Poisson Tucker Decomposition version of the Interaction-pattern Partitioned Topic Model

Bomin Kim

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1 Generative Process

To maintain single interaction pattern assignments (instead of admixture form which adds huge complexity in network history calculations), we assume each document $d \in [D]$ draws an interaction pattern c_d as below:

$$c_d \sim \text{Multinomial}(\frac{\psi_1}{\sum_c \psi_c}, \ldots, \frac{\psi_C}{\sum_c \psi_c}), \tag{1}$$

where

$$\psi_c \sim \Gamma(\frac{\gamma_0}{C}, \xi). \tag{2}$$

Next, we model the contents using Poisson Tucker Decomposition of Schein et al. (2016). First, each document $d \in [D]$ has Gamma weights

$$\pi_d \sim \Gamma(a, b). \tag{3}$$

Next, each interaction pattern $c \in [C]$ has the IP-specific topic distribution

$$\theta_{ck} \sim \Gamma(\epsilon_0, \epsilon_0), \tag{4}$$

and each topic $k \in [K]$ has the topic-word distribution

$$\phi_{kv} \sim \Gamma(\epsilon_0, \epsilon_0). \tag{5}$$

Then, the number of tokens of type v in document d is

$$w_{dv} \sim \text{Poisson}(\pi_d \sum_{c=1}^{C} \sum_{k=1}^{K} I_{dc}\theta_{ck}\phi_{kv}), \tag{6}$$

where I_{dc} is an indicator for $I(c = c_d)$. Note that Equation (6) above is identical to $w_{dv} \sim \text{Poisson}(\pi_d \sum_{k=1}^{K} \theta_{c_d k}\phi_{kv})$. Also note that $\boldsymbol{w}_d = (w_{d1}, \ldots, w_{dV})$ is a very sparse vector with $\text{sum}(\boldsymbol{w}_d) = N_d$.

2 Derivation

We first derive the sampling equation of π, θ and ϕ, respectively. First, we update π_d as below.

$$\pi_d|\text{rest} \sim \text{Gamma}(a + \boldsymbol{w}_{dc}, b + \sum_{v=1}^{V} \theta_{ck}\phi_{kv}), \tag{7}$$

where $\boldsymbol{w}_{dc} = \sum_{k=1}^{K}\sum_{v=1}^{V} w_{dvck}$ with $w_{dvck} \sim \text{Multinomial}(w_{dv}, \pi_{dc}\theta_{ck}\phi_{kv})$.

$$\theta_{ck}|\text{rest} \sim \text{Gamma}(\epsilon_0 + \boldsymbol{w}_{ck}, \epsilon_0 + \sum_{d=1}^{D} \pi_{dc} \sum_{v=1}^{V} \phi_{kv}), \tag{8}$$

where $\boldsymbol{w}_{ck} = \sum_{d=1}^{D}\sum_{v=1}^{V} w_{dvck}$ with $w_{dvck} \sim \text{Multinomial}(w_{dv}, \pi_{dc}\theta_{ck}\phi_{kv})$.

$$\phi_{kv}|\text{rest} \sim \text{Gamma}(\epsilon_0 + \boldsymbol{w}_{kv}, \epsilon_0 + \sum_{d=1}^{D} \pi_{dc}\theta_{ck}), \tag{9}$$

where $\boldsymbol{w}_{kv} = \sum_{d=1}^{D}\sum_{c=1}^{C} w_{dvck}$ with $w_{dvck} \sim \text{Multinomial}(w_{dv}, \pi_{dc}\theta_{ck}\phi_{kv})$.

Then, we need to use Gibbs update of c_d

$$\Pr(c_d = c|\text{rest}) =? \tag{10}$$