Suplementary Materials for "A Network Model for Dynamic Textual Communications with Application to Government Email Corpora"

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1. Normalizing constant of Gibbs measure

The non-empty Gibbs measure defines the probability of author a selecting the binary recipient vector $\boldsymbol{u}_{ad}$ as

$$P(\boldsymbol{u}_{ad}|\delta, \boldsymbol{\lambda}_{ad}) = \frac{\exp\Big\{\log\big(\mathrm{I}(\|\boldsymbol{u}_{ad}\|_1 > 0)\big) + \sum_{r\neq a}(\delta+\lambda_{adr})u_{adr}\Big\}}{Z(\delta, \boldsymbol{\lambda}_{ad})}.$$

To use this distribution efficiently, we derive a closed-form expression for $Z(\delta, \boldsymbol{\lambda}_{id})$ that does not require brute-force summation over the support of $\boldsymbol{u}_{ad}$ (*i.e.* $\forall \boldsymbol{u}_{ad} \in [0,1]^A$). We recognize that if $\boldsymbol{u}_{ad}$ were drawn via independent Bernoulli distributions in which $P(u_{adr} = 1|\delta, \boldsymbol{\lambda}_{ad})$ was given by logit$(\delta + \lambda_{adr})$, then

$$P(\boldsymbol{u}_{ad}|\delta, \boldsymbol{\lambda}_{ad}) \propto \exp\Big\{\sum_{r\neq a}(\delta+\lambda_{adr})u_{adr}\Big\}.$$

This is straightforward to verify by looking at

$$P(u_{adr} = 1|\boldsymbol{u}_{ad[-r]}, \delta, \boldsymbol{\lambda}_{ad}) = \frac{\exp(\delta+\lambda_{adr})}{\exp(\delta+\lambda_{adr})+1}.$$

We denote the logistic-Bernoulli normalizing constant as $Z^l(\delta, \boldsymbol{\lambda}_{ad})$, which is defined as

$$Z^l(\delta, \boldsymbol{\lambda}_{ad}) = \sum_{\boldsymbol{u}_{ad}\in[0,1]^A} \exp\Big\{\sum_{r\neq a}(\delta+\lambda_{adr})u_{adr}\Big\}.$$

Now, since

$$\exp\Big\{\log\Big(\mathrm{I}(\|\boldsymbol{u}_{ad}\|_1 > 0)\Big) + \sum_{r\neq a}(\delta+\lambda_{adr})u_{adr}\Big\} = \exp\Big\{\sum_{r\neq a}(\delta+\lambda_{adr})u_{adr}\Big\},$$

except when $\|\boldsymbol{u}_{ad}\|_1 = 0$, we note that

$$\begin{aligned} Z(\delta, \boldsymbol{\lambda}_{ad}) &= Z^l(\delta, \boldsymbol{\lambda}_{ad}) - \exp\Big\{\sum_{\forall u_{adr}=0}(\delta+\lambda_{adr})u_{adr}\Big\} \\ &= Z^l(\delta, \lambda_a^{(d)}) - 1. \end{aligned}$$

We can therefore derive a closed form expression for $Z(\delta, \boldsymbol{\lambda}_{ad})$ via a closed form expression for $Z^l(\delta, \boldsymbol{\lambda}_{ad})$. This can be done by looking at the probability of the zero vector under the logistic-Bernoulli model:

$$\frac{\exp\Big\{\sum_{\forall u_{adr}=0}(\delta+\lambda_{adr})u_{adr}\Big\}}{Z^l(\delta, \boldsymbol{\lambda}_{ad})} = \prod_{r\neq a}\Big(1 - \frac{\exp(\delta+\lambda_{adr})}{\exp(\delta+\lambda_{adr})+1}\Big).$$

Then, we have

$$\frac{1}{Z^l(\delta, \boldsymbol{\lambda}_{ad})} = \prod_{r\neq a}\frac{1}{\exp(\delta+\lambda_{adr})+1}.$$

Finally, the closed form expression for the normalizing constant under the non-empty Gibbs measure is

$$Z(\delta, \boldsymbol{\lambda}_{ad}) = \prod_{r\neq a}\big(\exp\{\delta+\lambda_{adr}\}+1\big) - 1.$$

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