A Network Model for Dynamic Textual Communications with Application to Government Email Corpora

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1 Tie Generating Process

We assume the following generative process for each document d in a corpus D:

1. (Data augmentation) For each sender $i \in \{1, ..., A\}$, create a list of receivers denoted by an indicator vector J_i according to the probability mass function of Gibbs measure:

$$J_i^{(d)} = \frac{1}{Z(J_i^{(d)})} \exp\Big(\frac{\sum_{j \in \mathcal{A}_{\setminus i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j \in \mathcal{A}_{\setminus i}} J_{ij}^{(d)}}\Big), \qquad J_i^{(d)} \in \mathcal{J} \tag{1}$$

   where $Z(J_i^{(d)}) = \sum\limits_{\forall J_i^{(d)} \in \mathcal{J}} \exp\Big(\frac{\sum_{j \in \mathcal{A}_{\setminus i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j \in \mathcal{A}_{\setminus i}} J_{ij}^{(d)}}\Big)$ is the partition function assuring that the probabilities sum to unity, and $\mathcal{J}$ be the sample space in which $J_i^{(d)}$ may exist (in this case, the sample space consists of all binary $(A-1)$ length vectors). δ is a tolerance parameter that is used to prevent form model degeneracy.

2. For every sender $i \in \mathcal{A}$, generate the time increments

$$\Delta T_{iJ_i} \sim \text{Exp}(\lambda_{iJ_i}^{(d)}). \tag{2}$$

3. Set timestamp, sender, and receivers simultaneously (NOTE: $t^{(0)} = 0$):

$$\begin{aligned} t^{(d)} &= t^{(d-1)} + \min(\Delta T_{iJ_i}), \\ i^{(d)} &= i_{\min(\Delta T_{iJ_i})}, \\ J^{(d)} &= J_{i^{(d)}}. \end{aligned} \tag{3}$$

2 Inference

$$
\begin{aligned}
&P(\mathcal{J}_{\text{a}}^{(d)}, \mathcal{T}_{\text{a}}^{(d)}, i_{\text{o}}^{(d)}, J_{\text{o}}^{(d)}, t_{\text{o}}^{(d)} | \mathcal{I}_{\text{o}}^{(<d)}, \mathcal{J}_{\text{o}}^{(<d)}, \mathcal{T}_{\text{o}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&= P\Big(\text{latent receivers generation}\Big) \times P\Big(\text{latent time generation}\Big) \times P\Big(\text{choose the observed}\Big) \\
&= \prod_{i \in \mathcal{A}} \Big(J_i^{(d)} \propto \exp\Big(\frac{\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big)\Big) \times \prod_{i \in \mathcal{A}} \Big(\Delta T_{iJ_i}^{(d)} \sim \text{Exp}(\lambda_{iJ_i}^{(d)})\Big) \times \prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} P\Big(\Delta T_{iJ_i}^{(d)} > \Delta T_{i_o^{(d)} J_o^{(d)}}\Big) \\
&= \Big(\prod_{i \in \mathcal{A}} \exp\Big(\frac{\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big)\Big) \times \Big(\prod_{i \in \mathcal{A}} \lambda_{iJ_i}^{(d)} e^{-\Delta T_{iJ_i}^{(d)} \lambda_{iJ_i}^{(d)}}\Big) \times \Big(\prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{iJ_i^{(d)}}^{(d)}}\Big) \\
&= \Big(\prod_{i \in \mathcal{A}} \exp\Big(\frac{\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big)\Big) \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)} e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{i_o^{(d)} J_o^{(d)}}^{(d)}}\Big) \times \Big(\prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} \lambda_{iJ_i}^{(d)} e^{-(\Delta T_{iJ_i}^{(d)} + \Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)}) \lambda_{iJ_i}^{(d)}}\Big),
\end{aligned} \tag{4}
$$

We can simplify this further by integreting out the latent time $\mathcal{T}_{\text{a}}^{(d)} = \{\Delta T_{iJ_i}^{(d)}\}_{i \in \mathcal{A}_{\backslash i_o^{(d)}}}$ in the last term as before, then we can simplify Equation (4) as below:

$$
\begin{aligned}
&P(\mathcal{J}_{\text{a}}^{(d)}, i_{\text{o}}^{(d)}, J_{\text{o}}^{(d)}, t_{\text{o}}^{(d)} | \mathcal{I}_{\text{o}}^{(<d)}, \mathcal{J}_{\text{o}}^{(<d)}, \mathcal{T}_{\text{o}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&= \Big(\prod_{i \in \mathcal{A}} \exp\Big(\frac{\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big)\Big) \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)}\Big) \times \Big(e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \sum\limits_{i \in \mathcal{A}} \lambda_{iJ_i^{(d)}}^{(d)}}\Big),
\end{aligned} \tag{5}
$$

where this joint distribution can be interpreted as 'probability of latent and observed edges from Bernoulli distribution × probability of the observed time comes from Exponential distribution × probability of all latent time greater than the observed time, given that the latent time also come from Exponential distribution.'

