半分全列举

Split and List

Contents

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Split and List

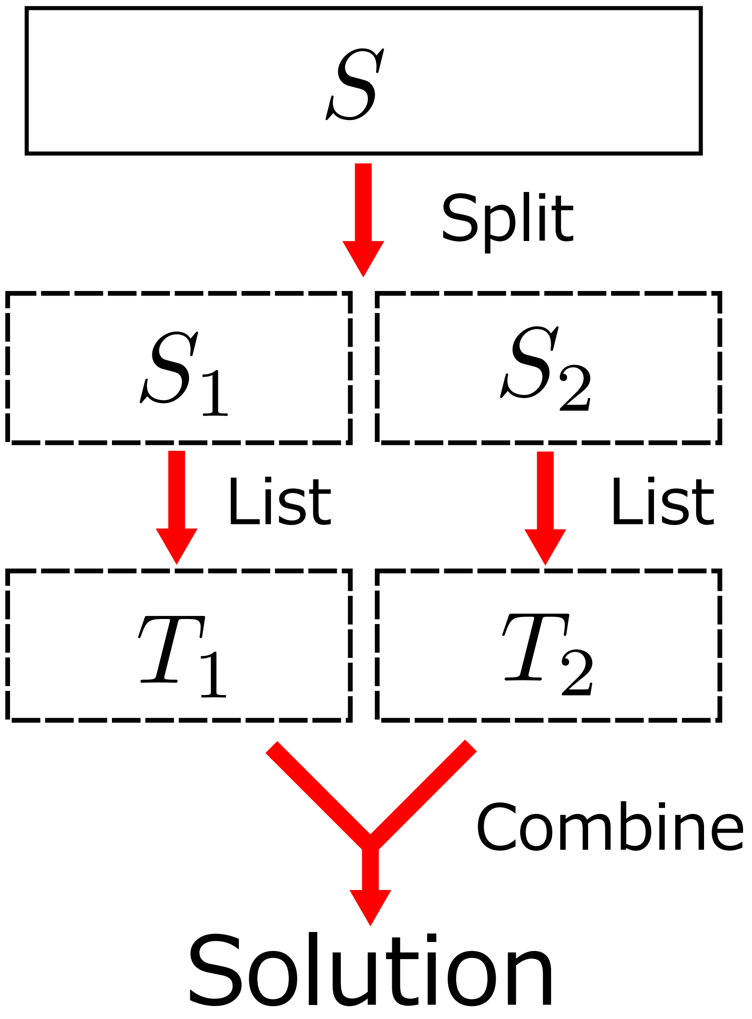
- The algorithm to split in half, which decrease $O(2^N)$ to $O(2^{\frac{N}{2}})$
- Source of name
- Twitter

Basic method

1. Split in half the set of number: S_1 and S_2.
2. List the all solutions for each S_1 and $S_2 \Rightarrow T_1$ and T_2.
3. Combine solutions of $x \in T_1$ and $y \in T_2$, with fast-time method.

Source: Split and list technique for solving hard problems

Image



Subset sum problems with Split and List

Subset sum problems with Split and List

1. Split in half the set of number: S_1 and S_2.
2. List the all sums of subset in S_1, enter them in T_1.
3. Do it of all subset in S_2, enter them in T_2.
4. For each a in T_1, determine that b exists in T_2 such that $a + b = K$, using binary search.

Image 1/6

5 7 17 9 25 19 28 16

Image 2/6

Split to S_1 and S_2.

