TECHNISCHE UNIVERSITÄT DRESDEN

INVERTED PENDULUM

Bolorkhuu Dariimaa

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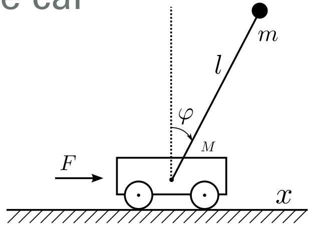
outline

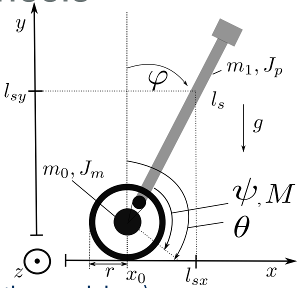
- introduction
- modelling
- control unit design
- mechanical buildup and electrical components
- assembly
- result
- conspectus and forecast

inverted pendelum on the car



- underactuated system
- 2 generelized coordinates (φ, x)
- DOF: 2 (position , angle of the pendelum)

inverted pendelum on wheels



- underactuated system
- 3 coordinates (ψ, φ, θ)
- 1 holonomic constraint
  $\theta = \psi + \varphi$
- DOF: 2 (position of the wheel, angle of the pendelum)

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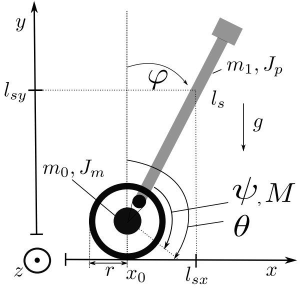
derivation of equations of motion



- generelized coordinates
  $[\psi, \varphi]^T =: [q_1, q_2]^T =: q$
- centre of gravity
  $l_{sx} = r(q_1 + q_2) + l_s \sin(q_2)$
  $l_{sy} = l_s \cos(q_2)$

- φ : absolut angle of the pendelum
- ψ : relativ angle between wheel and pendelum
- θ : rollangle of the wheel

kinetic energy

$$T(q,\dot{q}) = m_0 r^2 \frac{(\dot{q_1}+\dot{q_2})^2}{2} + J_m \frac{(\dot{q_1}+\dot{q_2})^2}{2} + J_p \frac{\dot{q_2}^2}{2} + \frac{m_1}{2}\left[(\frac{d}{dt} l_{sx})^2 + (\frac{d}{dt} l_{sy})^2\right]$$

potential energy

$$U(q) = m_1 g\, l_{sy}$$

Lagrangian function

$$L = T - U$$

Lagrangian equation

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_i}\right) - \frac{dL}{dq_i} = \tau_i, \qquad i = 1,2$$

matrix notation

$$\begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_1(q, \dot{q}) \\ C_2(q, \dot{q}) \end{bmatrix} + \begin{bmatrix} K_1(q) \\ K_2(q) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

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- mass-matrix

$$\begin{bmatrix} J^* & J^* + l_s m_1 r \cos(q_2) \\ J^* + l_s m_1 r \cos(q_2) & J^* + J_p + m_1 l_s^2 + 2 l_s m_1 r \cos(q_2) \end{bmatrix}$$

für $J^* = J_m + (m_0 + m_1) r^2$

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- centrifugal-/coriolis force

$$\begin{bmatrix} C_1(q,\dot{q}) \\ C_2(q,\dot{q}) \end{bmatrix} = \begin{bmatrix} -l_s m_1 \dot{q}_2^2 r \sin(q_2) \\ -l_s m_1 \dot{q}_2^2 r \sin(q_2) \end{bmatrix}$$

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- link torque via gravitation

$$\begin{bmatrix} K_1(q) \\ K_2(q) \end{bmatrix} = \begin{bmatrix} 0 \\ -g l_s m_1 \sin(q_2) \end{bmatrix}$$

matrix notation

$$\begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_1(q,\dot{q}) \\ C_2(q,\dot{q}) \end{bmatrix} + \begin{bmatrix} K_1(q) \\ K_2(q) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

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- link torque via gravitation

$$\begin{bmatrix} K_1(q) \\ K_2(q) \end{bmatrix} = \begin{bmatrix} 0 \\ -g l_s m_1 \sin(q_2) \end{bmatrix}$$

