INVESTIGATING A TORNADO MACHINE

EXTENDED ESSAY IN PHYSICS

RESEARCH QUESTION

How the diameter of the cross-section of the tornado at fixed height changes with the variation of power generated by a ventilator?

Subject: Physics

Word count: 3697

Session: May 2020

Contents

1. Introduction

Tornados are one of the deadliest and unresolved natural phenomena. For their formation, it is required that natural variables such as lifting mechanisms, moisture, and the difference in the pressure in the upper and lower level of atmosphere act accordingly in a specific time.

Researches that have been done in this field of work are important because tornadoes often result in catastrophic human and material losses. Tornados are impossible to stop but luckily, they can be examined and to the extent, we can predict their formation to minimize the risks and losses that exist.

For these reasons and because generally, I have an interest in making inventions, I have decided to make the tornado in laboratory conditions. In that way, I will examine its properties, see how it is generated and how does it behave when the parameters are changed.

Figure 1: Picture of tornado chamber

1.1. Making the tornado machine

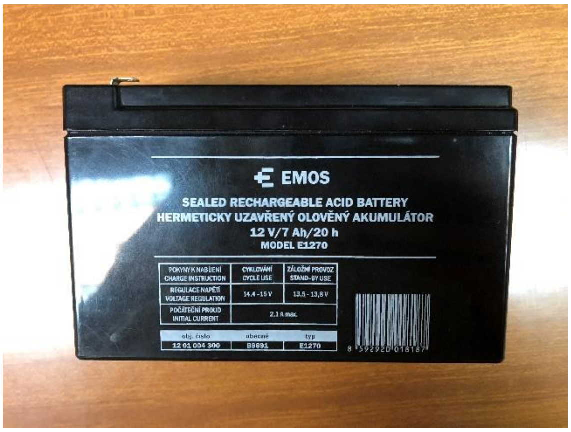
After researching the topic on the internet, I have decided to make a scheme of the tornado chamber. The details of construction are shown in the appendix. Here, there is the list of the key components of the tornado chamber.

<u>Key components</u>:

1. ***On the upper base is located ventilator COOLERMASTER BLADE MASTER PWM dimensions: 80 x 80 x 25mm***

<u>Figure 2</u>: Picture of ventilator

2. ***For the power supply I have used a battery cell EMOS with emf of 12V***



<u>Figure 3</u>: Picture of battery

3. Chamber – explained int the appendix in more detail

Dimensions:

Height of the chamber: $57.4cm \pm 0.1cm$

Length of the base: $31cm \pm 0.1cm$

Height of the slit: $47.5cm \pm 0.1cm$

Width of the slit: $1.7cm \pm 0.1cm$

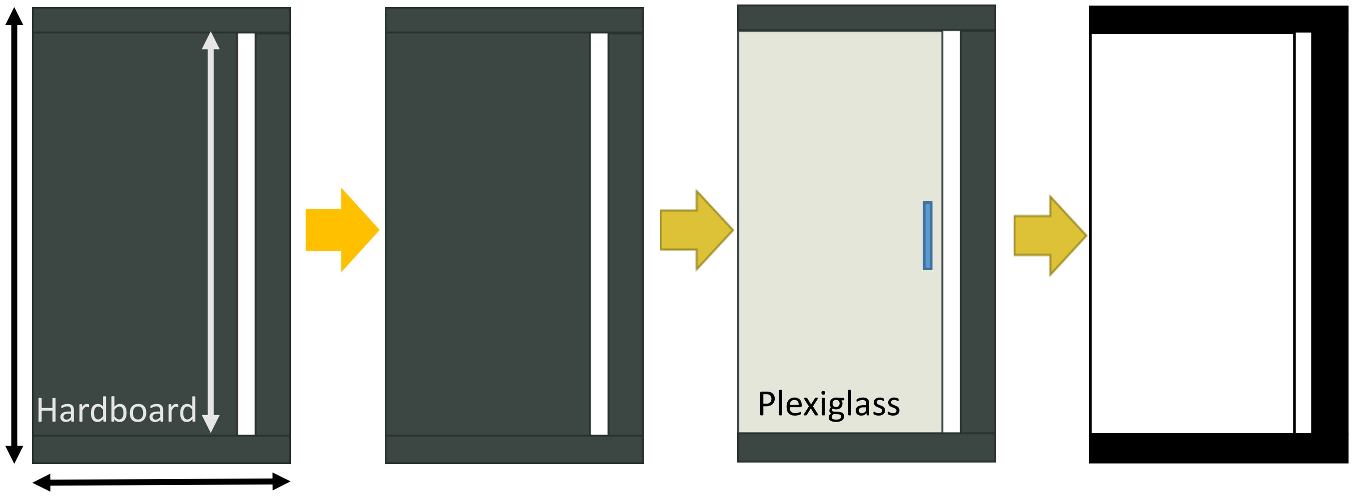


Figure 4: Shows cross-section area of each chamber wall and corresponding slit

1.2. Theoretical background

It is worth mentioning the Rankine vortex, which is a model that describes the fluid behaviour of a vortex and is often used to model a tornado (Katopodes, 2019):

$$v_\theta(r) = \begin{cases} \frac{\Gamma r}{2\pi R^2}, r \leq R \\ \frac{\Gamma}{2\pi r}, r > R \end{cases}$$

It describes the vorticity (local rotation of fluid) at the radial region of a tornado and, at where is the region of a free vortex, which allows for the velocity to decay at large distances. Unfortunately, the model of Rankine vortex will not be examined as it only serves as a theoretical description. Instead, the vortex chamber has been assembled to create the tornado in laboratory conditions. The ventilator put at the upper base is being used to create an upward flow of air molecules. Its blades are constructed in such a way so that it grabs the local air and throws it up away from the interior of the chamber. In that way, it creates a thrust because in the vicinity of the ventilator there is a small number of air molecules, hence a smaller gas pressure as opposed to greater pressure at the bottom of the chamber. From the ideal gas equation:

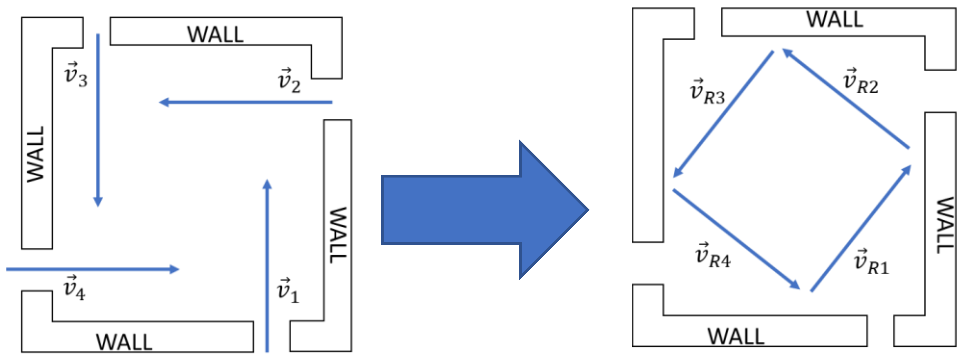
$$pV = nRT$$

we can see that the pressure is proportional to the number of moles. (an indicator of the number of molecules)

If we go down with the height to the base, there is a bigger number of air molecules which cause a bigger static pressure. Bernoulli equation states:

$$p_1 + \frac{\rho {v^2}_1}{2} + \rho g h_1 = p_2 + \frac{\rho {v^2}_2}{2} + \rho g h_2$$

Where p is pressure at one point, v is the speed of the fluid, p is the density of the fluid and h is the height at a given point. In other words, the difference in pressure throughout the fluid will cause a flow of that fluid. Fluids tend to move from a greater level/value of pressure to lower. In this case, fluid will start to move upwards because the pressure gradient is directed upwards as well. To secure the constant flow of air, I have made the slits at each right side of the chamber. 4. Air that is located outside the chamber (because of the difference in pressure in the chamber and outside) will begin to move into the vortex chamber. The outside molecules externally enter in the indicated directions (Figure 5). The interaction results in the indicated motion shown in the figure to the right explaining the circulation.



<u>Figure 5</u>: Shows each step of addition of vectors

It is important to say that the wall and the glass have their role. They do not allow the passage of air out but directs it into further circular flow (Figure 5). According to Feynman et. al. (1964, p.40-9) if we have a fluid which flow is cylindrical;

“For a circular path with its centre at the centre of cylinder, the line integral of velocity is”

$$\oint \vec{v} \cdot d\vec{s} = 2r\pi v$$

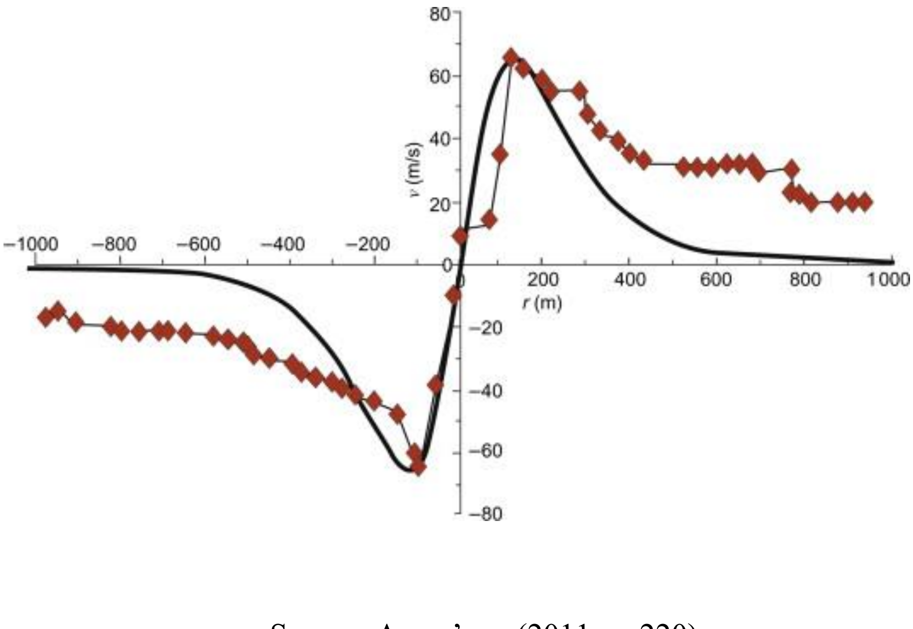
Let us denote the integral value with C, then:

$$v = \frac{C}{2r\pi}$$

The example of this fluid flow is water with circulation draining from a tank. Since the tornado has the same shape as the water draining from a tank (a cone) and they both are fluids, the same approach will be applied to this problem.

In the cross-section area, the radial velocity is inversely proportional to the radius of the cross-section area.   As we go closer to the centre of our vortex, we should be able to see that the velocity of the air is getting bigger and vice versa. This works in the region for r>R, where R is determined in such way that R is position where azimuthal rotation velocity is maximal (Figure 6). When r<R, the rotation speed should go to zero because there is no rotational motion on the axis itself. Then for azimuthal velocity a linear approximation is valid in which v (average azimuthal wind velocity) is proportional to r (distance from the centre). That is physically reasonable because when we move from one end of the diameter to the centre, particles of air need to change direction. To do that, they need to lower their velocity to zero. As a support to this statement, Arsen'yev, (2011) talks about the mathematical modelling of tornados and squall storms. From the experiment, the author got similar results of average azimuthal wind velocity inside an F2 tornado. We can see from the following picture that the velocity follows the trend of $\frac{1}{r}$ after the maximum velocity is achieved.

Figure 6: Average azimuthal wind velocity



Source: Arsen'yev (2011, p. 220)

To do this experiment efficiently, one must carefully choose a parameter that is observable and can be used for further analysis. The most noticeable part of the tornado is its cone shape. The horizontal cross-section (area of the tornado) changes as one goes along the vertical axis, meaning its diameter changes with height. By changing the power of the ventilator, the size of cross-section of diameter changes at a given point. Since the diameter is simple to analyse by a computer program, I have decided that I will study the relationship between the diameter of the horizontal cross-section and power at a given point of vertical axis. I decided to solve the problem of measurability of the diameter by illuminating it with a laser. Since the ventilator uses voltage and current, I thought I could easily measure those variables and change them by a variable resistor. The visibility of the tornado was made good by pointing a strong laser that disperses light horizontally at the tornado. The room where I did my experimenting was dark for better visibility and contrast.

