Dynamic Latent Factor Network Modeling (DLFM) of the United Nations Voting Behaviors

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Abstract

- Extension of additive-multiplicative latent factor model [2, 3] to dynamic/longitudinal networks
- Adaptively estimate the Gaussian covariance structure to infer the temporal dependence
- Allowance of varying number of nodes over time
- Our Findings:
  1. DLFM is effective at predicting and explaining networks that are highly-correlated across timepoints
  2. DLFM acheives faster convergence and accuracy in parameter estimates, compared to the existing method
  3. DLFM's visualization of the results is useful to understand the temporal trends in networks

UN Voting Data

- Original dataset [4] contains all roll-call votes in the United Nations General Assembly
- Response variable $\boldsymbol{Y}$ ($32 \times 23 \times 23$ - dim. array):
  - Voting similarity index (0-1) computed using 3 category vote data, only considering the important votes
  - Years: from 1983 to 2014
  - Nodes: 23 most active countries in 2004-2014
    USA UKG FRN GMY RUS UKR GRG SUD IRN TUR IRQ EGY SYR LEB ISR AFG CHN PRK ROK JPN IND PAK AUL
- Explanatory variables $\boldsymbol{X}$ ($32 \times 23 \times 23 \times 5$ - dim. array):
  1. Intercept: constant 1 (to estimate yearly baseline)
  2. log(distance$_{ij}$): log of the geographic distance between country i and country j
  3. Alliance$_{ijt}$: 1 if country i and country j have alliance in year t, and 0 otherwise
  4. Polity score difference$_{ijt}$: absolute difference in polity IV number between country i and country j, in year t
  5. Relative trade$_{ijt}$: measure of relative importance of the trade between country i and country j in year t, using

$$min\left(\frac{\text{Trade}_{ijt}}{\text{GDP}_{it}}, \frac{\text{Trade}_{ijt}}{\text{GDP}_{jt}}\right)$$

References

[1] Durante, D. and Dunson, D. B. (2014). Nonparametric bayes dynamic modelling of relational data. *Biometrika*, page asu040.

[2] Hoff, P., Fosdick, B., Volfovsky, A., and Stovel, K. (2014). amen: Additive and multiplicative effects modeling of networks and relational data. *R package version 0.999. URL: http://CRAN. R-project. org/package= amen*.

[3] Hoff, P. D. (2009). Multiplicative latent factor models for description and prediction of social networks. *Computational and mathematical organization theory*, 15(4):261–272.

[4] Voeten, E., Strezhnev, A., and Bailey, M. (2016). United nations general assembly voting data.

Modeling Framework

- For the symmetric matrices $\boldsymbol{Y}(t=1), ..., \boldsymbol{Y}(t=T)$,

$$Y_{N\times N}(t) = \sum_{p=1}^{P} X^p(t)\beta_p(t) + \Theta(t) + \Theta'(t) + U(t)D(t)U'(t) + E(t),$$

  where
  1. $X(t)$ and $\beta(t)$: fixed effects from predictors
  2. $\Theta(t)$ and $\Theta'(t)$: additive row/column random effect
  3. $U(t)D(t)U'(t)$: multiplicative random effect
     - $U(t)$ is $N \times R$ matrix where $U_i(t) = (u_{i1}(t), ..., u_{iR}(t))$
     - $D(t)$ is $R \times R$ matrix $(= \text{diag}(d_1(t), ..., d_R(t)))$
  4. $E(t)$: random noise matrix
- Gaussian process (GP) prior specfication [1]:
  1. For each covariate $p = 1, ..., P$;
  $$\{\beta_p(t)\}_{t=1}^T \sim \text{MVN}_T(0, \tau_p^\beta c_\beta), \text{ with } \tau_p^\beta \sim \text{IG}(a_\beta, b_\beta)$$
  2. For each node $i = 1, ..., N$;
  $$\{\theta_i(t)\}_{t=1}^T \sim \text{MVN}_T(0, \tau_i^\theta c_\theta), \text{ with } \tau_i^\theta \sim \text{IG}(a_\theta, b_\theta)$$
  3. For each dimension $r = 1, ..., R$;
  $$\{d_r(t)\}_{t=1}^T \sim \text{MVN}_T(0, \tau_r^d c_d), \text{ with } \tau_r^d \sim \text{IG}(a_d, b_d)$$
  4. For each node $i = 1, ..., N$; and dimension $r = 1, ..., R$;
  $$\{u_{ir}(t)\}_{t=1}^T \sim \text{MVN}_T(0, c_u)$$
  5. For each pair of (i, j) with $i > j$,
  $$\epsilon_{ij}(t) \sim \text{N}(0, \sigma_e^2), \text{ with } \sigma_e^2 \sim \text{IG}(a, b),$$

