



# Computation Efficient Mathematical Models for Energy Storage Valuation, Bidding, and Dispatch

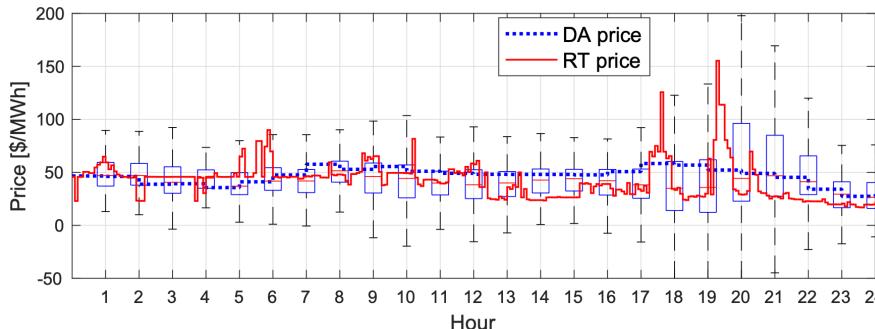
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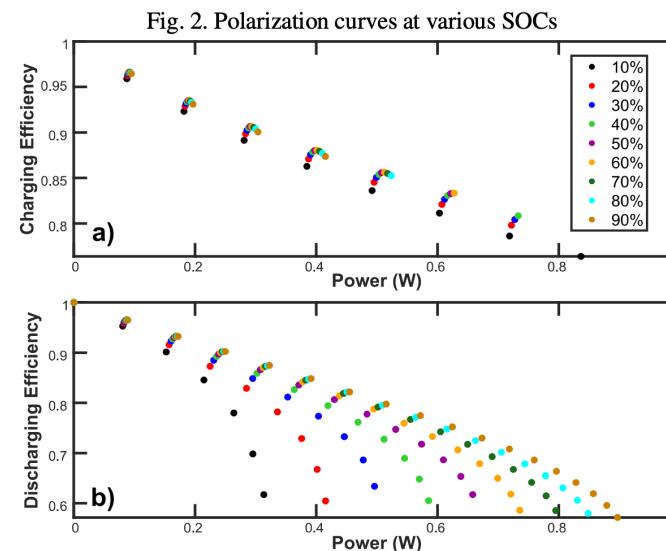
# Challenges

## Computation complexities in storage control and analytics

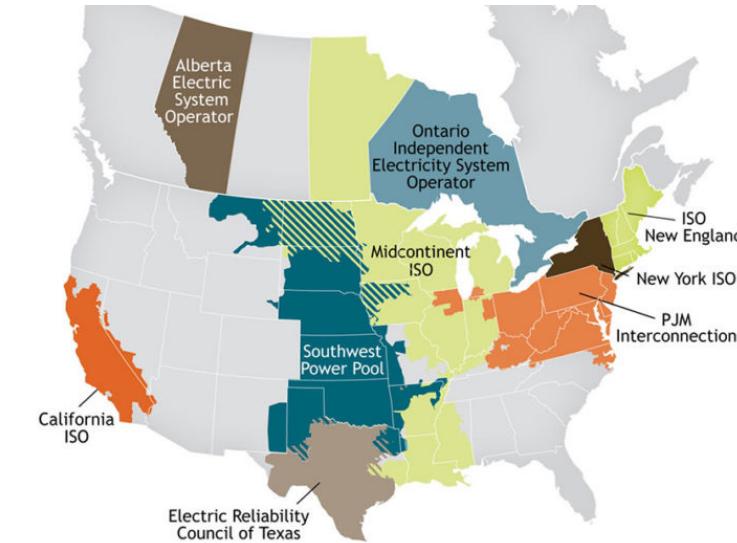
### Uncertainty Models



### Nonlinear Storage Models



### Dispatch Models



Xu, Bolun, Magnus Korpås, and Audun Botterud. "Operational Valuation of Energy Storage under Multi-stage Price Uncertainties." In 2020 59th IEEE Conference on Decision and Control (CDC), pp. 55-60. IEEE, 2020.

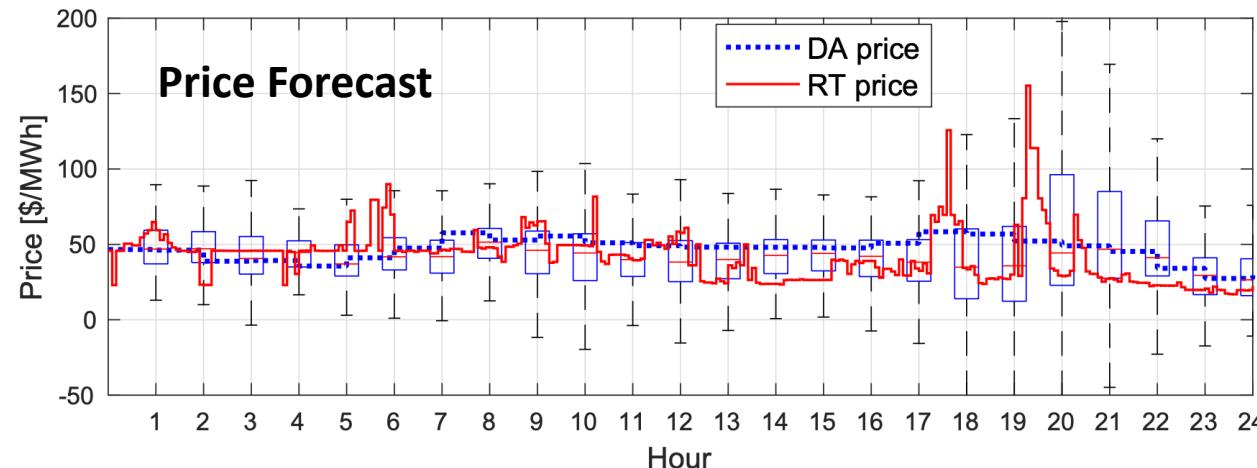
Chen, Yonghong, and Ross Baldick. "Battery storage formulation and impact on day ahead security constrained unit commitment." *IEEE Transactions on Power Systems* (2022).

