Risk Disparity

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Policy portfolios are a fixture of institutional investment management, but they may not serve the purposes for which they are intended. A policy portfolio serves primarily as an expression of an investor's return and risk preferences. Secondarily, it serves as a benchmark for determining the success or failure of active management. A clearly defined and easily replicable policy portfolio may indeed provide a useful gauge for judging active management, but it is a poor reflection of investor preferences. Peter Bernstein [2003 and 2007] raised this issue philosophically arguing that a policy portfolio's risk profile was inconstant and that it changed more radically and frequently than the typical investor's risk preferences. He went on to propose that investors manage their portfolios opportunistically rather than rigidly, but he did not provide specific guidance. This article offers empirical evidence of the inter-temporal disparity of a policy portfolio's risk profile, and it proposes a solution to this problem.

What Do Investors Want?

Investors typically seek to grow their assets and to limit exposure to significant drawdowns along the way. Unfortunately, these two goals conflict with each other. Policy portfolios serve as an expression of how investors balance these conflicting goals. The usual process by which investors achieve this balance is to estimate the expected returns, risk, and correlations of major asset classes and then to select a blend that offers the desired distribution of possible outcomes. Typically, investors do not examine the entire distribution. Instead, they summarize the distribution by its mean and standard deviation, assuming that returns are approximately log-normally distributed. Given this information and assumption, investors estimate the likelihood of various outcomes such as achieving a target growth rate or avoiding a drawdown of a certain magnitude.[1] They then select a portfolio of asset classes that best meets their objectives, given their current outlook for expected returns, risk, and correlations.[2]

The problem with this approach is that the return distribution implied by a particular portfolio at one point in time may be quite different at another point in time – and investors want the distribution, not the portfolio. The portfolio is simply a means to an end. The solution to the problem, therefore, is to revise the portfolio as needed to preserve the desired return distribution, or at least, to give the next best distribution – in other words to replace a rigid policy portfolio with a flexible investment policy.

Historically, investors have avoided portfolio revisions either because it was too expensive to do so, or because they lacked confidence they could do so successfully. These two impediments now pose less of a challenge as they might have in the past. It is now relatively inexpensive to change exposure to asset classes or risk factors, given the proliferation of ETF's, index funds, and index futures. And evidence, which I will later present, together with theory, suggests that investors may be able to anticipate macro-inefficiencies.[3]

Evidence of Inter-temporal Risk Disparity

Let's consider the following portfolio, which is intended to represent a policy portfolio for a typical institutional investor.

Asset Class[4]	Allocation
U.S. Large Cap	25%
U.S. Small Cap	15%
EAFE Equity	5%
Emerging Market Equity	5%
U.S. Treasuries	20%
U.S. Credit	15%
REITS	5%
Private Equity	10%

Exhibit 1 shows the annual volatility of this portfolio based on overlapping monthly returns, beginning January 1998 and ending February 2013.

