The Optimal Burger Flip

The Dynamics of a Uniform Cylinder

Nic Bolton

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Abstract

In this project, we use kinematic principles to analyze the motion of a burger as it is flipped by a spatula. By applying translational and rotational kinematics, we can determine the initial angular velocity required to flip the burger an odd number of times. We also use the concepts of torque and moment of inertia to account for the distribution of mass within the burger and the effect it has on its rotational motion.

Through this analysis, we can determine the optimal initial angular velocity required to flip the burger a specific number of times, providing valuable insights for when you're on the barbecue and are stumped on this. This can be used to improve the flipping technique and ensure that the burger is cooked evenly on both sides, while allowing you to show everyone how perfectly you can flip burgers.

Introduction

Flipping a burger is an art form that requires skill, precision, and a deep understanding of the underlying physics. In this project, we use rotational and translational kinematics, as well as torque and moment of inertia, to calculate the initial angular velocity needed to flip a burger a certain number of times. This work has potential applications in the culinary world, where chefs and cooks can use our results to optimize their burger-flipping techniques and impress their customers with their flipping prowess. In the following sections, we describe our computational setup, present our results, and discuss their implications.

The physical background needed to tackle this problem is the following. We need to use the general inertia tensor [1]:

$$I_{ij} = \int_V \rho(\vec{r})\left(\delta_{ij}\sum_k x_k^2 - x_i x_j\right) dV$$

so that we can find how the object moves about its axes. Once this is calculated, we can find our angular momentum which is just the diagonal components of this multiplied by the respective angular velocity:

$$\vec{L}_i = \vec{I}_i\vec{\omega}_i$$

All these expressions can be simplified, since in our project we are only going to rotate the burger around $\hat{x}$. We will use Euler's equation for the motion of a rigid body in a force field for this component as well:

$$I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z = N_x$$

Using this, we can find an expression for $\dot{\omega}_x$, and thus $\theta(t)$, which we can input into a rotational matrix for $\hat{x}$ to simulate our burger spinning around its principal axis along $\hat{x}$ [2].

$$\vec{r} = \lambda_\theta \vec{r}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

Theory

The setup to this problem was relatively simple. We modelled our burger as a perfect cylinder with a height H, radius R, and a uniform mass m. We can describe the density of the cylinder to be uniform with $\rho = m/\pi R^2 H$ which allows us to calculate our inertia tensor:

$$I = \begin{pmatrix} \frac{1}{12}m(3R^2+H^2) & 0 & 0 \\ 0 & \frac{1}{12}m(3R^2+H^2) & 0 \\ 0 & 0 & \frac{1}{2}mR^2 \end{pmatrix}$$

Our goal here is to find the angle about $\hat{x}$ as a function of time, along with the vertical position of the CM as a function of time. Since we just found a moment of inertia, we can put this into Euler's equation for rigid bodies on $\hat{x}$. Note, that since we are only rotating around this direction our angular velocities in the other directions are 0. Our equation simplifies to:

$$I_{xx}\dot{\omega}_x = N_x = 2R\vec{F}$$

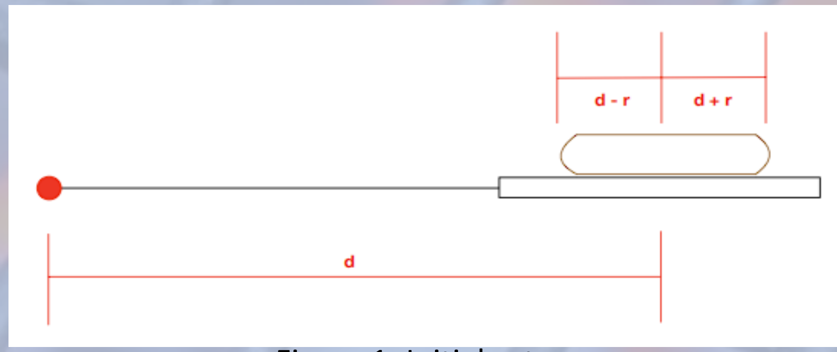

Figure 1: Initial setup

The total torque here is going to be the difference between the torques on both ends of our burger as shown. Using this result, we can arrive at $\theta(t)$:

$$\theta(t) = 2mR\omega_0 t/I_{xx}$$

We now have a result to describe the rotational motion of our burger, where ω_0 is the angular velocity that the burger will begin with. We now need to find the translational motion of the CM, which can be described with free falling motion. By combining free-falling motion with conservation of energy, we can arrive at:

