Stochastic Models in Cartoons

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Brownian Motion

$$dX_i(t) = \sigma\, dB_i(t),$$

Amount of change in character

Brownian Motion

$$dX_i(t) = \qquad\qquad \sigma\, dB_i(t),$$

proportional to std dev

Amount of change in character

Brownian Motion

$$dX_i(t) = \qquad\qquad \sigma \, dB_i(t),$$

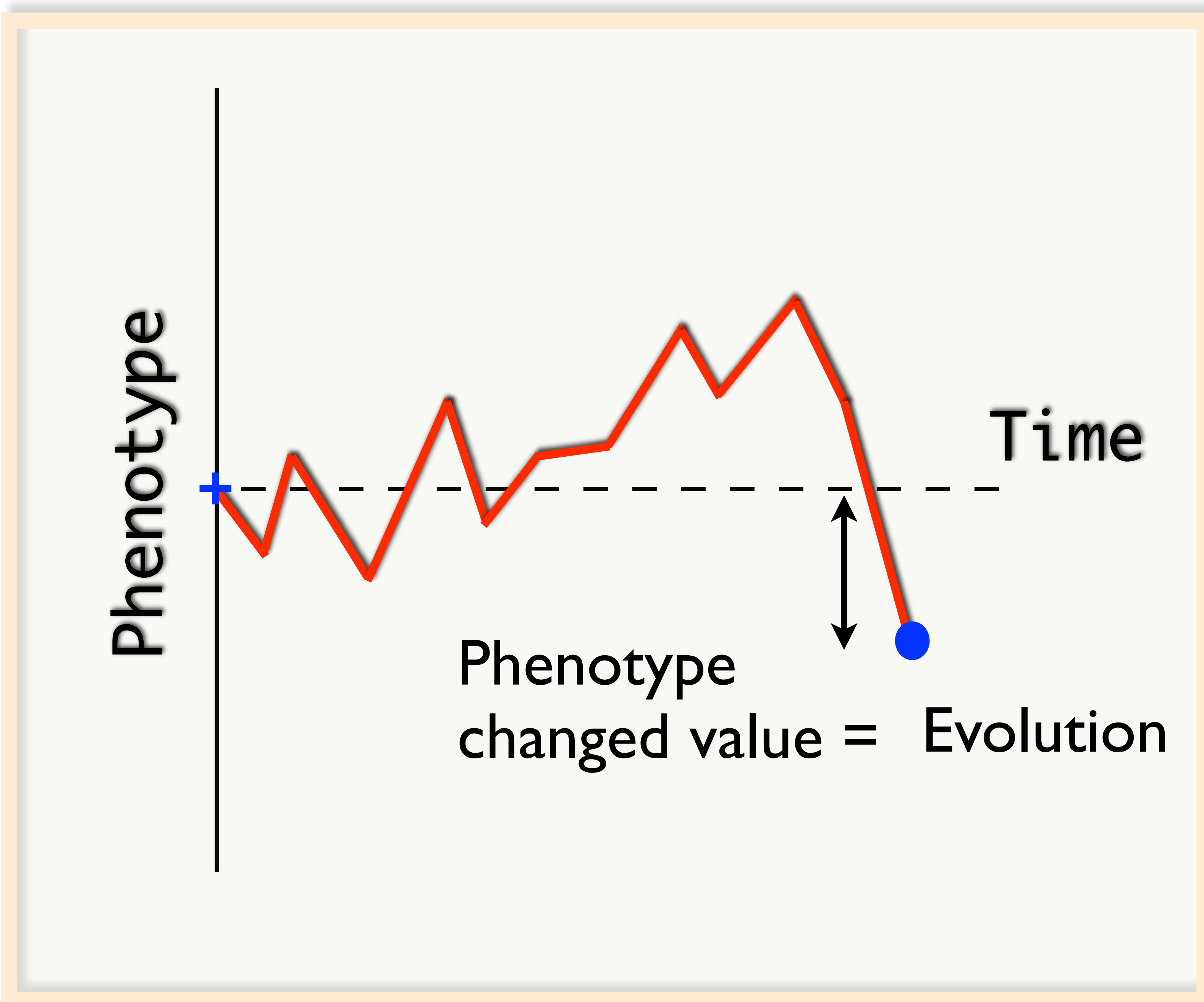
Amount of change in character

proportional to std dev

