bitDP

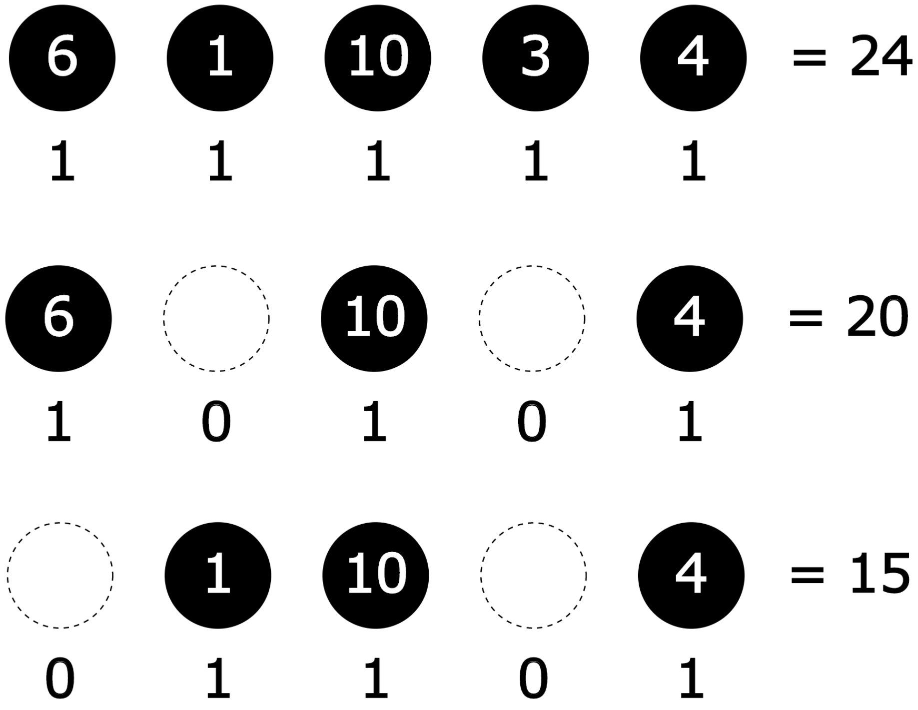
What's this?

- A kind of DP
- Simply the dp table has **subset** as index

Review

Subset is expressed as bit.

e.g: subset sum



Travelling Salesman Problem (TSP)

For a given weighted directed graph $G(V, E)$, find the distance of the shortest route that meets the following criteria:

- It is a closed cycle where it ends at the same point it starts.
- It visits each vertex exactly once.

(The graph which safisfies these condition is called Hamiltonian Cycle).

Input

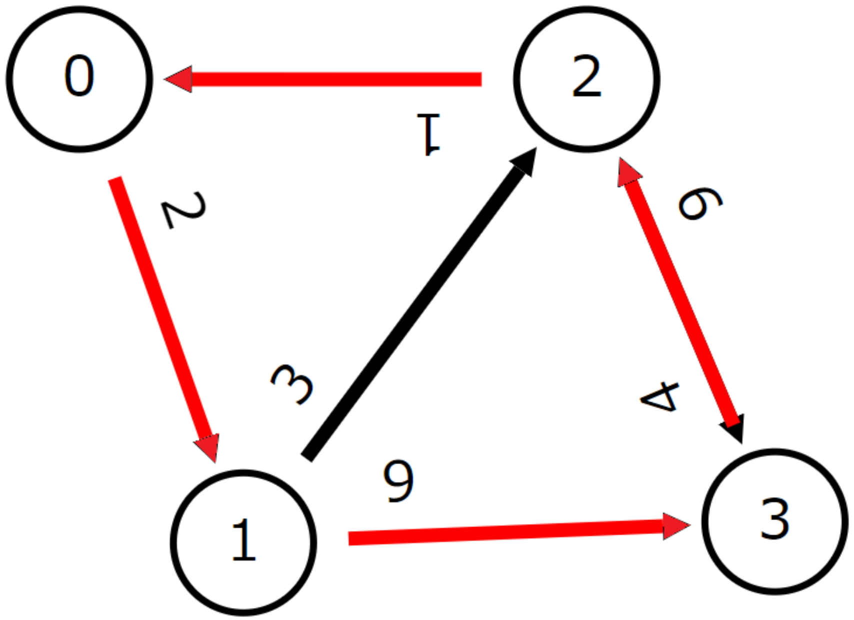
- $|V|, |E|$: vertex and edge size
- s_i, t_i, d_i: a path which links s_i to t_i, and has weight d_i.