Exhibit 1: Annualized Portfolio Volatility

January 1998 through February 2013

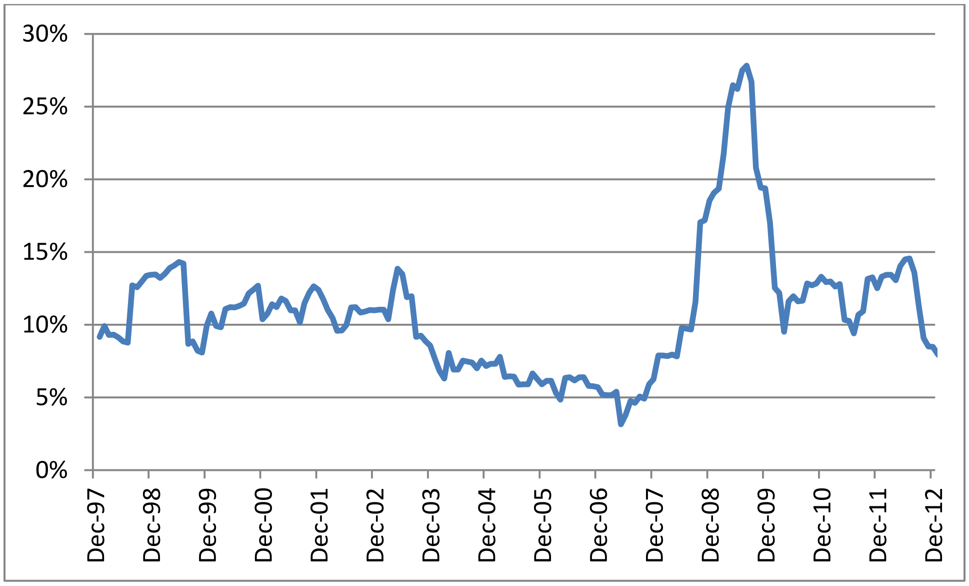


Exhibit 1 clearly reveals the inter-temporal disparity of annual portfolio volatility. Although average portfolio volatility was 10.79%, it ranged from a low of 3.15% as of the 12 months ending May 2007 to a high of 27.81% as of August 2009. These volatility differences imply stark differences in exposure to loss, even assuming the portfolio's expected return remains constant.[5] Exhibit 2 shows the likelihood of various losses as of the end of a one-year horizon as well as within a one-year horizon.[6]

Exhibit 2: Probability of Loss

Expected Return = 6.75%

	One Year End of Horizon		
	Annualized Standard Deviation		
Loss	3.15%	10.79%	27.81%
-5%	0.00%	13.43%	37.19%
-10%	0.00%	5.02%	29.53%
-20%	0.00%	0.25%	15.92%
	One Year Within Horizon		
	Annualized Standard Deviation		
Loss	3.15%	10.79%	27.81%
-5%	0.04%	42.58%	81.93%
-10%	0.00%	14.41%	64.49%
-20%	0.00%	0.62%	34.24%

Whereas this portfolio's average volatility implied that a 10% loss within a one-year horizon was about 14% likely, the one-year annualized volatility as of May 2007 implied a negligible chance of a 10% loss, and the one-year annualized volatility as of August 2009 implied a 64% likelihood of a 10% loss. I doubt that the typical investor's tolerance for loss varies nearly as much as these probabilities.

Even if we measure volatility over a three-year horizon, we again see substantial inter-temporal risk disparity. Annualized volatility ranged from a low of 5.45% to a high of 18.47%, implying that the likelihood of a 10% loss within a one-year horizon ranged from 0% to 44%, respectively.

How to Predict Portfolio Volatility

It is one thing to observe changes in portfolio volatility retrospectively. The challenge, however, is to anticipate these changes. One might think that implied volatility would be an effective precursor of portfolio volatility, because it represents a forward looking consensus view. But this measure is limited because we can only infer implied volatility from liquid options markets which typically do not exist for all of a portfolio's major components. And even if such markets did exist, this approach ignores correlations, which also change through time and are a key driver of portfolio volatility. One might therefore consider historical portfolio volatility, which does take correlations into account. But it too is of limited value because, unlike implied volatility, it is backward looking, and as we observed, highly unreliable.

Instead, I propose that investors use the absorption ratio to anticipate shifts in portfolio volatility and exposure to loss. This measure was introduced initially to measure and predict systemic risk in the financial markets.[7] The absorption ratio equals the fraction of the total variance of a set of asset returns explained or "absorbed" by a fixed number of eigenvectors, as shown in equation (1).

$$AR = \frac{\sum_{i=1}^{n} \sigma_{Ei}^2}{\sum_{j=1}^{N} \sigma_{Aj}^2} \tag{1}$$

where, *AR* is the absorption ratio, *n* is the number of assets, *N* is the number of eigenvectors in the numerator of absorption ratio, σ_{Ei}^2 is the variance of the i-th eigenvector , and σ_{Aj}^2 is the variance of the j-th asset

It captures the extent to which a set of assets is unified or tightly coupled. When assets are tightly coupled, they are collectively fragile in the sense that negative shocks travel more quickly and broadly than when assets are loosely linked. In contrast, a low absorption ratio implies that risk is distributed broadly across disparate sources; hence the assets are less likely to exhibit a unified response to bad news.