Jafari, Mehdi, Kara Rodby, John Leonard Barton, Fikile Brushett, and Audun Botterud. "Improved energy arbitrage optimization with detailed flow battery characterization." In 2019 IEEE Power & Energy Society General Meeting (PESGM), pp. 1-5. IEEE, 2019.

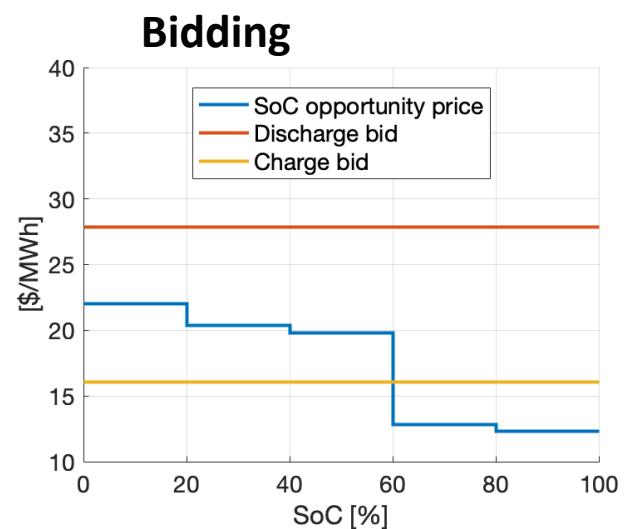
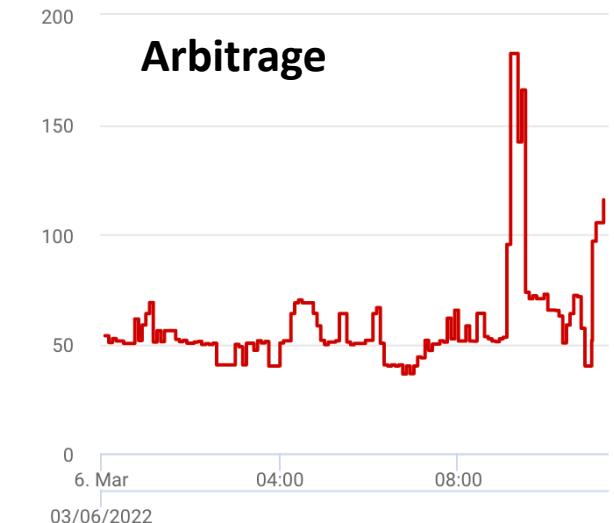
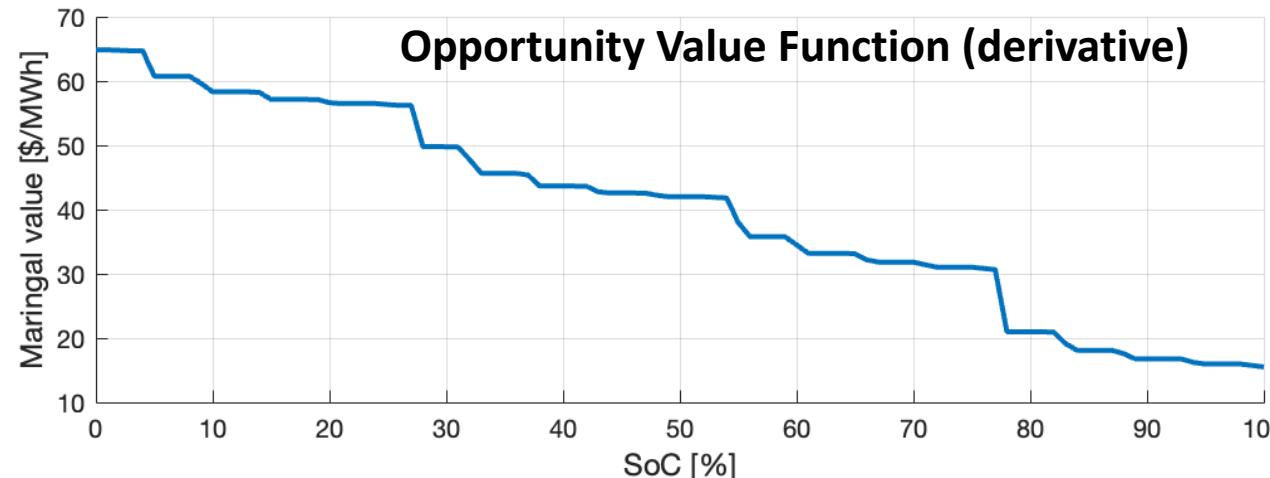
# Outline

- Opportunity cost function calculation using dynamic programming
- Incorporating nonlinear storage model
- Incorporating stochastic objectives
- Assess storage market model design