From theory, it is a lot more known about the velocity rather than the fact what gives the tornado such power and from where does it come. Although I have an artificial source of power I am generally interested in the relationship between the source and the diameter of the tornado.

Therefore, my research question is:

Research question:

<u>How the diameter of the cross-section of the tornado at a fixed height change with the variation of power generated by a ventilator?</u>

2. The experiment

2.1. Identifying the parameters

The first thing in the identification of variables is to see which ones are capable of being measured. The pressure, density of fluid and velocity will not be measured since there are no appropriate measurement tools for that. Also, they do not contribute to answering my research question. Ventilator uses current and voltage from a battery as a source of power. With the variable resistor, the power output can be changed and measured, using ammeter and voltmeter. Variables that are important in answering the research question will be divided into three sections of independent, dependent and controlled variables.

1. ***Independent: <u>Current and voltage</u>:***

They will be changed by changing the sliding contact form the variable resistor. The experiment will start at a maximum value of resistance and then gradually decreasing it. The voltmeter will be connected in parallel with the circuit and ammeter in series. They are independent because the sliding contact is independently changed. The angular speed of the ventilator is directly proportional to the power output. There were no tools to measure angular speed but with greater power output, the ventilator will be rotating faster.

2. ***Dependent: <u>The diameter of the cross section of the vortex</u>:***

Each time when power output was changed and thereby causing the change in the diameter, several pictures of the cross-section of the tornado were taken. The cross-section was made visible by using a powerful laser. The diameter of the cross-section will be analysed in the Logger Pro program. Diameter is a dependent variable because it depends on the power of the source.

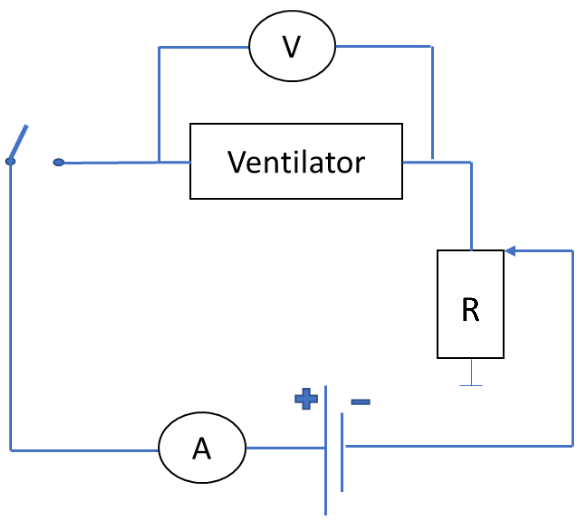
3. ***Controlled: <u>The height of the observed cross section, temperature, atmospheric pressure, composition of the circuit, the area of the chamber wall slit:</u>***

The height will be a controlled variable because I want to see how the diameter changes on the same level of height. Changing the height would make results unreliable because at different heights tornado has different properties even on the same power output. Height will be independently chosen based on the experience of taking the best quality pictures at a certain height. In that way, results can be nicely seen. The experiment will be done at room temperature so that it doesn't affect the results. The same elements of the circuit will be used throughout the whole experiment. When each picture would be taken, the ventilator will be turned off so that the heating effects of current can be negligible and thus prevent any changes in the power of my ventilator.

2.2. Experimental setup

For this experiment the following equipment have been used: 2 multimeters, variable resistor with maximum resistance of 125 Ω, battery cell of 12V, electrical conductors, a switch, laser (532nm, 50mW), bowl of water and fogger maker (Fogger maker is a device which needs to be put in the water in order to create a bursting of fog. This device is used to make tornado more visible)

Scheme of circuit:



<u>Figure 7</u>: Shows composition of circuit

In the circuit above (Figure 7), an ammeter is connected in series and voltmeter in parallel. On the right side, the variable resistor has been connected and by changing the sliding contact (form maximum resistance to a minimum) the current and the voltage supplied to the ventilator will be changed as well as the power it gives. The laser will be put on the stand at the wanted height. The pictures of the experimental setup can be seen in the Figure 8. below and in the appendix. The switch has been also put in the circuit because every time the measurement was done it was used to open the circuit so the heating effects of current would be neglected.

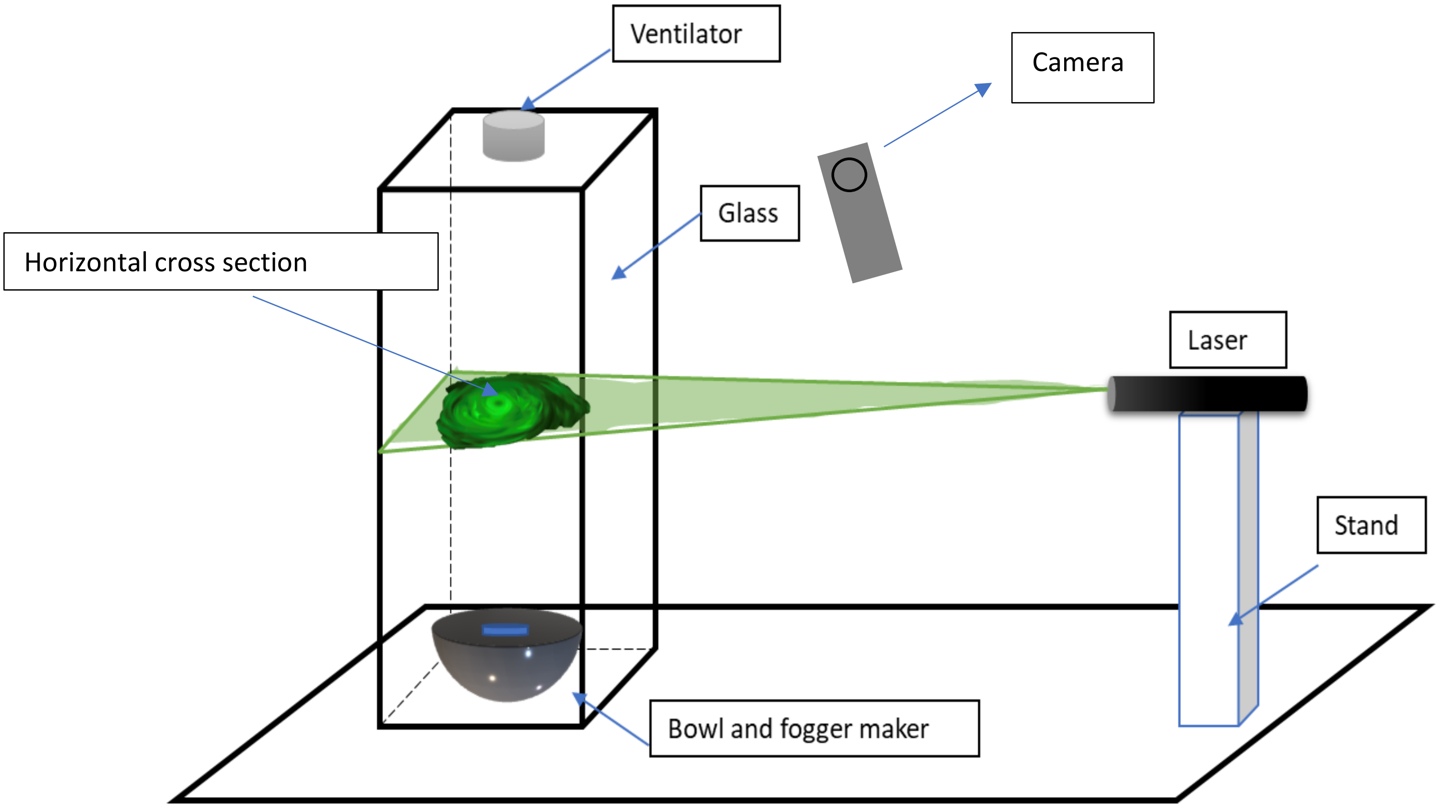


Figure 8: Shows the experimental setup

2.3. Measurements

2.3.1. Usage of camera

For the purpose of taking pictures iPhone 8 camera was used. Pictures were taken at the height of $50cm \pm 3cm$, and the angle between the camera and glass varied from $20° - 25°$.

2.3.2. Data and picture analysis

The pictures were plotted in Logger Pro program. The length of green laser line was experimentally measured to be 23.3cm. The ratio x to L is determined from the picture taken. the actual radius of the cross-section equals:

$$\frac{2r}{23.3} = \frac{x}{L}$$

by similarity reasons. Thus, I managed to measure the radius without disturbing tornado movement. The diameter was measured by comparing it with the calibrated length of a green laser line.

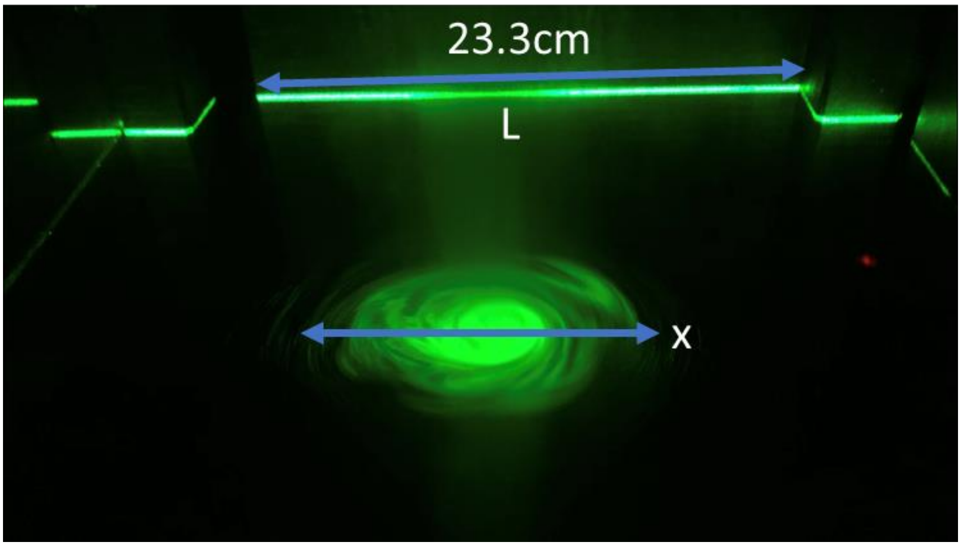


Figure 9: Shows how the measurements of diameter were made

For each picture, the distance was measured by taking the one point at which the diameter of tornado could be clearly seen and another one where the diameter is not so strongly established but can be regarded as a part of diameter. After that, the arithmetic mean was made of those two distances which is then plotted in the data Table1.

Picture (Figure 10) shows red and blue line which show how the measurement was done. While doing the experiment the lines were drawn through the centre at the same path.

For the better clarity, some pictures were taken at the other side of the vortex chamber.

