Experimental observation of resonance-assisted tunneling

Stefan Gehler, Steffen Löck, Susumu Shinohara, Arnd Bäcker, Roland Ketzmerick, Ulrich Kuhl, and Hans-Jürgen Stöckmann

(Dated: February 17, 2015)

About research group


Stefan Gehler


Steffen Löck


Susumu Shinohara


Arnd Bäcker


Roland Ketzmerick


Ulrich Kuhl *


Hans-Jürgen Stöckmann

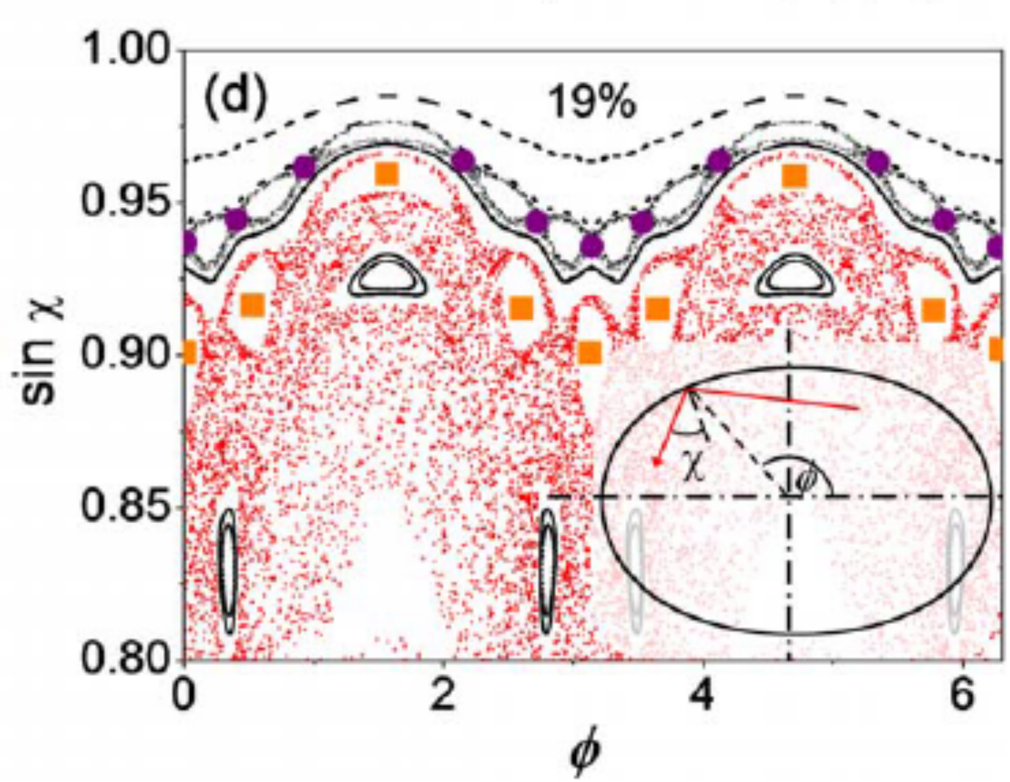
Dynamical tunneling

- Regular→Chaotic
- Regular→Chaotic→Regular

: Chaos-assisted tunneling

- Chaotic→Chaotic

: Tunneling between two chaotic regions



In this article, Regular to Chaotic is the case corresponding to RAT

Dynamical tunneling

Regular→Chaotic

$$\left|\psi_{reg}(t)\right|^2 \propto e^{-\gamma t}$$

For increasing wave number,

- For small wave numbers : direct regular-to-chaotic tunneling leads to an exponential decrease of γ with increasing wave number
- For larger wave numbers : resonance-assisted tunneling enhances the tunneling rates

Previous research

- Direct regular-to-chaotic tunneling

➢ Mushroom billiard (2008), BEC (2013)

