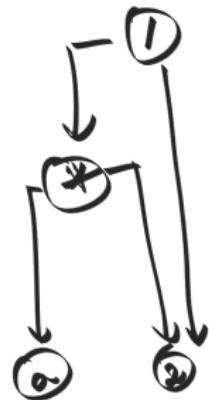


egg : Fast and extensible equality saturation

Siddharth Bhat

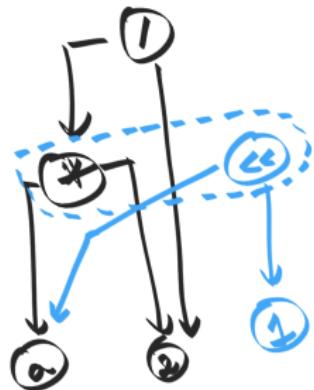
Monday, Jan 18 2021

egg : Fast and extensible equality saturation



$(\alpha * 2) / 2 :$
 $(1 \cap \alpha, 2) \cdot 2$

egg : Fast and extensible equality saturation (2)

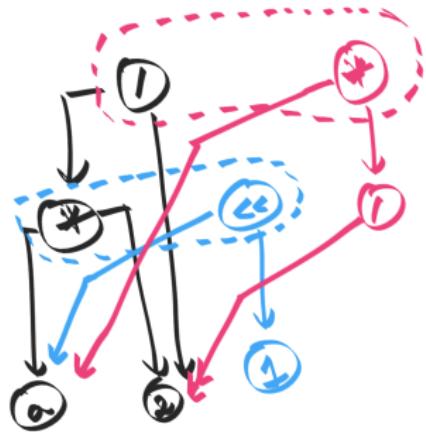


$(\alpha * 2) / 2 :$
 $(1 \cup (\alpha * 2), 2)$

$p * 2 \rightarrow p \ll 1$

$0 \cup (\alpha * 2), 2)$
 $(\ll \alpha, 1)$

egg : Fast and extensible equality saturation (3)



$(\alpha * 2) / 2 :$
 $(1 (* \alpha 2) 2)$

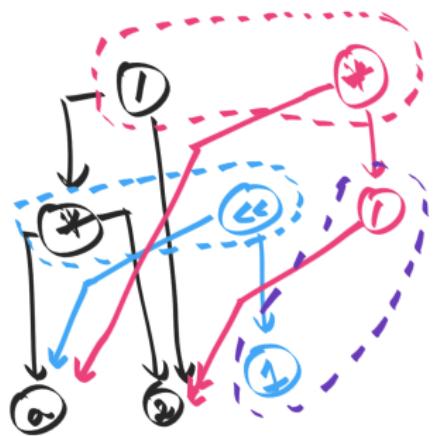
$p * 2 \rightarrow p \ll 1$

$(1 (* \alpha 2) 2)$
 $(\ll \alpha 1)$

$(p * \alpha) / 2 \rightarrow p * (\alpha / 2)$

$(1 (* \alpha 2) 2)$
 $(\frac{2}{\alpha} \alpha, 1) 2 2$

egg : Fast and extensible equality saturation (4)



$$\begin{array}{l} \pi/\pi \rightarrow 1 \\ (\# \alpha_1 \# \alpha_2) \\ \downarrow \end{array}$$

$$\begin{array}{l} (\alpha \# 2)/2: \\ (1 (\# \alpha_2) 2) \end{array}$$

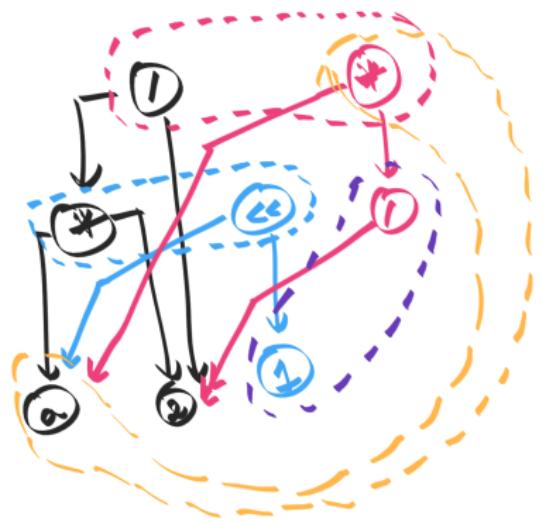
$$p \# 2 \rightarrow p \ll 1$$

$$\begin{array}{l} (1 (\# \alpha_2) 2) \\ (\ll \alpha_1 1) \end{array}$$

$$(p \# a)/2 \rightarrow p \# (a/2)$$

$$\begin{array}{l} (1 (\# \alpha_2) 2) \\ (\# \alpha_1 (1 \# 2) 2) \end{array}$$

egg : Fast and extensible equality saturation (5)