There is persuasive evidence that the absorption ratio reliably distinguishes fragile market conditions from resilient market conditions. Kritzman, Li, Page, and Rigobon [2011], for example, reported that an overwhelming preponderance of the largest daily, weekly, and monthly drawdowns in the U.S. equity market were preceded by a one-standard deviation spike in the absorption ratio. They also showed that annualized daily, weekly, and monthly returns were more than 10% lower on average following a one-standard deviation spike up in the absorption ratio compared to a one-standard deviation spike down. And Kinlaw, Kritzman, and Turkington [2012] showed that the left tail of the conditional distribution of U.S. equity returns was much fatter following increases in the absorption ratio than what one should expect from the full-sample mean and standard deviation. Moreover, the U.S. Treasury Department's Office of Financial Research [2012] studied the absorption ratio and suggested that it might be an effective early warning measure of financial crises. In its inaugural annual report, the OFR included the following assessment.

For the 1998 crisis, in which tight coupling of other markets to the Russian bond market caught LTCM by surprise, there was a gradual increase in the AR before the event and a gradual decrease after. Similarly, the AR rose gradually up to September 2008 but then jumped abruptly by more than 10 percent and remained elevated for two years. There was a similar pattern in 1929. Although the sample of four crises is small, the tendency for the AR to rise in advance of a crisis event suggests some promise as an early warning measure.

Although the absorption ratio was originally developed to measure and predict broad market fragility, it is suitable for measuring the fragility of any set of assets, including the components of an individual portfolio. I therefore apply the absorption ratio in two ways: to measure the intrinsic fragility of the indicated policy portfolio, and following Kritzman, Li, Page, and Rigobon [2011], to measure the extrinsic fragility of the U.S. equity market.[8] These two applications are more complementary than redundant in the following sense.

When we apply the absorption ratio to a portfolio, we capture the extent to which it becomes more unified. When the assets within it are highly unified, the portfolio is more vulnerable to drawdowns in any market in which it is invested because the factors that drive its returns tend to move more in concert.

In contrast, when we apply the absorption ratio to a broad market, such as the U.S. equity market, we capture the extent to which industries within the equity market are unified, irrespective of how the portfolio assets interact. As industries become more unified, the equity market becomes more susceptible to negative news. In other words, the equity market's absorption ratio informs us about external danger, whereas the portfolio's absorption ratio informs us of the portfolio's vulnerability, should the equity market, for example, experience a large loss.

Let us first focus on the intrinsic fragility of the indicated policy portfolio. I construct the portfolio's absorption ratio by performing a principal components analysis on the daily returns of the eight asset classes included in the portfolio. Then I measure the fraction of total variance absorbed by the first two eigenvectors. I estimate the covariance matrix using a rolling two-year window, and I exponentially decay the returns before I estimate the covariance matrix using a one-year half-life. The purpose for decaying the returns is to reduce sensitivity to arbitrary shifts in the estimation window and to recognize that investors are more sensitive to recent events than to distant events. I then calculate a standardized shift of the absorption ratio as follows. I first calculate the most recent 15-day average absorption ratio. Then I subtract from it the average absorption over the year immediately preceding this 15-day window. Finally, I divide the difference by the standard deviation of the absorption ratio as calculated over that trailing one-year window.[9]

I find that volatility differs significantly following a one-standard deviation shift up or down in the absorption ratio. The subsequent annualized three-month volatility is 1.5 percentage points higher following a shift up versus a shift down in the absorption ratio, and the subsequent 12 month volatility is 1.7 percentage higher.

I next compare the percentage differences in subsequent portfolio volatility with the percentage differences in the weighted average volatility of its component assets, given an indication of high versus low fragility. The differences of these differences capture the correlation effect on portfolio volatility. This analysis reveals that not only do the assets exhibit greater volatility following a shift up in the absorption ratio versus a shift down. They also become more highly correlated, thereby causing the difference in the portfolio's volatility to exceed the difference in the weighted average of the assets' volatility. [10]

I also examine the impact on the extreme left tail of portfolio returns contingent on whether the portfolio's absorption has spiked up or down. I find that the realized one percentile drawdowns that occurred following a shift up in the absorption ratio (implying a relatively fragile portfolio) were far greater than the one percentile drawdowns that occurred following a shift down in the absorption ratio (implying a relatively resilient portfolio). The one percentile loss during the 12 months following a spike down in the portfolio's absorption ratio was 13% compared to a 36% loss in the 12 months following a spike up in the portfolio's absorption ratio.