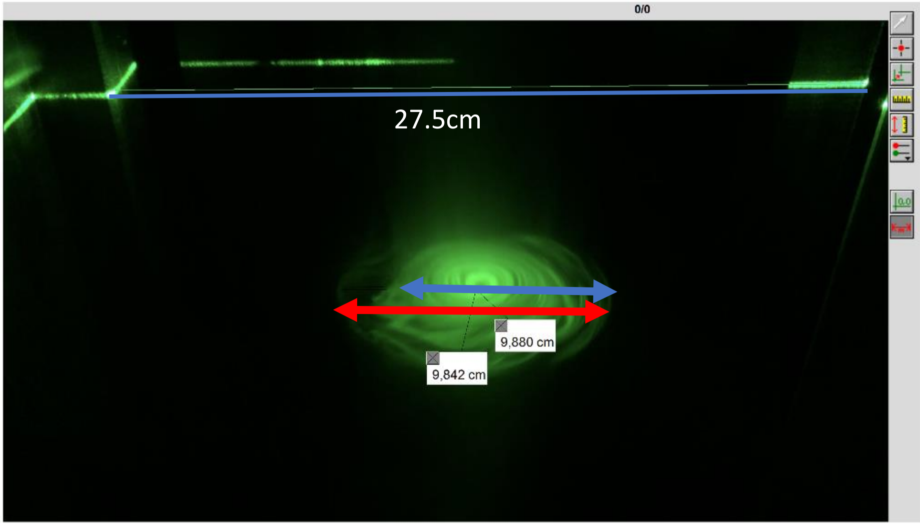


Figure 10: Shows how the measurements of diameter were made

2.4. The results of experiment

In Table 1. below there is gathered the data for the voltage, current, and diameter of the tornado. Uncertainties for current and voltage were experimentally estimated in the process of doing measurements. The uncertainty for diameter was deduced as a arithmetic mean of the experimental measurement. Each measurement has been done two times and the error has been estimated based on the spread of the data. The average of two measured diameters is presented in the following table:

Table 1.

The measured height of vortex= h: 41cm$\pm 0.2cm$			
i	U/V$\pm 0.01V$	I/A$\pm 1mA$	d/cm$\pm$2.50cm
1	6.66	0.049	9.6
2	6.83	0.050	9.5
3	7.01	0.053	9.9
4	7.21	0.056	10.0
5	7.45	0.059	9.9
6	7.72	0.063	10.5
7	7.99	0.068	13.6
8	8.27	0.072	15.2
9	8.62	0.078	16.7
10	9.03	0.085	20.3
11	9.31	0.092	20.7
12	9.73	0.099	22.1
13	10.15	0.110	19.9
14	10.55	0.117	23.3
15	11.19	0.133	25.5
16	11.97	0.152	26.0
17	12.52	0.167	26.8

Table 1: Shows the data I have gathered for voltage, current and diameter of vortex

Table 2.

Power will be calculated using the product formula of current and voltage:

$$P = UI$$

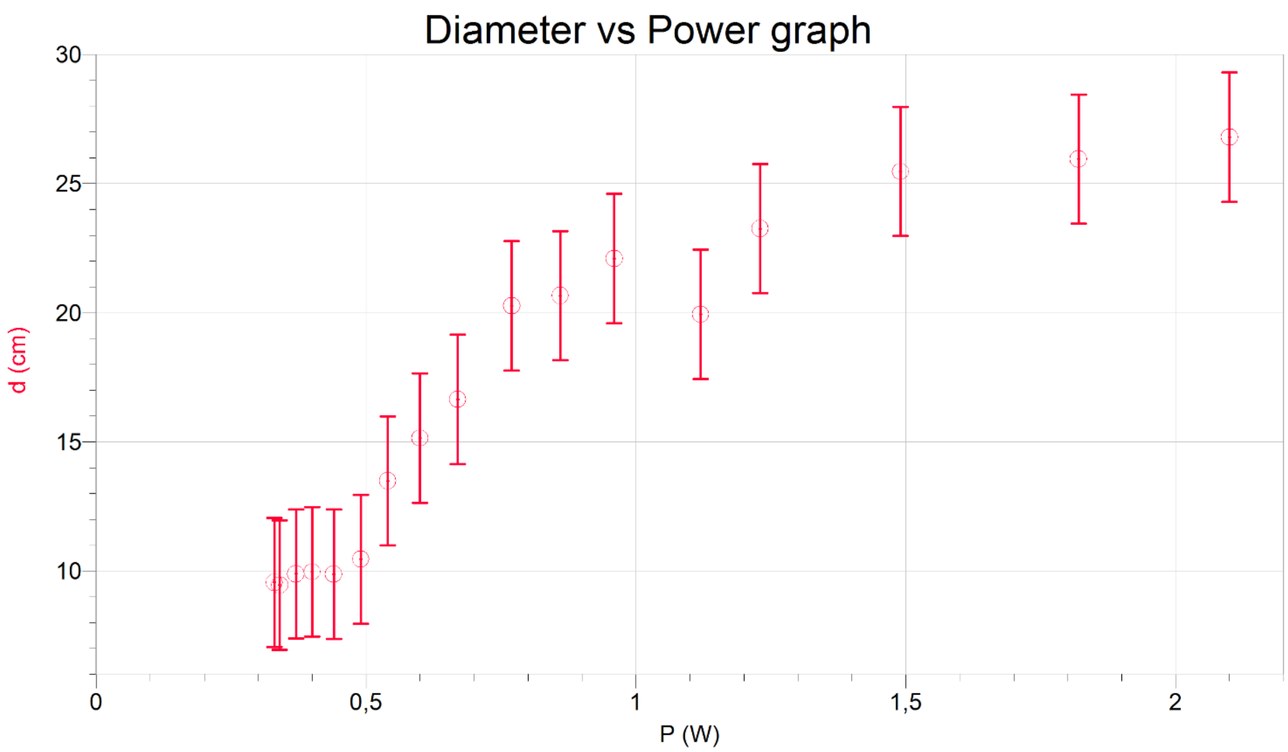
i	P/W
1	0.33
2	0.34
3	0.37
4	0.40
5	0.44
6	0.49
7	0.54
8	0.60
9	0.67
10	0.77
11	0.86
12	0.96
13	1.12
14	1.23
15	1.49
16	1.82
17	2.10

Table 2: Shows the data for calculated power

The process of calculation of the relative error and uncertainties are shown in appendix.

Now when all the necessary data and errors have been gathered, the data will be plotted in the Logger Pro program. In Graph 1. below we can see our data being plotted. On y-axis there is diameter and on the x-axis there is power.

Graph 1.



<u>Graph 1</u>: Shows plotted data for diameter and power

2.4.1. Finding the appropriate trendline curve

In the next step, the question is what type of function should be used to approximate this type of behaviour. As before Logger pro program will be used to see which ones will give the best correlation. There are three functions that can be used as a curve of the best fit:

$$y_1 = A(1 - e^{-Cx}) + B$$

$$y_2 = Ae^{-Cx} + B$$

$$y_3 = A \cdot 10^{BX} + C$$

Rewriting these equations using the values of coefficients and physical variables:

$$d_1 = 33.4(1 - e^{-1.6P}) - 5.4$$

$$d_2 = -33.4e^{-1.6P} + 28$$

$$d_3 = -33.4 \cdot 10^{-0.695P} + 28$$

2.4.2. Choosing the right trendline equation

After plotting the values and approximating them with definite trendline equations, it is needed to be decided which one is the best and right. They all have the same correlation of 98%, which is precise. In order to find the appropriate equation, some physical and mathematical concepts will be used to see if the curve will obey them. The hypothesis is that as the power of the ventilator increases, it is expected that when the power becomes enormously big, the diameter of the tornado will have the same maximum value that doesn't change. It has been deduced by common sense that tornado can only exist in the vortex chamber with finite dimensions and cannot grow bigger than his surroundings which is the vortex chamber. Therefore, it is expected that the right equation will yield the appropriate, finite value of diameter as power becomes infinitely large. That will be checked for each trendline using the limit when the power goes to infinity:

$$\lim_{P \to \infty} y$$

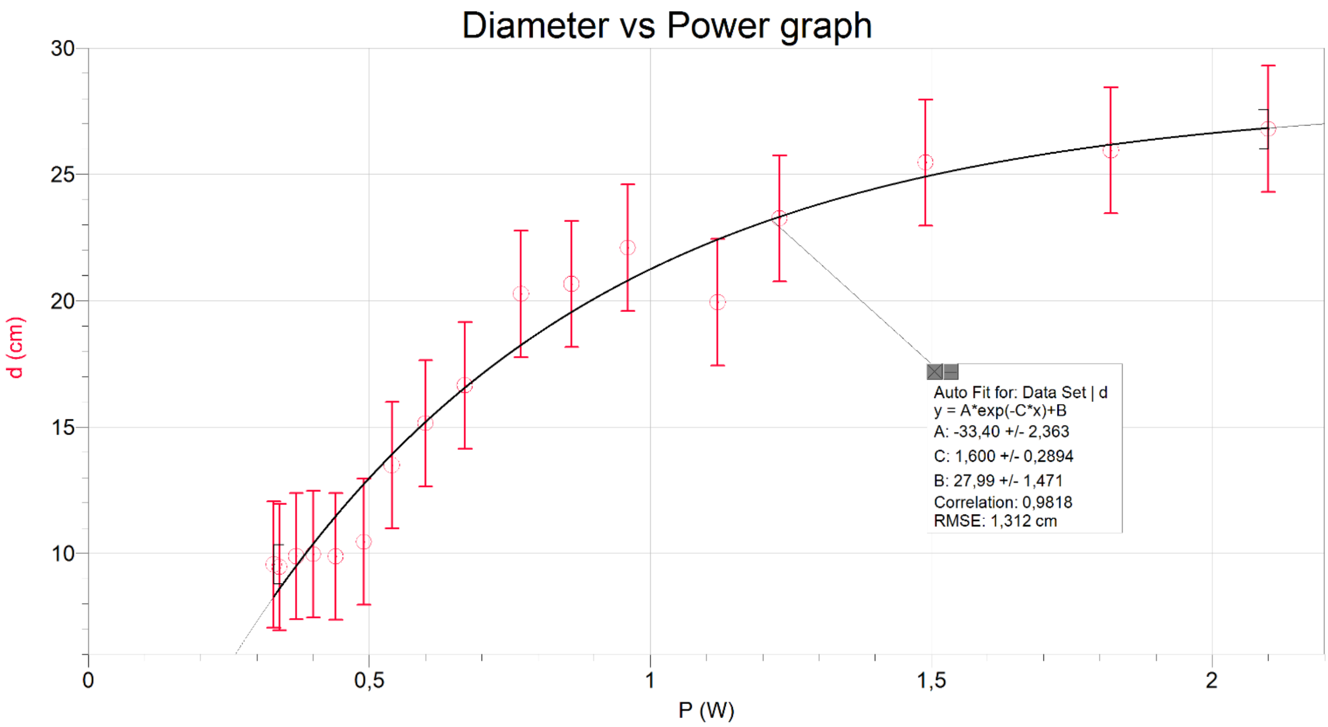
$$\lim_{P \to \infty} d = 28cm$$

This was a nice examination but the task of finding the right equation remains. For the next part, another physical concept will be used from observation and personal experience. Before even doing the measurements, it has been noticed that on the certain low speeds of ventilator/power, the tornado wouldn't form. That can be achieved only if the power of the ventilator or its angular speed is very low. After changing the resistance on a variable resistor, or, in order words, increasing the power, there was a certain point/value of power at which tornado would start to form. This can be mathematically explained as the x-intercept of the curve. If this is correct, then the curve should have x-intercept and that should be the minimum power needed to start the tornado. This will be found by equalizing trendline equations with zero because at that point the tornado hasn't been yet formed so its diameter is 0. For all functions the x-intercept is 0.11W. The functions given are the same which can be proved by algebraic transformations. Function d_2 is presented in the Graph 2. It is worth noticing that the condition:

$$P \geq P_{MIN}$$

must be achieved for the tornado to form.

Graph 2.



