (k',intersect ray s') of

- (Nothing,Just x) -> (Just x,s')

- (Just y,Just x) | x < y -> (Just x,s')

- \_ -> (k',sp)

+ where (k,s) = foldl' f (1/0.0,undefined) spheres

+ f (k', sp) s' = let !x = intersect ray s' in if x < k' then (x, s') else (k', sp)

radiance :: Ray -> Int -> STRefU s Word64 -> ST s Vec

radiance ray@(Ray o d) depth xi = case intersects ray of

- (Nothing,\_) -> return zerov

- (Just t,Sphere \_r p e c refl) -> do

+ (t,\_) | t == (1/0.0) -> return zerov

+ (t,Sphere \_r p e c refl) -> do

## Optimizing smallpt

2020-11-05

### Remove Maybe from intersect(s) (1.32x)

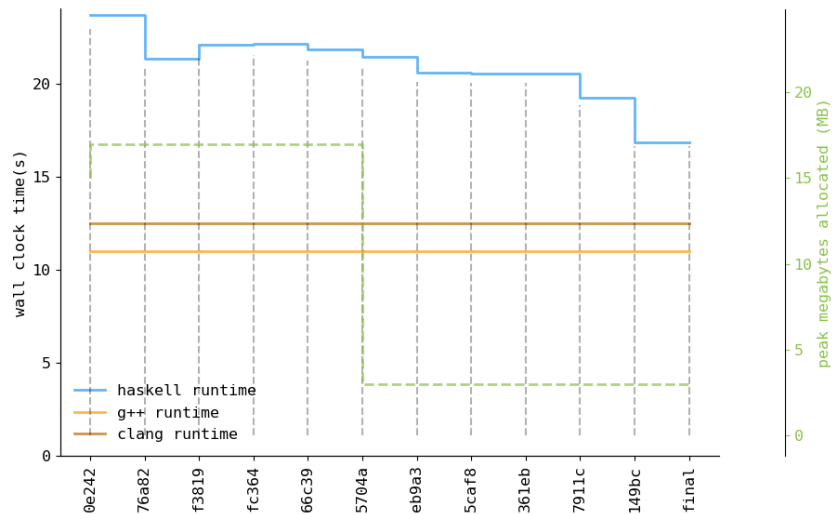
```
Remove Maybe from intersect(s) (1.32x)
| Old: Use Maybe Double to represent (was-hit?:bool, hit-distance: Double)
| New: use (1/0) to represent not (was-hit?)
intersect :: Ray -> Sphere -> Maybe Double
intersect :: Ray -> Sphere -> Double
intersect (Ray o d) (Sphere r p _e _c _refl) =
- if det<0 then Nothing else f (b-sdet) (b+sdet)
+ if det<0 then (1/0.0) else f (b-sdet) (b+sdet)
where op = p `subv` o
...
f a s = if a>eps then Just a else if s>eps then Just s else Nothing
f a s = if a>eps then a else if s>eps then s else (1/0.0)

intersects :: Ray -> (Maybe Double, Sphere)
intersects :: Ray -> (Double, Sphere)
intersects ray = (k, s)
where (k,s) = foldl' f (Nothing,undefined) spheres
f (k',sp) s' = case (k',intersect ray s') of
- (Nothing,Just x) -> (Just x,s')
- (Just y,Just x) | x < y -> (Just x,s')
- _ -> (k',sp)
+ where (k,s) = foldl' f (1/0.0,undefined) spheres
+ f (k', sp) s' = let !x = intersect ray s' in if x < k' then (x, s') else (k', sp)

radiance :: Ray -> Int -> STRefU s Word64 -> ST s Vec
radiance ray@(Ray o d) depth xi = case intersects ray of
- (Nothing,_) -> return zerov
- (Just t,Sphere _r p e c refl) -> do
+ (t,_) | t == (1/0.0) -> return zerov
+ (t,Sphere _r p e c refl) -> do
```

1. S
2. This is a far more performance critical version of what we saw with 'maximum' vs. 'max'.
3. innermost functions are of critical importance. remove Maybe which significantly reduces the boxing
4. Since a Ray that fails to intersect something can be said to intersect at infinity, Double already actually covers the structure at play
5. This also reduces allocation.