  where c_* is $T \times T$ correlation matrix corresponding to $* \in \{\beta, \theta, d, u\}$ obtained from the GP function

$$c_*(t, t') = \begin{cases} \exp(-\frac{|t-t'|}{\kappa_*}), & \text{covfc = Exponential} \\ \exp(-\frac{\|t-t'\|_2^2}{\kappa_*^2}), & \text{covfc = Sq. Exponential} \end{cases}$$

  Note: each value of κ_* and the corresponding proper covariance function is estimated.
- Bayesian Inference using Gibbs sampling (MCMC): Sequentially resample each of latent variables

$$\left(\sigma_e^2, \{\tau_p^\beta\}_{p=1}^P, \{\tau_i^\theta\}_{i=1}^N, \{\tau_r^d\}_{r=1}^R, \{\{\beta_p(t)\}_{t=1}^T\}_{p=1}^P, \{\{\theta_i(t)\}_{t=1}^T\}_{i=1}^N, \{\{d_r(t)\}_{t=1}^T\}_{r=1}^R, \{\{\{u_{ir}(t)\}_{t=1}^T\}_{i=1}^N\}_{r=1}^R\right)$$

  from the full conditional distributions

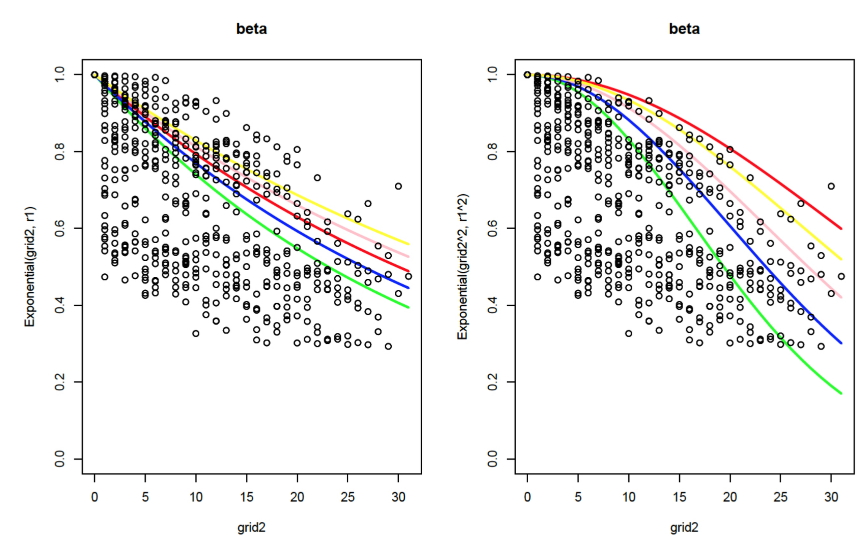
Adaptive sampling algorithm

1. Scanning with temporal independence

for *iter=1 to scan* **do**
  run MCMC with $\kappa_* = 0.001$ (which gives $c_* = I_T$)
end

2. Estimating covariance structure

for $*$ *in* (β, θ, d, u) **do**
  calculate the correlation $\rho^*_{t,t'}$ and compare the fittings
  - Exponential: $\log(\rho^*_{t,t'}) = -\frac{1}{\kappa}|t - t'|$
  - Sq. Exponential: $\log(\rho^*_{t,t'}) = -\frac{1}{\kappa^2}\|t - t'\|_2^2$
  construct $\hat{c}_*$ given $\hat{\kappa}_*$ and covfc with better fit
end