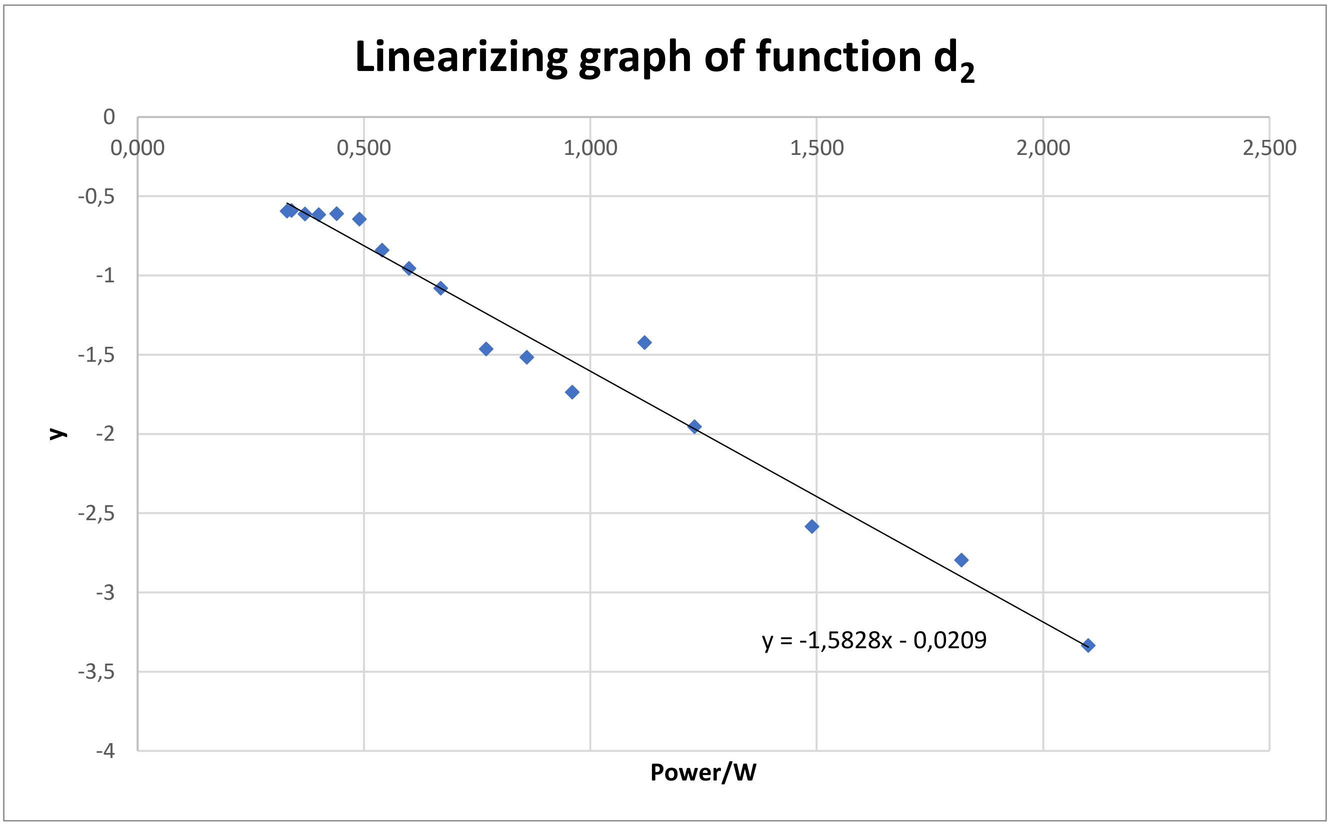
<u>Graph 2</u>: Shows plotted data for diameter and power

2.4.3. Linearizing the function

In order to identify more easily the relationship between the physical variables, and to confirm the validity of my results, we will linearize the data from the graph 2. The process of calculation is shown in appendix. The whole process was made in excel.

Graph 3.

From the graph, the strong negative correlation can be seen which means that the exponential fit function is the correct one to describe the relationship between the diameter and power.



Graph 3: Shows graph of linear function

The equation of the trendline is:

$$y = -1.58x - 0.0209$$

In this graph, y, represents:

$$y = \ln\frac{28 - d}{33.4}$$

On the x-axis, the power values are plotted. By graphing this relationship, the slope of the graph is the coefficient C, which is -1.6. y is linearly decreasing with power, meaning that diameter is exponentially increasing with power. Exponential dependence shows the asymptotical behaviour of the diameter of the cross-section with the increasing power of the ventilator. The limit is 28 which is exactly the dimension of the experimental setup. This is physically reasonable. To prove the validity of my statement, the answer can be found in the derivative of the function:

$$y = Ae^{-Cx} + B$$

Differentiating:

$$\frac{dy}{dx} = -ACe^{-Cx}$$

After substitution $Ae^{-Cx}$from the function into derivative formula:

$$\frac{dy}{dx} = C(B - y)$$

Rewriting the equation using our physical variables:

$$\frac{d(d)}{dP} = 1.6(27.99 - d)$$

This represents an intuitively pleasing result, which agrees with the previous statement. The change of the diameter is proportional to the difference between the asymptotical diameter and measured diameter. The change is positive which means that the diameter will be increasing, but consequently, the change will proportionally become smaller, and therefore the rate of change of the diameter will also decrease as well in accordance with asymptotical behaviour.

3. Evaluation and conclusion

3.1. Evaluation

The first thing that should be discussed is the accuracy of the measured data. When the pictures of the tornado were taken, the camera at the mobile phone was aligned with the glass at an angle that varied from 20 to 25 degrees. The pictures of the tornado are not circular as it is, yet more of an ellipse due to the imperfection of the camera. Unfortunately, I couldn't place the phone inside the tornado and due to that, the pictures of the tornado appear elliptically. The diameter I measured was the horizontal axis of ellipse shown on the pictures that were taken. The horizontal axis of ellipse corresponds to the diameter of the tornado cross-section. With this, the following problem was causing when picking the points when measuring the diameter. On some pictures, the diameter can be clearly established, while on others not. Changing the contrast on the camera has also affected the results because it makes the small particles of tornado visible to the camera. At high ventilator powers and by adjusting the contrast of the camera, it became apparent that particles already travel in the circle through the whole chamber in accordance with asymptotic behaviour detected. I estimated the error of the diameter from the arithmetic mean and absolute error since the diameter is not uniquely determined by the picture. I think it is fair because the trendline equation passes through all the error bars. In order to be more precise, I took the average of the two values of the measured diameter. By doing this I made my results more reliable because of the above approach. The next thing that should be talked about is the precision of the equipment. In order to measure the current and voltage ammeter was used. Relative errors for current and voltage are negligible since they do not induce any additional observable error of my measurements.

3.2. Conclusion

The goal of this research was to find out how does the diameter of the tornado change with variation of power at a given axial position (height). To find out the answer to this question, the tornado chamber has been assembled and the experiment was conducted. The tornado has been found to have a critical point (minimum power) for its formation and the limited diameter it can reach. Therefore, the answer to my research question is that the diameter of the horizontal cross section at a certain height depends exponentially on the power generated by the ventilator. This dependence was experimentally shown to be determined by the following expression:

$$d_2 = -33.4e^{-1.6P} + 28$$

This expression also shows the asymptotical behaviour of the diameter determined by physical constraints of the experimental setup. (28cm is the size of the apparatus).

4. APPENDIX

4.1. Construction of the tornado machine

The first step of making the tornado was to make sure that tornado is generated in closed volume. For that closed volume I have decided to make a vortex chamber. Obviously, it needed to be small where a small amount of wind (fluid) energy would be enough to start its formation. In vortex chamber, the rotation of the air mass is being generated and the vortex becomes visible by putting fogger maker into the water which generates steam. The chamber has been made by using hardboard and plexiglass through which we can see the tornado. The cross section of the chamber has the area of a rectangle. The base of the chamber is a square. On each right side of the chamber I have cut a “slit” through which air will be going in to. The hardboard has been painted black so the visibility of the tornado can be better. On the upper base a small circle has been cut equal to the diameter of the ventilator so it can effectively hoover the air in.

Dimensions:

Height of the chamber: $57.4cm \pm 0.1cm$

Length of the base: $31cm \pm 0.1cm$

Height of the slit: $47.5cm \pm 0.1cm$

Width of the slit: $1.7cm \pm 0.1cm$

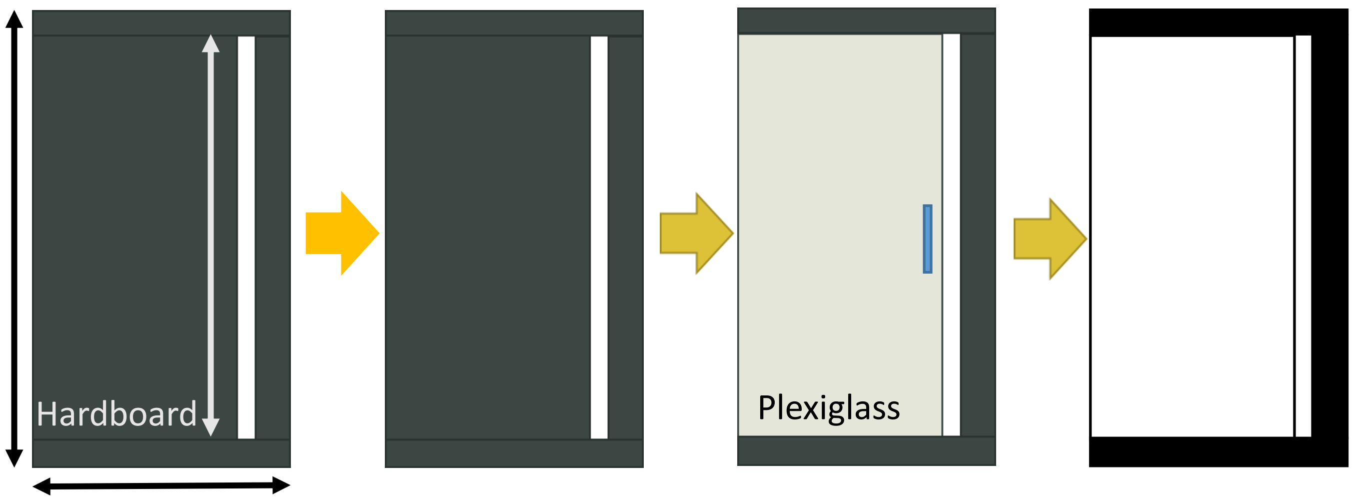


Figure 1: Shows each cross-section area of the chamber

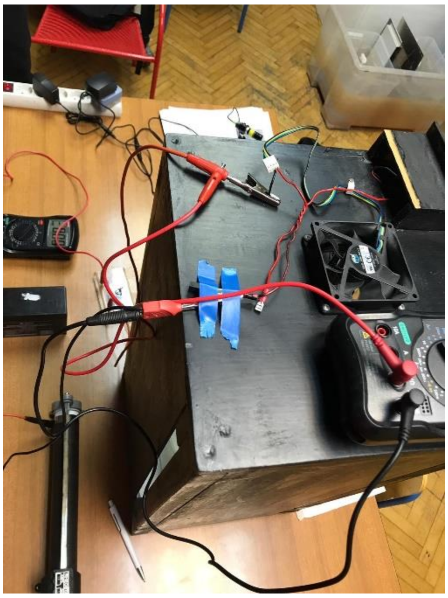
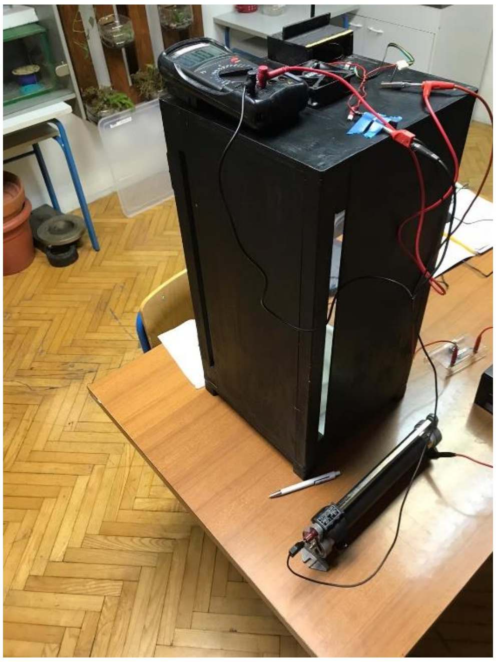
Table 3.