$$x/x \rightarrow 1$$

$$(*\alpha \{1^{22}\})$$

$$x*\perp \rightarrow \perp$$

$$(*\alpha \{1^{22}\})$$

or

$$(\alpha*2)/2:$$

$$(1 (*\alpha 2) 2)$$

$$p*2 \rightarrow p \ll 1$$

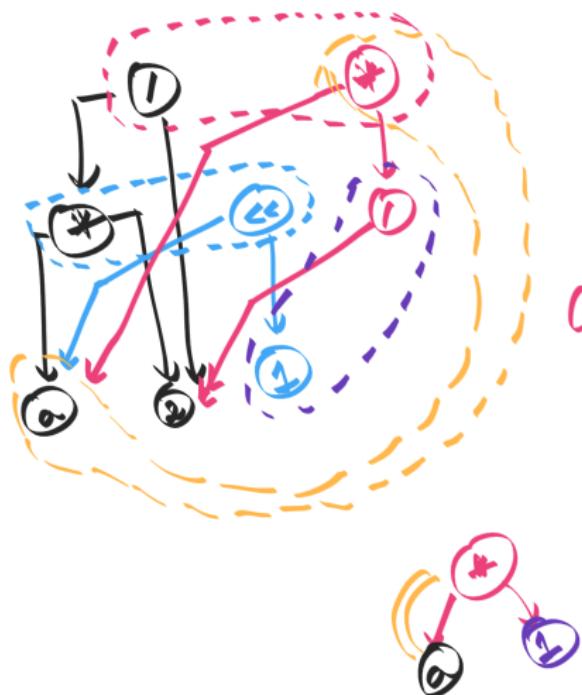
$$(1 (*\alpha 2) 2)$$

$$(p*\alpha)/2 \rightarrow p*(q/2)$$

$$(1 (*\alpha 2) 2)$$

$$(*\alpha 1 \{1^{22}\})$$

egg : Fast and extensible equality saturation (6)



$$\alpha/\alpha \rightarrow 1$$
$$(*\alpha (*1^2)^2)$$

$$\alpha*\alpha \rightarrow \alpha$$
$$(*\alpha (*1^2)^2)$$

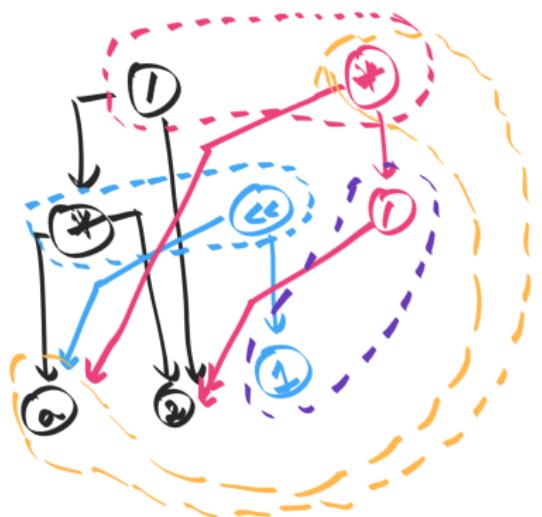
$$(\alpha*\alpha)/2 :$$
$$((1 (*\alpha 2) 2) 2)$$

$$p*2 \rightarrow p < 1$$
$$(1 (*\alpha 2) 2)$$

$$(\alpha*\alpha)/3 \rightarrow \alpha*(\alpha/3)$$

$$(1 (*\alpha 2) 2)$$
$$(*\alpha (*1^2)^2)$$

egg : Fast and extensible equality saturation (7)



$$\alpha/\alpha \rightarrow 1$$
$$(\alpha/\alpha, 1^{(2,2)})$$
$$\alpha/\alpha \rightarrow 1$$

$$(\alpha/\alpha)/2:$$
$$(1 (\alpha/\alpha) 2)$$

$$\alpha/\alpha \rightarrow \alpha$$
$$(\alpha/\alpha, 1^{(2,2)})$$
$$\alpha/\alpha \rightarrow \alpha$$

$$p*2 \rightarrow p << 1$$

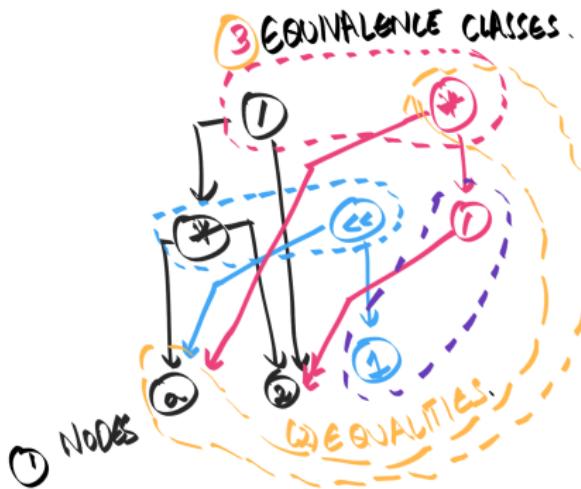
$$(1 (\alpha/\alpha) 2)$$
$$(\alpha/\alpha, 1)$$

$$(p\alpha)/\alpha \rightarrow p*(q/\alpha)$$

$$(1 (\alpha/\alpha) 2)$$
$$(\alpha/\alpha, 1^{(2,2)})$$



egg : Fast and extensible equality saturation (8)



$$\begin{aligned} \alpha / \alpha &\rightarrow 1 \\ (\# \alpha, 1^{(2,2)}) & \quad \quad \quad \downarrow \\ 0 & \end{aligned}$$

$$\begin{aligned} \alpha * 1 &\rightarrow n \\ (\# \alpha, 1^{(2,2)}) & \quad \quad \quad \downarrow \\ 0 & \end{aligned}$$

