A Network Model for Dynamic Textual Communications with Application to Government Email Corpora

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Interaction-Partitioned Topic Model (IPTM)

- Probablistic model for time-stamped textual communications ~~(e.g. emails, cosponsorship of bills, international sanctions)~~
- Integration of two generative models:
  - Latent Dirichlet allocation (LDA) for topic-based contents
  - Dynamic exponential random graph model (ERGM) for ties
- ~~IPTM assigns each topic to an "interaction pattern," which is governed by a set of dynamic network features~~

"who communicates with whom about what, and when?"

Content Generating Process: LDA (Blei et al., 2003)

- For each topic $k = 1, ..., K$:
  1. Topic-word distribution $\boldsymbol{\phi}^{(k)} \sim$ Dirichlet$(\beta, \mathbf{u})$
  2. Topic-IP distribution $c_k \sim$ Uniform$(1, C)$

- For each document $d = 1, ..., D$:

  3-1. Document-topic distribution $\boldsymbol{\theta}^{(d)} \sim$ Dirichlet$(\alpha, \boldsymbol{m})$

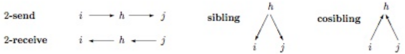
  3-2. For each word in a document $n = 1$ to $N^{(d)}$:
    - (a) Choose a topic $z_n^{(d)} \sim$ Multinomial$(\boldsymbol{\theta}^{(d)})$
    - (b) Choose a word $w_n^{(d)} \sim$ Multinomial$(\boldsymbol{\phi}^{(z_n^{(d)})})$

  3-3 Calculate the distribution of interaction patterns within a document:

$$p_c^{(d)} = \Big(\sum_{k:c_k=c} N^{(k|d)}\Big)/N^{(d)}, \tag{1}$$

Dynamic Network Features (Perry and Wolfe, 2012)

- $\boldsymbol{x}_t^{(c)}(i,j)$ is the network statistics at time t, for interaction pattern c
  - Degree: outdegree and indegree
  - Dyadic: send and receive
  - Triadic: 2-send, 2-receive, sibling and cosibling



- Partition the past 384 hours (=16 days) into 3 sub-intervals

$$[t-384h, t) = [t-384h, t-96h) \cup [t-96h, t-24h) \cup [t-24h, t),$$

then define the interval-based statistics for $l \in \{1,2,3\}$ and $c \in \{1,...,C\}$

$$\textbf{outdegree}_{t,l}^{(c)}(i) = \sum_{d: t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{i \to \forall j\} \quad \textbf{send}_{t,l}^{(c)}(i,j) = \sum_{d: t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{i \to j\}$$

$$\textbf{indegree}_{t,l}^{(c)}(j) = \sum_{d: t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{\forall i \to j\} \quad \textbf{receive}_{t,l}^{(c)}(i,j) = \sum_{d: t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{j \to i\}$$

Stochastic Intensity

- $\lambda_{ij}^{(d)}(t)$=P{for document d, $i \to j$ occurs in time interval $[t, t+dt)$}:

$$\lambda_{ij}^{(d)}(t) = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \boldsymbol{b}^{(c)T}\boldsymbol{x}_t^{(c)}(i,j)\Big\}, \tag{2}$$

  where $\lambda_0^{(c)}$ is the baseline hazards for the interaction pattern c and $\boldsymbol{b}^{(c)}$ is a vector of coefficients in $\boldsymbol{R}^p$.

- For multicast interactions – single sender i and multiple receivers J:

$$\lambda_{iJ}^{(d)}(t) = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \frac{1}{|J|}\sum_{j \in J}\boldsymbol{b}^{(c)T}\boldsymbol{x}_t^{(c)}(i,j)\Big\}, \tag{3}$$

  which is obtained by taking the average of $\boldsymbol{b}^{(c)T}\boldsymbol{x}_t^{(c)}(i,j)$ across the receivers.

- Probability of i sends a document to j (or J) is a mixture of contents and history of interactions

Tie Generating Process

1. For each sender $i \in \{1, ..., A\}$, choose a binary vector $J_i^{(d)}$ of length $(A-1)$, by applying Gibbs measure (Fellows and Handcock, 2017)

$$\mathsf{P}(J_i^{(d)}) = \frac{1}{Z(\delta, \log(\lambda_i^{(d)}))} \exp\Big\{\log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)\big) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}, \quad (4)$$

   where δ is a real-valued intercept controlling the recipient size and $Z(\delta, \log(\lambda_i^{(d)}))$ is the normalizing constant.

2. For each sender $i \in \mathcal{A}$, generate the time increments

$$\Delta T_{iJ_i} \sim \mathsf{Exp}(\lambda_{iJ_i}^{(d)}).$$

3. Set timestamp, sender, and receivers simultaneously:

$$t^{(d)} = t^{(d-1)} + \min(\Delta T_{iJ_i})$$
$$i^{(d)} = i_{\min(\Delta T_{iJ_i})}$$
$$J^{(d)} = J_{i^{(d)}}$$

Inference - Pseudocode

Algorithm 1 Markov Chain Monte Carlo (MCMC)