U/V	I/A	$U_{relative}$/%	$I_{relative}$/%	P/W	$P_{relative}$/%	ΔP/W
6,66	0,049	0,150	2,04	0,330	2,19	0,00723
6,83	0,05	0,146	2,00	0,340	2,15	0,00730
7,01	0,053	0,143	1,89	0,370	2,03	0,00751
7,21	0,056	0,139	1,79	0,400	1,92	0,00770
7,45	0,059	0,134	1,69	0,440	1,83	0,00805
7,72	0,063	0,130	1,59	0,490	1,72	0,00841
7,99	0,068	0,125	1,47	0,540	1,60	0,00862
8,27	0,072	0,121	1,39	0,600	1,51	0,00906
8,62	0,078	0,116	1,28	0,670	1,40	0,00937
9,03	0,085	0,111	1,18	0,770	1,29	0,00991
9,31	0,092	0,107	1,09	0,860	1,19	0,01027
9,73	0,099	0,103	1,01	0,960	1,11	0,01068
10,15	0,11	0,099	0,909	1,12	1,01	0,01129
10,55	0,117	0,095	0,855	1,23	0,949	0,01168
11,19	0,133	0,089	0,752	1,49	0,841	0,01253
11,97	0,152	0,084	0,658	1,82	0,741	0,01349
12,52	0,167	0,080	0,599	2,10	0,679	0,01425

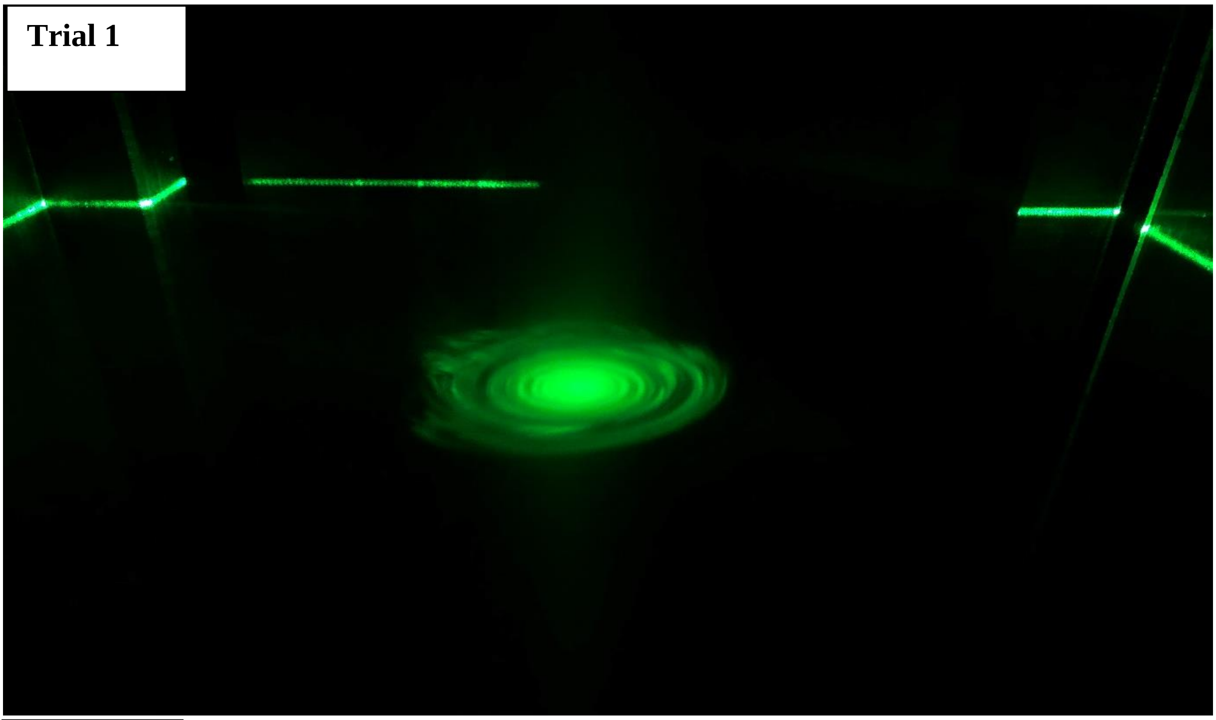
Table 3: Shows calculations of uncertainties and error