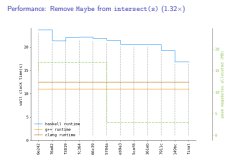
## Performance: Remove Maybe from intersect(s) (1.32×)



## Optimizing smallpt

2020-11-05

Performance: Remove Maybe from intersect(s) (1.32×)



# Hand unroll the fold in intersects (1.35x)

```
intersects :: Ray -> (Double, Sphere)
-intersects ray = (k, s)
-  where (k,s) = foldl' f (1/0.0,undefined) spheres
+intersects ray =
+  f (... (f (f (intersect ray sphLeft, sphLeft) sphRight) ...)
+  where
+    f (k', sp) s' = let !x = intersect ray s' in if x < k' then (x, s') else (k', sp)

-spheres :: [Sphere]
-spheres = let s = Sphere ; z = zeroV ; (.*) = mulvs ; v = Vec in
-  [ s 1e5 (v (1e5+1) 40.8 81.6)    z (v 0.75 0.25 0.25) DIFF --Left
-  , s 1e5 (v (-1e5+99) 40.8 81.6)  z (v 0.25 0.25 0.75) DIFF --Right
-  ...

+sphLeft, sphRight, ... :: Sphere
+sphLeft  = Sphere 1e5  (Vec (1e5+1) 40.8 81.6)    zeroV (Vec 0.75 0.25 0.25) DIFF
+sphRight = Sphere 1e5  (Vec (-1e5+99) 40.8 81.6)  zeroV (Vec 0.25 0.25 0.75) DIFF
+...
```

## Optimizing smallpt

2020-11-05

### Hand unroll the fold in intersects (1.35x)

```
Hand unroll the fold in intersects (1.35x)

intersects :: Ray -> (Double, Sphere)
-intersects ray = (k, s)
-  where (k,s) = foldl' f (1/0.0,undefined) spheres
+intersects ray =
+  f (... (f (f (intersect ray sphLeft, sphLeft) sphRight) ...)
+  where
+    f (k', sp) s' = let !x = intersect ray s' in if x < k' then (x, s') else (k', sp)

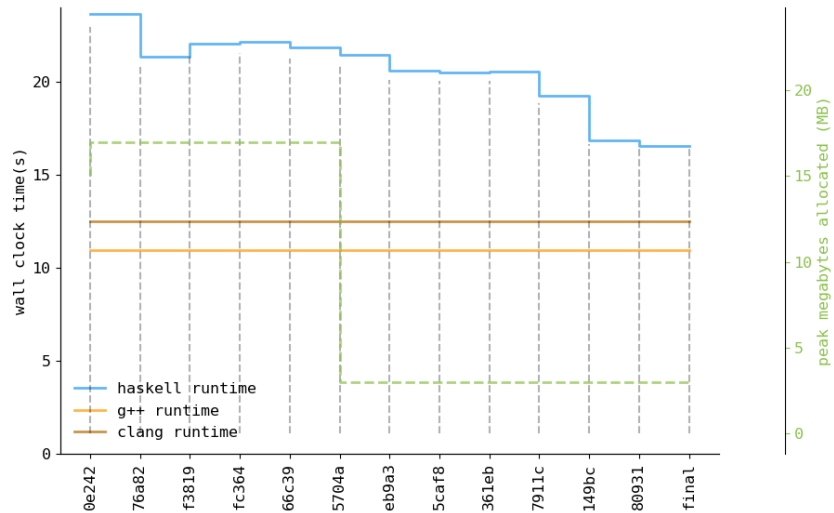
-spheres :: [Sphere]
-spheres = let s = Sphere ; z = zeroV ; (.*) = mulvs ; v = Vec in
-  [ s 1e5 (v (1e5+1) 40.8 81.6)    z (v 0.75 0.25 0.25) DIFF --Left
-  , s 1e5 (v (-1e5+99) 40.8 81.6)  z (v 0.25 0.25 0.75) DIFF --Right
-  ...

+sphLeft, sphRight, ... :: Sphere
+sphLeft  = Sphere 1e5  (Vec (1e5+1) 40.8 81.6)    zeroV (Vec 0.75 0.25 0.25) DIFF
+sphRight = Sphere 1e5  (Vec (-1e5+99) 40.8 81.6)  zeroV (Vec 0.25 0.25 0.75) DIFF
+...
```

1. D
2. 'intersects' is very hot
3. Loop unrolling
4. Many compilers do this for us, and there are special versions of it like Duff's Device.
5. Sadly GHC doesn't
6. Can do variants by hand.
7. RULE could handle each one specifically (only exactly that one)?