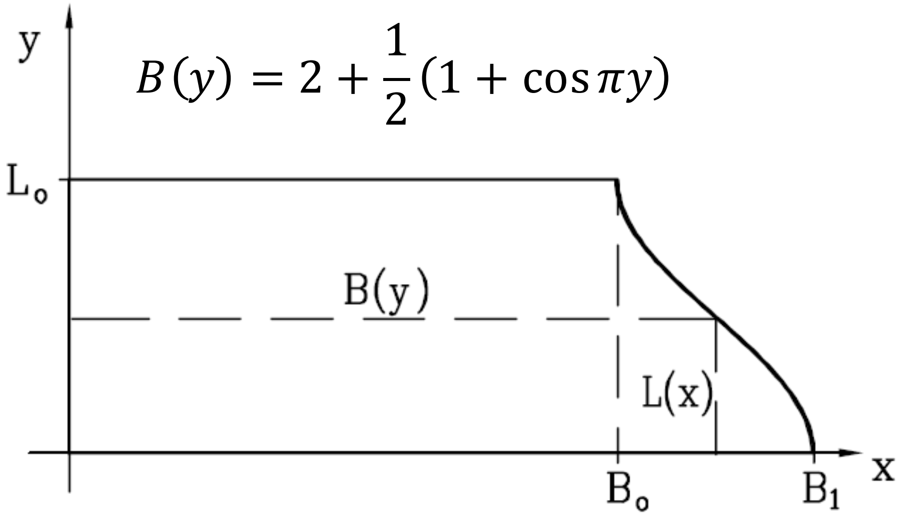
- RAT

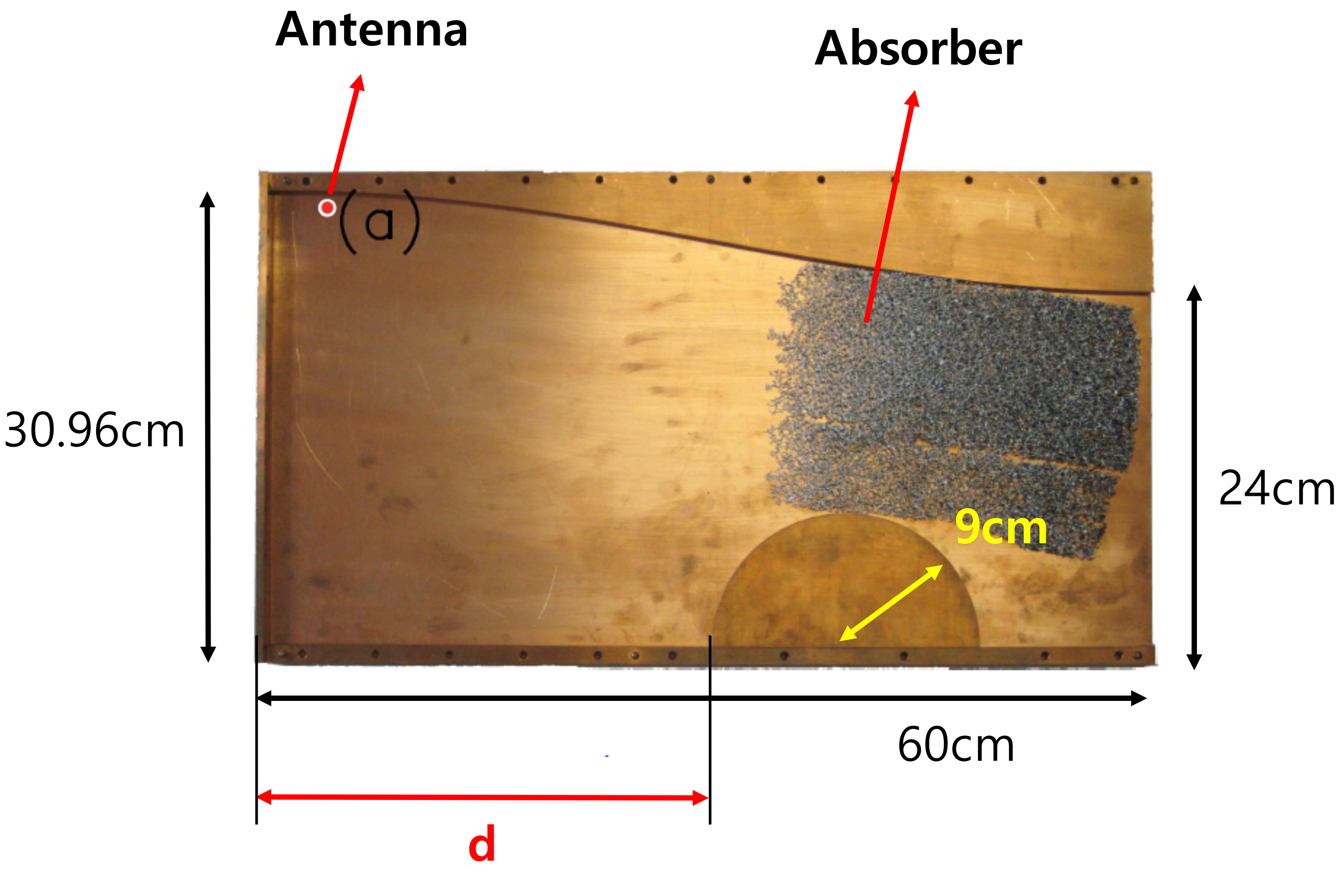
➢ H. Kwak (2013)