This evidence strongly suggests that the inter-temporal disparity of portfolio risk and exposure to loss depend importantly on the portfolio's intrinsic fragility as captured by the absorption ratio.

One might be tempted to believe that the absorption ratio is over-engineered, that one could obtain the same insights about portfolio fragility by measuring the average correlation of assets. This is not so. The average correlation fails to capture the relative importance of the assets. For example, it could be the case that assets with high volatilities experience an increase in correlation from one period to the next, whereas assets with low volatilities experience a decline in their correlations, such that the average correlation remains stable or even declines. The absorption ratio, by contrast, would likely increase, given this scenario, because high volatility assets are a more important determinant of portfolio risk than low volatility assets.[11]

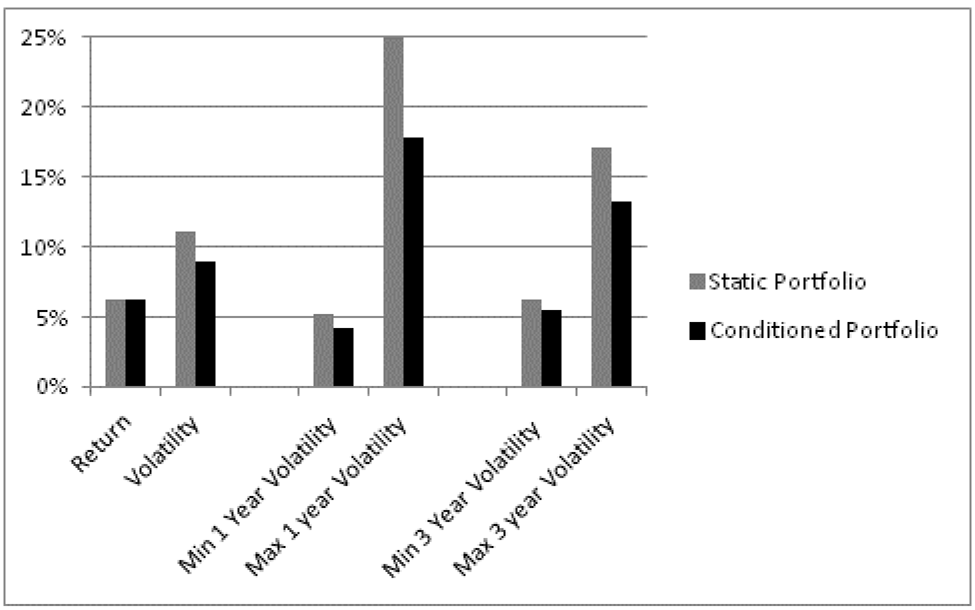
As mentioned previously, I also compute the absorption ratio based on the industries within the U.S. equity market to capture extrinsic fragility. I focus on the U.S. equity market, because most investors depend on their domestic equity market as the main growth engine for their portfolios. I compute the absorption ratio of the U.S. equity market following the calibration used by Kritzman, Li, Page, and Rigobon [2011]. They computed the absorption based on a covariance matrix estimated from exponentially decayed daily industry returns over a two-year window and included the 10 most important eigenvectors (top 20%) in the numerator. This calibration is consistent with the calibration I used to compute intrinsic fragility.

I next show how investors can employ these simple measures of intrinsic and extrinsic fragility to reduce a portfolio's inter-temporal risk disparity while improving its return to risk ratio.

How to Reduce Risk Disparity

I first test a rebalancing strategy that shifts some of the portfolio's equity exposure to Treasury bonds following an indication of heightened intrinsic fragility. Specifically, whenever I observe a one-standard deviation increase in intrinsic fragility, the next day I proportionately reduce the portfolio's liquid equity positions by one half and allocate these funds to U.S. Treasury bonds.[12] Exhibit 8 shows how this simple rebalancing rule affects the portfolio's return and risk profile.

