Dynamic Additive and Multiplicative Effects (DAME) Network Model with Application to the United Nations Voting Behaviors

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Abstract

- Network regression model for symmetric continuous-valued networks in discrete timepoints
- Extension of additive-multiplicative effects (AME) latent factor network model [2] to fit dynamic networks with time-varying coefficients
- Estimate the Gaussian covariance structure [1] to infer the temporal dependence
- Our Findings:
  1. DAME framework is effective at predicting and explaining networks that are highly-correlated across timepoints
  2. Inference method achieves faster convergence and accuracy in parameter estimates, compared to the existing method
  3. DAME's visualization is useful to understand any temporal trends in networks as well as nodes' latent positions
  4. Additive and multiplicative random effects can capture the second-order and third-order dependencies in networks such as reciprocity, transitivity, and clustering

UN Voting Data

- Original dataset [3] contains all roll-call votes in the United Nations General Assembly
- Response variable $\boldsymbol{Y}$ ($32 \times 23 \times 23$ - dim. array):
  - Voting similarity index (0-1) computed using 3 category vote data, only considering the important votes
  - Years: from 1983 to 2014 (32 timepoints)
  - Nodes: 23 most active countries in international relations
    USA UKG FRN GMY RUS UKR GRG SUD IRN TUR IRQ EGY SYR LEB ISR AFG CHN PRK ROK JPN IND PAK AUL
- Explanatory variables $\boldsymbol{X}$ ($32 \times 23 \times 23 \times 5$ - dim. array):
  1. Intercept: constant 1 (to estimate yearly baseline)
  2. log(distance$_{ij}$): log of the geographic distance between country i and country j
  3. Alliance$_{ijt}$: 1 if country i and country j have alliance in year t, and 0 otherwise
  4. Polity score difference$_{ijt}$: absolute difference in polity IV number between country i and country j, in year t
  5. Relative trade$_{ijt}$: measure of relative importance of the trade between country i and country j in year t, using
  $$min\left(\frac{\text{Trade}_{ijt}}{\text{GDP}_{it}}, \frac{\text{Trade}_{ijt}}{\text{GDP}_{jt}}\right)$$
  6. Common language$_{ij}$: indicator of whether country i and country j share the same language

Modeling Framework

- For the symmetric matrices $\boldsymbol{Y}(t=1), ..., \boldsymbol{Y}(t=T)$,

$$\boldsymbol{Y}_{N\times N}(t) = \sum_{p=1}^{P} X^p(t)\beta_p(t) + \Theta(t) + \Theta'(t) + U(t)D(t)U'(t) + E(t),$$

  where
  1. $X(t)$ and $\beta(t)$: fixed effects from predictors
  2. $\Theta(t)$ and $\Theta'(t)$: additive row/column random effect
  3. $U(t)D(t)U'(t)$: multiplicative random effect
     - $U(t)$ is $N \times R$ matrix where $U_i(t) = (u_{i1}(t), ..., u_{iR}(t))$
     - $D(t)$ is $R \times R$ matrix ($= \text{diag}(d_1(t), ..., d_R(t))$)
  4. $E(t)$: random noise matrix
- Gaussian process (GP) prior specfication [1]:
  1. For each covariate $p = 1, ..., P$;
  $$\{\beta_p(t)\}_{t=1}^T \sim \text{MVN}_T(0, \tau_p^\beta c_\beta), \text{ with } \tau_p^\beta \sim \text{IG}(a_\beta, b_\beta)$$
  2. For each node $i = 1, ..., N$;
  $$\{\theta_i(t)\}_{t=1}^T \sim \text{MVN}_T(0, \tau^\theta c_\theta), \text{ with } \tau^\theta \sim \text{IG}(a_\theta, b_\theta)$$
  3. For each dimension $r = 1, ..., R$;
  $$\{d_r(t)\}_{t=1}^T \sim \text{MVN}_T(0, c_d),$$
  4. For each node $i = 1, ..., N$; and dimension $r = 1, ..., R$;
  $$\{u_{ir}(t)\}_{t=1}^T \sim \text{MVN}_T(0, \tau_r^u I_T) \text{ with } \tau_r^u \sim \text{IG}(a_d, b_d)$$
  5. For each pair of (i, j) with $i > j$,
  $$\epsilon_{ij}(t) \sim \text{N}(0, \sigma_e^2), \text{ with } \sigma_e^2 \sim \text{IG}(a, b),$$