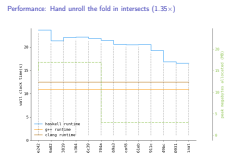
## Performance: Hand unroll the fold in intersects (1.35 $\times$ )



## Optimizing smallpt

2020-11-05

Performance: Hand unroll the fold in intersects (1.35 $\times$ )



# Custom datatype for intersects parameter passing

Old: Tuple with possibly-unevaluated Double and Sphere

New: Reference to a guaranteed-to-be-evaluated Double and Sphere

```
-intersects :: Ray -> (Double, Sphere)
```

```
+data T = T !Double !Sphere
```

```
+
```

```
+intersects :: Ray -> T
```

```
  intersects ray =  
-   f ( ... f (intersect ray sphLeft, sphLeft) sphRight) ... sphLite  
+   f ( ... f (T (intersect ray sphLeft) sphLeft) sphRight) ... sphLite  
  where  
-   f (k', sp) s' =  
-       let !x = intersect ray s' in if x < k' then (x, s') else (k', sp)  
+   f !(T k' sp) !s' =  
+       let !x = intersect ray s' in if x < k' then T x s' else T k' sp
```

```
radiance :: Ray -> Int -> Erand48 Vec  
radiance ray@(Ray o d) depth = case intersects ray of  
-  (!t,_) | t == 1/0.0 -> return 0  
-  (!t,!Sphere _r p e c refl) -> do  
+  (T t _) | t == 1/0.0 -> return 0  
+  (T t (Sphere _r p e c refl)) -> do  
    let !x = o + d .* t  
        !n = norm $ x - p  
        !nl = if dot n d < 0 then n else negate n
```

## Optimizing smallpt

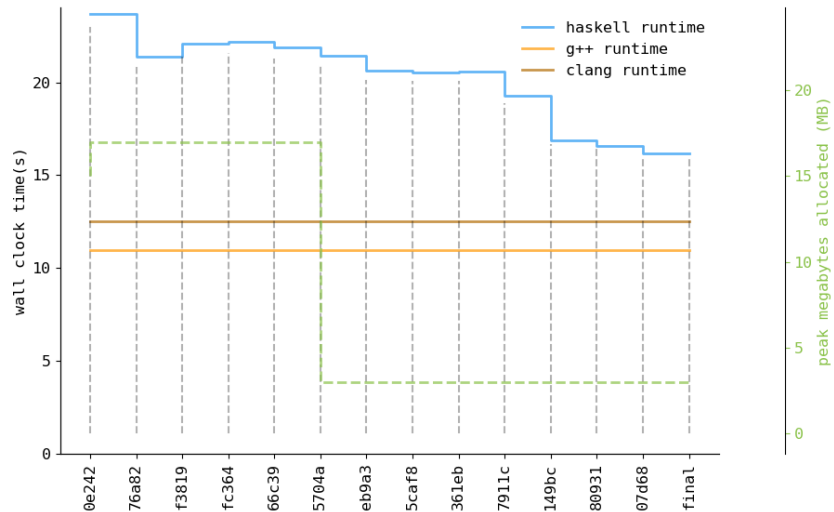
2020-11-05

### Custom datatype for intersects parameter passing

Old: Tuple with possibly-unevaluated Double and Sphere  
New: Reference to a guaranteed-to-be-evaluated Double and Sphere  
-intersects :: Ray -> (Double, Sphere)  
+data T = T !Double !Sphere  
+  
+intersects :: Ray -> T  
intersects ray =  
- f (...) f (intersect ray sphLeft, sphLeft) sphRight) ... sphLite  
+ f (...) f (T (intersect ray sphLeft) sphLeft) sphRight) ... sphLite  
 where  
- f (k', sp) s' =  
- let !x = intersect ray s' in if x < k' then (x, s') else (k', sp)  
+ f !(T k' sp) !s' =  
+ let !x = intersect ray s' in if x < k' then T x s' else T k' sp  
  
radiance :: Ray -> Int -> Erand48 Vec  
radiance ray@(Ray o d) depth = case intersects ray of  
- (!t,\_) | t == 1/0.0 -> return 0  
- (!t,!Sphere \_r p e c refl) -> do  
+ (T t \_) | t == 1/0.0 -> return 0  
+ (T t (Sphere \_r p e c refl)) -> do  
 let !x = o + d .\* t  
 !n = norm \$ x - p  
 !nl = if dot n d < 0 then n else negate n

1. D
2. We can optimize data passing.
3. Want: Data strict, but not unpacked.
4. Compiler knows its evaluated but no copying
5. A normal tuple lacks strictness information.
6. An unboxed tuple forces copying
7. Strict Tuple.
8. This exists in libraries of course, but we wanted to illustrate it.