- actuation torque

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} M \\ 0 \end{bmatrix}$$

partial linearization

new virtual input

$$a = \ddot{q}_1$$

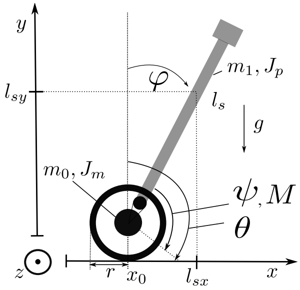
inside feedback

$$\tau_1 = \left[M_{11}(q) - M_{12}(q)M_{22}^{-1}(q)M_{21}(q)\right] a - M_{12}(q)M_{22}^{-1}(q)\left(C_2(q,\dot{q}) + K_2(q,\dot{q})\right) + C_1(q,\dot{q}) + K_1(q).$$

partial linearized system

$$\begin{aligned} \ddot{q}_1 &= a \\ \ddot{q}_2 &= -M_{22}^{-1}(q)\left(C_2(q,\dot{q}) + K_2(q,\dot{q}) + M_{21}(q)\, a\right). \end{aligned}$$

partial linearization



- state vector R^4
  $x := [x_1\, x_2\, x_3\, x_4]^T := [\psi\, \varphi\, \dot{\psi}\, \dot{\varphi}]^T$
- the input affin system
  with new input $u = a$

$$\dot{x} = f(x) + g(x)u$$

alternatively

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= a \\
\dot{x}_4 &= - \quad M_{22}^{-1}(x)\,(C_2(x,\dot{x}) + K_2(x,\dot{x}) + M_{21}(x)\,a)
\end{aligned}$$

linearization on the equilibrium point

equilibrium point

$$x_0 = [x_{1,0}\; x_{2,0}\; x_{3,0}\; x_{4,0}]^T = [0\,0\,0\,0]^T$$

on the state form

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \\ \dot{\tilde{x}}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & a_{42} & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ b_4 \end{bmatrix} \tilde{u}$$

$$a_{42} = \frac{g l_s m_1}{J_p + J_m + l_s^2 m_1 + 2 l_s m_1 r + (m_0 + m_1) r^2}$$

$$b_4 = -\frac{J_m + l_s m_1 r + (m_0 + m_1) r^2}{J_p + J_m + l_s^2 m_1 + 2 l_s m_1 r + (m_0 + m_1) r^2}$$

on the general form

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + b\tilde{u}, \qquad \tilde{x}(t_0) = x_0$$

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control unit design

kalmanian controllable matrix S

$$S = [b, Ab, A^2b, A^3b] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & b_4 & 0 & a_{42}b_4 \\ 1 & 0 & 0 & 0 \\ b_4 & 0 & a_{42}b_4 & 0 \end{bmatrix}.$$

determinant

$$\det(S) = -a_{42}^2 \cdot b_4^2, \qquad \Rightarrow \mathrm{Rg}(S) = 4.$$

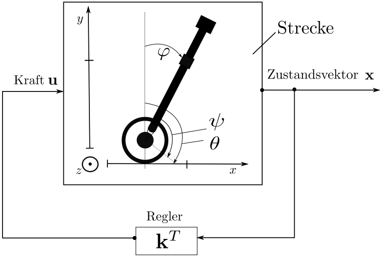
pole of the open plant

$$\det(sI - A) = 0$$

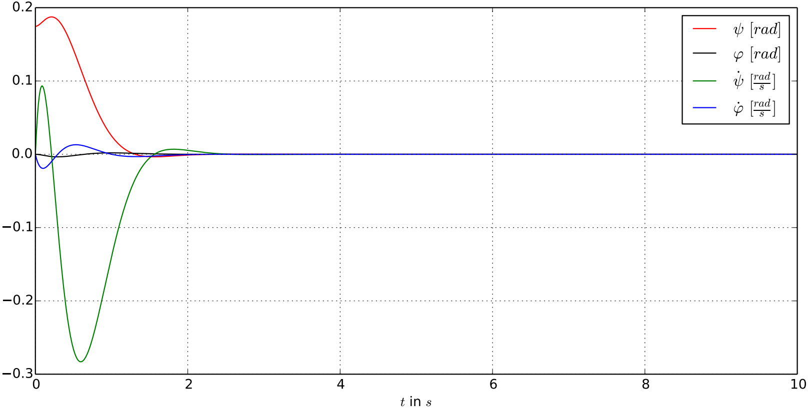
$$s_{1,2}^0 = 0, \qquad s_{3,4}^0 = \pm\sqrt{a_{42}}.$$

system is unstable, controller necessary

- desired pole
  $s_1 = s_2 = -5, \quad s_{3,4} = -3 \pm 3j$
- characteristic polynomial
  $\mathrm{CLCP} = (s+5)^2(s+3-3j)(s+3+3j)$
- characteristic polynomial
  of the controlled system
  $\mathrm{CLCP} = \det(sI - A - b(k_1, k_2, k_3, k_4))$
- detection of gain of the controller
  via coefficient comparison