# Storage Opportunity Cost



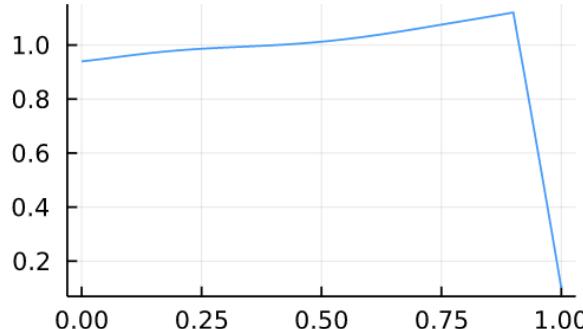
 **Storage Model**



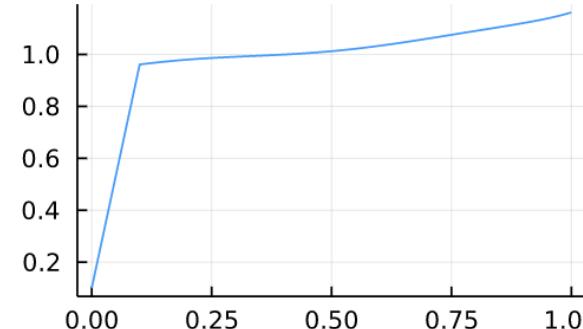
# Nonlinear Arbitrage Model

**Power rating, efficiency, discharge cost depends on SoC**

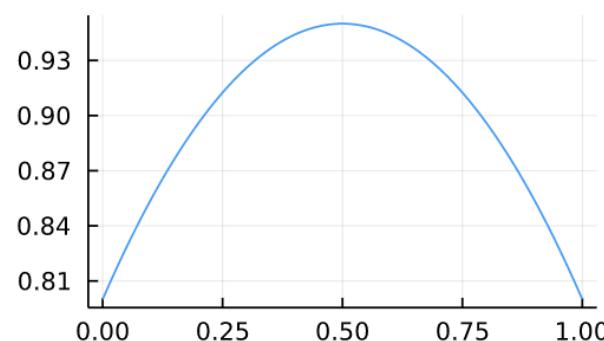
Charge Power Rating



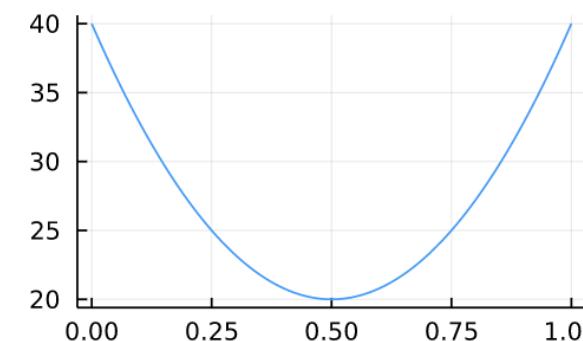
Discharge Power Rating



Efficiency



MC



**Objective:**

$$Q_{t-1}(e_{t-1}) = \max_{p_t, d_t} \lambda_t(d_t - p_t) - c(e_{t-1})d_t + Q_t(e_t)$$

**Power ratings:**

$$0 \leq d_t \leq D(e_{t-1}), \quad 0 \leq p_t \leq P(e_{t-1})$$

**SoC efficiency:**

$$e_t - e_{t-1} = -d_t/\eta^d(e_{t-1}) + b_t\eta^p(e_{t-1})$$

**SoC limits:**

$$0 \leq e_t \leq E$$

**No discharge during negative price:**

$$d_t = 0 \text{ if } \lambda_t < 0$$

# Solution Algorithm

**Analytical update of value function derivative**  $q_t(e) = \partial Q_t(e)/\partial e$

## Formulation

$$Q_{t-1}(e_{t-1}) = \max_{p_t, d_t} \lambda_t(d_t - p_t) - c(e_{t-1})d_t + Q_t(e_t)$$

subjects to the following constraints

$$0 \leq d_t \leq D(e_{t-1}), \quad 0 \leq p_t \leq P(e_{t-1})$$

$$d_t = 0 \text{ if } \lambda_t < 0$$

$$e_t - e_{t-1} = -d_t/\eta^d(e_{t-1}) + b_t\eta^p(e_{t-1})$$

$$0 \leq e_t \leq E$$

## Solution Algorithm

$$q_{t-1}(e) = \begin{cases} q_t(e + P\eta^p) & \text{if } \lambda_t \leq q_t(e + P\eta^p)\eta^p \\ \lambda_t/\eta^p & \text{if } q_t(e + P\eta^p)\eta^p < \lambda_t \leq q_t(e)\eta^p \\ q_t(e) & \text{if } q_t(e)\eta^p < \lambda_t \leq [q_t(e)/\eta^d + c]^+ \\ (\lambda_t - c)\eta^d & \text{if } [q_t(e)/\eta^d + c]^+ < \lambda_t \\ & \leq [q_t(e - D/\eta^d)/\eta^d + c]^+ \\ q_t(e - D/\eta^d) & \text{if } \lambda_t > [q_t(e - D/\eta^d)/\eta^d + c]^+. \end{cases}$$

Proof using KKT conditions:

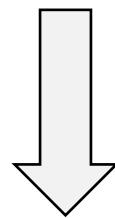
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# Markov-process Stochastic Model

Discretize historical prices and fit a Markov process  
 Time-varying transition probabilities

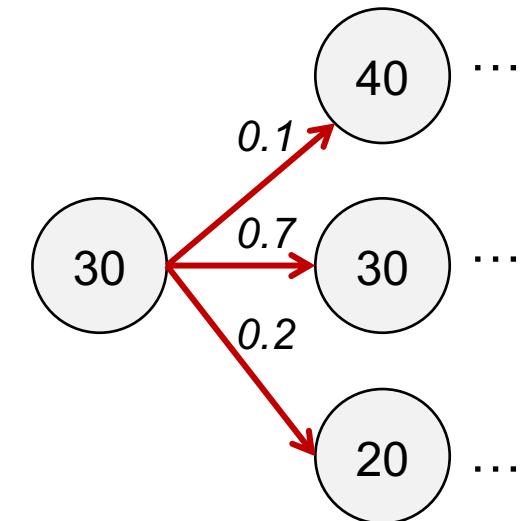
$$Q_{t-1}(e_{t-1} | \lambda_t) = \max_{b_t, p_t} \lambda_t \cdot (p_t - b_t) - cp_t + V_t(e_t | \lambda_t)$$

$$V_t(e_t | \lambda_t) = \mathbb{E}_{\lambda_{t+1}} [Q_t(e_t | \lambda_{t+1}) | \lambda_t]$$