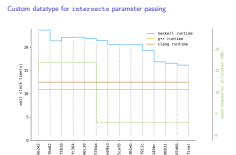
## Custom datatype for intersects parameter passing



## Optimizing smallpt

2020-11-05

Custom datatype for intersects parameter passing



# Optimize file writing

```
build-depends:
    base >= 4.12 && < 4.15
+    , bytestring ^>= 0.11

toInt :: Double -> Int
toInt x = floor $ clamp x ** recip 2.2 * 255 + 0.5
+toInt :: Double -> BB.Builder -- O(1) concatenation
+toInt x = BB.intDec (floor (clamp x ** recip 2.2 * 255 + 0.5)) <> BB.char8 ' '
...
withFile "image.ppm" WriteMode $ \hdl -> do
-    hPrintf hdl "P3\n%d %d\n%d\n" w h (255::Int)
-    for_ img \(Vec r g b) -> do
-        hPrintf hdl "%d %d %d " (toInt r) (toInt g) (toInt b)
+    BB.hPutBuilder hdl $
+        BB.string8 "P3\n" <> -- efficient builders for ASCII
+        BB.intDec w <> BB.char8 ' ' <> BB.intDec h <> BB.char8 '\n' <>
+        BB.intDec 255 <> BB.char8 '\n' <>
+        (mconcat $ fmap \(Vec r g b) -> toInt r <> toInt g <> toInt b) img)
```

## Optimizing smallpt

2020-11-05

### Optimize file writing

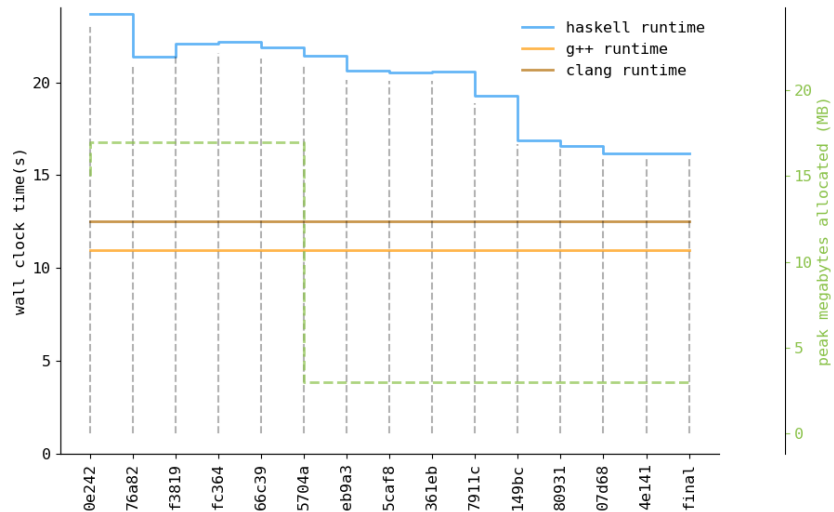
```
Optimize file writing

build-depends:
    base >= 4.12 && < 4.15
+    , bytestring ^>= 0.11

toInt :: Double -> Int
toInt x = floor $ clamp x ** recip 2.2 * 255 + 0.5
+toInt :: Double -> BB.Builder -- O(1) concatenation
+toInt x = BB.intDec (floor (clamp x ** recip 2.2 * 255 + 0.5)) <> BB.char8 ' '
...
withFile "image.ppm" WriteMode $ \hdl -> do
-    hPrintf hdl "P3\n%d %d\n%d\n" w h (255::Int)
-    for_ img \(Vec r g b) -> do
-        hPrintf hdl "%d %d %d " (toInt r) (toInt g) (toInt b)
+    BB.hPutBuilder hdl $
+        BB.string8 "P3\n" <> -- efficient builders for ASCII
+        BB.intDec w <> BB.char8 ' ' <> BB.intDec h <> BB.char8 '\n' <>
+        BB.intDec 255 <> BB.char8 '\n' <>
+        (mconcat $ fmap \(Vec r g b) -> toInt r <> toInt g <> toInt b) img)
```

1. D
2. Strings are inefficient
3. 'bytestring' has some efficient writing code, so we just convert to that for a modest gain.