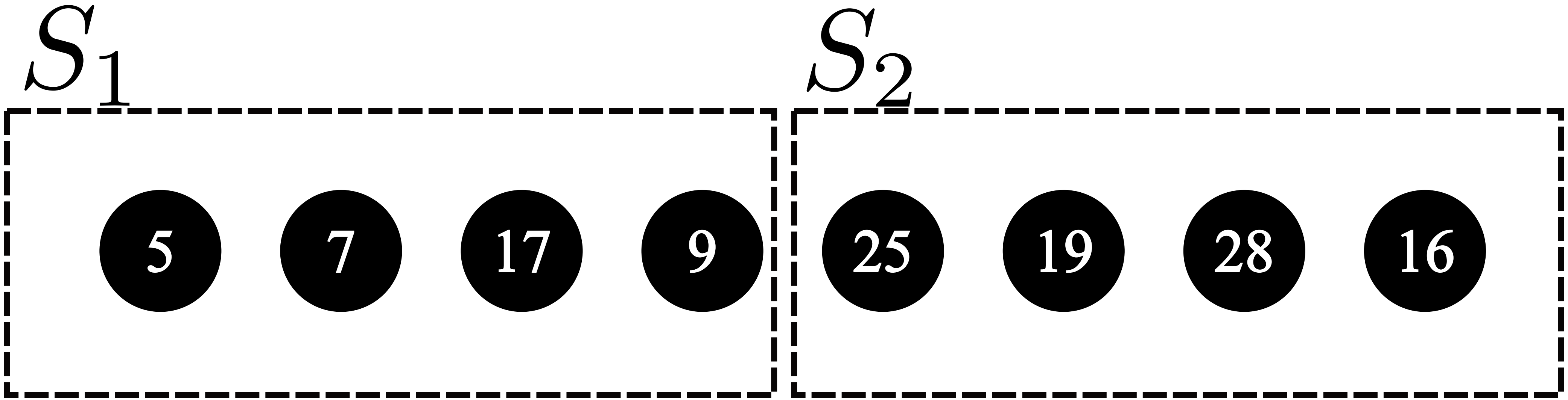


Image 3/6

List to T_1 from S_1

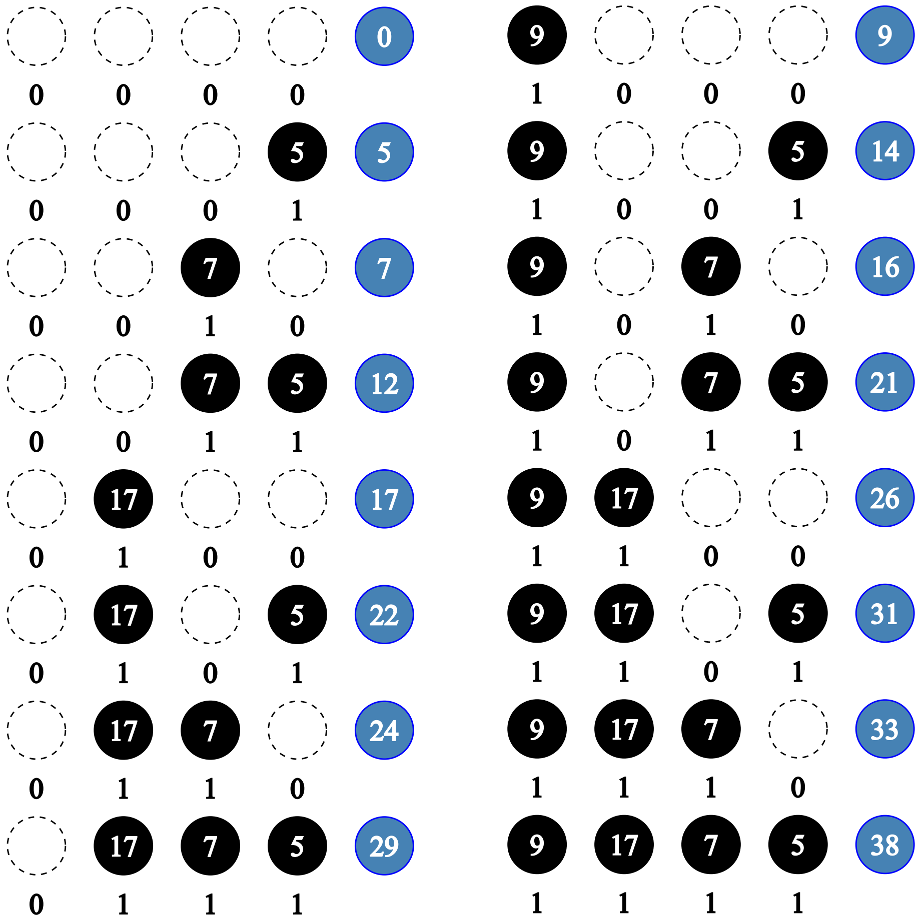


Image 4/6

List to T_2 from S_2