Markov process

$\rho_{i,j,t}$  - probability of moving from price node  $i$  to  $j$   
 from time period  $t$  to  $t+1$

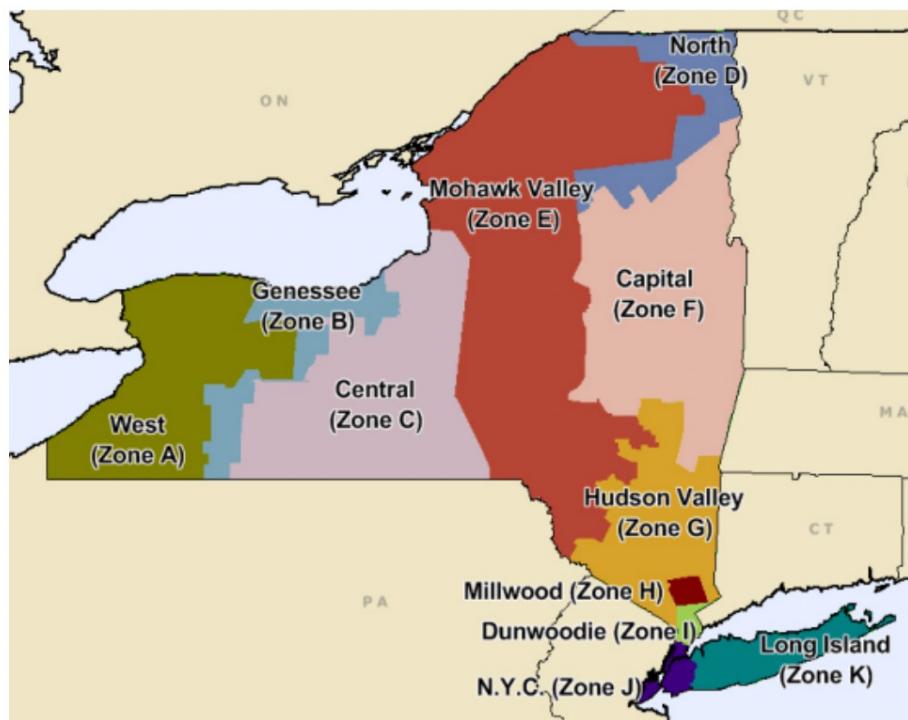


$$Q_{t-1,i}(e_{t-1}) = \max_{b_t, p_t} \pi_{t,i} \cdot (p_t - b_t) - cp_t + V_{t,i}(e_t)$$

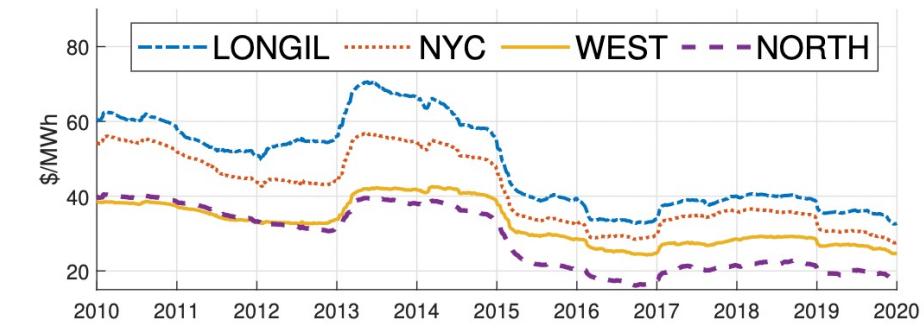
$$V_{t,i}(e_t) = \sum_{j \in \mathcal{N}} \rho_{i,j,t} \cdot Q_{t,j}(e_t)$$

# New York State Case Study

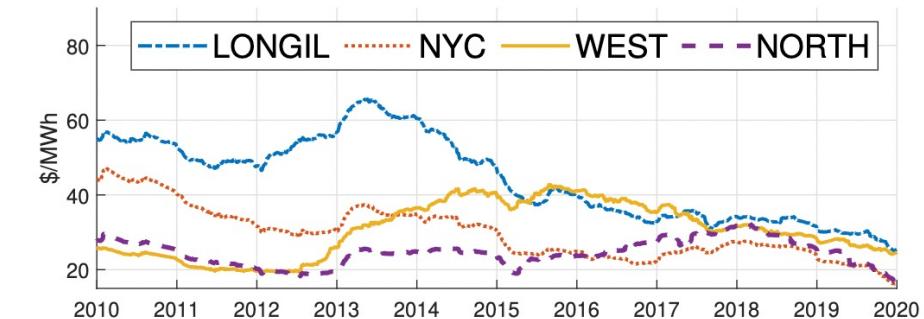
Train price model using 2017-2018 data and test on 2019



(a) New York State Price Zones.



(b) 30 day moving average price.