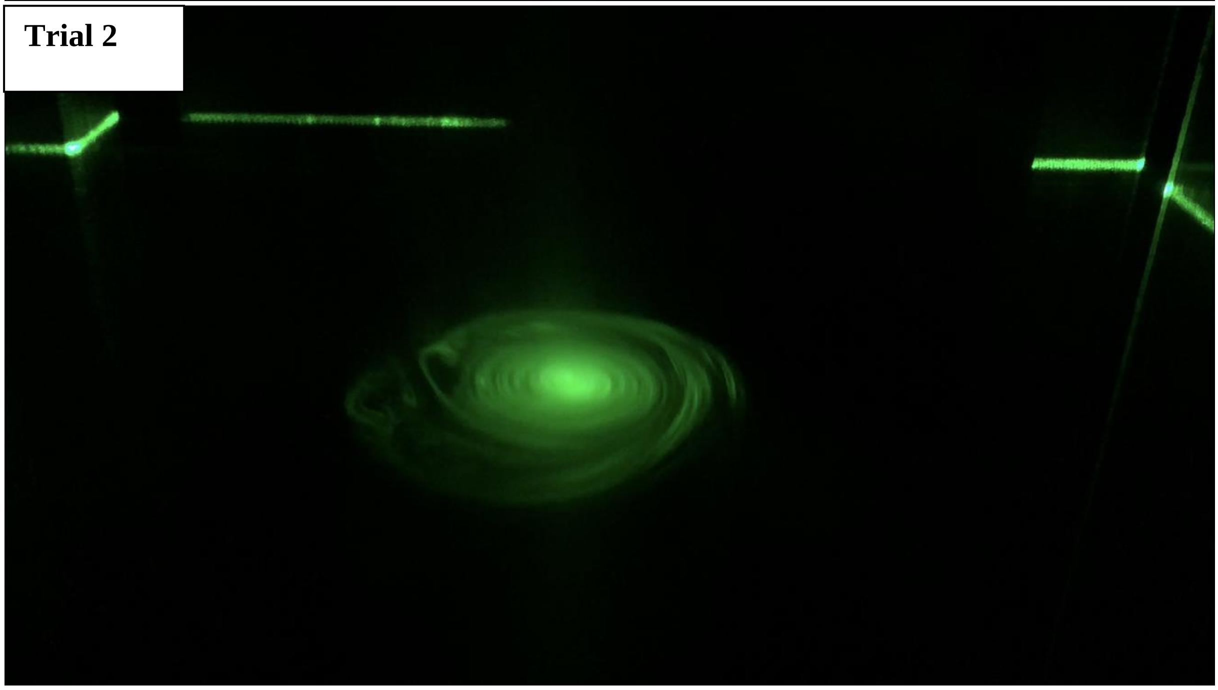
4.2. Photos of the experimental setup

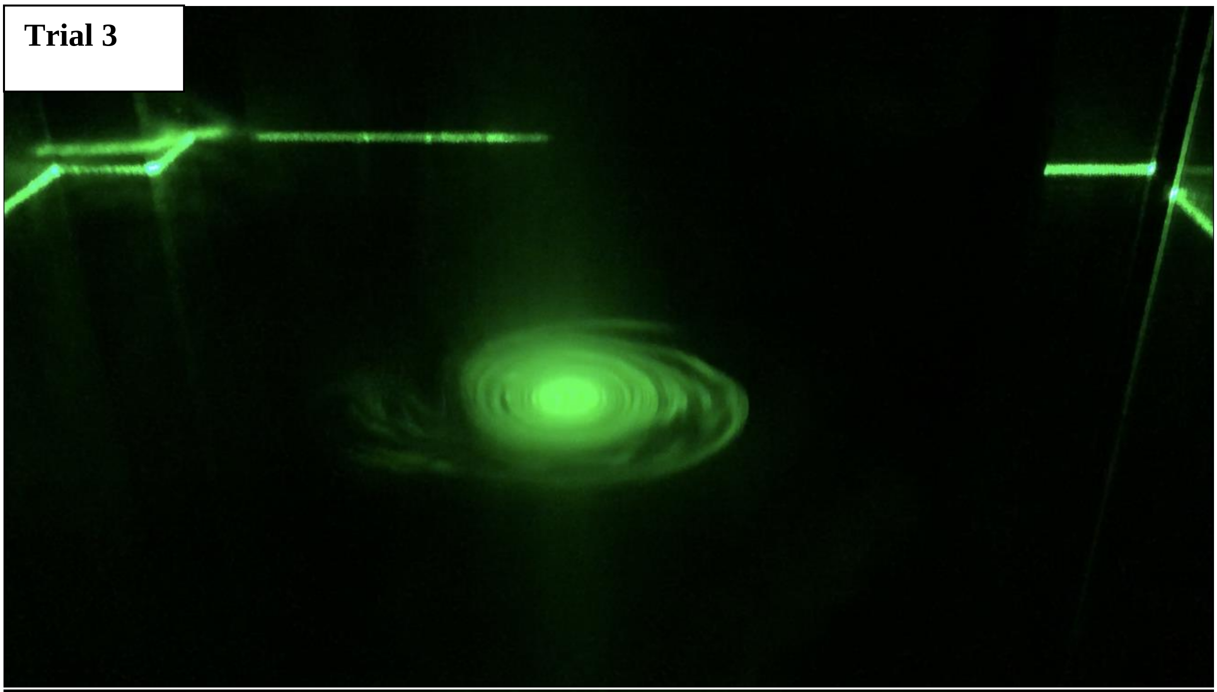


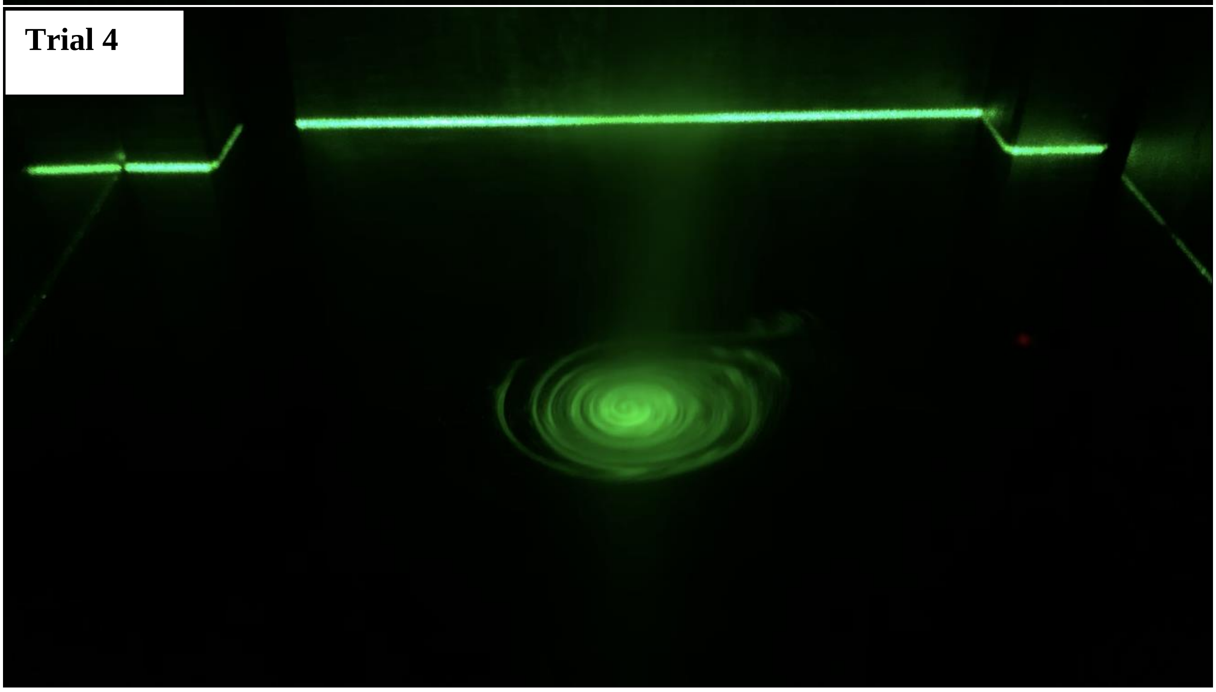



4.3. Photos of horizontal cross-section of tornado






Trial 3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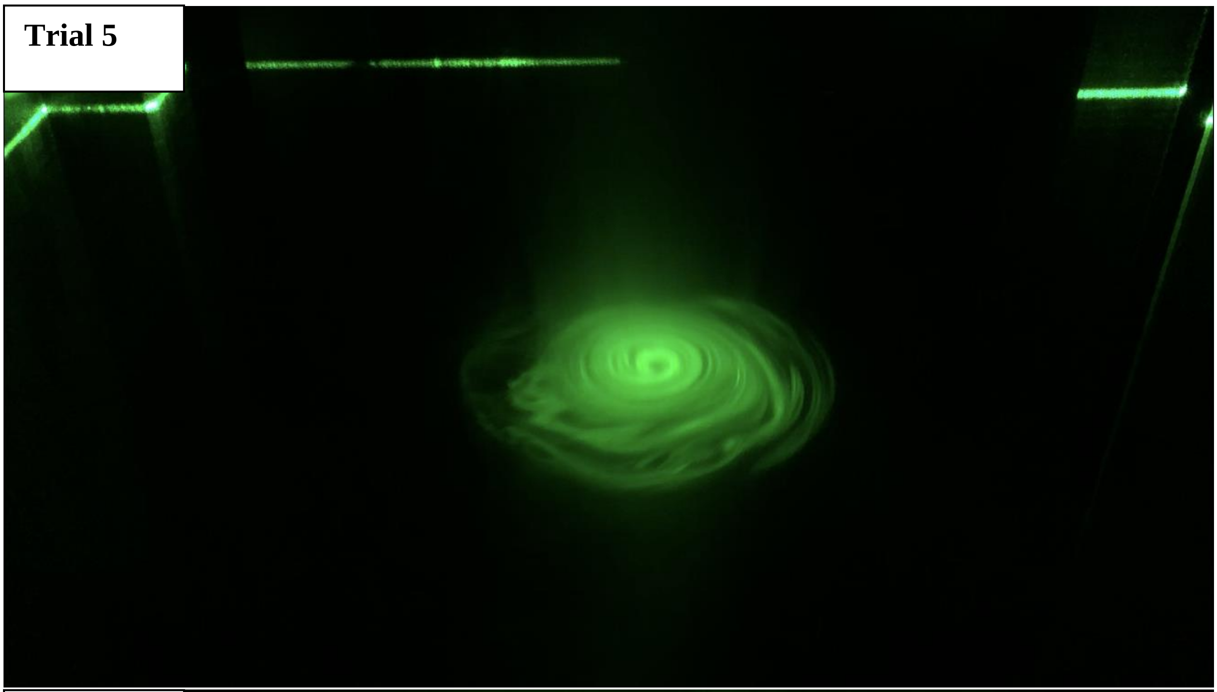
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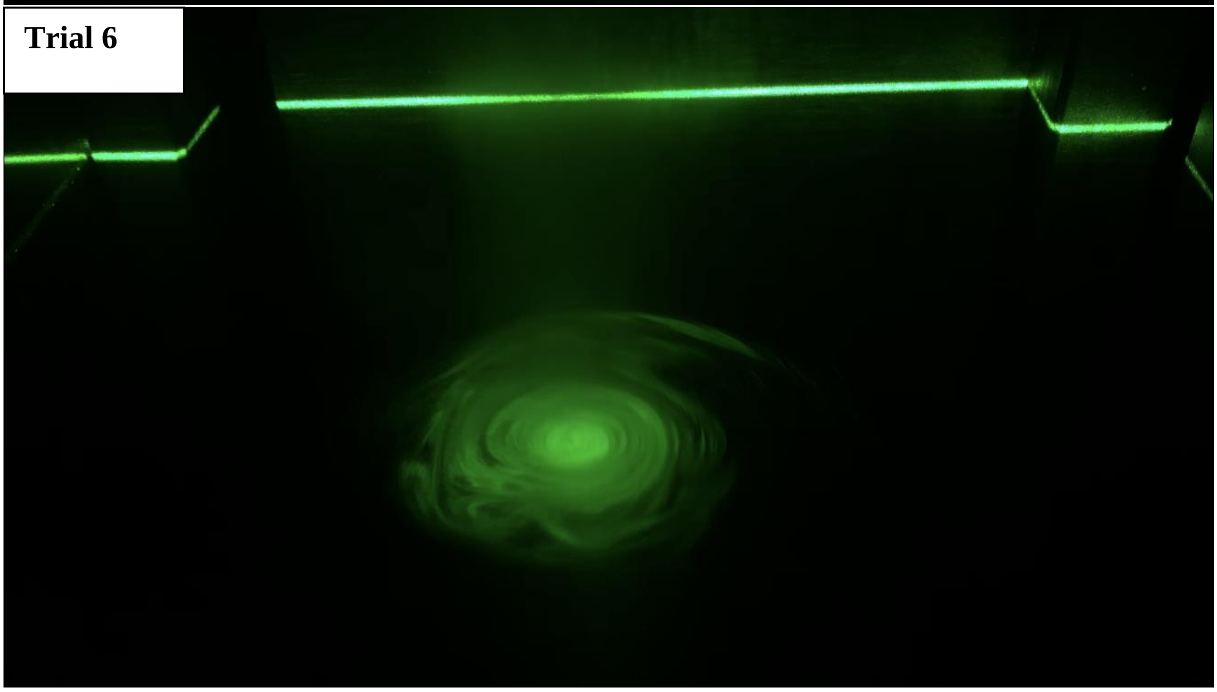
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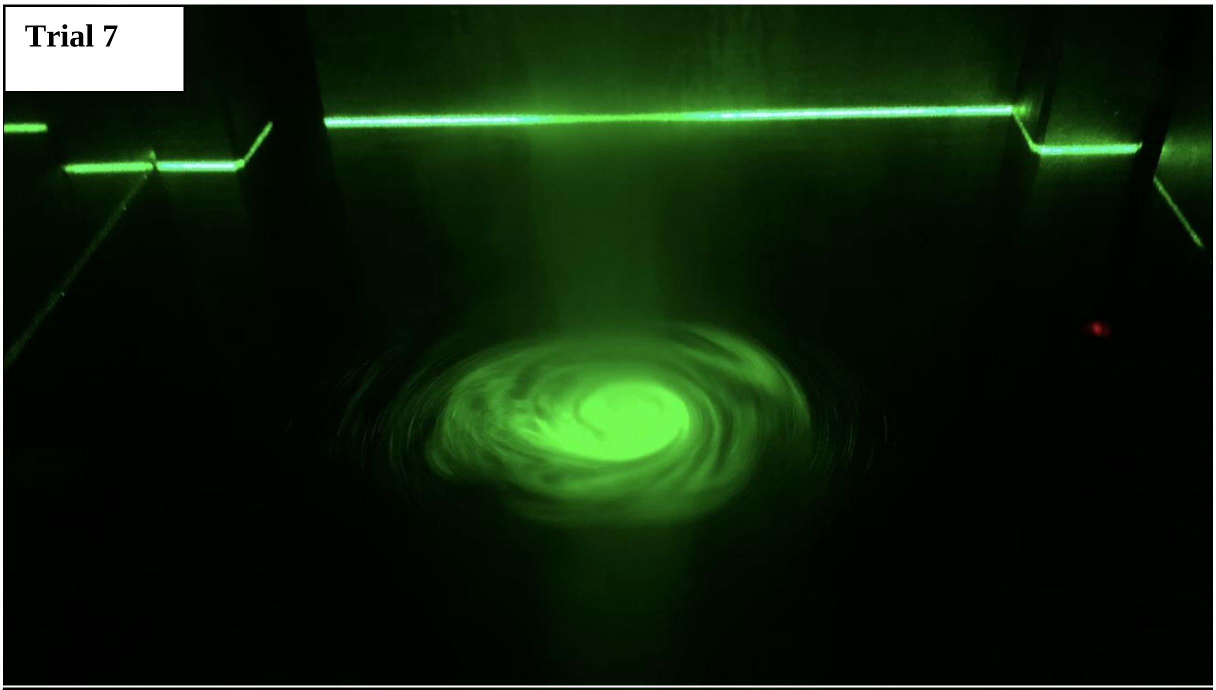
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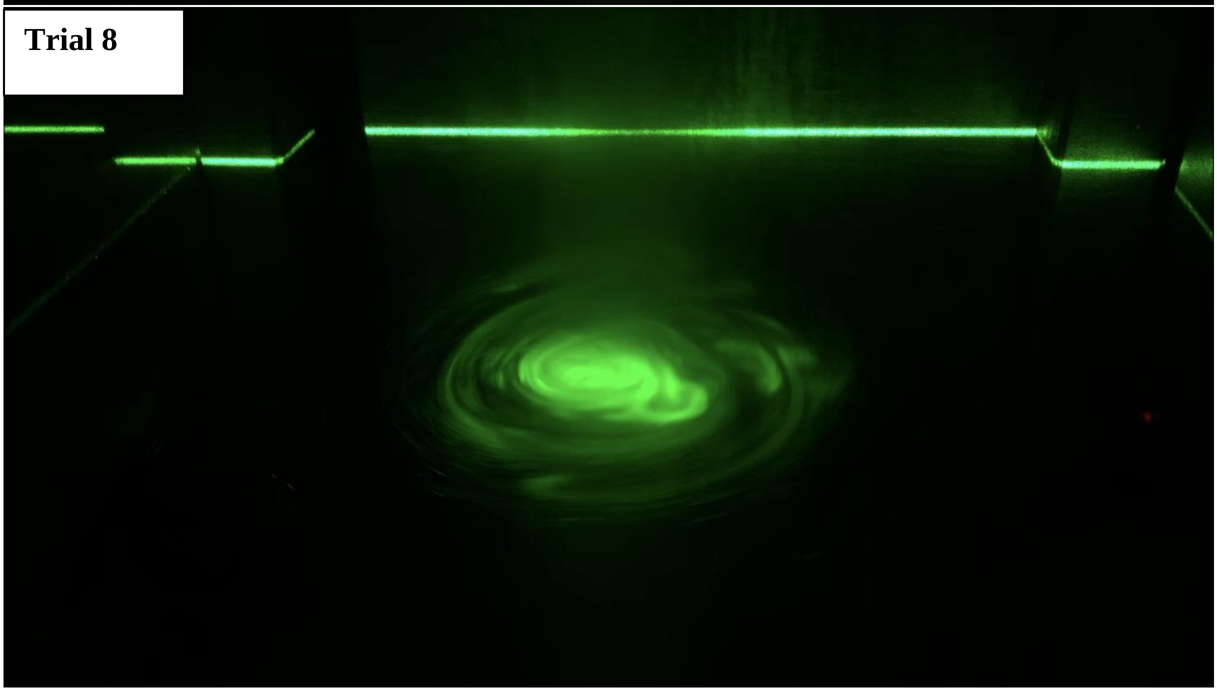
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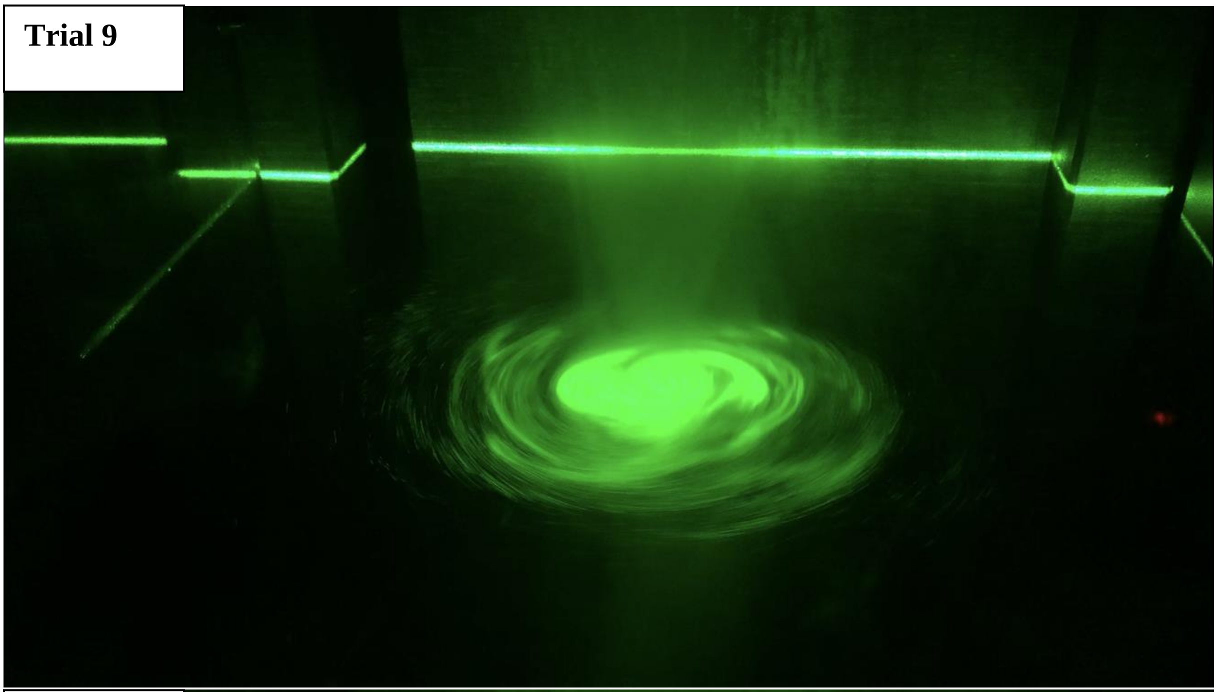