  where c_* is $T \times T$ correlation matrix corresponding to $* \in \{\beta, \theta, d, u\}$ obtained from the GP function
  $$c_*(t, t') = \exp(-\frac{|t-t'|}{\kappa_*})$$
  Note: each value of κ_* and the corresponding proper covariance function is estimated.
- Bayesian Inference using Gibbs sampling: Sequentially resample each of latent variables
$$\Big(\sigma_e^2, \{\tau_p^\beta\}_{p=1}^P, \tau^\theta, \{\tau_r^u\}_{r=1}^R, \{\{\beta_p(t)\}_{t=1}^T\}_{p=1}^P, \{\{\theta_i(t)\}_{t=1}^T\}_{i=1}^N, \{\{d_r(t)\}_{t=1}^T\}_{r=1}^R, \{\{\{u_{ir}(t)\}_{t=1}^T\}_{i=1}^N\}_{r=1}^R\Big)$$
  from the full conditional distributions
- Metropolis-Hastings algorithm for Gaussian process length parameters c_*

Estimate Covariance structure

1. Scanning with temporal independence

for *iter=1 to scan* **do**
  run MCMC with $\kappa_* = 0.001$ (which gives $c_* = I_T$)
end

2. Estimating covariance structure

for $*$ *in* $(\beta_1, ..., \beta_P, \theta, d, u)$ **do**
  calculate the correlation $\rho^*_{t,t'}$ and compare the fittings
  - Exponential: $\log(\rho^*_{t,t'}) = -\frac{1}{\kappa}|t-t'|$
  - Sq. Exponential: $\log(\rho^*_{t,t'}) = -\frac{1}{\kappa^2}||t-t'||_2^2$
  construct $\hat{c}_*$ given $\hat{\kappa}_*$ and covfc with better fit
end

3. Running with temporal dependence

for *iter=1 to nsample* **do**
  run MCMC with estimated $\hat{c}_* = (\hat{c}_\beta, \hat{c}_\theta, \hat{c}_d, \hat{c}_u)$
end
Summarize the results only using the new samples with estimated temporal dependence structure