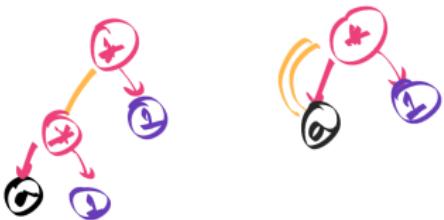
$$\begin{aligned} (\alpha * 2) / 2 &\rightarrow \\ (1 (\# \alpha 2), 2) & \end{aligned}$$

$$p * 2 \rightarrow p \ll 1$$

$$\begin{aligned} 0 (\# \alpha 2), 2 & \\ (2 \ll \alpha, 1) & \end{aligned}$$

$$(p \alpha a) / 3 \rightarrow p \# (a / 3)$$

$$\begin{aligned} (1 (\# \alpha 2), 2) & \\ (\# \alpha, 1^{(2,2)}) & \end{aligned}$$



Evaluation: Herbie

$$\text{sqrt}(x+1) - \text{sqrt}(x) \rightarrow 1/(\text{sqrt}(x+1) + \text{sqrt}(x))$$

Herbie detects inaccurate expressions and finds more accurate replacements. The red expression is inaccurate when $x > 1$; Herbie's replacement, in blue, is accurate for all x .

Evaluation: Herbie

sqrt(x+1) - sqrt(x) → 1/(sqrt(x+1) + sqrt(x))

Herbie detects inaccurate expressions and finds more accurate replacements. The red expression is inaccurate when $x > 1$; Herbie's replacement, in blue, is accurate for all x .

$$\sqrt{x+1} - \sqrt{x} = \frac{(\sqrt{x+1} - \sqrt{x})(\sqrt{x+1} + \sqrt{x})}{\sqrt{x+1} + \sqrt{x}} = \frac{(x+1) - x}{\sqrt{x+1} + \sqrt{x}} = \frac{1}{\sqrt{x+1} + \sqrt{x}}$$

Evaluation: Herbie

$$\text{sqrt}(x+1) - \text{sqrt}(x) \rightarrow 1/(\text{sqrt}(x+1) + \text{sqrt}(x))$$

Herbie detects inaccurate expressions and finds more accurate replacements. The red expression is inaccurate when $x > 1$; Herbie's replacement, in blue, is accurate for all x .

$$\sqrt{x+1} - \sqrt{x} = \frac{(\sqrt{x+1} - \sqrt{x})(\sqrt{x+1} + \sqrt{x})}{\sqrt{x+1} + \sqrt{x}} = \frac{(x+1) - x}{\sqrt{x+1} + \sqrt{x}} = \frac{1}{(\sqrt{x+1} + \sqrt{x})}$$

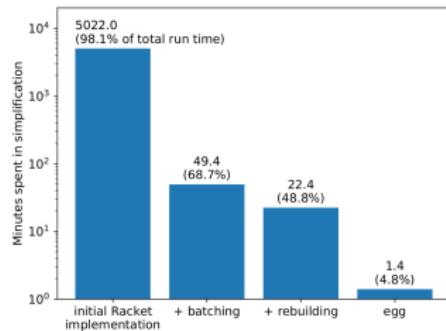


Fig. 12. Herbie sped up its expression simplification phase by adopting egg-inspired features like batched simplification and rebuilding into its Racket-based e-graph implementation. Herbie also supports using egg itself for additional speedup. Note that the y-axis is log-scale.

Herbie: Using egg — rules

<https://github.com/uwplse/herbie/blob/master/egg-herbie/src/rules.rs>

```
;; projections
("real-part", "(re real (complex real ?x ?y))", "?x")
;; why duplication of real and complex rules?
("associate-+r+.c", "(+ c ?a (+ c ?b ?c))", "(+ c (+ c ?a ?b) ?c)")
("associate-+r+", "(+ f64 ?a (+ f64 ?b ?c))", "(+ f64 (+ f64 ?a ?b) ?c)")
;; branch simplification
("if-true", "(if real (TRUE real) ?x ?y)", "?x")
```

Herbie: Using egg — analysis

<https://github.com/uwplse/herbie/blob/master/egg-herbie/src/math.rs>

```
define_language! {
    pub enum Math {
        "+" = Add([Id; 3]),
        "-" = Sub([Id; 3]),
        ...
    }
}
```

Herbie: Using egg — analysis

```
https://github.com/uwplse/herbie/blob/master/egg-herbie/src/math.rs
define_language! {
    pub enum Math {
        "+" = Add([Id; 3]),
        "-" = Sub([Id; 3]),
        ...
    }
}

pub struct ConstantFold { ... }

pub type Constant = num_rational::BigRational;
```

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    }
}

pub struct ConstantFold { ... }

pub type Constant = num_rational::BigRational;

impl Analysis<Math> for ConstantFold {
    type Data = Option<Constant>; // data for each equiv-class
}
```

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    fn make(egraph: &EGraph, enode: &Math) -> Self::Data {
```

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        let x = |id: &Id| egraph[*id].data.as_ref(); // access Data
    }
}
```

Herbie: Using egg — analysis

<https://github.com/uwplse/herbie/blob/master/egg-herbie/src/math.rs>

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    fn make(egraph: &EGraph, enode: &Math) -> Self::Data {
        let x = |id: &Id| egraph[*id].data.as_ref(); // access Data

        match enode {
            Math::Add([_p, a, b]) => Some(x(a)? + x(b)?),
            ...
        }
    }
}
```