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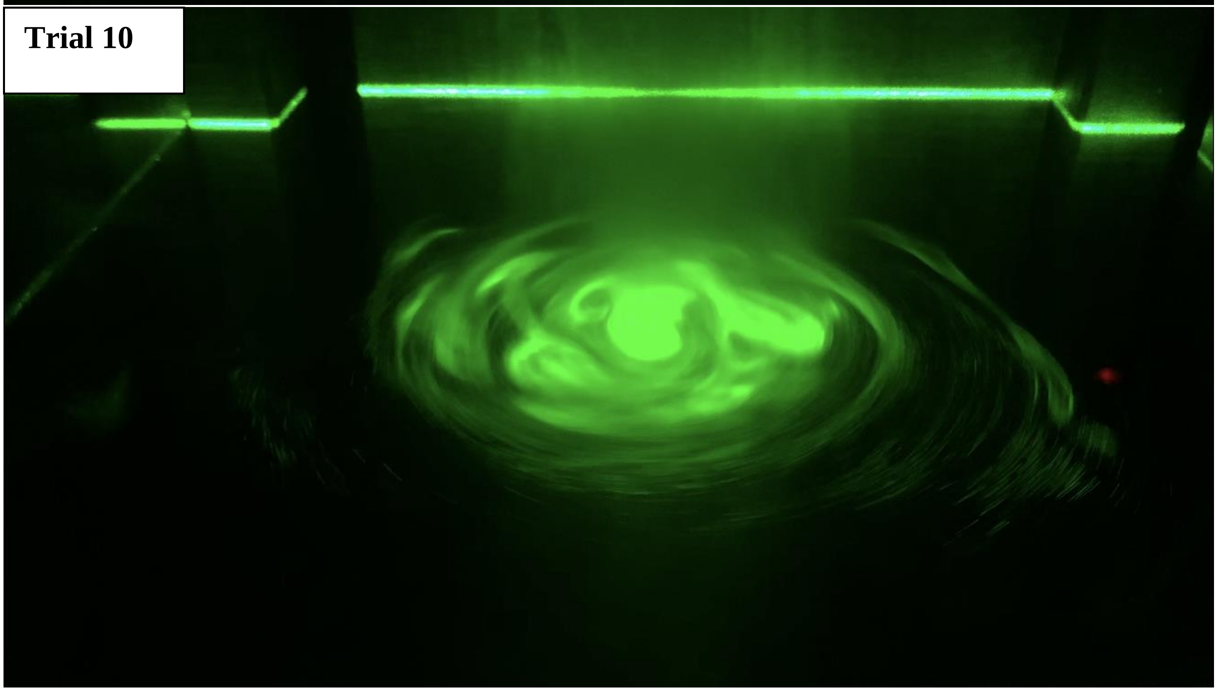
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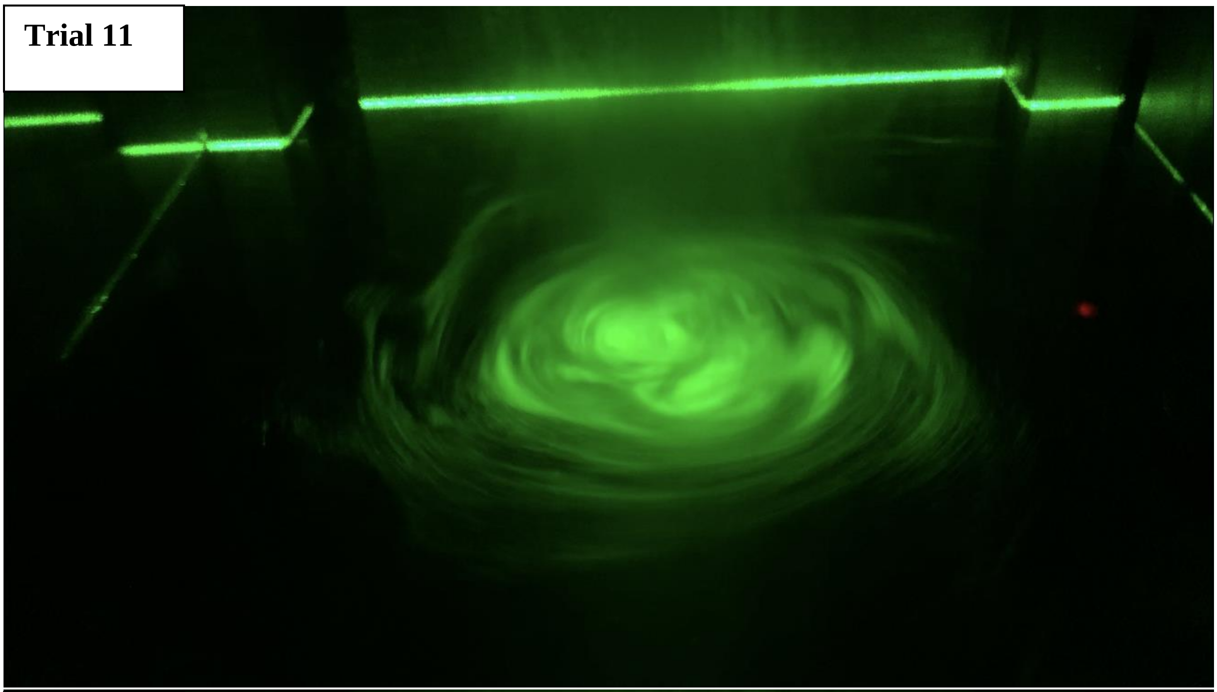
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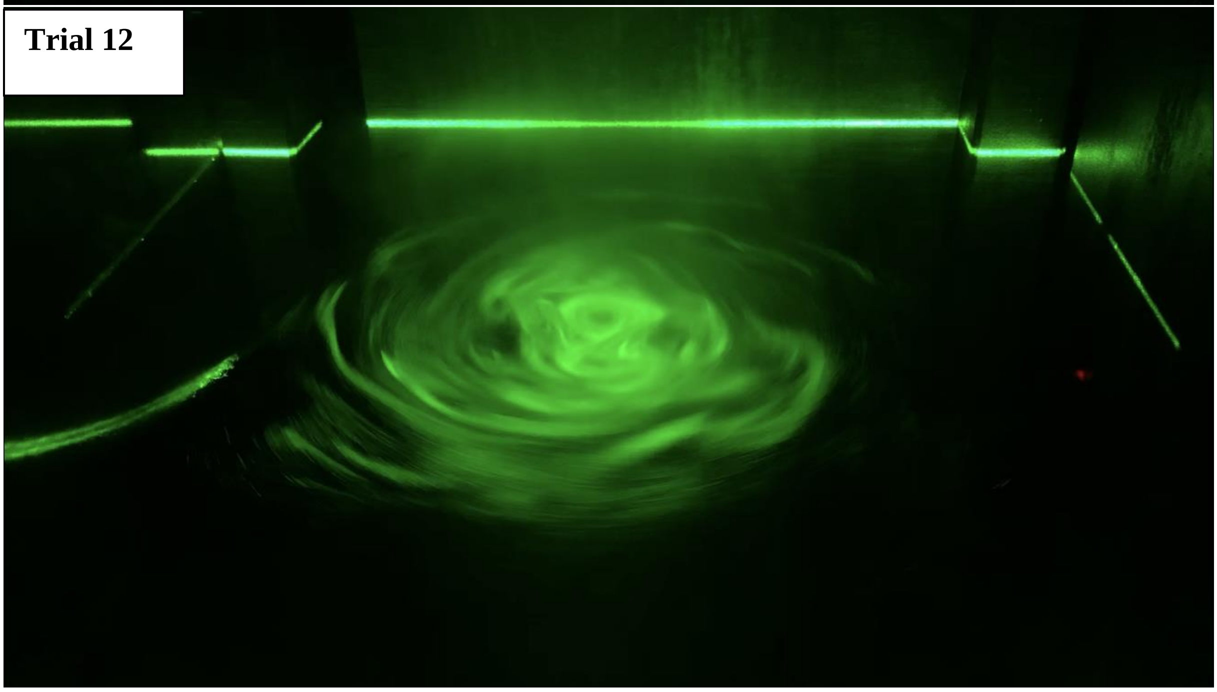
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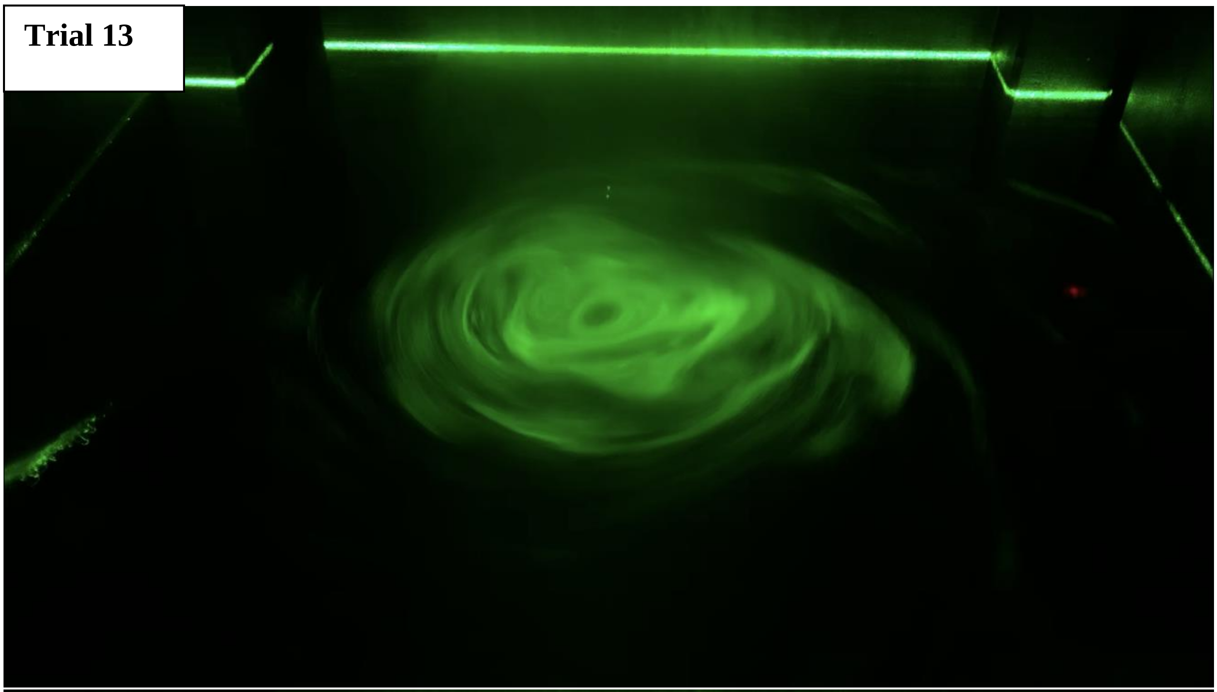
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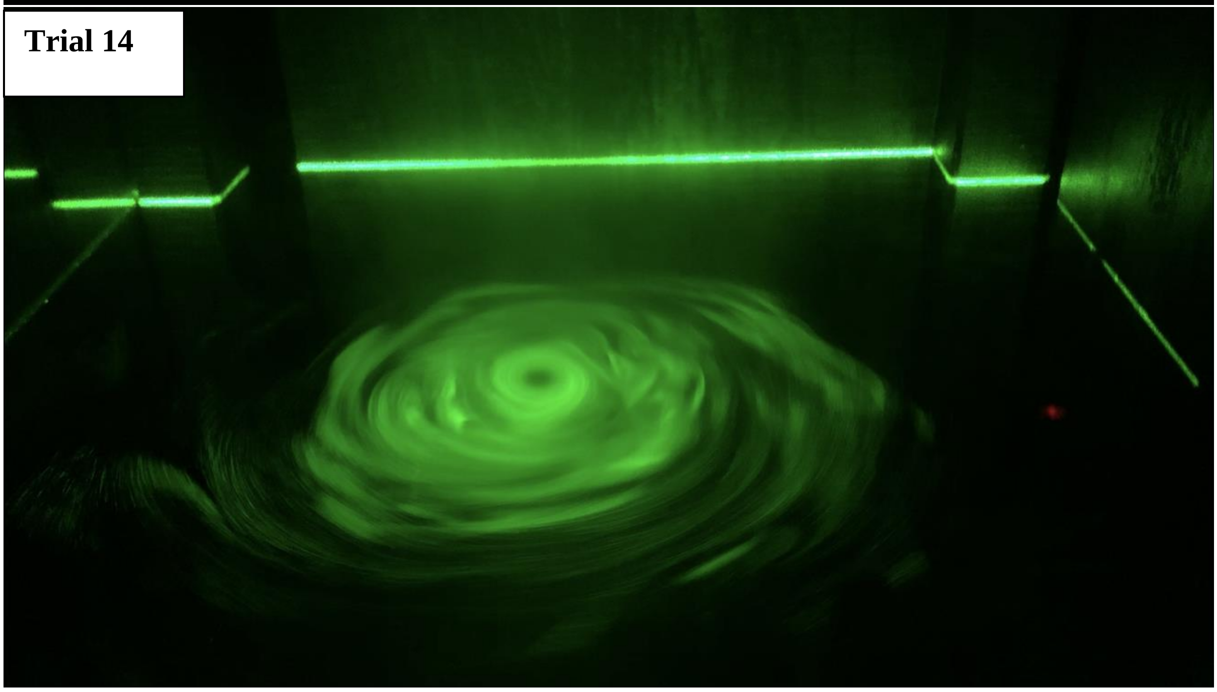
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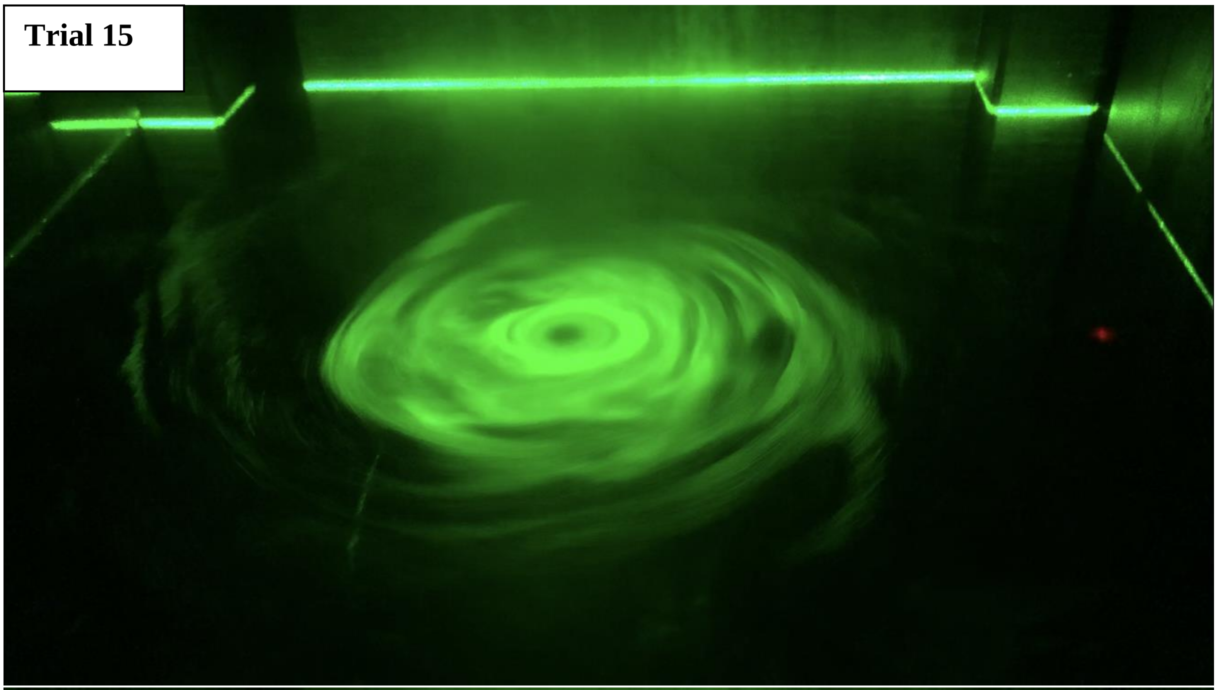
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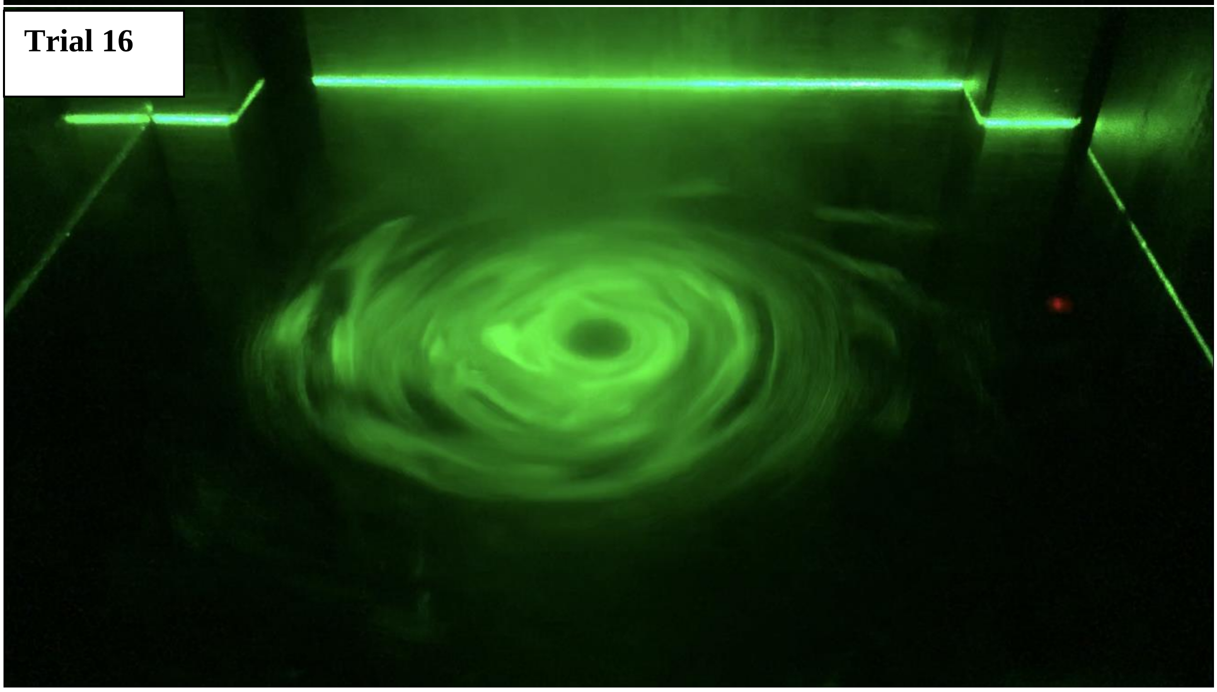