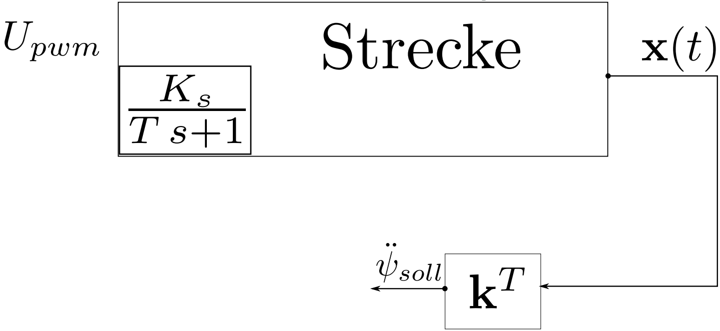


simulation

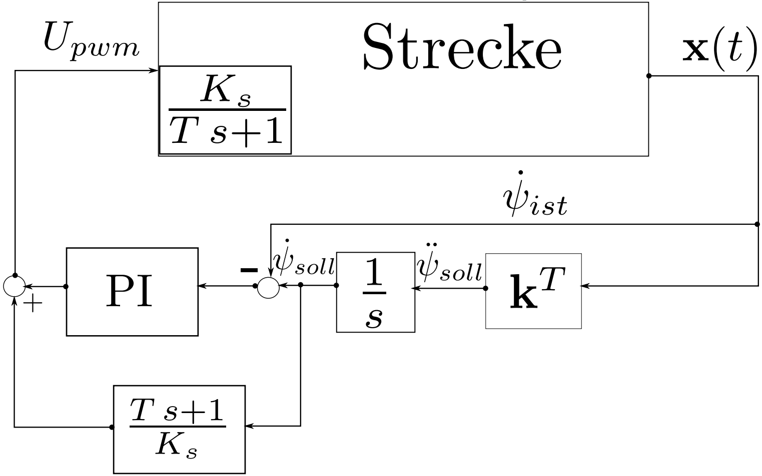


0.2
0.1
0.0
−0.1
−0.2
−0.3
0
2
4
6
8
10
t in s
ψ $[rad]$
φ $[rad]$
$\dot{\psi}$ $[\frac{rad}{s}]$
$\dot{\varphi}$ $[\frac{rad}{s}]$

structure of the control loop



structure of the control loop



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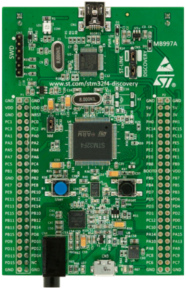
used microcontroller STM32F407



good characteristics

- systemclock 168MHz ARM-Core
- 1MB Flash, 512kByte Ram
- ploating point unit
  (software, hardware)
- many pins
- opensource
  - compiler
  - linker
  - flasher

detriments

- big electricity requirement
  by way of comparison to AVR-microcontroller

imagesource: st.com

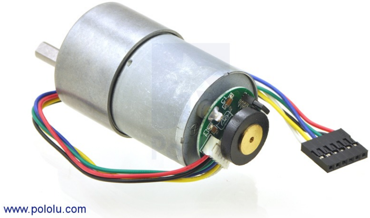
used motors

good characteristics

- operating voltage 12V
- torque 0.7767Nm
- built-in 64 encoder
- gear with 30:1 ratio
- max. 5A current at adherence of motor shaft

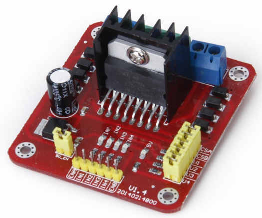
detriments

- different gain at the direction



imagesource: pololu.com

motordriver L298N H-Bridge



good characteristics

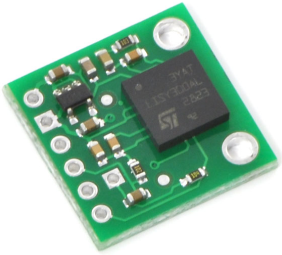
- operating voltage 15V
- max. current ca. 1A
- little thermically reliable
- for 2 motors

detriments

- none shunt-resistor

imagesource: amazon.de user: Generic

used gyroskop LISY300AL



good characteristics

- input voltage 5V
- internal 3.3V voltage controller
- very sensitive
- sensitivity 3.3mV/ ∘ /s