Herbie: Using egg — analysis

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impl Analysis<Math> for ConstantFold {
    type Data = Option<Constant>; // data for each equiv-class

    fn make(egraph: &EGraph, enode: &Math) -> Self::Data {

        let x = |id: &Id| egraph[*id].data.as_ref(); // access Data

        match enode {
            Math::Add([_p, a, b]) => Some(x(a)? + x(b)?),
            Math::Sqrt([_p, a]) => {
                let a = x(a)?;
                if *a.numer() > BigInt::from(0) && *a.denom() > BigInt::from(0) {
                    let s1 = a.numer().sqrt();
                    let s2 = a.denom().sqrt();
                    let is_perfect = &(s1 * &s1) == a.numer()
                        && &(s2 * &s2) == a.denom();
                    if is_perfect { Some(Ratio::new(s1, s2)) } else { None }
                } else { None }
            }
        }
    }
}
```

Herbie: Using egg — Lattice

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```
// commutative, associative, idempotent, has identity element (Nothing)
fn merge(&self, to: &mut Self::Data, from: Self::Data) -> bool {
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// commutative, associative, idempotent, has identity element (Nothing)
fn merge(&self, to: &mut Self::Data, from: Self::Data) -> bool {
    match (&to, from) {
        (None, None) => false,
```

Herbie: Using egg — Lattice

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        (None, None) => false,
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fn merge(&self, to: &mut Self::Data, from: Self::Data) -> bool {
    match (&to, from) {
        (None, None) => false,
        (Some(_), None) => false, // no update needed
        (None, Some(c)) => {
            *to = Some(c);
            true
        }
    }
}
```

Herbie: Using egg — Lattice

```
// commutative, associative, idempotent, has identity element (Nothing)
fn merge(&self, to: &mut Self::Data, from: Self::Data) -> bool {
    match (&to, from) {
        (None, None) => false,
        (Some(_), None) => false, // no update needed
        (None, Some(c)) => {
            *to = Some(c);
            true
        }
        (Some(a), Some(ref b)) => {
            // bad analysis detected!
            if a != b { log::warn!("Bad merge detected: {} != {}", a, b); }
            false
        }
    }
}
```

Herbie: Using egg — Extraction

```
pub struct Extracted { pub best: RecExpr, pub cost: usize, }
```

Herbie: Using egg — Extraction

```
pub struct Extracted { pub best: RecExpr, pub cost: usize, }
pub struct IterData { pub extracted: Vec<(Id, Extracted)>, }
impl IterationData<Math, ConstantFold> for IterData {
```

Herbie: Using egg — Extraction

```
pub struct Extracted { pub best: RecExpr, pub cost: usize, }
pub struct IterData { pub extracted: Vec<(Id, Extracted)>, }
impl IterationData<Math, ConstantFold> for IterData {
    fn make(runner: &Runner) -> Self {
        let mut extractor = Extractor::new(&runner.egraph, AstSize);
```

Herbie: Using egg — Extraction

```
pub struct Extracted { pub best: RecExpr, pub cost: usize, }

pub struct IterData { pub extracted: Vec<(Id, Extracted)>, }
impl IterationData<Math, ConstantFold> for IterData {

    fn make(runner: &Runner) -> Self {
        let mut extractor = Extractor::new(&runner.egraph, AstSize);