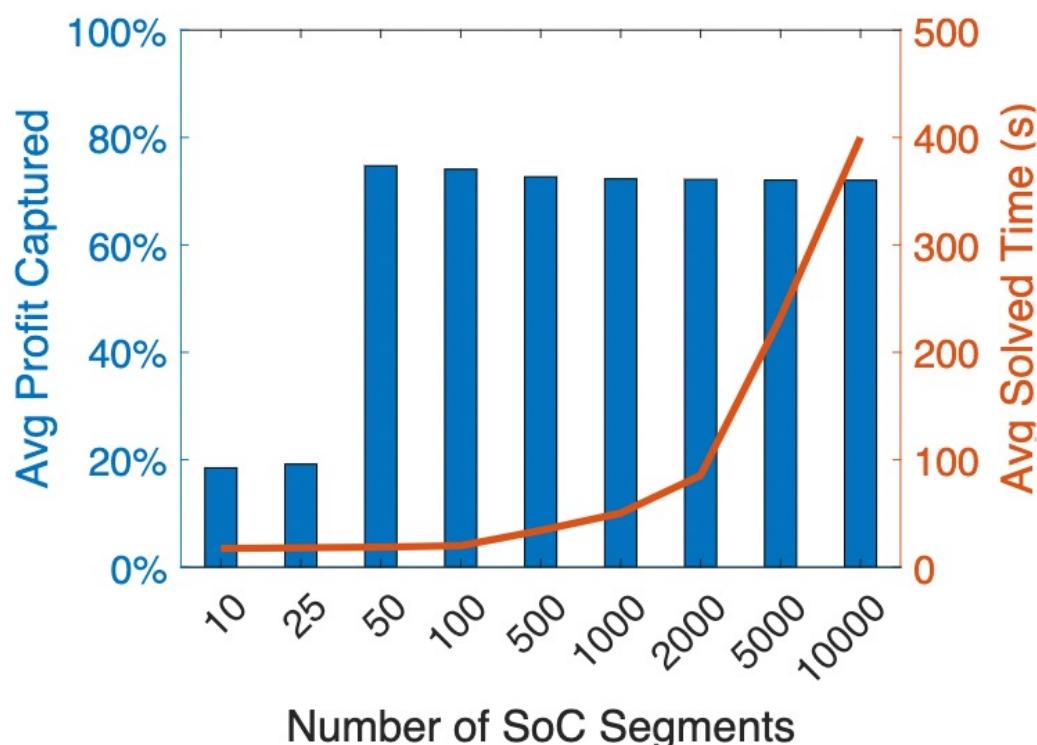


(c) 30 day moving average daily price deviations.

# New York State Case Study

**60% to 90% profit ratio with extreme computation speed**

**Computation time for full year solution**  
 – 105,120 time steps



Zone	P2E	Prorated Profit Ratio [%]			
		0 MC	10 MC	30 MC	50 MC
NYC	1	59.9	66.1	71.8	78.5
	0.5	67.2	72.0	78.7	84.3
	0.25	76.2	78.9	85.3	90.8
LONGIL	1	56.0	59.0	62.1	62.3
	0.5	63.5	65.1	66.7	67.4
	0.25	72.7	72.5	71.7	72.0
NORTH	1	58.4	63.5	70.1	75.3
	0.5	69.5	74.6	81.1	83.6
	0.25	79.4	83.7	90.2	88.0
WEST	1	67.1	70.9	75.2	78.2
	0.5	74.1	77.3	80.1	81.8
	0.25	82.2	84.2	85.8	86.7

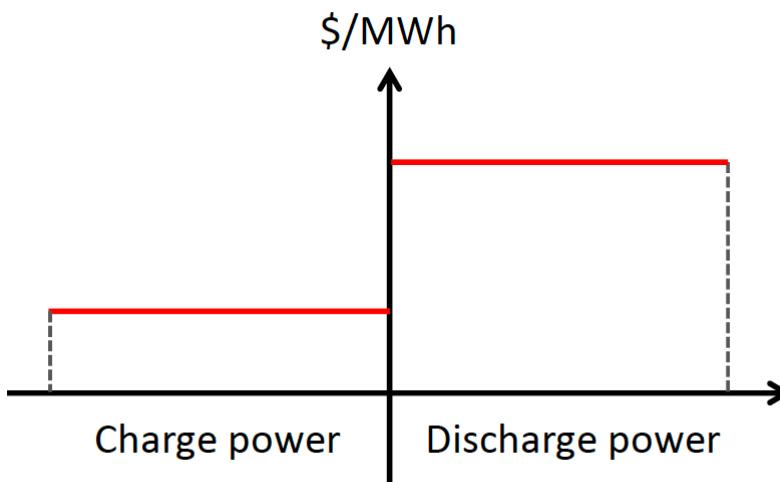
P2E – power to energy ratio, MC – marginal cost  
 Trained using 2017-2018, tested on 2019

