Location of stagnation radius

Start with the (dimensionless) momentum and entropy equations

$$\frac{dp}{dr} + \rho u \frac{du}{dr} = -qu - \frac{\rho}{r^2}$$
$$\frac{d\log p}{dr} - \gamma \frac{d\log \rho}{dr} = \frac{q\left[(\gamma-1)\rho(u^2+1)-2\gamma p\right]}{2p\rho u}$$

Combining these equations to obtain

$$-\gamma \frac{p}{\rho}\frac{d\rho}{dr} - \rho u \frac{du}{dr} = \frac{q\left[(\gamma-1)\rho(u^2+1)-2\gamma p\right]}{2\rho u} + qu + \frac{\rho}{r^2}.$$

Evaluating this at equation at the stagnation radius, r_{st}, gives

$$-\frac{\gamma-1}{2}\frac{d\rho}{dr}\bigg|_{\mathrm{r_{st}}} = \lim_{r->r_{\mathrm{st}}} \frac{q\left[(\gamma-1)\rho(u^2+1)-2\gamma p\right]}{\rho u} + \frac{\rho(r_{\mathrm{st}})}{r_{\mathrm{st}}^2}$$

The limit may be evaluated using L'Hopital's rule. After some algebraic manipulation I obtain that

$$r_{\mathrm{st}} = -\frac{\gamma+1}{\gamma-1}\nu^{-1},$$

where

$$\nu = \frac{d\log \rho}{d\log r}\bigg|_{r_{\mathrm{st}}}.$$

Ansatz for the velocity profile

If we set the velocity to 0, the energy conservation equation gives

$$\frac{c_s^2}{\gamma-1} = \frac{1}{2} + \frac{1}{r}h(r/r_{\mathrm{st}}),$$

and

$$h(x) = 1 - x\frac{3-\eta}{2-\eta}\frac{x^{2-\eta}-1}{x^{3-\eta}-1}$$

Now let us take the following ansatz for the c_s

$$\frac{c_s^2}{\gamma-1} = \frac{1}{2} + \frac{1}{r}\left(h(r/r_{\mathrm{st}}) - \frac{1}{2}h(r/r_{\mathrm{st}})^2\right)$$

Plugging this back into energy conservation equation gives

$$u = -\frac{h(r/r_{\mathrm{st}})}{\sqrt{r}}$$

The density follows from mass conservation

$\rho = \frac{r^{3-\eta} - r_{\rm st}^{3-\eta}}{r^2 u (3-\eta)}$

Then

$\left.\frac{d\log\rho}{d\log r}\right|_{r_{\rm st}} = -\frac{(4\eta - 1)}{6}$

> Written with StackEdit.