Set initial values $\mathcal{Z}^{(0)}$, $\mathcal{C}^{(0)}$, and $(\mathcal{B}^{(0)}, \delta^{(0)})$

for *o=1 to O* **do**

  for *d=1 to D* **do**

    for $i \in \mathcal{A}_{\backslash i_o^{(d)}}$ **do**

      for $j \in \mathcal{A}_{\backslash i}$ **do**

        Sample the latent edge $J_{ij}^{(d)}$ via Gibbs sampling

      end

    end

    for *n=1 to* $N^{(d)}$ **do**

      Sample the topic assignments via Gibbs sampling

      $z_n^{(d)} \sim \text{Multinomial}(p^{\mathcal{Z}})$

    end

  end

  for *k=1 to K* **do**

    Sample the interaction pattern assignments via Gibbs sampling

    $C_k \sim \text{Multinomial}(p^{\mathcal{C}})$

  end

  for *n=1 to* n_B **do**

    Sample the interaction pattern parameters $\mathcal{B}$ via Metropolis-Hastings

  end

  for *n=1 to* n_δ **do**

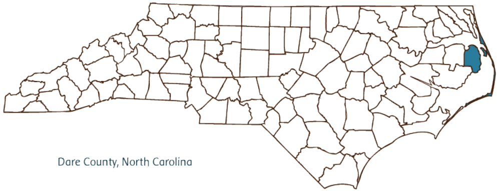
    Sample the receiver size parameter δ via Metropolis-Hastings

  end

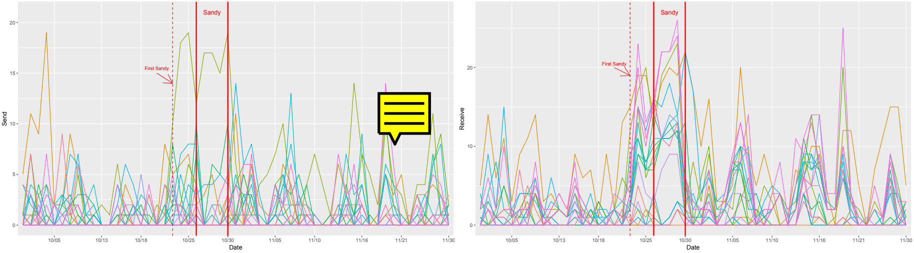
end

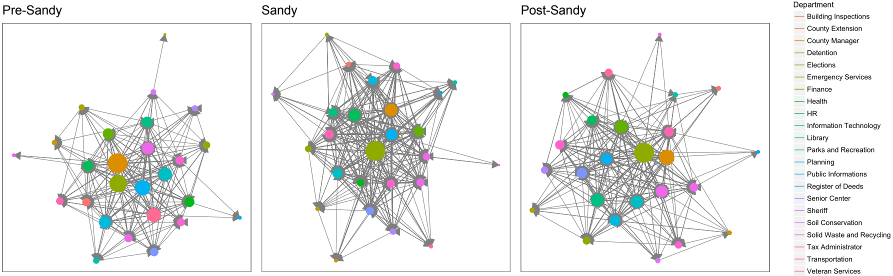
Data: North Carolina Dare county email data

- $D = 1456$ emails between $A = 27$ county government managers, covering 2 month periods (October 1 - November 30) in 2013



Effect of Hurricane Sandy on Email Exchange





IPTM Result

$\hat{\boldsymbol{b}}_p^{(c)}$	IP = 1	IP = 2
intercept	-3.264	-7.217
outdegree$[t-1d,t)$	0.025	1.520
outdegree$[t-3d,t-1d)$	0.538	-4.776
outdegree$[t-16d,t-3d)$	-0.167	0.255
indegree$[t-1d,t)$	-1.435	-4.743
indegree$[t-3d,t-1d)$	0.952	-1.529
indegree$[t-16d,t-3d)$	-0.276	0.279
send$[t-1d,t)$	1.639	-0.001
send$[t-3d,t-1d)$	0.054	-4.223
send$[t-16d,t-3d)$	0.972	3.765
receive$[t-1d,t)$	-0.380	-4.940
receive$[t-3d,t-1d)$	-1.625	-1.076
receive$[t-16d,t-3d)$	-0.389	-2.490
2-send$[t-1d,t)$	2.185	0.477
2-send$[t-3d,t-1d)$	0.919	2.364
2-send$[t-16d,t-3d)$	-0.071	0.154
2-receive$[t-1d,t)$	1.020	1.189
2-receive$[t-3d,t-1d)$	-0.168	3.971
2-receive$[t-16d,t-3d)$	0.029	0.098
sibling$[t-1d,t)$	-1.443	-0.608
sibling$[t-3d,t-1d)$	-1.289	-1.405
sibling$[t-16d,t-3d)$	-0.239	0.019
cosibling$[t-1d,t)$	0.390	4.586
cosibling$[t-3d,t-1d)$	0.792	-2.063
cosibling$[t-16d,t-3d)$	-0.103	-0.693

Table: Effect of dynamic statistics on email exchange

Topic	IP = 1	IP = 2
	will	-7.217
	director	1.520
	manteo	-4.776
	-0.167	0.255
	-1.435	-4.743
	0.952	-1.529
	-0.276	0.279
	1.639	-0.001
	-0.071	0.154
	1.020	1.189
	-0.168	3.971

Table: Effect of dynamic statistics on email exchange

Conclusion

- Joint modeling of ties (sender, receiver, time) and contents
- Allowance of multicast – multiple senders and/or receivers
- Possible application to