# Dispatch and Bidding Analysis

## Include Storage into Single-Period Economic Dispatch

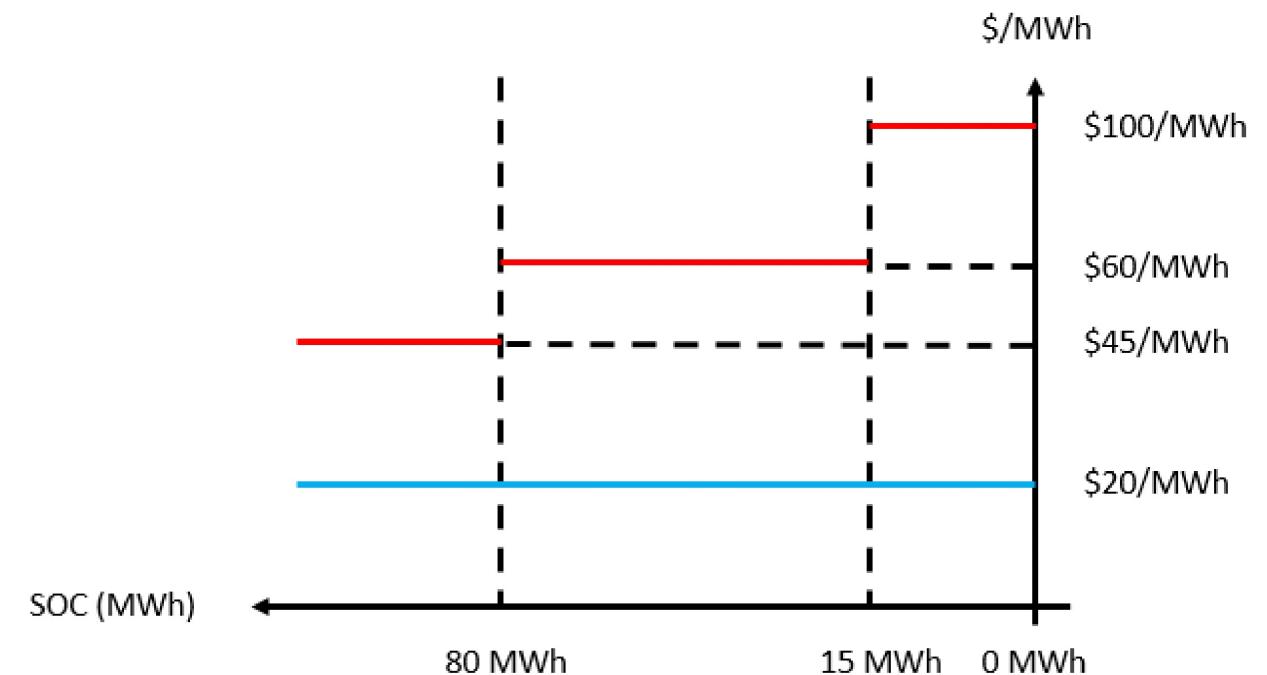
### Existing Single Segment Bids

Storage submits one bid to charge and one bid to discharge



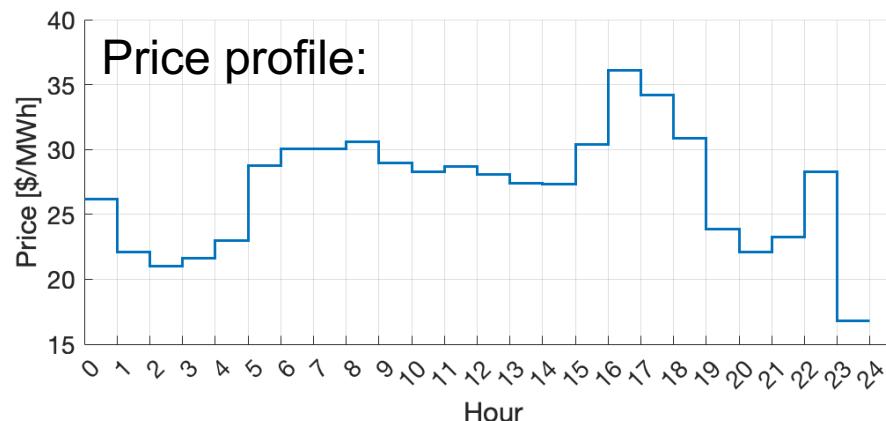
### SoC-segment Market Model

Charge and discharge bids depend on SoC (being proposed in CAISO)



# Optimal Bid Generation

Generate bids based on price forecasts and storage parameters

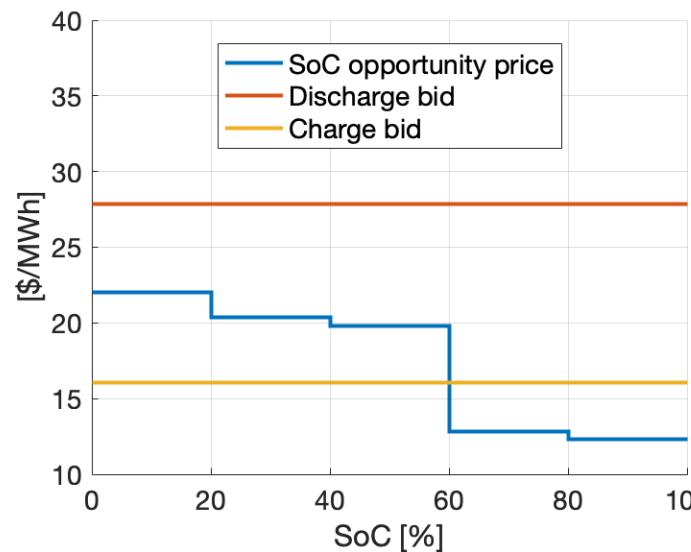


Storage parameters:

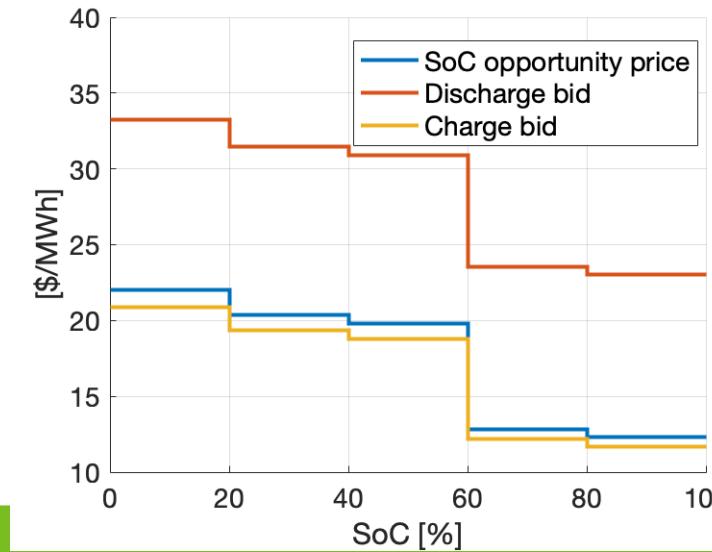
- 4 Hour duration
- 85% round-trip eff.
- 10\$/MWh discharge cost

Solve dynamic programming  
(<1ms computation time)

1-seg bid (existing bids)



5-seg bid (SoC bids)



# Optimal Bid Generation

Use dynamic programming value function to generate optimal bids

Profit maximization:

$$Q_{t-1}(e_{t-1}) = \max_{b_t, p_t} \lambda_t(p_t - b_t) - cp_t + Q_t(e_t)$$

Revenue                      Cost – to be engineered into bids

SoC constraints:

$$e_t - e_{t-1} = -p_t/\eta + b_t\eta \quad \longrightarrow \quad \frac{\partial e_t}{\partial p_t} = -\frac{1}{\eta} \quad \frac{\partial e_t}{\partial b_t} = \eta$$