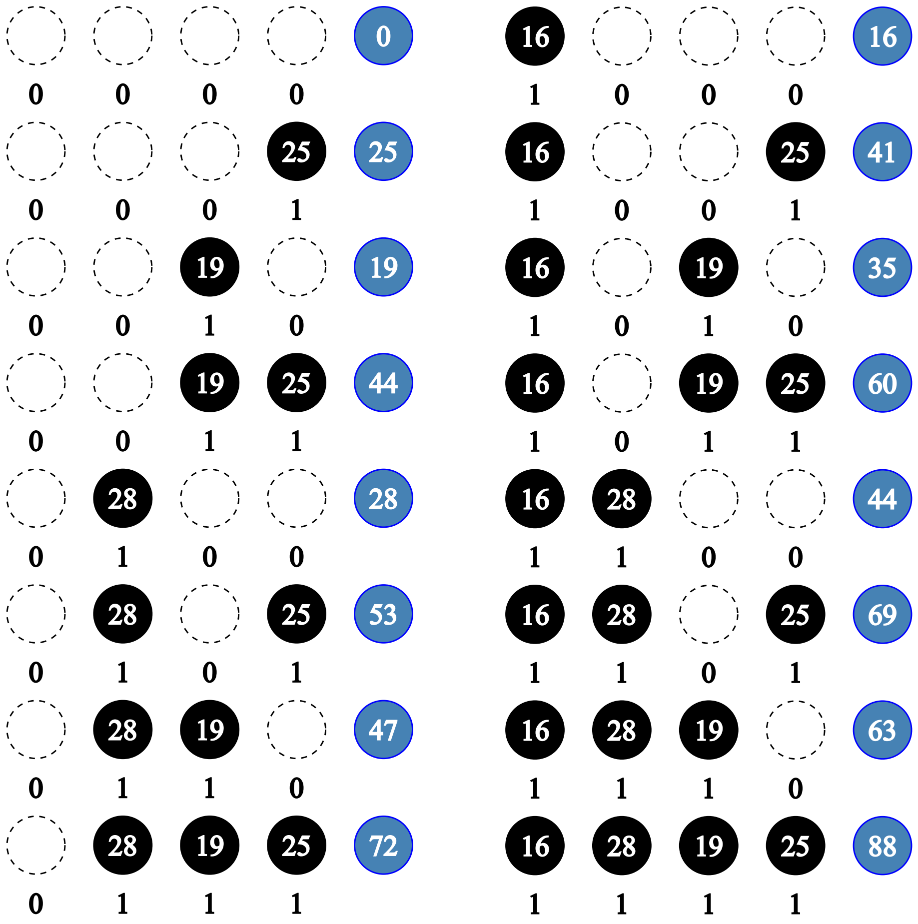


Image 5/6

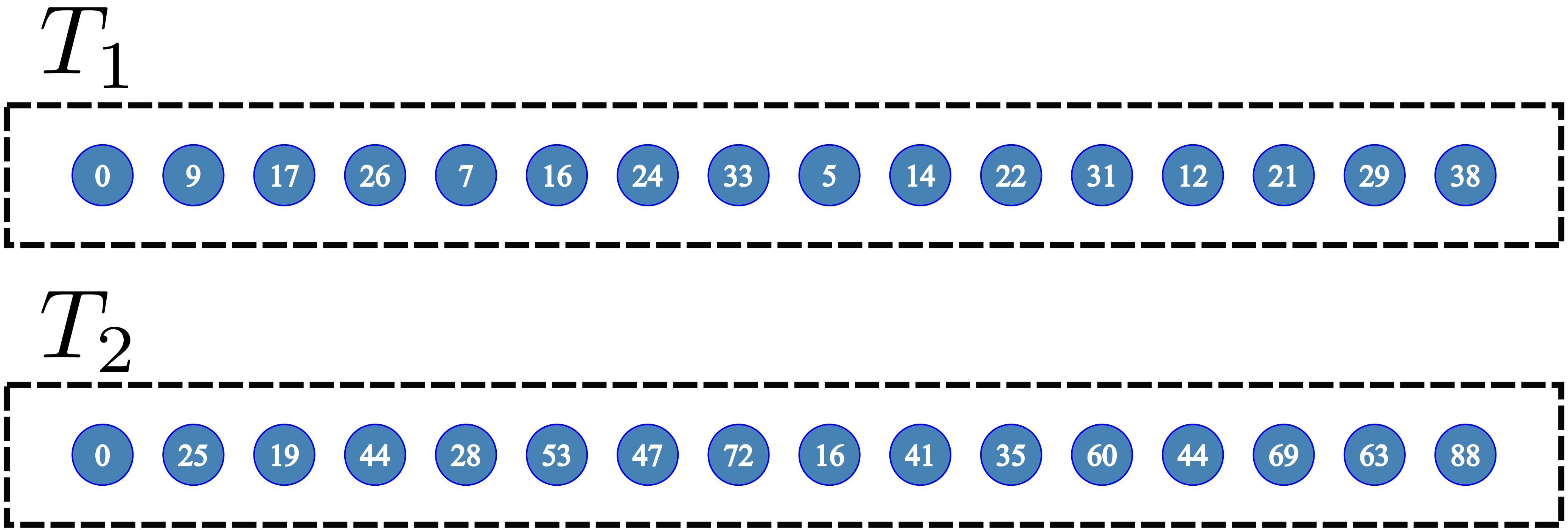
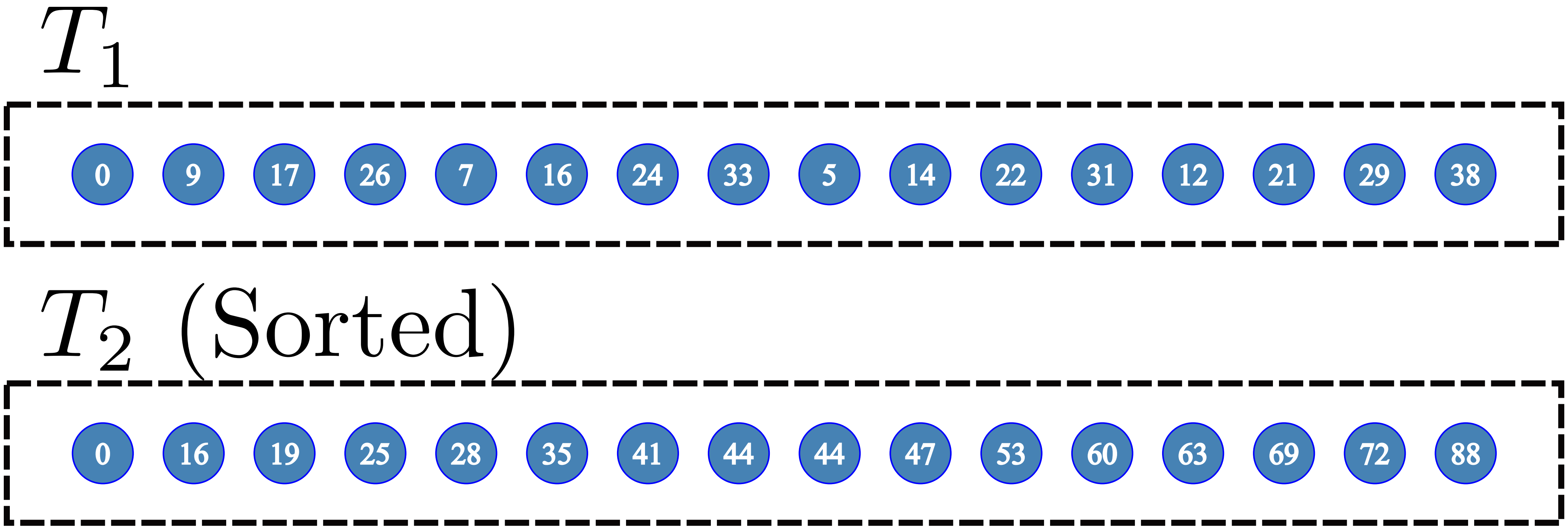

T_1
0 9 17 26 7 16 24 33 5 14 22 31 12 21 29 38
T_2
0 25 19 44 28 53 47 72 16 41 35 60 44 69 63 88

Image 6/6

- We want to find $(T_1[i], T_2[j])$ such that $T_1[i] + T_2[j] = K$.
- Sort and binary search.