        let extracted = runner
            .roots
            .iter()
            .map(|&root| {
                let (cost, best) = extractor.find_best(root);
                let ext = Extracted { cost, best };
                (root, ext)
            })
            .collect();
        Self { extracted }
    }
}
```

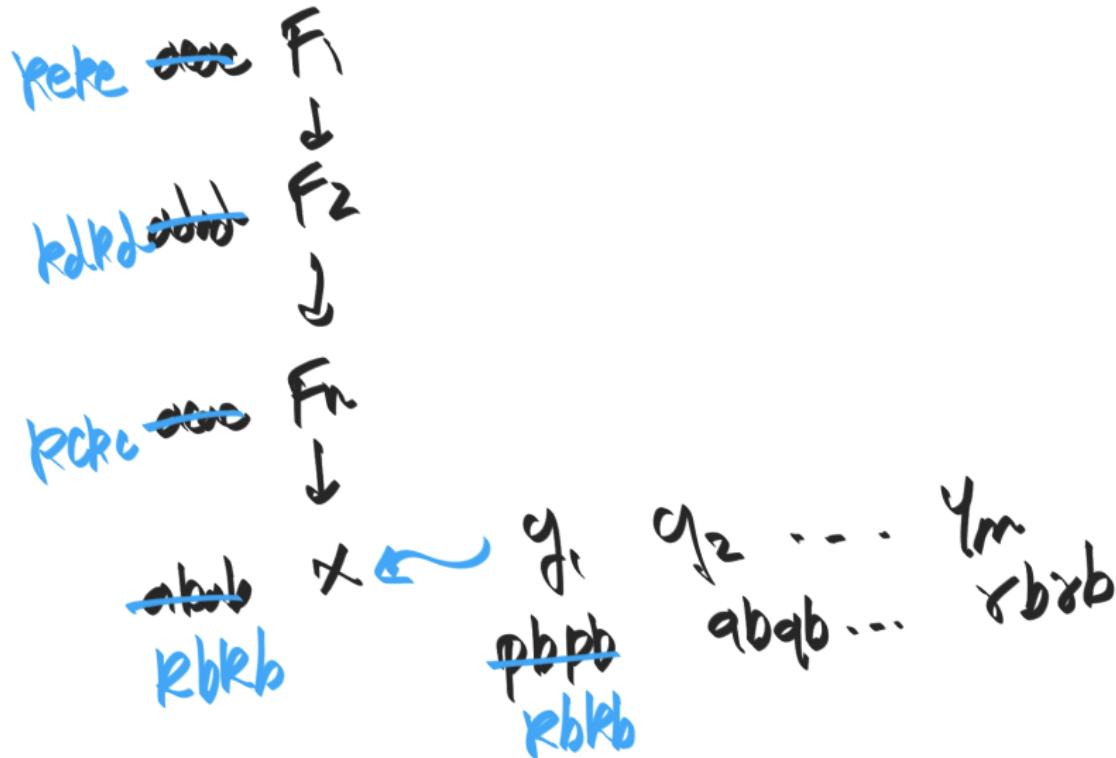
- ▶ Read Phase: Find all matching rewrites
- ▶ Write Phase: Perform rewrites
- ▶ Canonicalize Phase: Merge equivalent nodes.

Speedup over prior art: Merging

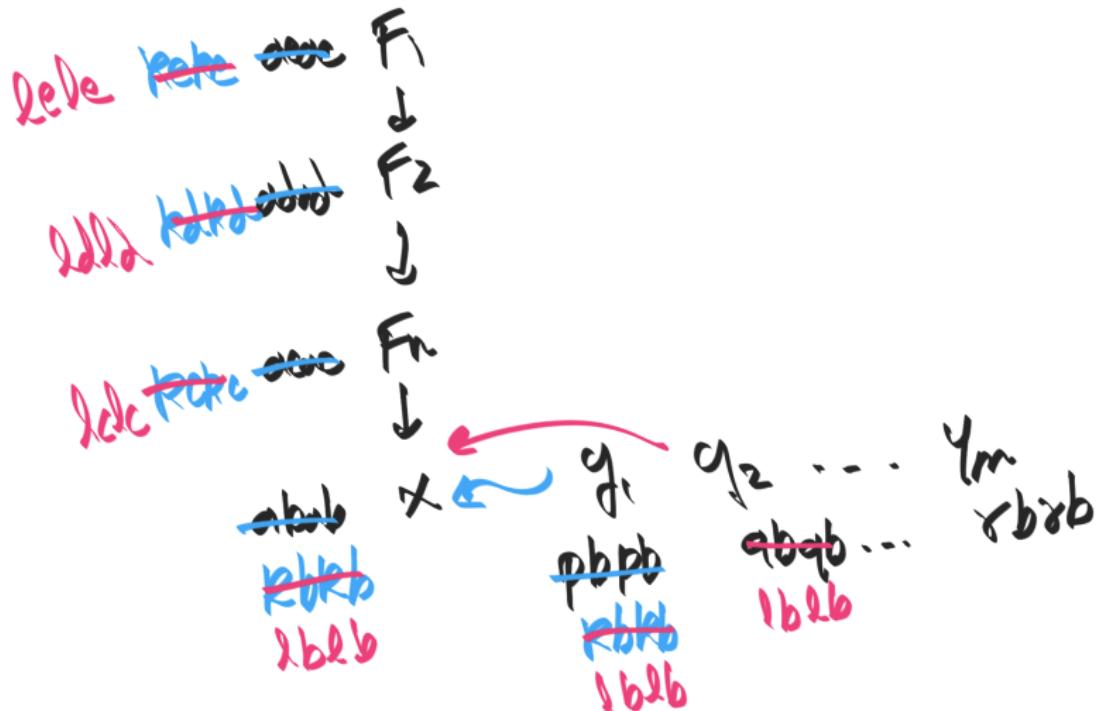
000c F_1
↓
000d F_2
↓
000c F_n
↓
abab X

$q_1 \ q_2 \ \dots \ q_m$
 $pbpb \ abqb \ \dots \ rbrb$

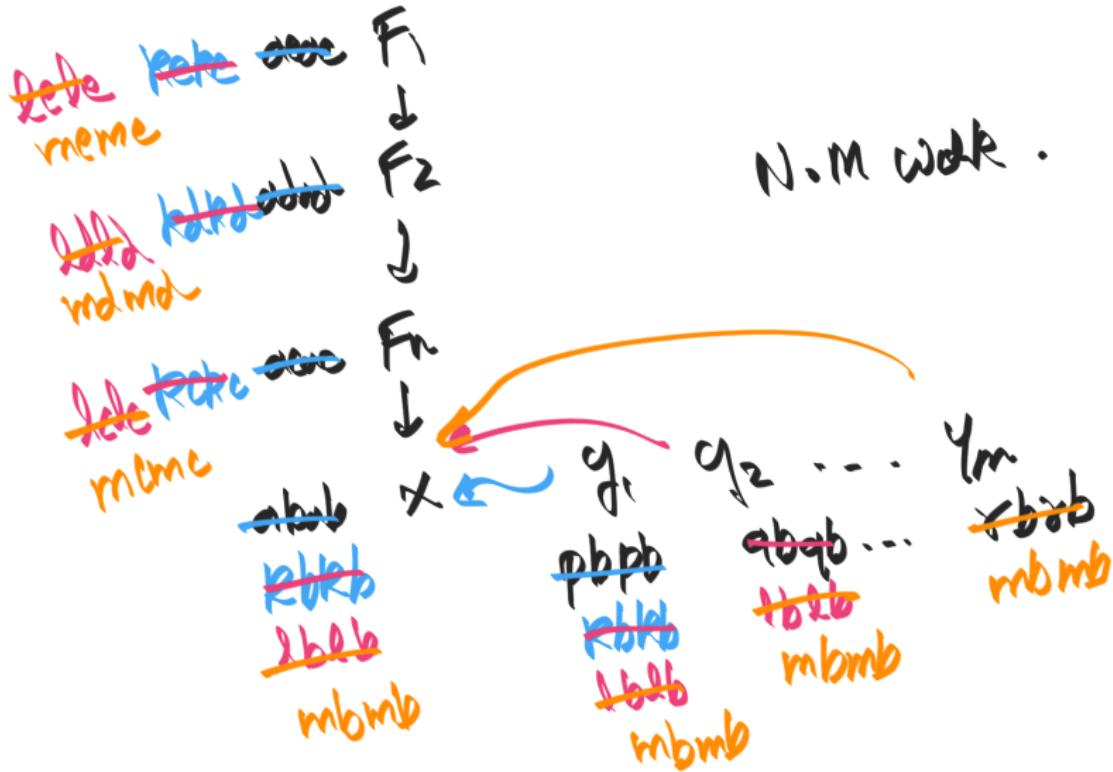
Speedup over prior art: Merging



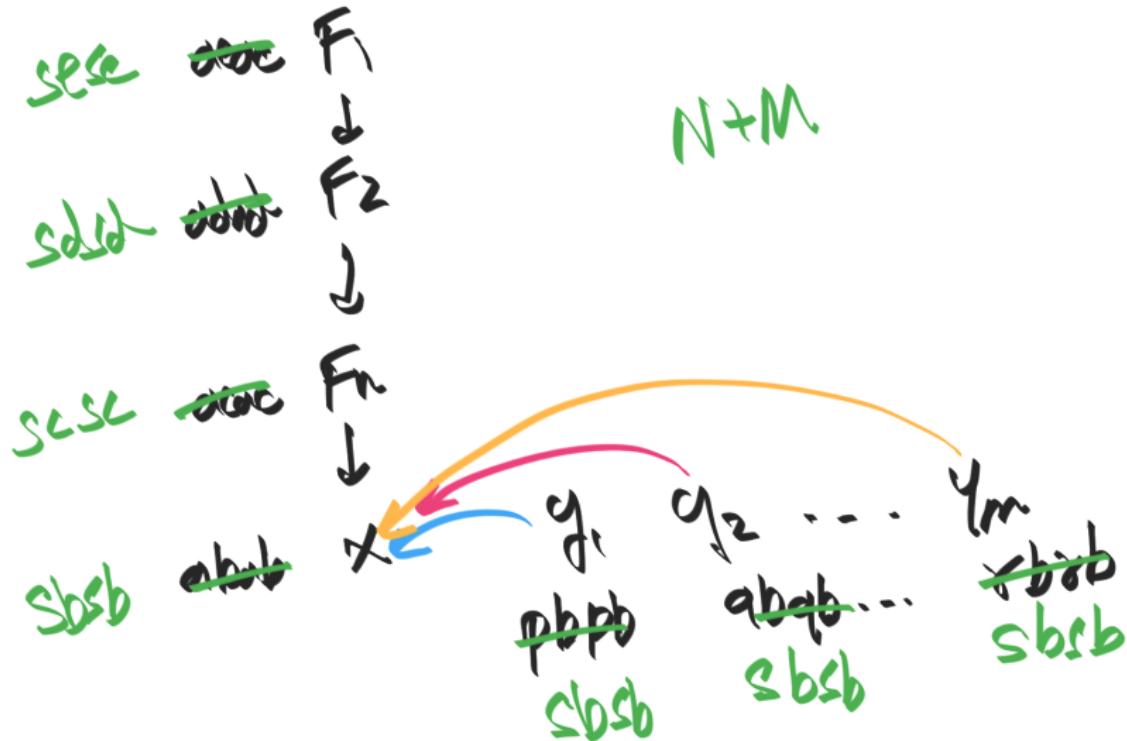
Speedup over prior art: Merging



Speedup over prior art: Merging



Speedup over prior art: Merging



Speedup over prior art: Merging

3.2.1 *Examples of Rebuilding.* Deferred rebuilding speeds up congruence maintenance by amortizing the work of maintaining the hashcons invariant. Consider the following terms in an e-graph: $f_1(x), \dots, f_n(x), y_1, \dots, y_n$. Let the workload be $\text{merge}(x, y_1), \dots, \text{merge}(x, y_n)$. Each merge may change the canonical representation of the $f_i(x)$ s, so the traditional invariant maintenance strategy could require $O(n^2)$ hashcons updates. With deferred rebuilding the merges happen before the hashcons invariant is restored, requiring no more than $O(n)$ hashcons updates.

Analysis, formally

- ▶ Abstract interpretation of equivalence classes.
- ▶ For each node, provide function $\alpha : \text{node} \rightarrow L$ (Abstraction function)
- ▶ (L, \cap) is a join-semilattice.
- ▶ egg provides for each equivalence class $\text{class} \mapsto \bigcap_{\text{node} \in \text{class}} \alpha(\text{node}) \in L$

Lambda calculus in egg : Language