Storage cost curve:

$$C = cp_t - Q(e_t) \quad \longrightarrow \quad \begin{array}{ll} \text{Discharge curve} & \text{Charge curve} \\ \frac{\partial C}{\partial p_t} = c + \frac{1}{\eta} q_t(e_t) & \frac{\partial C}{\partial b_t} = \eta q_t(e_t) \end{array}$$

# Arbitrage Case Study

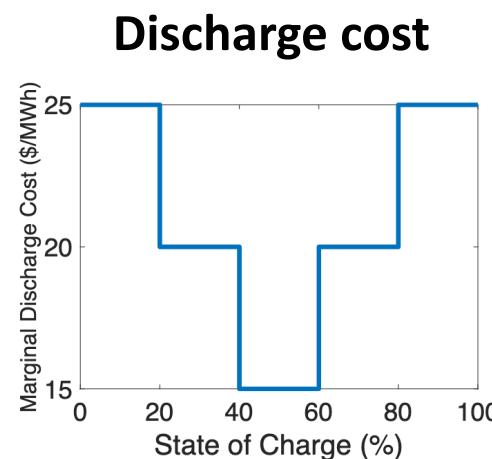
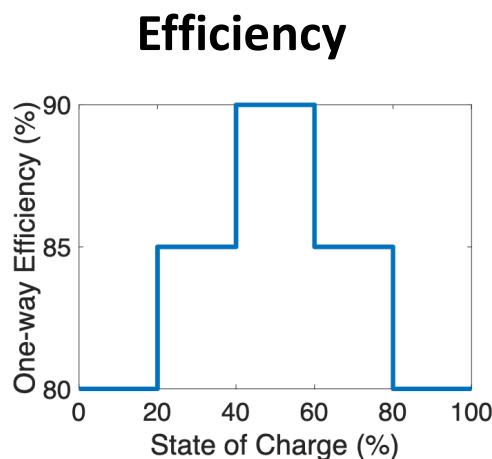
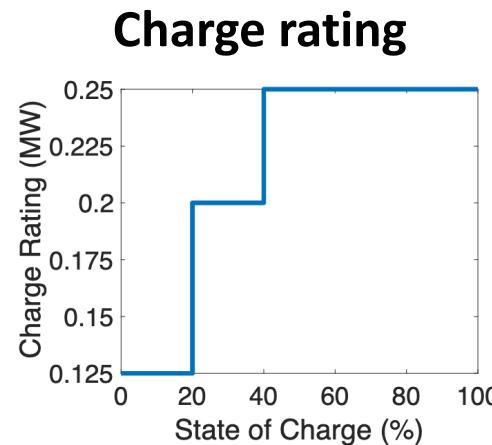
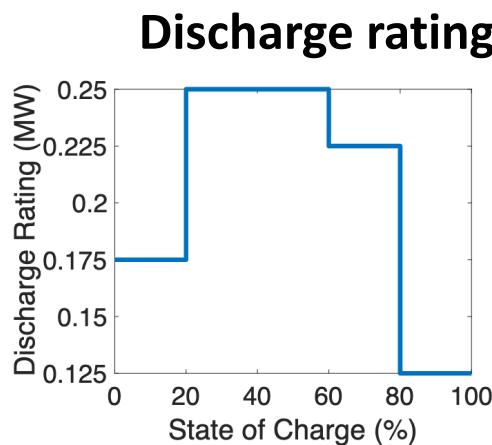
## CA Walnut 2016 / Price-taker / Perfect forecast

- Hour-ahead bidding
  - Storage submits same bids for one hour, cleared 12 times (5-minute)
- Linear storage models (constant parameters)
  - 5-segment model improves profits by 11%
- Solution time shown for full year simulation
  - RTD-5: 5-segment model
  - RTD-1: 1-segment model

Storage Model	Market Model	Revenue	Cost	Profit	Profit Ratio [%]	Solution Time (s)
Lin	Multi	23042	3770	19272	100	53
	RTD-5	22856	4100	18757	97.3	15
	RTD-1	20606	3699	16908	87.7	15

# Arbitrage Case Study

## Market Dispatch Performance for Nonlinear Storage Model

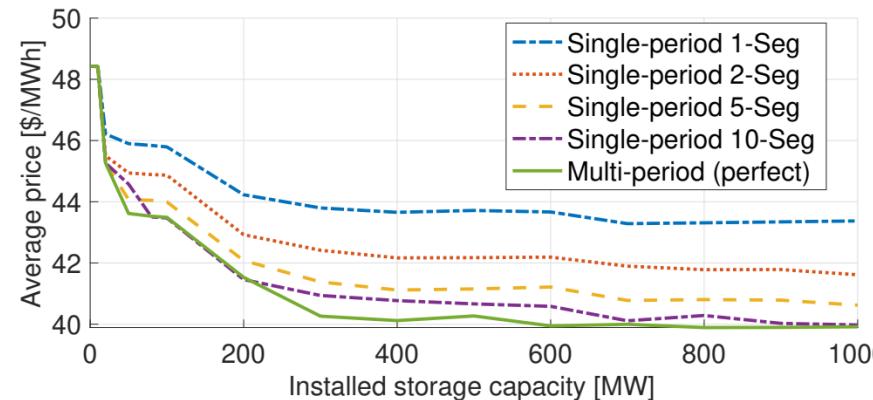


Storage Model	Market Model	Revenue	Cost	Profit	Profit Ratio [%]	Solution Time (s)
NLA	Multi	20803	3228	17575	100	288
	RTD-5	17328	2707	14621	83.2	
	RTD-1	12468	2389	10079	57.3	
NLB	Multi	21233	3148	18085	100	192
	RTD-5	20372	2995	17377	96.1	15
	RTD-1	17544	2834	14710	81.3	15
NLL	Multi	20957	3047	17910	100	262
	RTD-5	19092	2922	16170	90.3	15
	RTD-1	12997	2652	10345	57.8	15
NLF	Multi	21102	3321	17781	100	261
	RTD-5	18340	2759	15581	87.6	15
	RTD-1	16125	2498	13627	76.6	15
NLC	Multi	22081	3310	18772	100	100
	RTD-5	21450	3382	18068	96.2	
	RTD-1	18529	3056	15472	82.4	