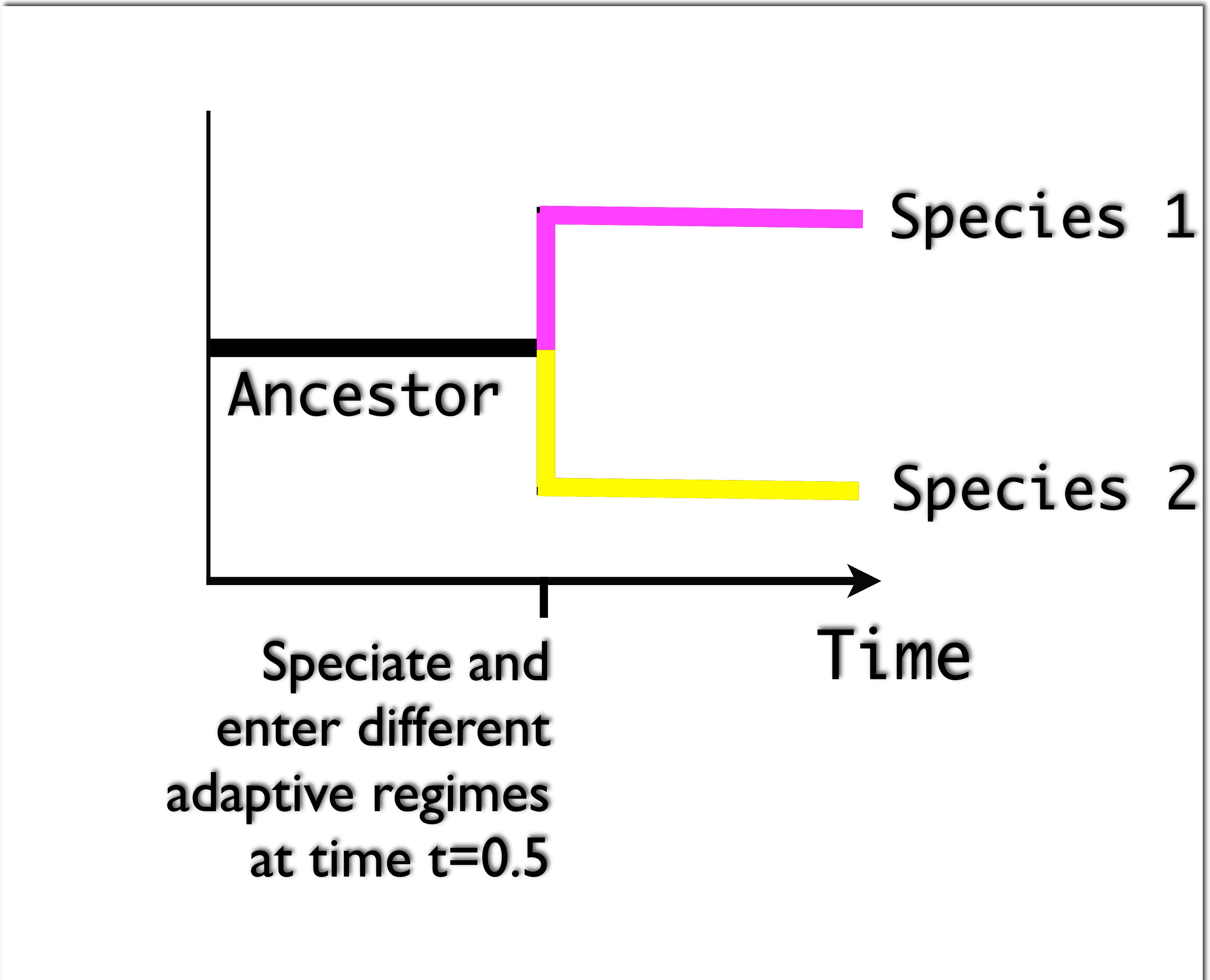
and a random amount of change

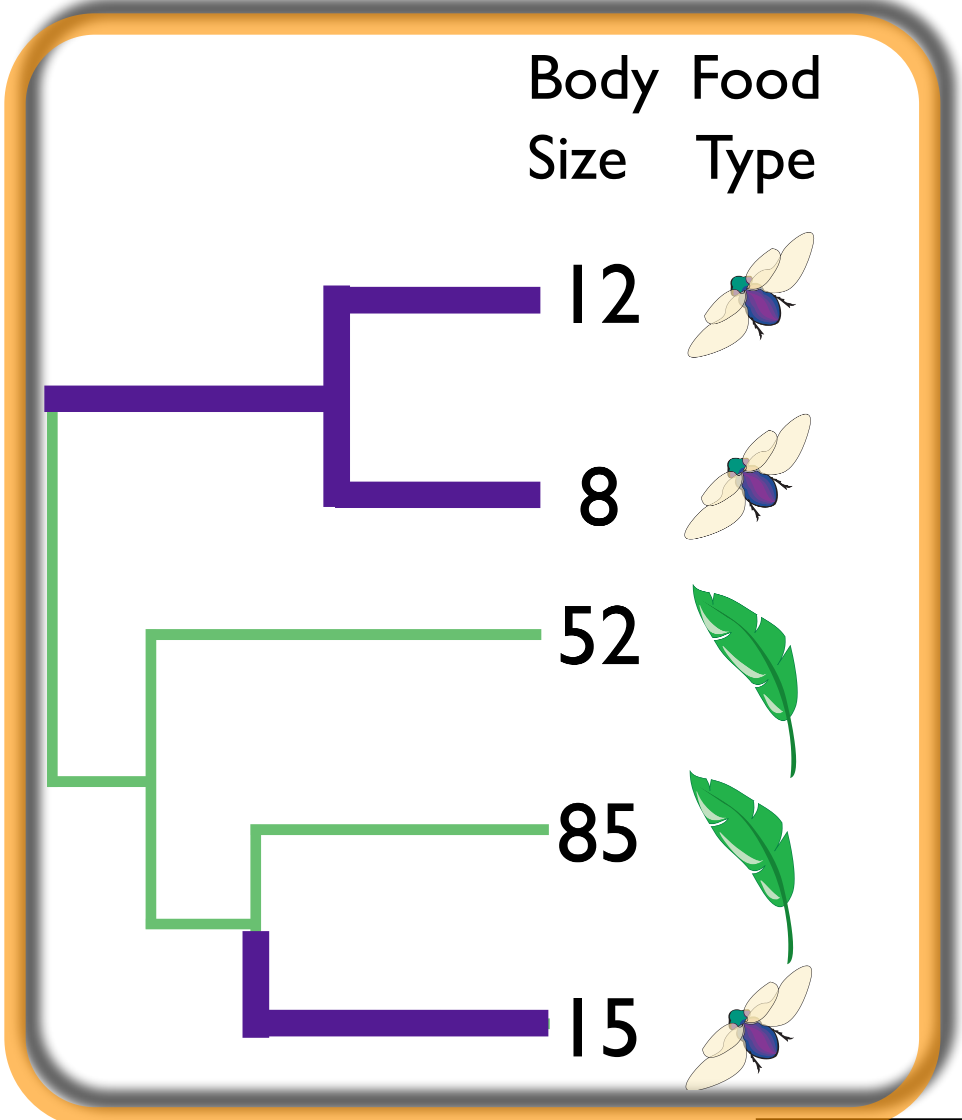
Let’s create some cartoons!

We can build up simulations of evolution



BM and OU models make different predictions





Recent theoretical developments have made it possible to perform comparative analyses using an explicit evolutionary model

Brownian Motion

$$dX_i(t) = \sigma \, dB_i(t), \quad t_i^{j-1} \leq t \leq t_i^j.$$

Orstein Uhlenbeck Process

$$dX_i(t) = \alpha \left(\beta_i^j - X_i(t) \right) dt + \sigma \, dB_i(t)$$

Ornstein Uhlenbeck Process

A model for evolution with selection

time interval t: $t_i^{j-1} \le t \le t_i^j.$

$$dX_i(t) = \alpha \left(\beta_i^j - X_i(t)\right) dt + \sigma\, dB_i(t),$$

Hansen (1997)

Ornstein Uhlenbeck Process

$$dX_i(t) = \alpha\left(\beta_i^j - X_i(t)\right)dt + \sigma\, dB_i(t),$$

Amount of change in character

Deterministic (**selection**)

Stochastic (**drift**)