Exhibit 3: Conditioning on Intrinsic Portfolio Fragility



It is clear from Exhibit 3 that armed with information about a portfolio's intrinsic fragility it is possible to reduce its inter-temporal risk disparity without sacrificing its long-term growth rate. By responding to heightened intrinsic fragility, the spread in one-year volatility falls from 19.79 percentage points to 13.65 percentage points, and the spread in three-year volatility falls from 11.01 percentage points to 7.77 percentage points. Moreover, the portfolio's annualized growth rate increases slightly from 6.21% to 6.25%, and its annualized daily standard deviation declines significantly from 11.13% to 8.97%, thereby improving the return to risk ratio from 0.56 to 0.70.[13]

Now let us turn to the extrinsic fragility of the U.S. stock market. I apply the same rebalancing rule in response to indications of extrinsic equity market fragility as I did to indications of heightened intrinsic fragility. Exhibit 9 shows the results of responding to signals of extrinsic fragility.

Exhibit 4: Conditioning on Extrinsic Market Fragility

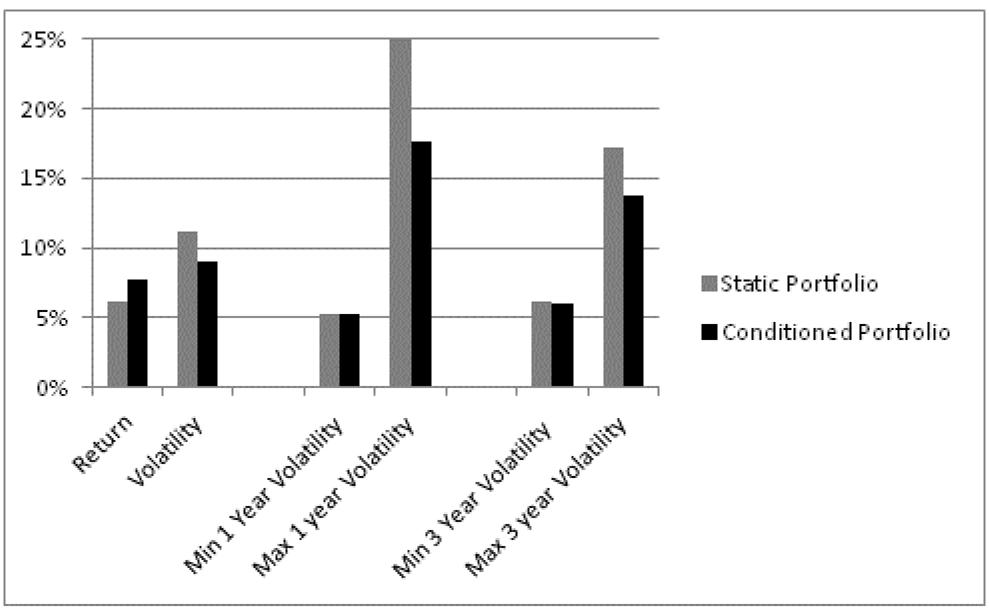


Exhibit 4 shows that responding to extrinsic fragility by itself reduces the portfolio's inter-temporal disparity about as much as by responding to intrinsic fragility by itself. But it produces a significantly higher long-term growth rate, 7.69% versus 6.25%, at the same level of risk, 8.97%. Thus, this rebalancing strategy increases the return to risk ratio even further to 0.86. But why choose? Both signals of fragility significantly improve the return and risk profile of the portfolio, and they are not entirely redundant. In fact, the intrinsic and extrinsic signals are only 66% correlated. This suggests the tantalizing prospect, that in combination, they could perform even better than they do independently, which is what I next explore.

I test a rebalancing rule that exchanges one half of the portfolio's liquid equity exposure for Treasury bonds, whenever there is an independent one-standard deviation spike either in intrinsic or extrinsic fragility, and shifts the entire liquid equity position to Treasury bonds whenever both measures of fragility spike simultaneously. The results of this rebalancing rule are displayed in Exhibit 5.