3. Running with temporal dependence

for *iter=1 to nsample* **do**
  run MCMC with estimated $\hat{c}_* = (\hat{c}_\beta, \hat{c}_\theta, \hat{c}_d, \hat{c}_u)$
end

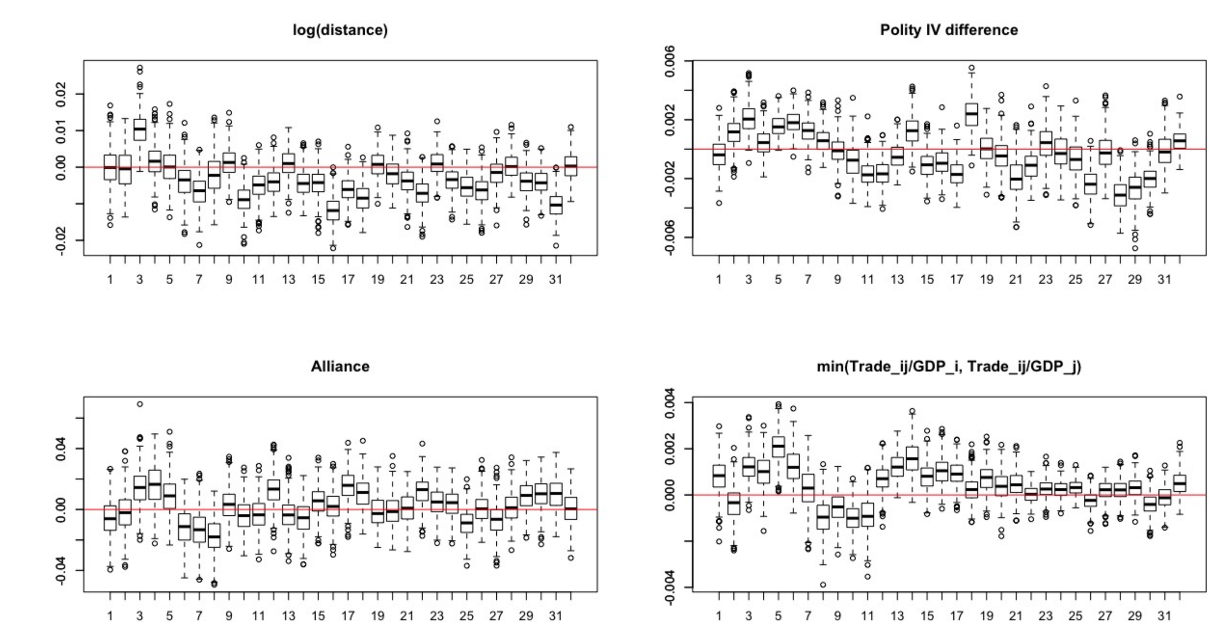
Summarize the results only using the new samples with estimated temporal dependence structure

