# A Network Model for Dynamic Textual Communications with Application to Government Email Corpora

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## Interaction-Partitioned Topic Model (IPTM)

- Probablistic model for time-stamped textual communications (e.g. emails, cosponsorship of bills, international sanctions)
- Integration of two generative models:
  - Latent Dirichlet allocation (LDA) for topic-based contents
  - Dynamic exponential random graph model (ERGM) for ties
- IPTM assigns each topic to an "interaction pattern," which is governed by a set of dynamic network features

"who communicates with whom about what, and when?"

## Content Generating Process: LDA (Blei et al., 2003)

- For each topic k = 1, ..., K:
  - 1. Topic-word distribution  $\phi^{(k)} \sim \text{Dirichlet}(\beta, \mathbf{u})$
  - 2. Topic-IP distribution  $c_k \sim \mathsf{Uniform}(1,C)$
- For each document d = 1, ..., D:
  - 3-1. Document-topic distribution  $\theta^{(d)} \sim \text{Dirichlet}(\alpha, m)$
  - 3-2. For each word in a document n=1 to  $N^{(d)}$ :
    - (a) Choose a topic  $z_n^{(d)} \sim \mathsf{Multinomial}(\pmb{\theta}^{(d)})$
    - (b) Choose a word  $w_n^{(d)} \sim \text{Multinomial}(\phi^{(z_n^{(d)})})$
  - 3-3 Calculate the distribution of interaction patterns within a document:

$$p_c^{(d)} = \left(\sum_{k:c_k = c} N^{(k|d)}\right) / N^{(d)},\tag{1}$$



## Dynamic Network Features (Perry and Wolfe, 2012)

- $x_1^{(c)}(i,j)$  is the network statistics at time t, for interaction pattern c
  - Degree: outdegree and indegree
  - Dyadic: send and receive
  - Triadic: 2-send, 2-receive, sibling and cosibling

• Partition the past 384 hours (=16 days) into 3 sub-intervals

$$[t-384h,t)=[t-384h,t-96h)\cup[t-96h,t-24h)\cup[t-24h,t),$$

then define the interval-based statistics for  $l \in \{1, 2, 3\}$  and  $c \in \{1, ..., C\}$ 

$$\begin{aligned} & \text{outdegree}_{t,l}^{(c)}(i) = \sum\limits_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{i \rightarrow \forall j\} & \text{send}_{t,l}^{(c)}(i,j) = \sum\limits_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{i \rightarrow j\} \\ & \text{indegree}_{t,l}^{(c)}(j) = \sum\limits_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{\forall i \rightarrow j\} & \text{receive}_{t,l}^{(c)}(i,j) = \sum\limits_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{j \rightarrow i\} \end{aligned}$$

### Stochastic Intensity

•  $\lambda_{i,i}^{(d)}(t) = P\{\text{for document } d, i \to j \text{ occurs in time interval } [t, t + dt):$ 

$$\lambda_{ij}^{(d)}(t) = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\left\{\lambda_0^{(c)} + \boldsymbol{b}^{(c)T} \boldsymbol{x}_t^{(c)}(i,j)\right\},\tag{2}$$

where  $\lambda_0^{(c)}$  is the baseline hazards for the interaction pattern c and  $b^{(c)}$  is a vector of coefficients in  $\mathbf{R}^p$ .

• For multicast interactions – single sender i and multiple receivers J:

$$\lambda_{iJ}^{(d)}(t) = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\left\{\lambda_0^{(c)} + \frac{1}{|J|} \sum_{j \in J} \boldsymbol{b}^{(c)T} \boldsymbol{x}_t^{(c)}(i,j)\right\},\tag{3}$$

which is obtained by taking the average of  $m{b}^{(c)T}m{x}_t^{(c)}(i,j)$  across the receivers.

• Probability of i sends a document to j (or J) is a mixture of contents and history of interactions

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### Tie Generating Process

1. For each sender  $i \in \{1,...,A\}$ , choose a binary vector  $J_i^{(d)}$  of length (A-1), by applying Gibbs measure (Fellows and Handcock, 2017)

$$\mathsf{P}(J_i^{(d)}) = \frac{1}{Z(\delta, \log(\lambda_i^{(d)}))} \exp\Big\{ \log \Big( \mathsf{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0) \Big) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)} \Big\}, \ \ \textbf{(4)}$$

where  $\delta$  is a real-valued intercept controlling the recipient size and  $Z(\delta, \log(\lambda_i^{(d)}))$  is the normalizing constant.

2. For each sender  $i \in \mathcal{A}$ , generate the time increments

$$\Delta T_{iJ_i} \sim \mathsf{Exp}(\lambda_{iJ_i}^{(d)}).$$

3. Set timestamp, sender, and receivers simultaneously:

$$\begin{split} t^{(d)} &= t^{(d-1)} + \min(\Delta T_{iJ_i}) \\ i^{(d)} &= i_{\min(\Delta T_{iJ_i})} \\ J^{(d)} &= J_{i^{(d)}} \end{split}$$



#### Inference - Pseudocode

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Algorithm 1 Markov Chain Monte Carlo (MCMC)
Set initial values \mathcal{Z}^{(0)}, \mathcal{C}^{(0)}, and (\mathcal{B}^{(0)}, \delta^{(0)})
for o=1 to O do
    for d=1 to D do
         for i \in \mathcal{A}_{\backslash i^{(d)}} do
             for i \in A_{\setminus i} do
                  Sample the latent edge J_{ij}^{(d)} via Gibbs sampling
             end
         end
         for n=1 to N^{(d)} do
             Sample the topic assignments via Gibbs sampling
             z_n^{(d)} \sim \mathsf{Multinomial}(p^{\mathcal{Z}})
         end
    end
    for k=1 to K do
         Sample the interaction pattern assignments via Gibbs sampling
         C_k \sim \mathsf{Multinomial}(p^{\mathcal{C}})
    end
    for n=1 to n_B do
         Sample the interaction pattern parameters \mathcal{B} via Metropolis-Hastings
    end
    for n=1 to n_{\delta} do
         Sample the receiver size parameter \delta via Metropolis-Hastings
    end
```

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## Data: North Carolina Dare county email data

• D=1456 emails between A=27 county government managers, covering 2 month periods (October 1 - November 30) in 2013



## Effect of Hurricane Sandy on Email Exchange



#### **IPTM** Result

### Conclusion

- Joint modeling of ties (sender, receiver, time) and contents
- Allowance of multicast multiple senders and/or receivers
- Possible application to