$$z(t) = -\frac{1}{2}gt^2 + \sqrt{2gh}\,t$$

Where h is determined by how high the burger is desired to be thrown. Using the fact that we want a period of $T = (2n+1)\pi/\dot{\theta}$ so that the burger lands on the opposite side it began on, we can solve for our optimal initial angular velocity:

$$\omega_0 = \frac{(2n+1)I_{xx}\,\pi}{4mR}\sqrt{\frac{g}{2h}}$$

Finally, our last quantity, the angular momentum can be described as a constant:

$$L_x = I_{xx}\omega_0$$

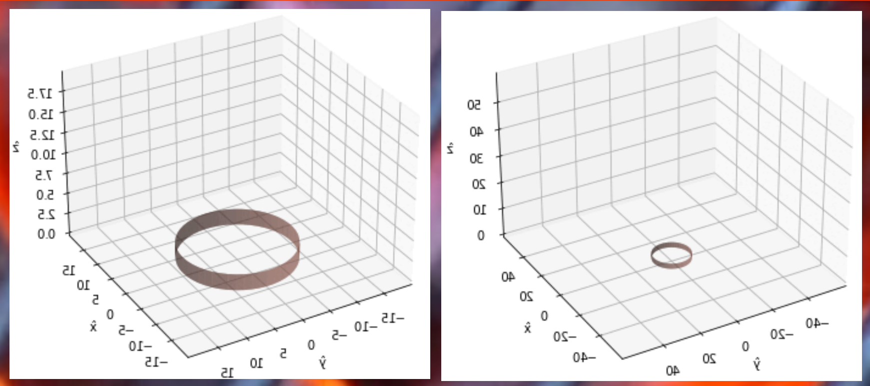

Figure 2: Simulation of burger flipping 1 time with h= 20 m (left). Burger flipping 7 times with h = 60m (right).

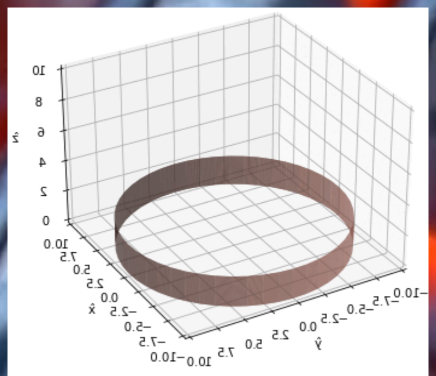

Figure 3: Initial conditions that lead to unrealistic flipping of burger (h = 7.5m with 5 rotations).

Discussion

Programming our experiment in Python was done without having to use any numerical integrator, since we found direct equations for z and θ with dependence on time. The initial conditions were set, and we studied the motion of the burger using ipywidgets and matplotlib.animation.

Now, there are two results that are produced based off our derivations. We can either have the simulation work, and the burger lands perfectly flat on the opposite side it started on (seen in Figure 2). We could also have a scenario where we do not give the burger enough of an initial velocity in the center of mass. This results in the rotation of the burger to hit the spatula on the way up, so that it would not flip, but be interrupted by the object (we can see that parts of the burger dip into $-\hat{z}$ in Figure 3).

We want the edge that is experiencing the lower magnitude of torque to always be above $-H/2$. Using Taylor expansion, it was found that the condition that assures this is:

$$\frac{h}{2n+1} > \frac{1}{4}\pi R$$

Whenever this condition is true, the burger will have a perfect flip.

Conclusion

After running these simulations with the derived equations and condition, it was found that the behavior of the burger was as expected. Given the properties of the burger, a maximum height, and a number of flips, the simulation could be run successfully (Figure 2).

The average burger weighs $\sim 0.11kg$, has a radius of $\sim 0.057m$ and a height of $\sim 0.025m$ [3]. This would mean that someone would have to flip their burger with an initial angular velocity of $\sim 0.11\ rad/s$ to achieve the optimal flip. Since our burger can be considered uniform, it is very stable while rotating around $\hat{x}$. The only instability in this case, would be if the condition above was not followed, and you would have a horrible flip that would leave your spectators unimpressed.

References

[1] Thornton, S.T.; Marion, J.B., *Classical Dynamics of Particles and Systems*, 5th ed, Brooks/Cole, 2004

[2] Wikipedia Contributors. Rotation matrix https://en.wikipedia.org/wiki/Rotation_matrix

[3] How to Make a Perfect Burger: A Step-by-Step Guide https://foodnetwork.com/how-to/articles/how-to-make-a-perfect-burger-a-step-by-step-guide