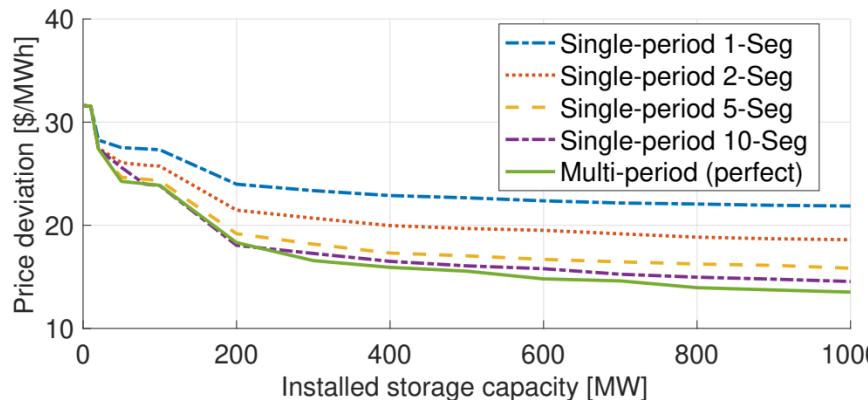
# Price Influencer Case Study

**ISO-NE test system, Real-time market, 4 hour storage**

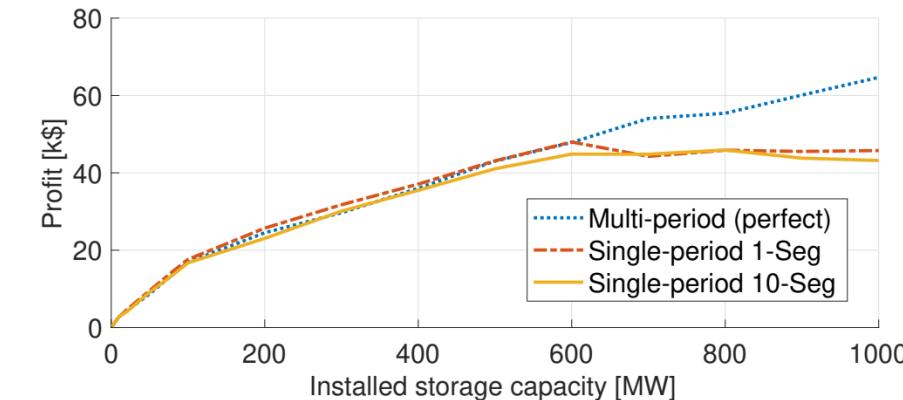
<https://arxiv.org/pdf/2207.07221.pdf>



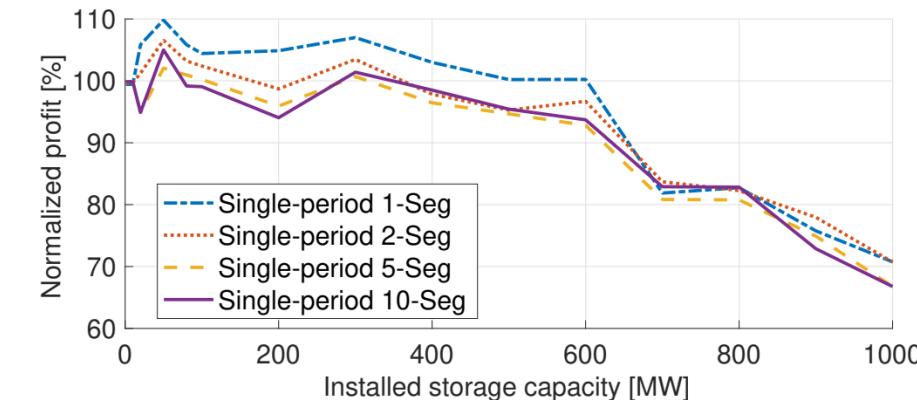
(a) Average real-time prices



(b) Scenario-average price standard deviations



(a) Average daily profits



(b) Normalized profit to multi-period case

# Conclusion and Future Directions

- Computation efficient dynamic programming provides a foundation for storage control, bidding, and integration analysis
- Improve solution to nonlinear storage model
- Use machine learning to aid stochastic control/valuation
- Agent-based analysis of energy storage market integration

# Thank you!

<https://bolunxu.github.io/>



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Joshua Jaworski



Xin Qin



Di Wu



Gabe Murtaugh

## References:

Xu, Bolun, Magnus Korpås, and Audun Botterud. "Operational Valuation of Energy Storage under Multi-stage Price Uncertainties." In *2020 59th IEEE Conference on Decision and Control (CDC)*, pp. 55-60. IEEE, 2020.

Zheng, Ningkun, Joshua Jerzy Jaworski, and Bolun Xu. "Arbitraging Variable Efficiency Energy Storage using Analytical Stochastic Dynamic Programming." *IEEE Transactions on Power Systems* (2022).

Zheng, Ningkun, and Bolun Xu. "Impact of Bidding and Dispatch Models over Energy Storage Utilization in Bulk Power Systems." IREP Bulk Power System Dynamics and Control Conference 2022.

Ningkun Zheng, Xin Qin, Di Wu, Gabe Murtaugh, and Bolun Xu. "Energy Storage State-of-Charge Market Model", <https://arxiv.org/pdf/2207.07221.pdf>

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