Exhibit 5: Conditioning on Intrinsic and Extrinsic Fragility

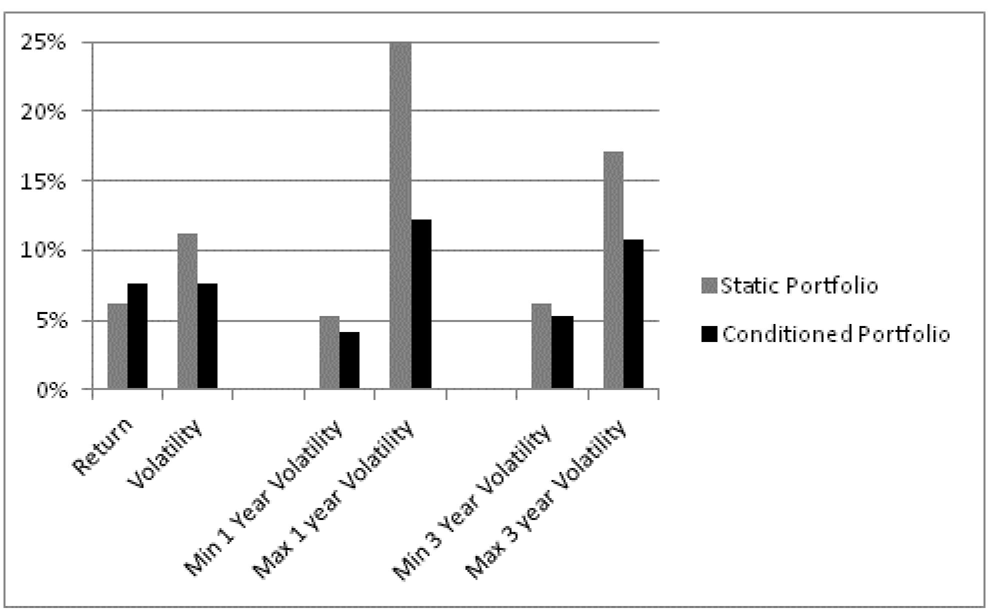
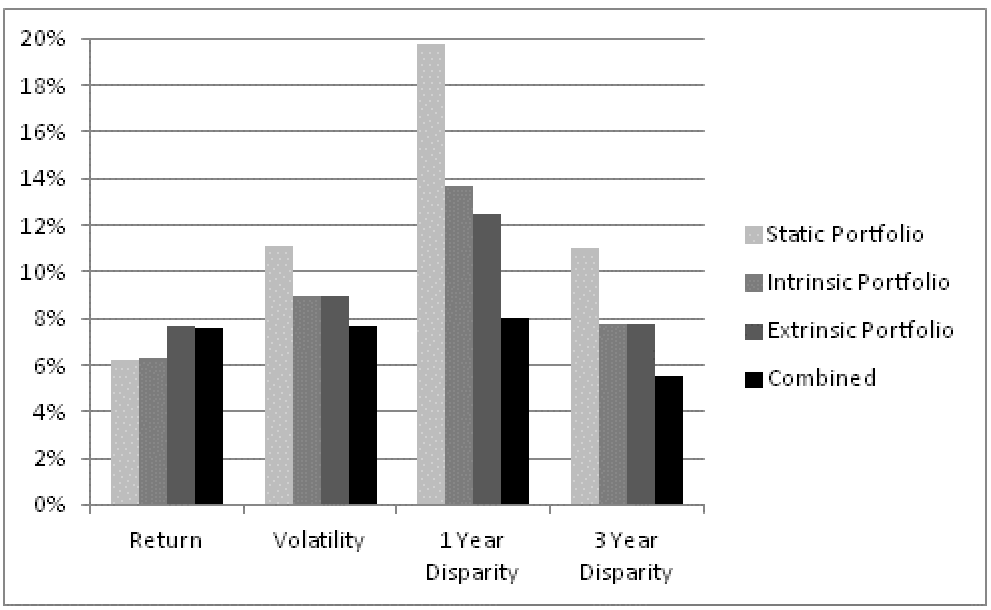


Exhibit 5 shows that joint conditioning improves the portfolio's return to risk ratio and reduces its inter-temporal risk disparity substantially beyond what we could achieve by independent conditioning. It produces about the same growth rate (7.61%) as extrinsic conditioning (7.69%) but with a lower standard deviation a lower standard deviation (7.64% versus 8.97%), thus increasing the return to risk ratio from 0.56 for the static portfolio to 1.00, and it reduces both one-year and three-year risk disparity by more than half compared to the static portfolio. Exhibit 6 compares these metrics across all strategies.

