Review on Parametric Parametric OF Survival Model

Likelihood function

Recall from Ben's "Parametric Overreported Failure (OF) Survival Model" that the indicator of observation i being truly censored (given that the subset of non-censored failure outcomes are being misclassified) is

$$\alpha_i = I(C_i = 1 | \widetilde{C}_i = 0), \tag{1}$$

such that $\Pr(\alpha_i = 1) = \Pr(C_i = 1 | \widetilde{C}_i = 0)$ and $\Pr(\alpha_i = 0) = 1 - \Pr(\alpha_i = 1) = \Pr(C_i = 0 | \widetilde{C}_i = 0)$.

Although we only define miscalssification/correct-classification for $\widetilde{C}_i = 0$ case, we have to note that

$$\Pr(C_i = 0 | \widetilde{C}_i = 1) = 0 \tag{2}$$

and the opposite

$$\Pr(C_i = 1 | \widetilde{C}_i = 1) = 1 - \Pr(C_i = 0 | \widetilde{C}_i = 1) = 1. \tag{3}$$

From here, I am not sure at all......but just tried (Is the function based on C_i or $\widetilde{C}_i$?)

Reflecting this to the likelihood of Survival model, $f(t_i)$ should consists of those with $C_i = 1$:

$$\Pr(C_i = 1 | \widetilde{C}_i = 0) * S(t_i)?f(t_i)? + \Pr(C_i = 1 | \widetilde{C}_i = 1) * f(t_i)$$
$$= \Pr(\alpha_i = 1) * S(t_i)?f(t_i)? + f(t_i) \tag{4}$$

And the survival function $S(t_i)$ for the case of $C_i = 0$ is therefore

$$\Pr(C_i = 0 | \widetilde{C}_i = 0) * S(t_i) + \Pr(C_i = 0 | \widetilde{C}_i = 1) * S(t_i)?f(t_i)?(\text{does not matter})$$
$$= (1 - \Pr(\alpha_i = 1)) * S(t_i) \tag{5}$$

where

$$\Pr(\alpha_i = 1) = \frac{\exp(\mathbf{Z}\gamma)}{1 + \exp(\mathbf{Z}\gamma)}. \tag{6}$$