2.1 Inference on the augmented data $\mathcal{J}_{\text{a}}$

Given the observed sender of the document $i_o^{(d)}$, we sample the latent receivers for each sender $i \in \mathcal{A}_{\backslash i_o^{(d)}}$. Here we illustrate how each sender-receiver pair in the document d is updated.

Define $\mathcal{J}_i^{(d)}$ be the $(A-1)$ length random vector of indicators (0/1) with its realization being $J_i^{(d)}$, representing the latent receivers corresponding to the sender i in the document d. For each sender i, we are going to resample $J_{ij}^{(d)}$, which is the j^{th} element of the receiver vector $J_i^{(d)}$, one at a time with random order.

$$
\begin{aligned}
&P(\mathcal{J}_{ij}^{(d)} = J_{ij}^{(d)} | \mathcal{J}_{i\backslash j}^{(d)}, i_{\text{o}}^{(d)}, J_{\text{o}}^{(d)}, t_{\text{o}}^{(d)}, \mathcal{I}_{\text{o}}^{(<d)}, \mathcal{J}_{\text{o}}^{(<d)}, \mathcal{T}_{\text{o}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \boldsymbol{\delta}) \\
&\propto P(\mathcal{J}_i^{(d)} = J_i^{(d)}, \mathcal{J}_{i\backslash j}^{(d)}, i_{\text{o}}^{(d)}, J_{\text{o}}^{(d)}, t_{\text{o}}^{(d)} | \mathcal{I}_{\text{o}}^{(<d)}, \mathcal{J}_{\text{o}}^{(<d)}, \mathcal{T}_{\text{o}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \boldsymbol{\delta}) \\
&\propto \exp\Big(\frac{\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big) \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)}\Big) \times \Big(e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{iJ_i^{(d)}}^{(d)}}\Big) \\
&\propto \exp\Big(\frac{\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big) \times \Big(e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{iJ_i^{(d)}}^{(d)}}\Big),
\end{aligned} \tag{6}
$$

where we replace typical use of $(-d)$ to $(< d)$ on the right hand side of the conditional probability, due to the fact that $d^{(th)}$ document only depends on the past documents, not on the future ones.

To be more specific, since $J_{ij}^{(d)}$ could be either 1 or 0, we divide into two cases as below:

$$\begin{aligned} &P(\mathcal{J}_{ij}^{(d)} = 1 | \mathcal{J}_{\backslash ij}^{(d)}, i_{\text{O}}^{(d)}, J_{\text{O}}^{(d)}, t_{\text{O}}^{(d)}, \mathcal{I}_{\text{O}}^{(<d)}, \mathcal{J}_{\text{O}}^{(<d)}, \mathcal{T}_{\text{O}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\ &\propto \exp\Big(\frac{\sum_{j\in\mathcal{A}_{\backslash i}} J_{i[+j]}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j\in\mathcal{A}_{\backslash i}} J_{i[+j]}^{(d)}} - \Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{iJ_{i[+j]}^{(d)}}^{(d)}\Big), \end{aligned} \tag{7}$$

where $J_{i[+j]}^{(d)}$ meaning that the j^{th} element of $J_i^{(d)}$ is fixed as 1. On the other hand,

$$\begin{aligned} &P(\mathcal{J}_{ij}^{(d)} = 0 | \mathcal{J}_{\backslash ij}^{(d)}, i_{\text{O}}^{(d)}, J_{\text{O}}^{(d)}, t_{\text{O}}^{(d)}, \mathcal{I}_{\text{O}}^{(<d)}, \mathcal{J}_{\text{O}}^{(<d)}, \mathcal{T}_{\text{O}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\ &\propto \exp\Big(\frac{\sum_{j\in\mathcal{A}_{\backslash i}} J_{i[-j]}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j\in\mathcal{A}_{\backslash i}} J_{i[-j]}^{(d)}} - \Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{iJ_{i[-j]}^{(d)}}^{(d)}\Big), \end{aligned} \tag{8}$$

where $J_{i[-j]}^{(d)}$ meaning similarly that the j^{th} element of $J_i^{(d)}$ is fixed as 0.

Now we can use multinomial sampling using the two probabilities, Equation (7) and Equation (8). Again, we would calculate the probabilities in the log space to prevent from numerical underflow.

2.2 Inference on $\mathcal{Z}$

same as before but edge probability part changed to $\Big(\prod_{i\in\mathcal{A}} \exp\Big(\frac{\sum_{j\in\mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j\in\mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big)\Big)$

2.3 Inference on $\mathcal{C}$

same as before but edge probability part changed to $\Big(\prod_{i\in\mathcal{A}} \exp\Big(\frac{\sum_{j\in\mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j\in\mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big)\Big)$

2.4 Inference on $\mathcal{B}$ and δ

same as before but edge probability part changed to $\Big(\prod_{i\in\mathcal{A}} \exp\Big(\frac{\sum_{j\in\mathcal{A}_{\backslash i}} J_{ij}^{(d)} \lambda_{ij}^{(d)} - \delta}{\sum\limits_{j\in\mathcal{A}_{\backslash i}} J_{ij}^{(d)}}\Big)\Big)$