imagesource: pololu.com

XBEE for wireless communication

good characteristics

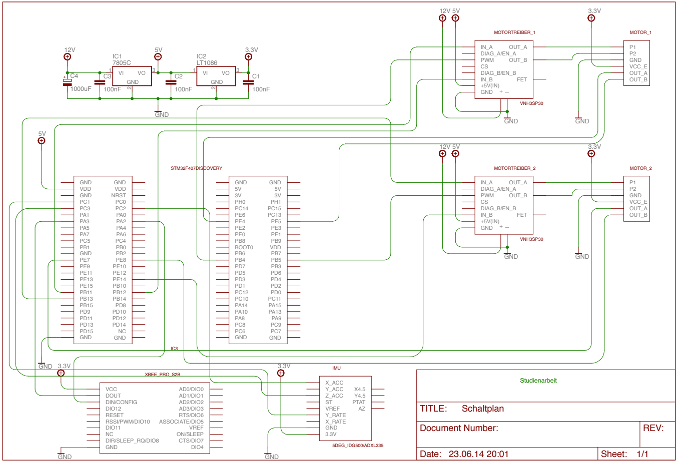
- operating voltage 3.3V
- serial communikation via USART
- 1600m getatable according to datasl
- data throughput 1Mbps

detriments

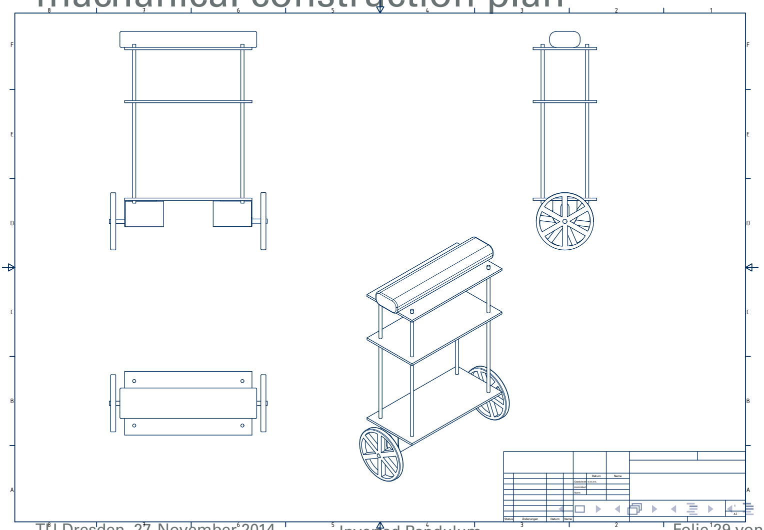
- need 2 piece

imagesource:
de.rs-online.com

circuit layout



machanical construction plan



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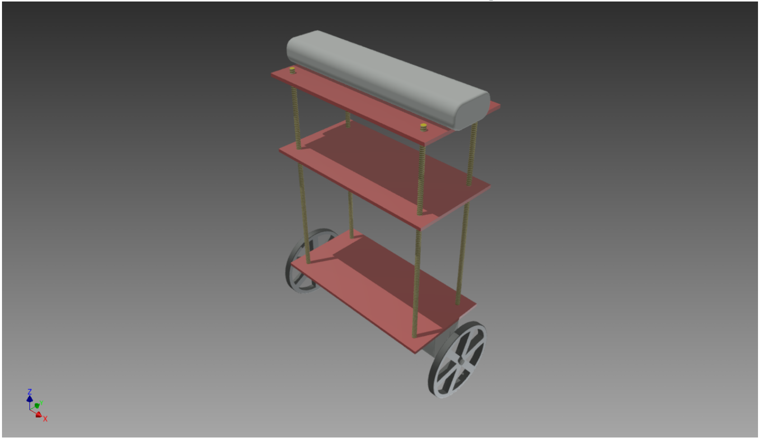
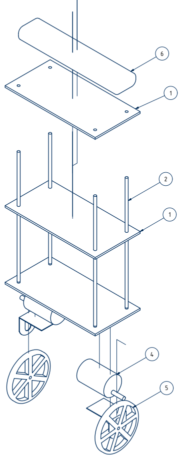


machanical construction plan



TEILELISTE			
OBJEKT	ANZAHL	BAUTEILNUMMER	BESCHREIBUNG
1	3	Platte	
2	4	Gewindestange	
3	2	Motorblech	
4	2	Motor	
5	2	Rad	
6	1	Batterie	