Exhibit 6: Summary Comparison



Conditioning only on intrinsic fragility marginally increases the portfolio's long-term growth rate, but it significantly reduces its volatility and inter-temporal risk disparity. Conditioning only on extrinsic fragility improves the portfolio's long-term growth rate and one-year risk disparity while providing about the same improvement to the portfolio's volatility and three-year risk disparity. Finally, conditioning on both intrinsic and extrinsic fragility offers about the same improvement to long-term growth as does conditioning on extrinsic fragility, but it substantially reduces the portfolio's volatility and risk disparity beyond the effects of independent conditioning. Most of the improvement arises from a reduction in equity exposure during the sell-off of the Dot Com Bubble and prior to the collapse of equity prices that occurred during the Global Financial Crisis.

I offer one more piece of evidence in support of the notion that conditioning on fragility helps to distinguish relatively benign environments from relatively dangerous environments. I compute the risk premium of the policy portfolio relative to U.S. Treasury bonds during three regimes: a safe regime in which neither the portfolio nor the external environment displayed significant fragility; a somewhat dangerous regime in which either the portfolio or the external environment was fragile but not both; and a very dangerous regime in which both the portfolio and the external environment were simultaneously fragile.

Regime	Risk Premium
Safe	5.0%
Somewhat Dangerous	2.2%
Very Dangerous	-8.7%

The safe regimes were periods during which the investor would have maintained the initial weights of the policy portfolio. During the somewhat dangerous regimes, when either the portfolio or the equity market exhibited evidence of fragility, the investor would have shifted half of the portfolio's liquid equity exposure to Treasury bonds. During the very dangerous regimes, when both the portfolio and the equity market simultaneously displayed evidence of fragility, the investor would have shifted the entire liquid equity position to Treasury bonds. The ability of these signals to anticipate high, low, and negative risk premium regimes suggests that they could help investors to capture the policy portfolio's upside potential, while protecting it during periods when it is especially vulnerable to losses.[14]

One could certainly devise more sophisticated rebalancing rules to exploit information about intrinsic and extrinsic fragility. For example, the rules I tested respond only to signals of heightened fragility. It may be more beneficial to scale portfolio risk symmetrically; that is, to scale up portfolio risk following indications of resilience and to scale it down following indications of fragility. And perhaps there are other measures that predict portfolio risk more effectively. But it is not my purpose here to construct a sophisticated and complex rebalancing strategy. Rather, I wish to demonstrate that merely by responding to simple and readily observable indicators, investors can stabilize risk more effectively than by rigidly adhering to fixed portfolio weights.

The Disparity of Risk Parity

Risk disparity and the process I have proposed for addressing this problem apply not only to policy portfolios but to any risky strategy that maintains relatively fixed weights, including the popular strategy called risk parity. This strategy proposes that investors construct portfolios so that each of its assets contributes equally to total portfolio risk. This strategy requires investors to lever up the risk of relatively low risk assets and to reduce exposure to relatively high risk assets. It is designed to impose cross-sectional risk parity within the portfolio. But even if a portfolio derives its risk equally from the components within it, the portfolio will still experience significant disparity in its risk through time. The strategy that I have described is designed to mitigate inter-temporal disparity in risk, and it can be applied to risk parity strategies as well as to policy portfolios. Investors who seek cross-sectional risk parity could maintain risk parity across assets and risk parity through time by anticipating shifts in the relative riskiness of assets. Investors do not have to choose between cross-sectional risk parity and inter-temporal risk parity.

Conclusion

Peter Bernstein posed a formidable challenge to the time-honored practice of establishing a policy portfolio for the purpose of expressing an investor's return and risk preferences. He correctly pointed out that a portfolio's risk profile is not constant through time, and he therefore argued that investors manage their portfolios more opportunistically.