Varying number of nodes

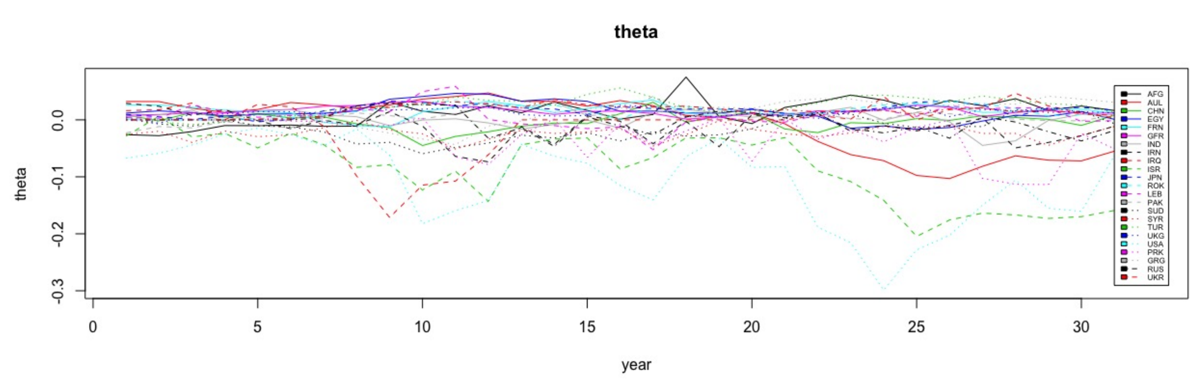
- New node can join or existing node can disappear at any timepoint
  (ex. countries not existed: RUS ~1988, UKR ~1986, GRG ~1989; countries in war: IRQ 1995-2003)
- Allow "structural NA's" remain as NA, while pure missing values imputed using posterior estimates
  1. Reduce bias in fixed effect estimates β
  2. Avoid meaningless random effect estimates Θ and U
  3. Provide flexibility in fitting the model to larger networks (limitation: with known in-and-out structure)

Results

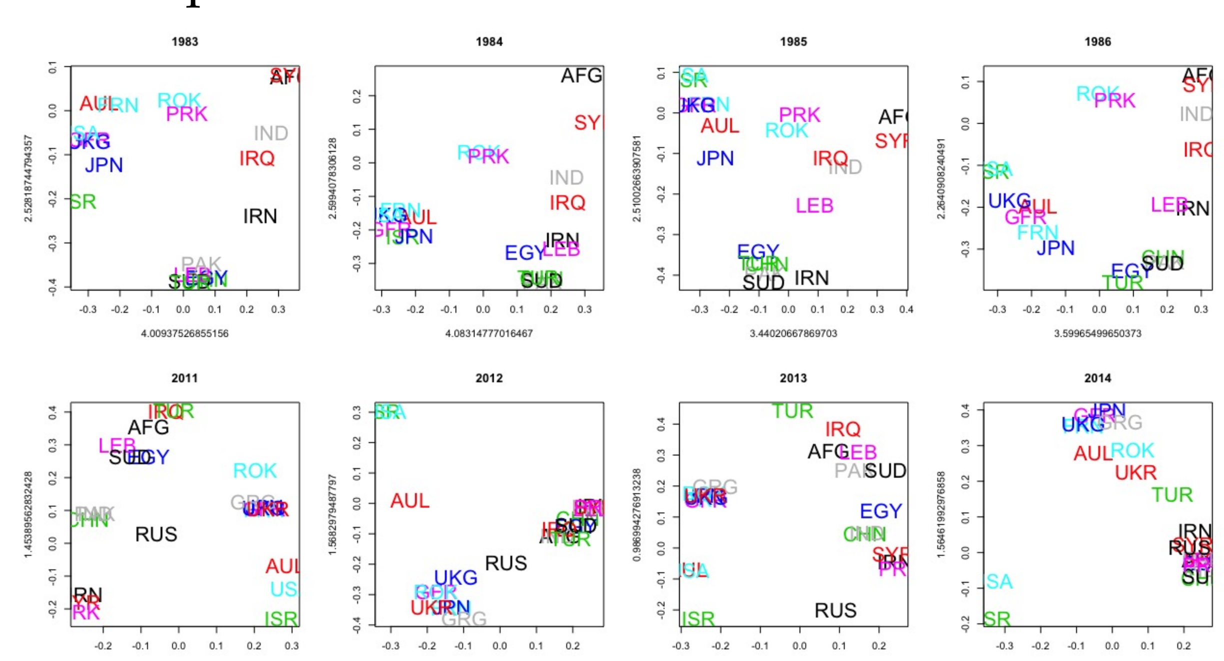
- Fixed effect estimates β



- Additive random effect estimates Θ



- Multiplicative random effect estimates U and D



Conclusions

1. DLFM estimates of fixed effects and random effects show interesting foreign policy trends in UN voting behaviors, revealing noticeable difference in the Cold War era and post-Cold War era
2. Computationally outperform the multiple fittings of static model (R package 'AMEN' [2])

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