explo-1

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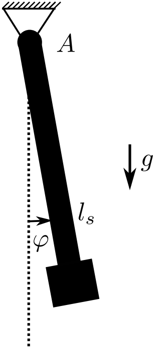
parameter idetification: moment of inertia J_p



vibration test

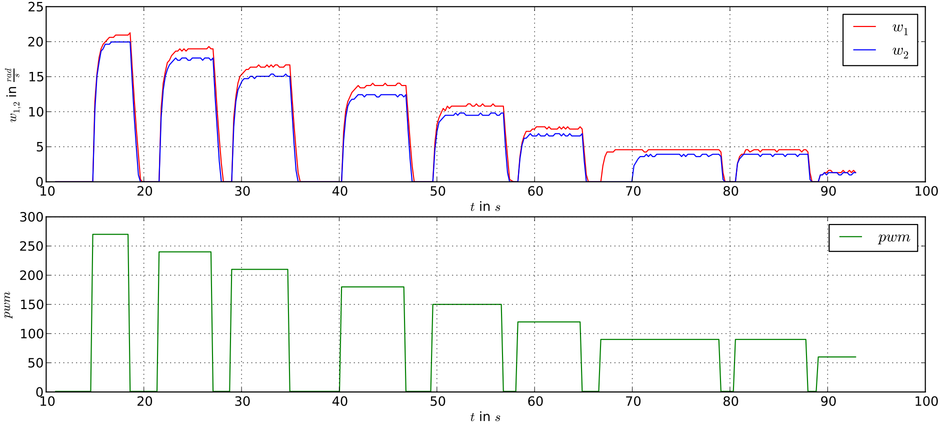
- difficult to measure
- $J_p = J_A - m_1 l_s^2$
  $$\ddot{\varphi} + \underbrace{\frac{l_s m_1 g}{J_A}}_{w_0^2} \sin(\varphi) = 0 \text{ für } \sin(\varphi) \approx \varphi$$
- $J_p = J_A - m_1 l_s^2 = \frac{l_s m_1 g T^2}{4\pi^2} - m_1 l_s^2$
- measured $T \approx 0.9s$ $\quad J_p \approx 0.0091 kgm^2$

moment of inertia J_m of the wheel

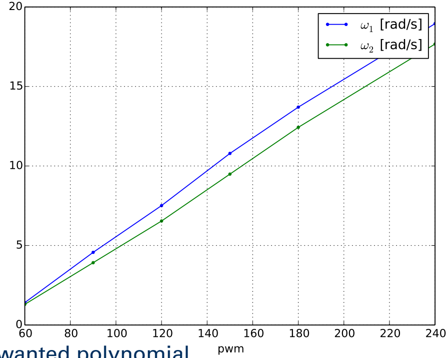
imagesource: pololu.com

consider as fine solid cylinder

- $J_m = 2\frac{mr^2}{2} = mr^2 = 6.48 \cdot 10^{-5} kgm^2$


$w_{1,2}$ in $\frac{rad}{s}$
t in s
w_1
w_2
pwm
t in s

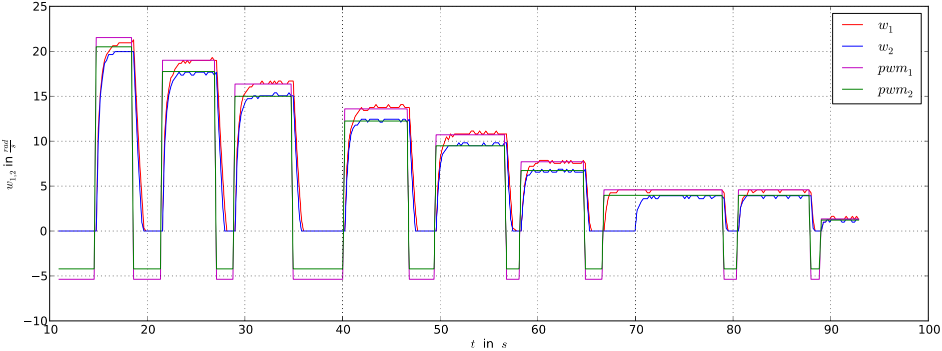
method of least squares



samples

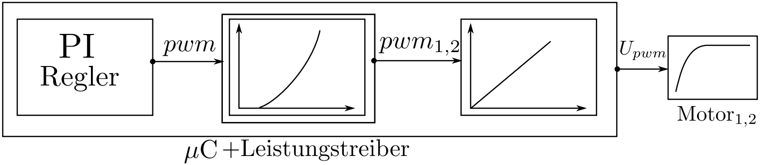
wanted polynomial

$$
\begin{aligned}
\mathrm{pwm}_1 &= 9.80666 \cdot 10^{-2} \cdot \mathrm{pwm} - 4.241 \\
\mathrm{pwm}_2 &= 9.18355 \cdot 10^{-2} \cdot \mathrm{pwm} - 4.295.
\end{aligned}
$$