Varying number of nodes

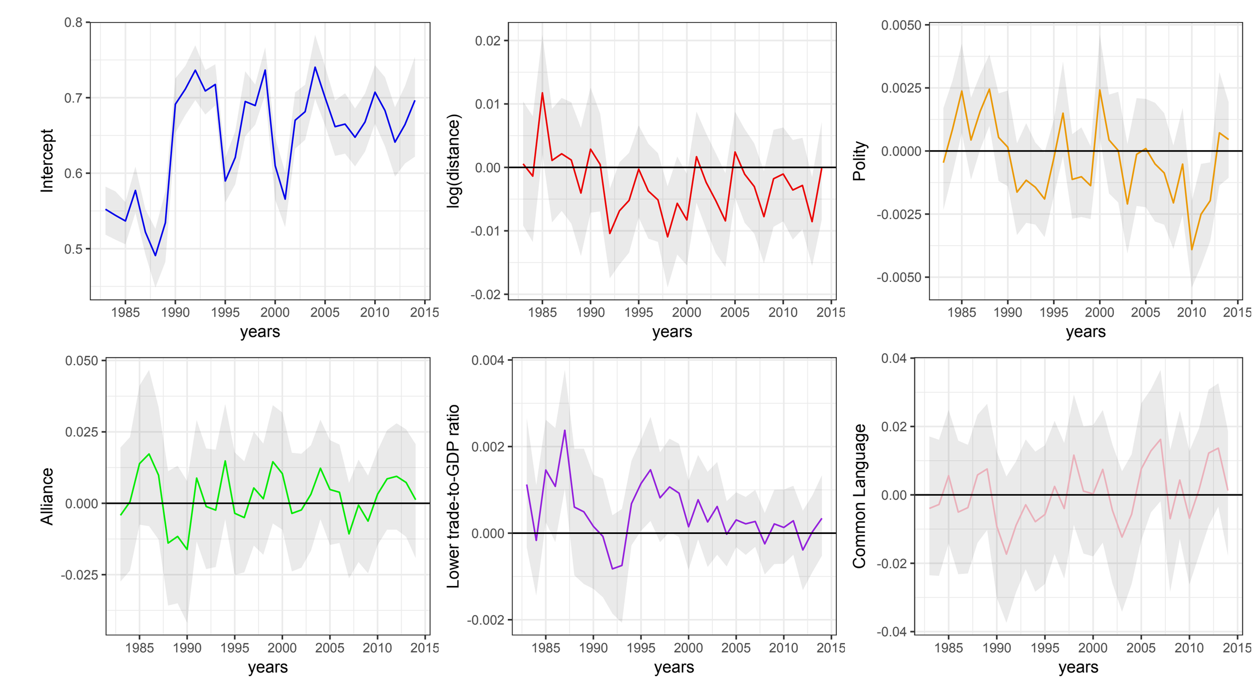
- New node can join or existing node can disappear at any timepoint (ex. countries not existed: RUS ~1991, UKR ~1991, GRG ~1992; countries in war: IRQ 1995-2003; countries joined UN later: ROK ~1990, PRK ~1990)
- Allow "structural NA's" remain as NA, while pure missing values imputed using posterior estimates
  1. Reduce bias in fixed effect estimates β
  2. Avoid meaningless random effect estimates Θ and U
  3. Provide flexibility in fitting the model to larger networks

Conclusions

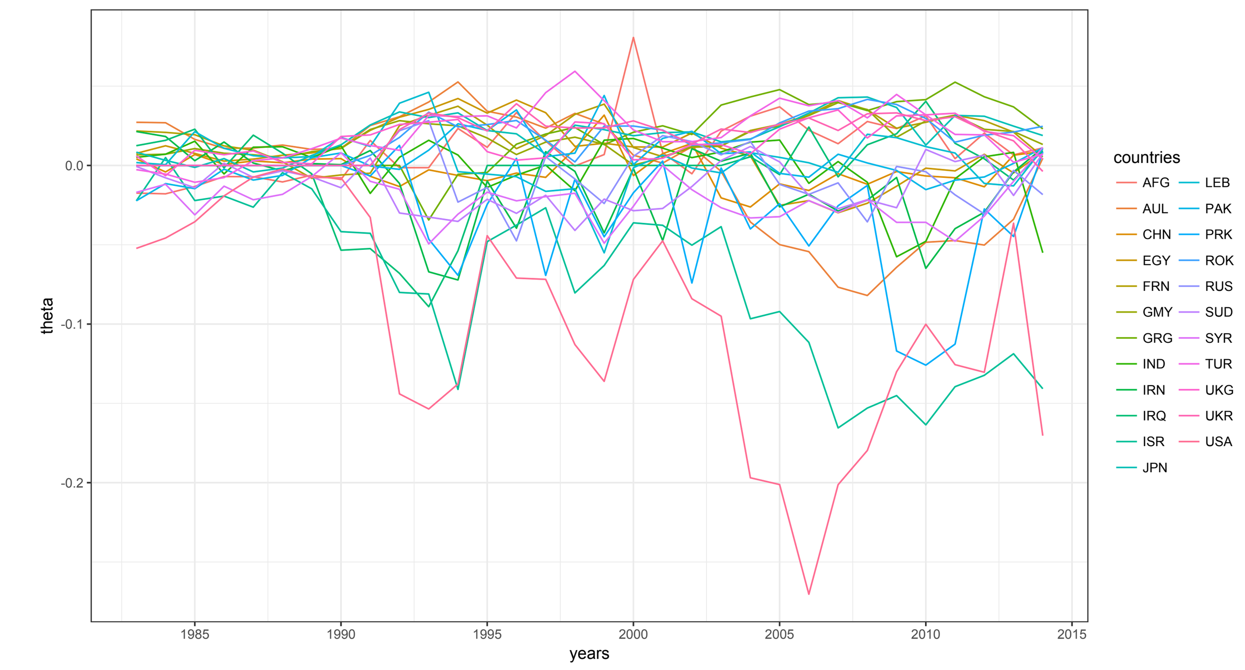
1. DAME results show interesting foreign policy trends in UN voting behaviors, revealing noticeable difference during the Cold War and post-Cold War
2. Better predictive performance compared to the yearly fittings of static model [2] and multiplicative effect only model [1]