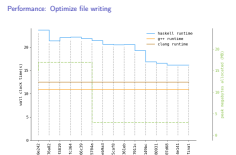
## Performance: Optimize file writing



## Optimizing smallpt

2020-11-05

Performance: Optimize file writing



Use LLVM backend (1.87x)

```
+package smallpt-opt
+  ghc-options: -fllvm
```

## Optimizing smallpt

2020-11-05

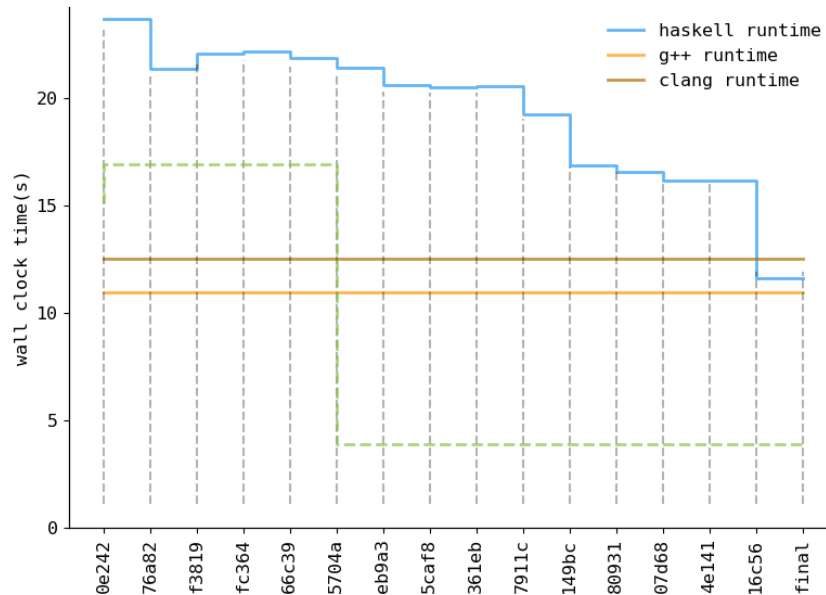
└ Use LLVM backend (1.87x)

Use LLVM backend (1.87x)

```
+package smallpt-opt
+ ghc-options: -fllvm
```

1. **S**
2. Finally, this particular code is quite numeric heavy.
3. There are optimizations for numeric heavy code we're missing in GHC.
4. LLVM has an extensive library of laws to optimize low level numeric ops.
5. LLVM is too low-level to understand haskell as haskell.
6. LLVM makes decisions with the tacit assumption that the assembly came from a C-like language, which is often to the detriment of a Haskell-like language.
7. In this case, as the code is "fortran-like", LLVM wins.

# The view from the mountaintop



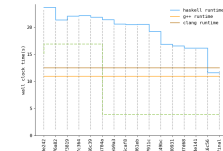
## Optimizing smallpt

2020-11-05

The view from the mountaintop

1. D

The view from the mountaintop



## Avoid CPU ieee754 slow paths

```
intersect :: Ray -> Sphere -> Double
intersect (Ray o d) (Sphere r p _e _c _refl) =
  if det<0
-   then 1/0.0
+   then 1e20
  else
    ...
-   in if a>eps then a else if s>eps then s else 1/0.0
+   in if a>eps then a else if s>eps then s else 1e20
...
radiance :: Ray -> Int -> Erand48 Vec
radiance ray@(Ray o d) depth = case intersects ray of
-   (T t _) | t == 1/0.0 -> return 0
+   (T 1e20 _) -> return 0
...
```

## Optimizing smallpt

2020-11-05

Avoid CPU ieee754 slow paths

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```

1. D
2. We used +Inf to match the Maybeness
3. C++ code set 1e20 s the horizon
4. Mechanical sympathy is important.
5. Know how the CPU (abstractly) executes - slow path / fast path.