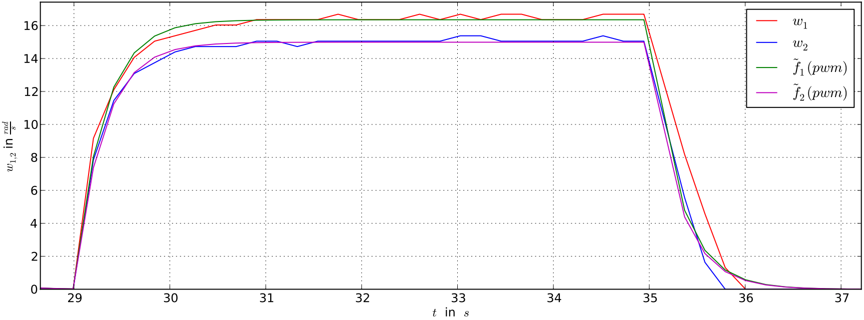


w_1
w_2
pwm_1
pwm_2
$w_{1,2}$ in $\frac{rad}{s}$
t in s
25
20
15
10
5
0
−5
−10
10
20
30
40
50
60
70
80
90
100

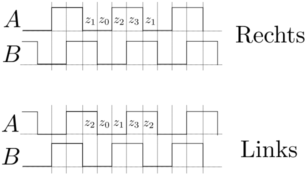
concept of the PWM-conversion



approximated funktion T = 0.3s



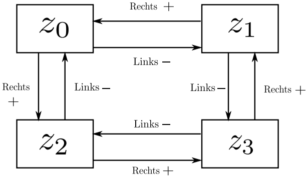
analysis of the encoder



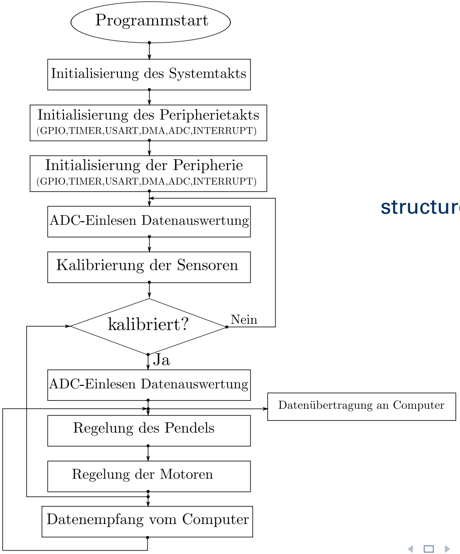
- $\pm 90°$ phase-delayed signal
- allocation of the suites
  $A \in \{0, 1\}, \quad B \in \{0, 1\}$
-

state	A	B	code
z_0	0	0	0
z_1	0	1	1
z_2	1	0	2
z_3	1	1	3

- right: $z_1 \rightarrow z_0 \rightarrow z_2 \rightarrow z_3 \rightarrow z_0 \ldots$
  left : $z_2 \rightarrow z_0 \rightarrow z_1 \rightarrow z_3 \rightarrow z_2 \ldots$



structure chart of the software



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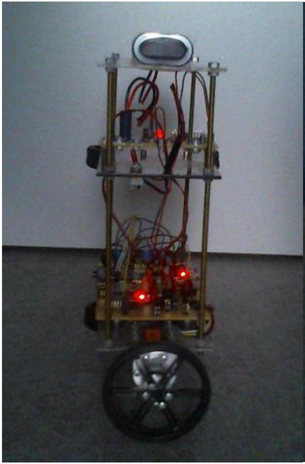
result



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conspectus and forecast

conspectus:

the state controller is very sensitive towards parameter imprecision. using an I-Controller extension the stabilisation works better. the friction of link-room is neglectable. the modelling of friction is necessary.

forecast:

using LQR controller the stabilisation gets better. the optimization is reque-red for the inside PI-control loop.

thanks for your attention!