⟶ "Recently the coupling matrix element between two modes coupled by a nonlinear resonance chain was very nicely observed experimentally in the near integrable regime using a deformable asymmetric microcavity. The experimental observation of the enhancement of regular-to-chaotic tunneling rates due to RAT, however, has remained open."

Experimental setup

- Microwave resonator : Desymmetrized cosine billiard





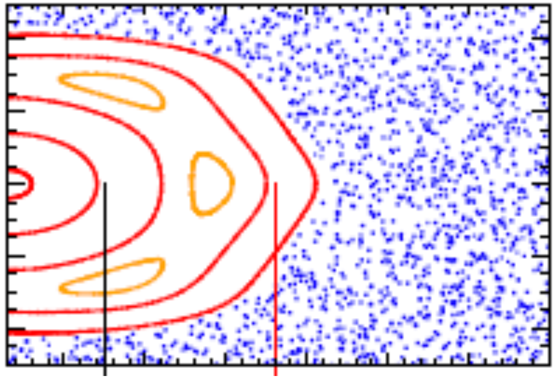
Experimental setup

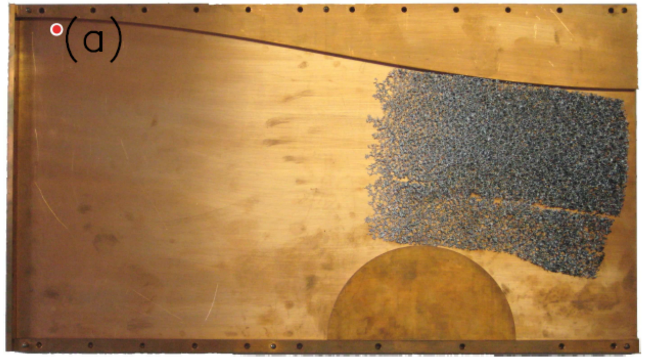
- Broad band foam absorber

→ Open the system

→ The chaotic region of the closed system correspond to the continuum

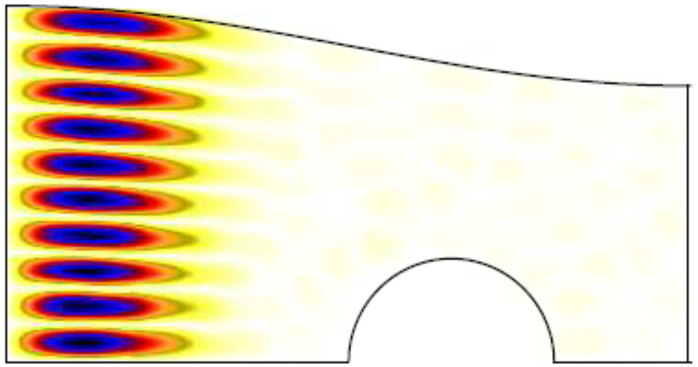
→ Observation of regular-to-chaotic tunneling by measuring the line width of a regular mode