Constraints

- $2 \leq |V| \leq 15$
- $0 \leq d_i \leq 1,000$
- There are no multiedge

Source: AOJ DPL_2_A

Example



from	to	weight
0	1	2
1	2	3
1	3	9
2	0	1
2	3	6
3	2	4

Naive solution: brute-forth

1. Generate all permutations of vertex list.
2. Check whether the route exists, and sum up the distance it visited.
3. minimum sum of distance.

The time complexity is $O(|V|!)$

DP

- Def.

$$dp[S][v] = \text{the shortest path 0 to v when it has been visited vertexes } S$$

- Init.
  - $dp[\emptyset][0] = 0$
  - $\text{others} = \text{INF}$

Note that we can decide the start vertex bacaue of circlation of graph and it is in $\emptyset$ of dp-init.
The start vertex is enterd in S when the graph is closed as cyclic graph.

- Trans.
  $dp[S][v]$ already known,
  If v and u are adjacent,
    - $dp[S \cup u][u] = \min(src, dp[S][v] + d_{vu})$
- Ans.
  $dp[V][0]$

Here, d_{vu} is the weight from vertex v to u.

- Ofcourse, we cannot use subset as index of array.
- Instead, use **bit**

- Def.

$dp[S][v] = \text{the shortest path 0 to v when it has been visited vertexes } S$

- Init.
  - $dp[0][0] = 0$
  - $\text{others} = \text{INF}$
- Trans.

