# Poisson Tucker Decomposition version of the Interaction-pattern Partitioned Topic Model

Bomin Kim

March 15, 2018

## 1 Generative Process

To maintain single interaction pattern assignments (instead of admixture form which adds huge complexity in network history calculations), we assume each document $d \in [D]$ draws an interaction pattern $c_d$ as below:

$$c_d \sim \text{Multinomial}(\boldsymbol{\psi}), \tag{1}$$

where

$$\psi \sim \text{Dirichlet}\Big(\xi, (\frac{1}{C}, \ldots, \frac{1}{C})\Big). \tag{2}$$

Alternatively, following degree-corrected GPIRM we can use

$$c_d \sim \text{Multinomial}(\frac{\psi_1}{\sum_c \psi_c}, \ldots, \frac{\psi_C}{\sum_c \psi_c}), \tag{3}$$

where

$$\psi_c \sim \Gamma(\frac{\gamma_0}{C}, \xi). \tag{4}$$

Next, we model the contents using Poisson Tucker Decomposition of Schein et al. (2016). First, each document $d \in [D]$

$$\pi_{dc} = \begin{cases} \sim \Gamma(a_c, b) & \text{if } c_d = c \\ 0 & \text{if } c_d \neq c \end{cases} \tag{5}$$

Q: If we want this positive real numbers instead of indicator (such as degree-corrected GPIRM). Is this how you did? If so, how to derive the conditionals for Gibbs sampling of $c_d$? If not, how to specify the prior for $\pi_{dc}$ so as to achieve/enforce single membership constraints?

Then, each interaction pattern $c \in [C]$ has the IP-specific topic distribution

$$\theta_{ck} \sim \Gamma(\epsilon_0, \epsilon_0), \tag{6}$$

and each topic $k \in [K]$ has the topic-word distribution

$$\phi_{kv} \sim \Gamma(\epsilon_0, \epsilon_0). \tag{7}$$

Then, the number of tokens of type $v$ in document $d$ is

$$w_{dv} \sim \text{Poisson}(\sum_{c=1}^{C}\sum_{k=1}^{K} \pi_{dc}\theta_{ck}\phi_{kv}), \tag{8}$$

which is identical to $w_{dv} \sim \text{Poisson}(\pi_{dc_d}\sum_{k=1}^{K}\theta_{c_d k}\phi_{kv})$. Also note that $\boldsymbol{w}_d = (w_{d1}, \ldots, w_{dV})$ is a very sparse vector with $\text{sum}(\boldsymbol{w}_d) = N_d$.

## 2 Derivation

We first derive the sampling equation of $\pi$, $\theta$ and $\phi$, respectively. First, for $c = c_d$, we update $\pi_{dc}$ as below.

$$\pi_{dc}|\text{rest} \sim \text{Gamma}(a_c + \boldsymbol{w}_{dc}, b + \sum_{v=1}^{V}\theta_{ck}\phi_{kv}), \tag{9}$$

where $\boldsymbol{w}_{dc} = \sum_{k=1}^{K}\sum_{v=1}^{V} w_{dvck}$ with $w_{dvck} \sim \text{Multinomial}(w_{dv}, \pi_{dc}\theta_{ck}\phi_{kv})$.

$$\theta_{ck}|\text{rest} \sim \text{Gamma}(\epsilon_0 + \boldsymbol{w}_{ck}, \epsilon_0 + \sum_{d=1}^{D}\pi_{dc}\sum_{v=1}^{V}\phi_{kv}), \tag{10}$$

where $\boldsymbol{w}_{ck} = \sum_{d=1}^{D}\sum_{v=1}^{V} w_{dvck}$ with $w_{dvck} \sim \text{Multinomial}(w_{dv}, \pi_{dc}\theta_{ck}\phi_{kv})$.

$$\phi_{kv}|\text{rest} \sim \text{Gamma}(\epsilon_0 + \boldsymbol{w}_{kv}, \epsilon_0 + \sum_{d=1}^{D}\pi_{dc}\theta_{ck}), \tag{11}$$

where $\boldsymbol{w}_{kv} = \sum_{d=1}^{D}\sum_{c=1}^{C} w_{dvck}$ with $w_{dvck} \sim \text{Multinomial}(w_{dv}, \pi_{dc}\theta_{ck}\phi_{kv})$.

Then, we need to use Gibbs update of $c_d$

$$\Pr(c_d = c|\text{rest}) =? \tag{12}$$