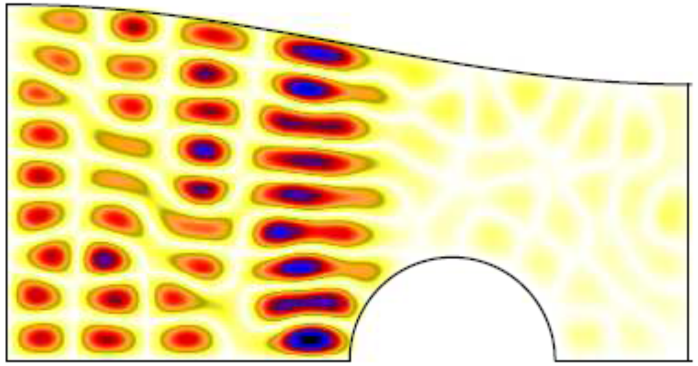


Experiment 1 : RAT induced by parameter variation

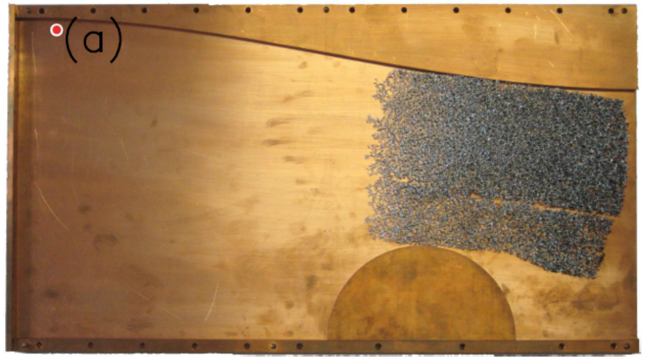
- They first show the signature of RAT from the parametric dependence of two close-by regular modes.
- Complex reflection amplitude : $S_{11}(\nu) = 1 - i\sum_k \frac{Re(\lambda_k)\psi_k(\overrightarrow{r_1})\psi_k(\overrightarrow{r_1})}{\nu-\nu_k-\Delta_k+\frac{i}{2}\Gamma_k}$
- λ_k : complex valued coupling coefficient of the antenna
- Measure S11 for several half disk positions d, while all other parameters are fixed.