I document the inter-temporal disparity of portfolio risk, and I propose that investors replace rigid policy portfolios with flexible investment policies. Specifically, I describe how to measure intrinsic portfolio fragility, and I show how a simple rebalancing rule helps to mitigate the inter-temporal disparity of portfolio volatility while at the same time improving the portfolio's return to risk ratio. In addition, I measure the extrinsic fragility of the equity market, and I show how it too can be used to scale portfolio risk advantageously. Finally, I combine signals of both intrinsic and extrinsic fragility, showing that in combination, they perform even better than either one does independently. I argue that by monitoring both intrinsic and extrinsic fragility investors may be able to institute a flexible investment policy that produces a substantially more stable risk profile than a policy portfolio that adheres rigidly to a fixed set of asset weights.

I thank Juan Vargas for computational assistance and Timothy Adler, Frank Fabozzi, William Kinlaw, Yuanzhen Li, and David Turkington for helpful comments.

ENDNOTES

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[1] Of course, investors could refine this approach by considering higher moments or other features of the return distribution. See, for example, Cremers, Kritzman, and Page [2005].

[2] This approach assumes that asset classes are properly defined, which would require them to be intrinsically homogeneous and extrinsically heterogeneous. Alternatively, investors may allocate across factors and then map these factors onto the appropriate investment vehicles.

[3] Samuelson [1998] offered a theoretical argument in support of macro-inefficiency. What is now known as the Samuelson dictum states that markets are relatively micro-efficient because a smart investor who spots mispriced securities trades to exploit the inefficiency and by so doing corrects it. However, when an aggregation of securities such as an asset class is mispriced and a smart investor trades to exploit it, his actions are insufficient to revalue the entire asset class. Macro-inefficiencies typically require an exogenous shock to jolt many investors to trade in concert in order to revalue an entire asset class. Hence, macro-inefficiencies persist sufficiently long for investors to act upon them.

[4] I use the following indexes as proxies for the indicated asset classes: US large cap – S&P 500; US small cap – Russell 2000; EAFE equity – MSCI equity; emerging market equity – MSCI EM; US Treasuries – Barcap US Treasuries all maturities, US credit – Barcap US credit all maturities; REITS – NAREIT; and private equity – LPX.

[5] For purposes of this analysis, I assume that the portfolio's long-term growth rate remains constant at 6.21%, which corresponds to an arithmetic average of 6.75%. These are the values that occurred for this portfolio for the period from January 1998 through February 2013.

[6] The likelihood of a within-horizon loss is given by a first passage probability as follows:

$P = N[(\ln(1+L)-\mu T)/(\sigma\sqrt{T})] + N[(\ln(1+L)+\mu T)/(\sigma\sqrt{T})]\,(1+L)^{2\mu/\sigma\text{^}2}$, where P = probability, L = loss, μ = expected return, σ =standard deviation, T = time horizon, and N() = cumulative normal distribution function. See, for example, Karlin, S., and H. Taylor [1975].

[7] See, for example, Kritzman, Li, Page, and Rigobon [2011].

[8] This distinction presents a semantic challenge. One could reasonably argue that fragility across several asset classes is a better gauge of extrinsic fragility than fragility within a single asset class. But I use the term intrinsic fragility to refer to a condition that is specific to a particular portfolio. And I use the term extrinsic fragility to refer to

the condition of a broad and important market such as the U.S. equity market. In any event, I measure two distinct sources of fragility.

[9] This calibration is consistent with the calibration used by Kritzman, Li, Page and Rigobon [2011].

[10] The average percentage difference in portfolio volatility in the three months following shifts up and down in the absorption ratio was 9.8% versus 8.3% for the weighted average volatility of the assets. In the subsequent 12 months following shifts up and down in the absorption ratio the percentage difference in portfolio volatility was 10.2% versus 8.5% for the weighted average volatility of the assets.

[11] Pukthuanthong and Roll [2009] provide a formal analysis of the distinction between average correlation and their measure of market integration which they base on principal components analysis.

[12] I base the signal on a one-standard deviation move, because this calibration is consistent with the extant literature. I suspect that one could improve the strength of the signal by testing alternative thresholds, but I believe that the results are more persuasive without data mining.

[13] I do not account for transaction costs, because they would be quite low and because they would impact the static and dynamic strategies similarly.

[14] One might argue that any strategy that increased average exposure to bonds during this measurement period would appear favorable because bonds performed unusually well. Bonds did perform well on average over this sample period, but these signals of fragility successfully anticipated periods within the sample when bonds performed worse than the portfolio.

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