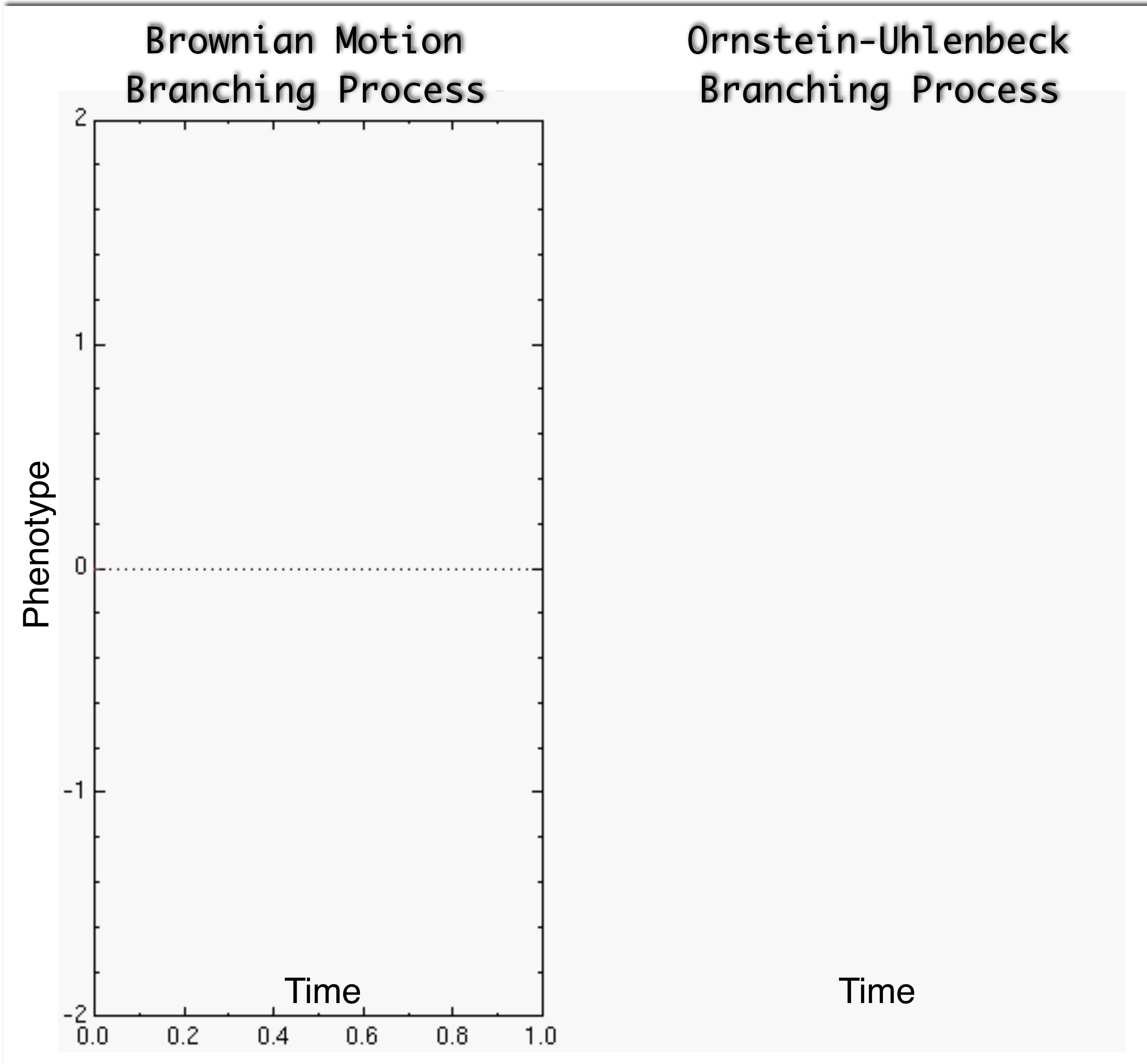
OU in the limit as $\alpha \to 0$

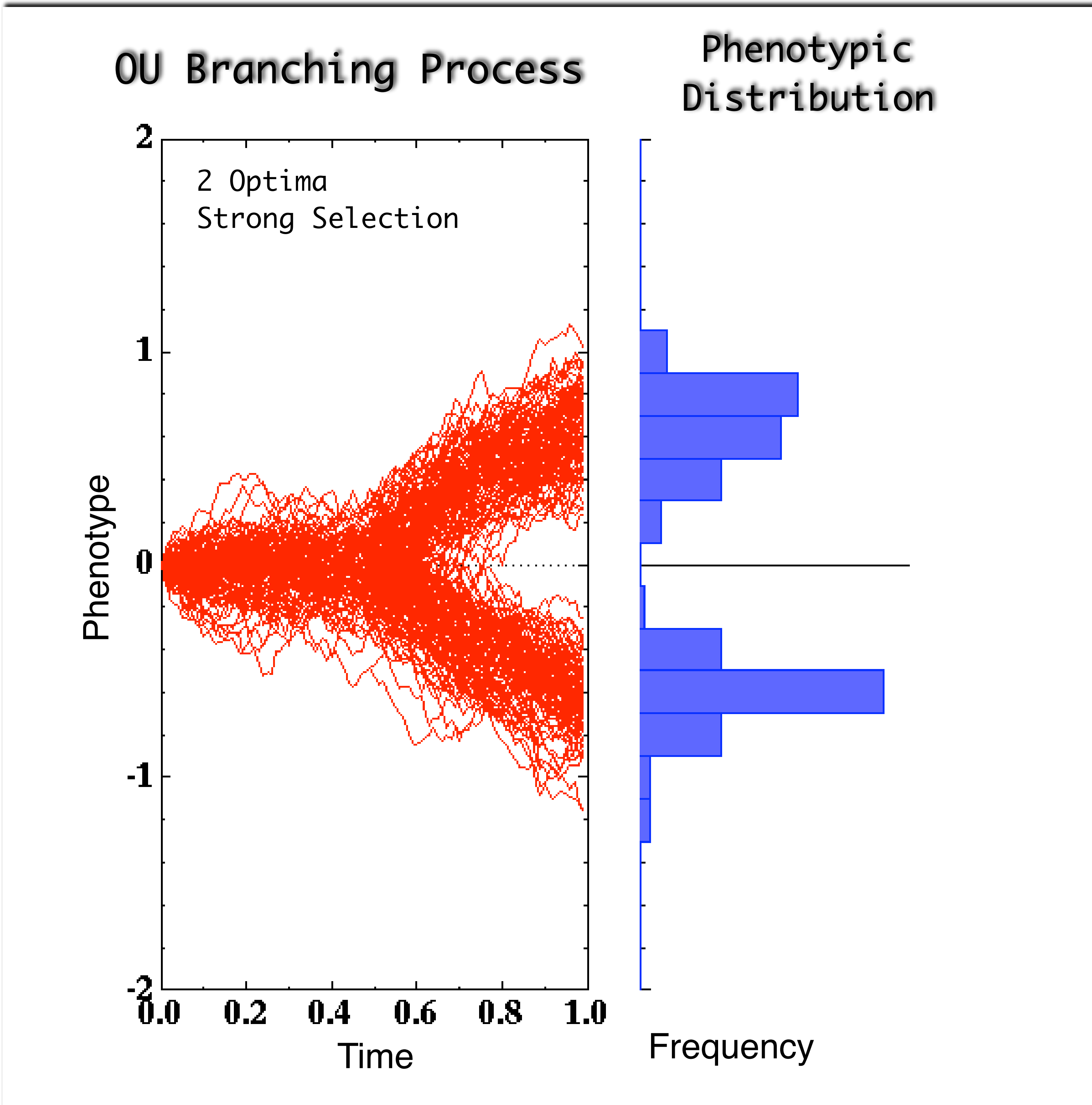
$$dX_i(t) = \cancel{\alpha}^{\,0} (\beta_i^j - X_i(t))\, dt + \sigma\, dB_i(t),$$

OU in the limit as α → 0

$$dX_i(t) = \xcancel{\cancel{\alpha}\,(\beta_i^j - X_i(t))}\,dt + \sigma\,dB_i(t),$$

BM and OU models make different predictions





OU Branching Process
Phenotypic Distribution
2 Optima
Strong Selection
Phenotype
Time
Frequency
2
1
0
-1
-2
0.0
0.2
0.4
0.6
0.8
1.0

Thus, with:

a set of interspecific data,
a phylogeny, and
a little biological insight,

we can explore alternative evolutionary scenarios

and potentially make a statement about how characters evolved!