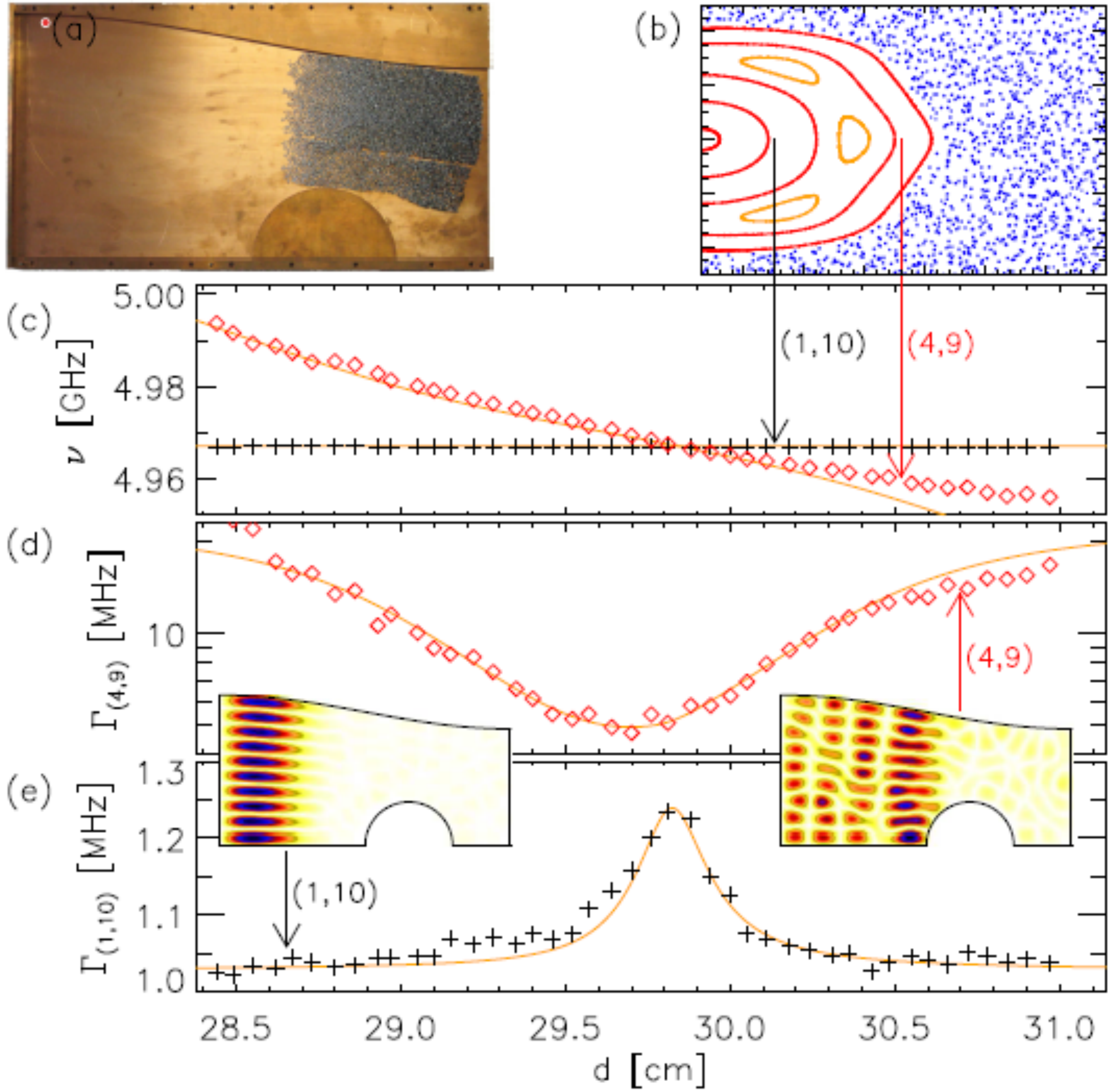


(n,m)=(1,10)



(n′,m′)=(4,9)





→ mode (1,10) and mode (4,9) are coupled due to the nonlinear 3:1 resonance chain

→ assuming eigenenergy of mode (1,10) is independent of d

→ The width increase of 0.2MHz

Quantitative description

- **Matrix model**

$$H = \begin{pmatrix} E_1 - i\dfrac{\gamma_1}{2} & V_{3:1} \\ V_{3:1} & E_4(d) - i\dfrac{\gamma_4}{2} \end{pmatrix}$$

- E_1 and E_4 : the eigenenergies of the uncoupled modes (and still assuming E_1 is independent of d)
- γ_1, γ_4 : related to antenna coupling and wall absorption → independent of d
- $V_{3:1}$: coupling due to the 3:1 nonlinear resonance

- Diagonalization → linewidths of the two modes → agrees with increase of the width
- For a full analysis of RAT, tunneling into the chaotic region have to be considered

Quantitative description

- Effective Hamiltonian : $H = H_0 - iWW^{\dagger}$
- $H = \begin{pmatrix} E_1 - i\frac{\gamma_1}{2} & V_{3:1} & 0 \\ V_{3:1} & E_4(d) - i\frac{\gamma_4}{2} & -iV_{dir,4} \\ 0 & -iV_{dir,4} & E_{ch}(d) - i\frac{\gamma_{ch}}{2} \end{pmatrix}$

