A Network Model for Dynamic Textual Communications with Application to Government Email Corpora

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Interaction-Partitioned Topic Model (IPTM)

- Probablistic model for time-stamped textual communications (e.g. emails, cosponsorship of bills, international sanctions)
- Integration of two generative models:
  - Latent Dirichlet allocation (LDA) for topic-based contents
  - Dynamic exponential random graph model (ERGM) for ties
- IPTM assigns each topic to an "interaction pattern," which is governed by a set of dynamic network features

"who communicates with whom about what, and when?"

Content Generating Process: LDA (Blei et al., 2003)

- For each topic $k = 1, ..., K$:
  1. Topic-word distribution $\boldsymbol{\phi}^{(k)} \sim$ Dirichlet$(\beta, \mathbf{u})$
  2. Topic-IP distribution $c_k \sim$ Uniform$(1, C)$

- For each document $d = 1, ..., D$:

  3-1. Document-topic distribution $\boldsymbol{\theta}^{(d)} \sim$ Dirichlet$(\alpha, \boldsymbol{m})$

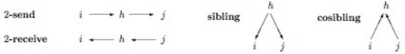
  3-2. For each word in a document $n = 1$ to $N^{(d)}$:
  (a) Choose a topic $z_n^{(d)} \sim$ Multinomial$(\boldsymbol{\theta}^{(d)})$
  (b) Choose a word $w_n^{(d)} \sim$ Multinomial$(\boldsymbol{\phi}^{(z_n^{(d)})})$

  3-3 Calculate the distribution of interaction patterns within a document:

$$p_c^{(d)} = \Big(\sum_{k:c_k=c} N^{(k|d)}\Big)/N^{(d)}, \qquad (1)$$

Dynamic Network Features (Perry and Wolfe, 2012)

- $\boldsymbol{x}_t^{(c)}(i,j)$ is the network statistics at time t, for interaction pattern c
  - Degree: outdegree and indegree
  - Dyadic: send and receive
  - Triadic: 2-send, 2-receive, sibling and cosibling



- Partition the past 384 hours (=16 days) into 3 sub-intervals

$$[t-384h,t) = [t-384h,t-96h) \cup [t-96h,t-24h) \cup [t-24h,t),$$

then define the interval-based statistics for $l \in \{1,2,3\}$ and $c \in \{1,...,C\}$

$$\textbf{outdegree}_{t,l}^{(c)}(i) = \sum_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{i \rightarrow \forall j\} \quad \textbf{send}_{t,l}^{(c)}(i,j) = \sum_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{i \rightarrow j\}$$

$$\textbf{indegree}_{t,l}^{(c)}(j) = \sum_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{\forall i \rightarrow j\} \quad \textbf{receive}_{t,l}^{(c)}(i,j) = \sum_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{j \rightarrow i\}$$

Stochastic Intensity

- $\lambda_{ij}^{(d)}(t)$=P{for document d, $i \to j$ occurs in time interval $[t, t+dt)$:

$$\lambda_{ij}^{(d)}(t) = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \boldsymbol{b}^{(c)T}\boldsymbol{x}_t^{(c)}(i,j)\Big\}, \tag{2}$$

where $\lambda_0^{(c)}$ is the baseline hazards for the interaction pattern c and $\boldsymbol{b}^{(c)}$ is a vector of coefficients in $\boldsymbol{R}^p$.

- For multicast interactions – single sender i and multiple receivers J:

$$\lambda_{iJ}^{(d)}(t) = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \frac{1}{|J|}\sum_{j \in J} \boldsymbol{b}^{(c)T}\boldsymbol{x}_t^{(c)}(i,j)\Big\}, \tag{3}$$

which is obtained by taking the average of $\boldsymbol{b}^{(c)T}\boldsymbol{x}_t^{(c)}(i,j)$ across the receivers.