```
define_language! {  
    enum Lambda {  
        Bool(bool),  
        Num(i32),  
  
        "var" = Var(Id),  
  
        "+" = Add([Id; 2]),  
        "=" = Eq([Id; 2]),  
  
        "app" = App([Id; 2]),  
        "lam" = Lambda([Id; 2]),  
        "let" = Let([Id; 3]),  
        "fix" = Fix([Id; 2]),  
  
        "if" = If([Id; 3]),  
  
        Symbol(egg::Symbol),  
    }  
}
```

Lambda calculus in egg : Analysis

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }
```

Lambda calculus in egg : Analysis

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }
impl Analysis<Lambda> for LambdaAnalysis {
    type Data = Data;
```

Lambda calculus in egg : Analysis

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }
impl Analysis<Lambda> for LambdaAnalysis {
    type Data = Data;
    fn make(egraph: &EGraph, enode: &Lambda) -> Data {
        let f = |i: &Id| egraph[*i].data.free.iter().cloned();
        let mut free = HashSet::default();
```

Lambda calculus in egg : Analysis

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }

impl Analysis<Lambda> for LambdaAnalysis {
    type Data = Data;
    fn make(egraph: &EGraph, enode: &Lambda) -> Data {
        let f = |i: &Id| egraph[*i].data.free.iter().cloned();
        let mut free = HashSet::default();
        match enode {
            Lambda::Var(v) => { // var
                free.insert(*v);
            }
        }
    }
}
```

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```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }

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        let mut free = HashSet::default();

        match enode {
            Lambda::Var(v) => { // var
                free.insert(*v);
            }
            Lambda::Let([v, a, b]) => { // let v = a in b(v)
                free.extend(f(b));
                free.remove(v);
                free.extend(f(a));
            }
        }
    }
}
```

Lambda calculus in egg : Analysis

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                free.remove(v);
                free.extend(f(a));
            }
            // \v. a(v)
            Lambda::Lambda([v, a]) | Lambda::Fix([v, a]) => {
                free.extend(f(a));
                free.remove(v);
            }
            _ => enode.for_each(|c| free.extend(&egraph[c].data.free)),
        }
    }
}
```

Lambda calculus in egg : Analysis

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            // \v. a(v)
            Lambda::Lambda([v, a]) | Lambda::Fix([v, a]) => {
                free.extend(f(a));
                free.remove(v);
            }
            _ => enode.for_each(|c| free.extend(&egraph[c].data.free)),
        }

        let constant = eval(egraph, enode);
        Data { constant, free }
    }
}
```

Lambda calculus in egg : Dynamic rewrites

```
-- v2=x is not free in ( $\lambda x. x + 1$ )
x_bound = let v1 = ( $\lambda x. x * 2$ ) in ( $\lambda x. x + 1$ )
x_bound = let v1 = e           in ( $\lambda v2. \text{body}$ )
```

Lambda calculus in egg : Dynamic rewrites

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How to push let into lambda?

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How to push let into lambda?

```
x_bound' =  $\lambda x. \text{let } v1 = (\lambda x. x * 2) \text{ in } x + 1$ 
x_bound' =  $\lambda v2. \text{let } v1 = e \text{ in body}$ 
```

Lambda calculus in egg : Dynamic rewrites

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x_bound' =  $\lambda v2. \text{let } v1 = e \text{ in } \text{body}$ 
```

```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free = let v1 = x*2 in ( $\lambda x. x + 1$ )
x_free = let v1 = e   in ( $\lambda v2. \text{body}$ )
```

Lambda calculus in egg : Dynamic rewrites

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-- v2=x is free in e=x*2
x :: Int; x = 42
x_free'_wrong =  $\lambda x. \text{let } v1 = x * 2 \text{ in } x + 1$ 
```

How to push let into lambda?

Lambda calculus in egg : Dynamic rewrites

```
-- v2=x is not free in (|x. x + 1)
x_bound = let v1 = (\x. x*2) in (\x.    x+1)
x_bound = let v1 = e           in (\v2. body)
```

How to push let into lambda?

```
x_bound' = \x.  let v1 = (\x. x*2) in x + 1
x_bound' = \v2. let v1 = e           in body
```

```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free = let v1 = x*2 in (\x.    x+1)
x_free = let v1 = e   in (\v2. body)
```

```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free'_wrong = \x. let v1 = x*2 in x + 1
```

How to push let into lambda?

```
x :: Int; x = 42
x_free' = \fresh. let v1 = e   in (let v2 = fresh in body )
x_free' = \fresh. let v1 = x*2 in (let x = fresh in (x + 1))
```

Lambda calculus in egg : Dynamic rewrites

```
-- v2=x is not free in (|x. x + 1)
x_bound = let v1 = (\x. x*2) in (\x.    x+1)
x_bound = let v1 = e           in (\v2. body)
```