  $dp[S][v]$ already known,

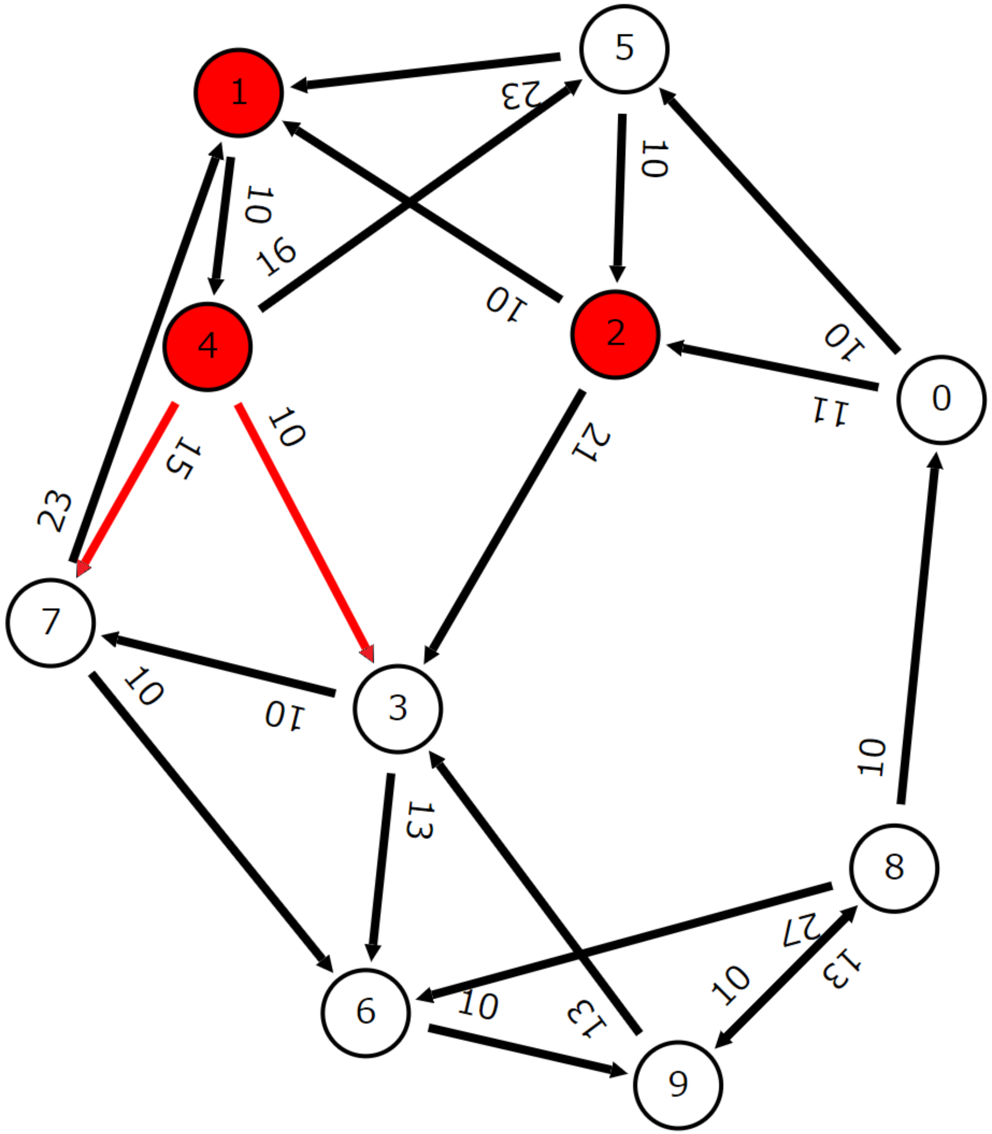
  If v and u are adjacent,
  - $dp[S \mid (1 << u)][u] = \min(src, dp[S][v] + d_{vu})$
- Ans.
  - $dp[(1 << |V|) - 1][0]$

$dp[S][v]$ should be known when iteration from small to large of S and v.

So, we can use for-loop!

Supplement: Image of transition

- Now:
  - $dp[(0000010110)_2][4]$
- Transition:
  - $dp[(00000\underline{1}1110)_2][3]$
  - $dp[(00\underline{1}0010110)_2][7]$



Code: Iterative

```
dp[0][0] = 0;
for (int S = 0; S < (1 << V); S++) {
  for (int v = 0; v < V; v++) {
    if (dp[S][v] >= INF) continue;
    for (auto& to : edge[v]) {
      int u = to.first;
      int d = to.second;
      if ((S >> u) & 1) continue;
      dp[S | (1 << u)][u] = min(dp[S | (1 << u)][u], dp[S][v] + d);
    }
  }
}
// the answer is dp[(1 << V) - 1][0].
```

Code: Recursive

- We can change iterative to recursive method, but it is a little hard **as this definition of DP**.
- Easyness to write DP as iterative or recursive depends on a DP definition.

- Def.

$dp[S][v] =$ the shortest path $\underline{\text{v to goal-vertex}}$ when it has been visited vertexes S

```cpp
int dfs(int S, int v)
{
  if (dp[S][v] >= 0) return dp[S][v];
  int ret = INF;
  for (auto& to : edge[v]) {
    int u = to.first;
    int d = to.second;
    if ((S >> u) & 1) continue;
    ret = min(ret, dfs(S | (1 << u), u) + d);
  }
  return dp[S][v] = ret;
}
```

- In generally, if we don't know the direction of transition with iteration, we have no choice but recursive method.

Importance: Constraint

$n = |V|$

The time complexity of TSP is $O(n^2 2^n)$

n = 18 is barely OK

Summary

- Some constraints is very small!
  ⇒ maybe bitDP
- bitDP has subset as index, which expressed as bit.
- If you cannot iterate because of the direction of the transition, you use recursive method.