- Probability of i sends a document to j (or J) is a mixture of contents and history of interactions

Tie Generating Process

1. For each sender $i \in \{1, ..., A\}$, choose a binary vector $J_i^{(d)}$ of length $(A-1)$, by applying Gibbs measure (Fellows and Handcock, 2017)

$$\mathsf{P}(J_i^{(d)}) = \frac{1}{Z(\delta, \log(\lambda_i^{(d)}))} \exp\Big\{\log\big(\mathsf{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)\big) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}, \quad (4)$$

   where δ is a real-valued intercept controlling the recipient size and $Z(\delta, \log(\lambda_i^{(d)}))$ is the normalizing constant.

2. For each sender $i \in \mathcal{A}$, generate the time increments

$$\Delta T_{iJ_i} \sim \mathsf{Exp}(\lambda_{iJ_i}^{(d)}).$$

3. Set timestamp, sender, and receivers simultaneously:

$$t^{(d)} = t^{(d-1)} + \mathsf{min}(\Delta T_{iJ_i})$$
$$i^{(d)} = i_{\mathsf{min}(\Delta T_{iJ_i})}$$
$$J^{(d)} = J_{i^{(d)}}$$

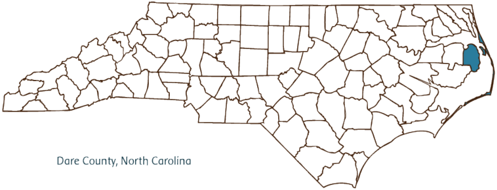
Inference - Pseudocode

Algorithm 1 Markov Chain Monte Carlo (MCMC)

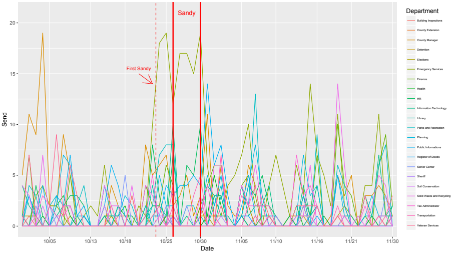
```
Set initial values $\mathcal{Z}^{(0)}$, $\mathcal{C}^{(0)}$, and $(\mathcal{B}^{(0)}, \delta^{(0)})$
for o=1 to O do
    for d=1 to D do
        for $i \in \mathcal{A}_{\setminus i_o^{(d)}}$ do
            for $j \in \mathcal{A}_{\setminus i}$ do
                Sample the latent edge $J_{ij}^{(d)}$ via Gibbs sampling
            end
        end
        for n=1 to $N^{(d)}$ do
            Sample the topic assignments via Gibbs sampling
            $z_n^{(d)} \sim \text{Multinomial}(p^{\mathcal{Z}})$
        end
    end
    for k=1 to K do
        Sample the interaction pattern assignments via Gibbs sampling
        $C_k \sim \text{Multinomial}(p^{\mathcal{C}})$
    end
    for n=1 to $n_B$ do
        Sample the interaction pattern parameters $\mathcal{B}$ via Metropolis-Hastings
    end
    for n=1 to $n_\delta$ do
        Sample the receiver size parameter $\delta$ via Metropolis-Hastings
    end
end
```

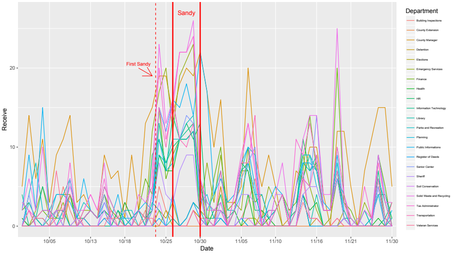
Data: North Carolina Dare county email data

- $D = 1456$ emails between $A = 27$ county government managers, covering 2 month periods (October 1 - November 30) in 2013



Effect of Hurricane Sandy on Email Exchange





IPTM Result

Conclusion

- Joint modeling of ties (sender, receiver, time) and contents
- Allowance of multicast – multiple senders and/or receivers
- Possible application to