## Fix differences with C++ version

```
-         if depth>2
+         if depth'>2 -- depth' = depth + 1
...

```

## Optimizing smallpt

2020-11-05

- └ Fix differences with C++ version

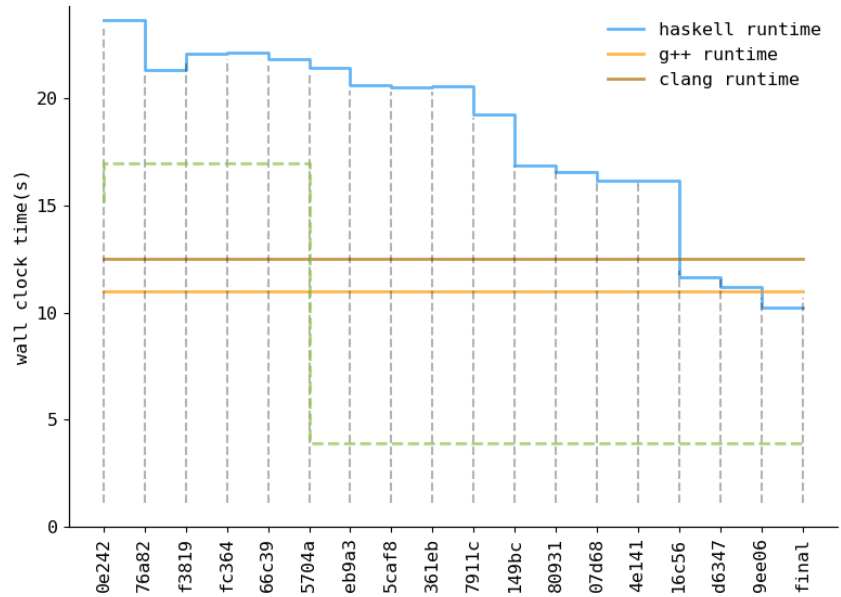
Fix differences with C++ version

```
-         if depth>2
+         if depth>2 -- depth = depth + 1
...

```

1. **S**
2. Since the sha1 of the output didn't match the C++ version we started investigating.
3. clang++, g++ actually produce different sha1s
4. unincremented depth was being used in one branch, causing us to do more work
5. now confident to say we're doing the same computation as C++

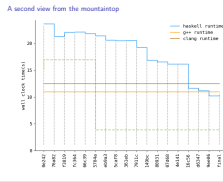
## A second view from the mountaintop



## Optimizing smallpt

2020-11-05

A second view from the mountaintop



## Takeaways

- ▶ The unrolling in 'intersects' is ugly.
- ▶ (We feel) the maintainability of this code hasn't been significantly harmed.
- ▶ We're faster than clang++ and within 6% of g++
- ▶ Haven't exhausted the optimization opportunities.
- ▶ GHC could learn to do several of these optimizations for us.
- ▶ Others are just good Haskell style.
- ▶ Clean Haskell is often performant Haskell.
- ▶ Repository stepping through each optimization is available at ...

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1. D

## Raw data

- All test were on an otherwise idle Equinix Metal c3.small.x86 (Intel Xeon E-2278G with 32GiB RAM, Ubuntu 20.04).

0e242	23.6586	23.658	23.7251	23.6954	23.676	23.656	23.673	23.7139	23.6598	23.7641
76a82	21.3573	21.384	21.3516	21.3304	21.3658	21.3776	21.3564	21.3843	21.3401	21.3683
f3819	22.1036	22.0754	22.1034	22.0864	22.0692	22.1004	22.0584	22.1181	22.0728	22.0972
fc364	22.1211	22.114	22.1205	22.1101	23.0621	22.1101	22.1163	22.133	22.1491	22.1464
66c39	21.8626	21.8684	21.8977	21.9043	21.893	21.8483	21.8335	21.8869	21.8848	21.8335
5704a	21.4682	21.4692	21.4411	21.473	21.4783	21.4613	21.4818	21.4507	21.4388	21.4933
eb9a3	20.6134	20.6014	20.6527	20.596	20.6034	20.6174	20.5965	20.594	20.5967	20.5892
5caf8	20.5209	20.535	20.5312	20.5289	20.5338	20.5717	20.5387	20.5386	20.5262	20.5488
361eb	20.5551	20.5485	20.5602	20.552	20.5668	20.555	20.5573	20.5564	20.5599	20.5823
7911c	19.2532	19.2664	19.2955	19.2565	19.2517	19.2722	19.3284	19.2611	19.2623	19.2596
149bc	16.8551	16.8551	16.8788	16.9067	16.8683	16.8685	16.8651	16.9186	16.8589	16.8604
80931	16.5752	16.5739	16.5831	16.5918	16.5678	16.5785	16.6128	16.5682	16.5816	16.577
07d68	16.1819	16.1672	16.1829	16.2267	16.1716	16.1854	16.1806	16.1949	16.1917	16.1784
4e141	16.2206	16.1816	16.2002	16.1799	16.1813	16.1781	16.1929	16.243	16.1705	16.1877
16c56	11.6334	11.6166	11.6837	11.6504	11.6227	11.6135	11.5949	11.5966	11.6013	11.64
d6347	11.1632	11.218	11.1741	11.1802	11.1849	11.1755	11.1729	11.2155	11.1718	11.2089
9ee06	10.2131	10.2154	10.1994	10.2105	10.2028	10.2344	10.2008	10.2497	10.2226	10.3042
gcc	10.97	10.97	10.97	10.99	10.98	10.97	10.97	10.98	10.98	10.98
clang	12.53	12.51	12.5	12.53	12.51	12.5	12.52	12.48	12.53	12.55