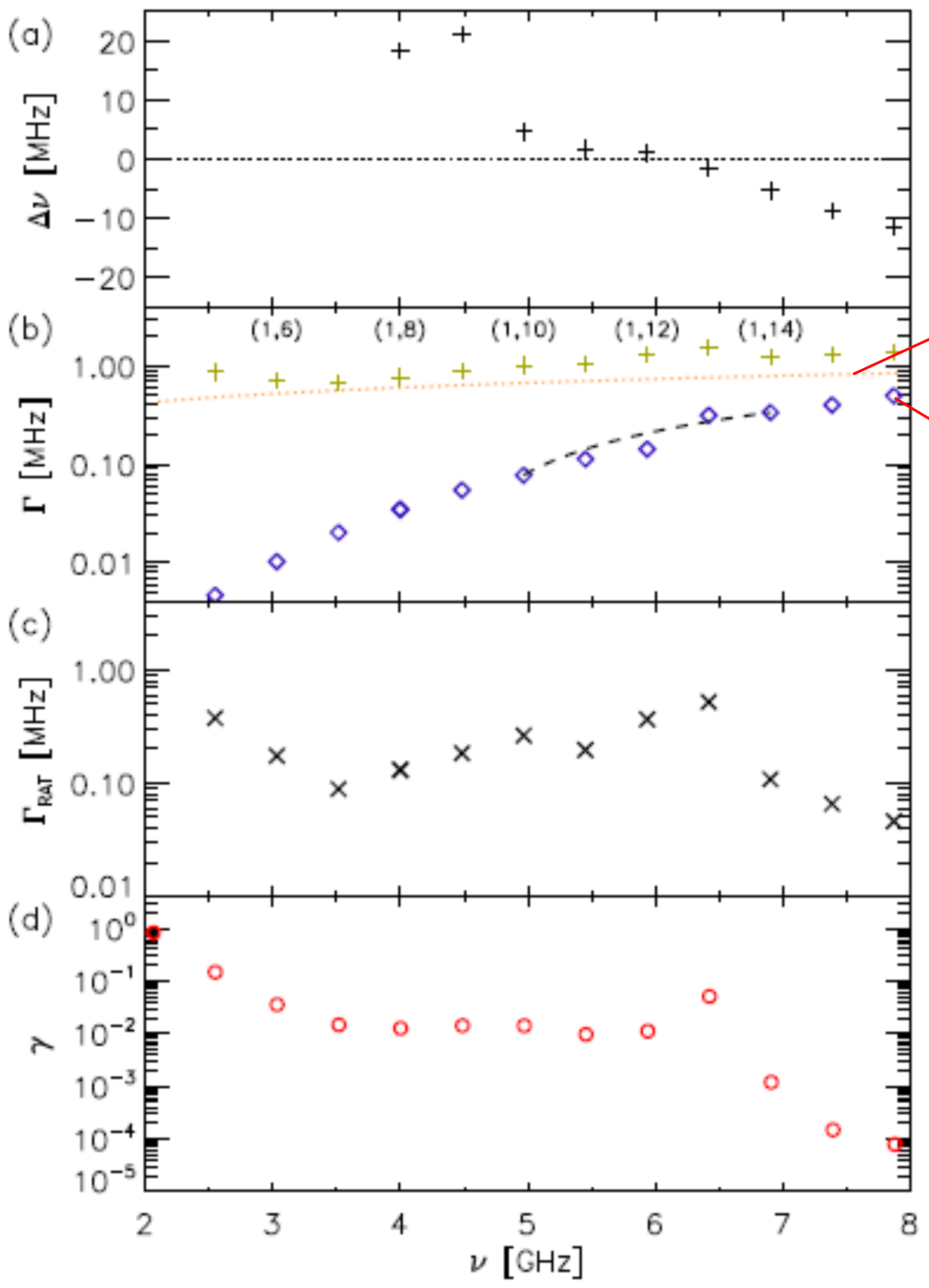
→ After the diagonalization, they compared the eigenenergies with the experimentally measured one

→ Fitting the experimental data → $V_{3:1} = 2.3m^{-2}$ ($V_{3:1,cl} = 0.51m^{-2}$)

Experiment 2 : RAT plateau and peak structure

- Fixed half disk position d=30.0cm
- Regular mode width contains different contributions

➢ $\Gamma_{(n,m)} = \Gamma_{RAT} + \Gamma_{wall} + \Gamma_{antenna}$



Γ_{wall}

$$\Gamma_{(n,m)} = \Gamma_{RAT} + \Gamma_{wall} + \Gamma_{antenna}$$

$\Gamma_{antenna}$

(a) $\Delta\nu = \nu_{(1,m)} - \nu_{(4,m-1)}$ for m=8, ⋯,16

(b) Γ_{wall}, $\Gamma_{antenna}$

(c) Γ_{RAT}

(d) Numerical dimensionless tunneling rates $\gamma_{(1,m)}$

Summary

- Experimentally observed RAT using an opened microwave billiard
- Quantitative description is made by 3 x 3 matrix model $\rightarrow$ coupling element $V_{3:1}$
- RAT with characteristic plateau and peak structure