How to push let into lambda?

```
x_bound' = \x.  let v1 = (\x. x*2) in x + 1
x_bound' = \v2. let v1 = e           in body
```

```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free = let v1 = x*2 in (\x.    x+1)
x_free = let v1 = e   in (\v2. body)
```

```
-- v2=x is free in e=x*2
x :: Int; x = 42
x_free'_wrong = \x. let v1 = x*2 in x + 1
```

How to push let into lambda?

```
x :: Int; x = 42
x_free' = \fresh. let v1 = e   in (let v2 = fresh in body )
x_free' = \fresh. let v1 = x*2 in (let x = fresh in (x + 1))
```

Lambda calculus in egg : Dynamic rewrites

```
x_free = let v1 = e    in (\v2. body)
x_free' = \fresh. let v1 = e    in (let v2 = fresh in body  )

rw!("let-lam-diff";
"(let ?v1 ?e (lam ?v2 ?body))" => // let v1 = e in \v2 . body
{ CaptureAvoid { // use an Applier called CaptureAvoid
    fresh: var("?fresh"), v2: var("?v2"), e: var("?e"),
    // \v2. let v1 = e in body
    if_not_free: "(lam ?v2 (let ?v1 ?e ?body))".parse().unwrap(),
    // \fresh. let v1 = e in let v2 = fresh in body
    if_free: "(lam ?fresh (let ?v1 ?e (let ?v2 (var ?fresh) ?body)))"
        .parse().unwrap(),
}
if is_not_same_var(var("?v1"), var("?v2"))),
```

Lambda calculus in egg : Dynamic rewrites

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }
impl Analysis<Lambda> for LambdaAnalysis {
    type Data = Data;
    ...
}
```

Lambda calculus in egg : Dynamic rewrites

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }
impl Analysis<Lambda> for LambdaAnalysis {
    type Data = Data;
    ...
    { CaptureAvoid { // rewrite pattern
        fresh: var("?fresh"), v2: var("?v2"), e: var("?e"),
        if_not_free: "(lam ?v2 (let ?v1 ?e ?body))".parse().unwrap(),
        if_free: "(lam ?fresh (let ?v1 ?e (let ?v2 (var ?fresh) ?body)))"
            .parse().unwrap(),
    }}}
```

Lambda calculus in egg : Dynamic rewrites

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }
impl Analysis<Lambda> for LambdaAnalysis {
    type Data = Data;
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    { CaptureAvoid { // rewrite pattern
        fresh: var("?fresh"), v2: var("?v2"), e: var("?e"),
        if_not_free: "(lam ?v2 (let ?v1 ?e ?body))".parse().unwrap(),
        if_free: "(lam ?fresh (let ?v1 ?e (let ?v2 (var ?fresh) ?body)))"
            .parse().unwrap(),
    } }
    struct CaptureAvoid {
        fresh: Var, v2: Var, e: Var,
        if_not_free: Pattern<Lambda>,
        if_free: Pattern<Lambda>,
    }
}
```

Lambda calculus in egg : Dynamic rewrites

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }
impl Analysis<Lambda> for LambdaAnalysis {
    type Data = Data;
    ...
    { CaptureAvoid { // rewrite pattern
        fresh: var("?fresh"), v2: var("?v2"), e: var("?e"),
        if_not_free: "(lam ?v2 (let ?v1 ?e ?body))".parse().unwrap(),
        if_free: "(lam ?fresh (let ?v1 ?e (let ?v2 (var ?fresh) ?body)))"
            .parse().unwrap(),
    } }
    struct CaptureAvoid {
        fresh: Var, v2: Var, e: Var,
        if_not_free: Pattern<Lambda>,
        if_free: Pattern<Lambda>,
    }
    impl Applier<Lambda, LambdaAnalysis> for CaptureAvoid {
        fn apply_one(&self, egraph: &mut EGraph, eclass: Id, subst: &Subst) ...
            let e = subst[self.e];
            let v2 = subst[self.v2];
            let v2_free_in_e = egraph[e].data.free.contains(&v2);
    }
}
```

Lambda calculus in egg : Dynamic rewrites

```
struct Data { free: HashSet<Id>, constant: Option<Lambda>, }
impl Analysis<Lambda> for LambdaAnalysis {
    type Data = Data;
    ...
    { CaptureAvoid { // rewrite pattern
        fresh: var("?fresh"), v2: var("?v2"), e: var("?e"),
        if_not_free: "(lam ?v2 (let ?v1 ?e ?body))".parse().unwrap(),
        if_free: "(lam ?fresh (let ?v1 ?e (let ?v2 (var ?fresh) ?body)))"
            .parse().unwrap(),
    } }
    struct CaptureAvoid {
        fresh: Var, v2: Var, e: Var,
        if_not_free: Pattern<Lambda>,
        if_free: Pattern<Lambda>,
    }
    impl Applier<Lambda, LambdaAnalysis> for CaptureAvoid {
        fn apply_one(&self, egraph: &mut EGraph, eclass: Id, subst: &Subst) ...
            let e = subst[self.e];
            let v2 = subst[self.v2];
            let v2_free_in_e = egraph[e].data.free.contains(&v2);
            if v2_free_in_e { ... } else { ... }
    }
}
```

Conclusion

- ▶ Use equalities to find equivalent programs.
- ▶ Extract out the best program using a local analysis.
- ▶ Analysis + Rewrites in a single unified framework.