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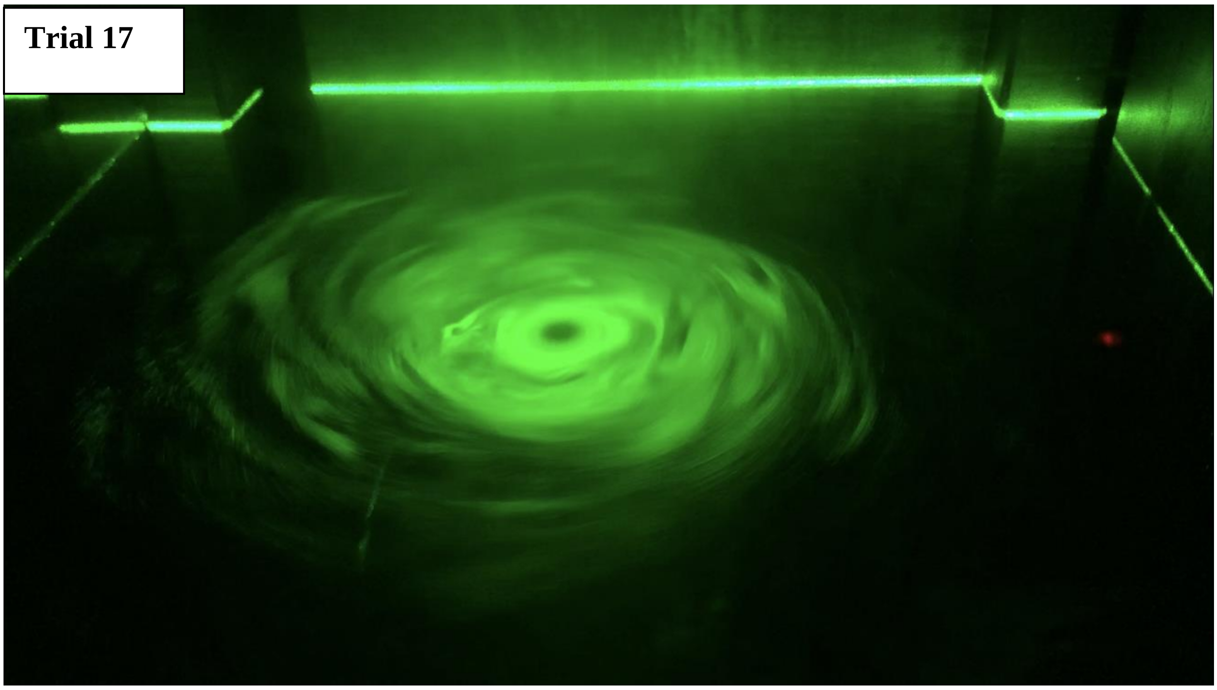

4.4. Process of linearization

$$d = -33.4e^{-1.6P} + 27.99$$

$$P = -\frac{1}{1.6}\ln\left(\frac{27.99 - d}{33.4}\right)$$

$$let\ y\ be\ the\ natural\ logarithm$$

$$y = \ln\left(\frac{27.99 - d}{33.4}\right)$$

Then,

$$y = -1.6P$$

5. Bibliography

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3. Katopodes, N., D., (2019), Free-Surface Flow, Chapter 7 - Vorticity Dynamics, Pages 516-565, Butterworth-Heinemann, ISBN 9780128154892, https://doi.org/10.1016/B978-0-12-815489-2.00007-1., (http://www.sciencedirect.com/science/article/pii/B9780128154892000071)