Code 1/3

```
bool subsetSum(vector<int> S, int K)
{
  vector<int> S1, S2;
  // Split to S1 and S2.
  for (int i = 0; i < S.size(); i++) {
    if (i < S.size()/2) S1.push_back(S[i]);
    else S2.push_back(S[i]);
  }
```

Code 2/3

```
// List from S1 and S2 to T1 and T2.
vector<int> T1, T2;
for (int b = 0; b < (1 << S1.size()); b++) {
  int sum = 0;
  for (int i = 0; i < S1.size(); i++) {
    if ((b >> i) & 1) sum += S1[i];
  }
  T1.push_back(sum);
}
for (int b = 0; b < (1 << S2.size()); b++) {
  int sum = 0;
  for (int i = 0; i < S2.size(); i++) {
    if ((b >> i) & 1) sum += S2[i];
  }
  T2.push_back(sum);
}
sort(T2.begin(), T2.end());
```

Code 3/3

$$T_1[i] + T_2[j] = K$$
$$\Leftrightarrow T_2[j] = K - T_1[i]$$

```
  for (int i = 0; i < T1.size(); i++) {
    auto itr = lower_bound(T2.begin(), T2.end(), K - T1[i]);
    if (itr != T2.end() && T1[i] + *itr == K) return true;
  }

  return false;
}
```

Code: Using map

The type map is made of binary search tree.

```
  ...

  // List from S1 to mp.
  // mp[T] = the number of T.
  map<int, int> mp;
  for (int b = 0; b < (1 << S1.size()); b++) {
    int sum = 0;
    for (int i = 0; i < S1.size(); i++) {
      if ((b >> i) & 1) sum += S1[i];
    }
    mp[sum]++;
  }
  for (int b = 0; b < (1 << S2.size()); b++) {
    int sum = 0;
    for (int i = 0; i < S2.size(); i++) {
      if ((b >> i) & 1) sum += S2[i];
    }
    if (mp.count(K - sum)) return true;
  }
  return false;
}
```

Knapsack Problems with Split and List

Knapsack Problems with Split and List

1. Split in half the set of number: S_1 and S_2.
2. List the all combination of items in S_1, enter them in T_1.
3. Do it of all items in S_2, enter them in T_2.

Code 1/4

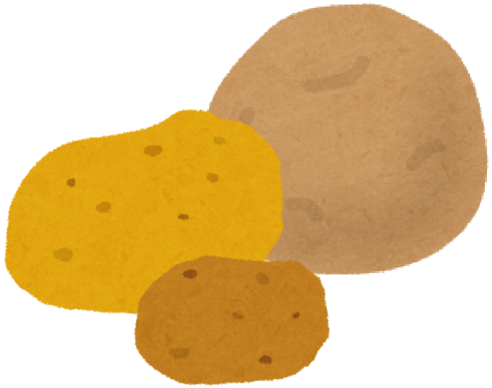
```
typedef long long ll;
typedef pair<ll, ll> P;
constexpr ll LLINF = 1100000000000000000LL;

// S[i] := (weight, value)
ll knapsack(vector<P>& S, ll W)
{
  // Split S to S1 and S2
  vector<P> S1, S2;
  for (int i = 0; i < S.size(); i++) {
    if (i < S.size()/2) S1.push_back(S[i]);
    else S2.push_back(S[i]);
  }
```

Code 2/4

```
// List S1 and S2 to T1 and T2.
vector<P> T1, T2;
for (int b = 0; b < (1 << S1.size()); b++) {
  P item(0, 0);
  for (int i = 0; i < S1.size(); i++) {
    if ((b >> i) & 1) {
      item.first  += S1[i].first;
      item.second += S1[i].second;
    }
  }
  T1.push_back(item);
}
for (int b = 0; b < (1 << S2.size()); b++) {
  P item(0, 0);
  for (int i = 0; i < S2.size(); i++) {
    if ((b >> i) & 1) {
      item.first  += S2[i].first;
      item.second += S2[i].second;
    }
  }
  T2.push_back(item);
}
```

4. Removing unnecessary thing:

item-i		item-j
		
weight: w_i	$\leq$	weight: w_j
value: v_i	$>$	value: v_j

- We can remove item-j. (weight is heavier but value smaller).
- After removing them, it satisfies that $w_i < w_j \Rightarrow v_i < v_j$
  - If sorted, $w_i < w_{i+1} \Rightarrow v_i < v_{i+1}$

Extract necessary things.