Results

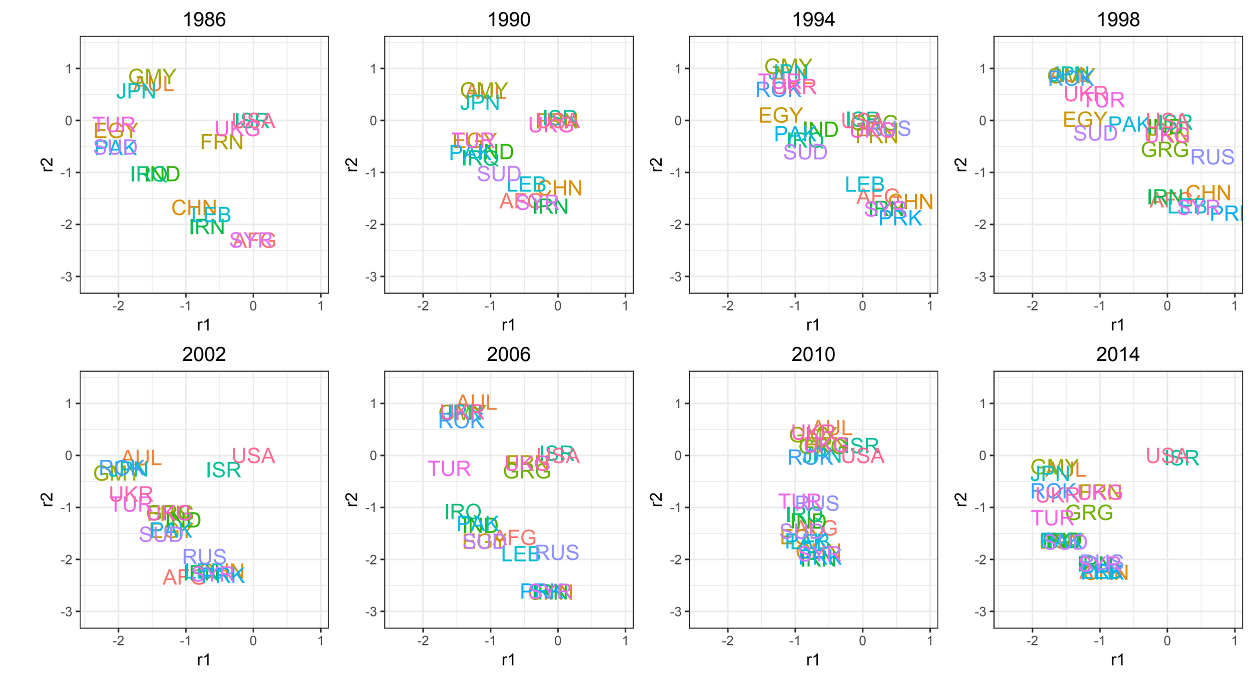
- Fixed effect estimates β



- Additive random effect estimates Θ



- Multiplicative random effect estimates U and D



References

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[3] Voeten, E., Strezhnev, A., and Bailey, M. (2016). United nations general assembly voting data.

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