## Optimizing smallpt

2020-11-05

- Raw data

Raw data

► All tests were on an otherwise idle Equinox Metal c3.small.x86 (Intel Xeon E-2279G with 32GB RAM, Ubuntu 20.04).

	2016	2015	2014	2013	2012	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001	2000	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975	1974	1973	1972	1971	1970	1969	1968	1967	1966	1965	1964	1963	1962	1961	1960	1959	1958	1957	1956	1955	1954	1953	1952	1951	1950	1949	1948	1947	1946	1945	1944	1943	1942	1941	1940	1939	1938	1937	1936	1935	1934	1933	1932	1931	1930	1929	1928	1927	1926	1925	1924	1923	1922	1921	1920	1919	1918	1917	1916	1915	1914	1913	1912	1911	1910	1909	1908	1907	1906	1905	1904	1903	1902	1901	1900	1899	1898	1897	1896	1895	1894	1893	1892	1891	1890	1889	1888	1887	1886	1885	1884	1883	1882	1881	1880	1879	1878	1877	1876	1875	1874	1873	1872	1871	1870	1869	1868	1867	1866	1865	1864	1863	1862	1861	1860	1859	1858	1857	1856	1855	1854	1853	1852	1851	1850	1849	1848	1847	1846	1845	1844	1843	1842	1841	1840	1839	1838	1837	1836	1835	1834	1833	1832	1831	1830	1829	1828	1827	1826	1825	1824	1823	1822	1821	1820	1819	1818	1817	1816	1815	1814	1813	1812	1811	1810	1809	1808	1807	1806	1805	1804	1803	1802	1801	1800	1799	1798	1797	1796	1795	1794	1793	1792	1791	1790	1789	1788	1787	1786	1785	1784	1783	1782	1781	1780	1779	1778	1777	1776	1775	1774	1773	1772	1771	1770	1769	1768	1767	1766	1765	1764	1763	1762	1761	1760	1759	1758	1757	1756	1755	1754	1753	1752	1751	1750	1749	1748	1747	1746	1745	1744	1743	1742	1741	1740	1739	1738	1737	1736	1735	1734	1733	1732	1731	1730	1729	1728	1727	1726	1725	1724	1723	1722	1721	1720	1719	1718	1717	1716	1715	1714	1713	1712	1711	1710	1709	1708	1707	1706	1705	1704	1703	1702	1701	1700	1699	1698	1697	1696	1695	1694	1693	1692	1691	1690	1689	1688	1687	1686	1685	1684	1683	1682	1681	1680	1679	1678	1677	1676	1675	1674	1673	1672	1671	1670	1669	1668	1667	1666	1665	1664	1663	1662	1661	1660	1659	1658	1657	1656	1655	1654	1653	1652	1651	1650	1649	1648	1647	1646	1645	1644	1643	1642	1641	1640	1639	1638	1637	1636	1635	1634	1633	1632	1631	1630	1629	1628	1627	1626	1625	1624	1623	1622	1621	1620	1619	1618	1617	1616	1615	1614	1613	1612	1611	1610	1609	1608	1607	1606	1605	1604	1603	1602	1601	1600	1599	1598	1597	1596	1595	1594	1593	1592	1591	1590	1589	1588	1587	1586	1585	1584	1583	1582	1581	1580	1579	1578	1577	1576	1575	1574	1573	1572	1571	1570	1569	1568	1567	1566	1565	1564	1
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