Instead of removing necessary things, we extract necessary things.

$$T_2(\text{Sorted in weight})\ (w_0, v_0), \ldots, (w_{i-1}, v_{i-1}), \boxed{(w_i, v_i)}$$

Push if $v_{n_j} < v_i$

$$T_2' \quad (w_{n_0}, v_{n_0}), \ldots, (w_{n_{j-1}}, v_{n_{j-1}}), (w_{n_j}, v_{n_j})\ \boxed{}$$

Code 3/4

```
// Extract from T2 to T2p
sort(T2.begin(), T2.end());
vector<P> T2p;
if (!T2.empty()) T2p.push_back(T2[0]);
for (int i = 1; i < T2.size(); i++) {
  if (T2p.back().second < T2[i].second) {
    T2p.push_back(T2[i]);
  }
}
```

5. For each item a in T_1, find that item b in T_2 such that $(\text{weight of } a) + (\text{weight of } b) \geq W$, using binary search.
   The answer is maximum value of combinations by a and b.

Code 4/4

$$(\text{weight of } T_1[i]) + (\text{weight of } T_2'[j]) \leq W$$
$$\Leftrightarrow (\text{weight of } T_2'[j]) \leq W - (\text{weight of } T_1[i])$$

```
  ll ret = -1;
  for (int i = 0; i < T1.size(); i++) {
    if (T1[i].first > W) continue;
    auto itr = lower_bound(T2p.begin(), T2p.end(), make_pair(W - T1[i].first, LLINF)) - 1;
    ret = max(ret, T1[i].second + itr->second);
  }
  return ret;
}
```

Supplement: comparation of pair

- $(a_1, a_2) < (b_1, b_2) \Leftrightarrow a_1 < b_1 \vee (a_1 = b_1 \wedge a_2 < b_2)$
- So, $(x, y) < (x, \mathrm{INF})$ for all y.

Supplement: lower_bound with pair

- If you want to find the first element $a[i]$ such that $a[i].\mathrm{first} > X$:

```
auto itr = lower_bound(a.begin(), a.end(), make_pair(X, INF));
```

- If you want to find the last element $a[i]$ such that $a[i].\mathrm{first} \leq X$:

```
auto itr = lower_bound(a.begin(), a.end(), make_pair(X, INF)) - 1;
```

Sammary

- Split and List is a method of splitting to two set and listing by them.
- Split and List is allowed to solve in the time complexity $O(2^{\frac{N}{2}})$

Addition: construction of vector

```cpp
vector<int> S1, S2;
// Split to S1 and S2.
for (int i = 0; i < S.size(); i++) {
  if (i < S.size()/2) S1.push_back(S[i]);
  else S2.push_back(S[i]);
}
```

Using constructor:

```
vector<int> S1(S.begin(), S.begin() - S.size()/2);
vector<int> S2(S.begin() - S.size()/2, S.end());
```

It is convenient!

Exercise

- ARC017 C: 無駄なものが嫌いな人
  - English statement is next page.
  - Hint is next next page.
- Educational Codeforces Round 32 E. Maximum Subsequence

ARC017: People who dislike extra things.

Statement

I dislike extra thing. It is no use thinking the usual knapsack problem.
I want to put items in the knapsack, whose sum of weight is **exactly** equal to the size of knapsack.
I want you to enumerate the number of how to choose items.

Input:

```
N X
w1
w2
:
wN
```

- N: the number of items. $1 \leq N \leq 32$.
- X: the size of knapsack. $1 \leq X \leq 10^9$.
- wi: the weight of i-th item. $1 \leq w_i \leq 5 \times 10^7$.

Output

Print the number of how to choose items whose sum of weight is equal to X.

Sample is here

Hint

- This is not knapsack problem, but **subset sum problem**.
- Make a backet (or histgram) by the type of map <long long, long long>.