THE UNIVERSITY OF SYDNEY

AERO3261 PROPULSION

SEMESTER 1 2024

ASSIGNMENT 2

Performance of Turbofan Engines

GROUP 20

Iestyn HUGHES	**530705835**
Anton DEMARK	**510514576**
Wenjin HUANG	**500045844**
Thomas BULLOCK	**530754154**
Kyamuwendo GANYA	**530706142**

Executive Summary

The following report details the process of developing and analysing a hybrid-electric propulsion system for a commercial aircraft, namely the NASA STARC-ABL, intended to follow on from the Airbus A320 and Boeing 737 series. The objective was to optimise performance, whilst remaining within thermal and environmental constraints. The performance of both separate and mixed exhaust turbofan engines was investigated, as well as the degree of hybridisation, to ascertain the optimum engine for the design brief. The CFM56 engine was selected as a baseline and GASTURB was used to complete trade-off and sensitivity studies ,in which design parameters were altered to observe the impact of performance. Turbine inlet temperature (TIT), fan pressure ratio (FPR) and bypass pressure ratio (BPR) were parameters that were altered in the process. Take-off was the condition in which the chosen design point was evaluated. Then, the performance in this condition was weighed against that in cruise condition. Following this, variations in performance parameters were evaluated alongside throttle setting changes, for the off-design segment. In light of the findings, a design point was selected aligning with the stipulated requirements for the cruise condition. The selected engine was a separate exhaust turbofan, with a 1.8m fan diameter and 20% DoH. In terms of design point parameters, there was an FPR of 1.65, a BPR of 7, a TIT of 1575K and a TSFC of 0.631 kg/hr/daN. The separate exhaust turbofan overall had a higher performance than the mixed configuration and the selected degree of hybridisation further enhanced performance.

Contents

List of Figures

List of Tables

Nomenclature

For concision, shorthand acronyms and symbols were used throughout this report as recorded in Table 0.1.

Table 0.1: Nomenclature and Units

Symbol	**Description**	**Units**
A	Area	m^2
BCF	Blade Cooling Fraction	-
BLI	Boundary Layer Ingestion	-
BPR	Bypass ratio	-
d_{fan}	Fan Diameter	m
DoH	Degree of Hybridisation	-
F	Thrust	N
$F_{\mathrm{N,I}}$	Net Thrust Installed	N
F_{N}	Net Thrust	N
F_{spec}	Specific Thrust	N/kg
FAR $\left(\frac{\dot{m}_{\mathrm{fuel}}}{\dot{m}_{\mathrm{air}}}\right)$	Fuel-to-Air Ratio	-
FPR or π_{fan}	Fan Pressure Ratio	-
HPCPR or π_{HPC}	High Pressure Compressor Ratio	-
HPT	High Pressure Turbine	-
IFPR or $\pi_{\mathrm{fan,inner}}$	Inner Fan Pressure Ratio	-
IPC	Intermediate Pressure Compressor	-
IPCPR or π_{IPC}	Intermediate Pressure Compressor Pressure Ratio	-
LPT	Low Pressure Turbine	-
$\dot{m}_{\mathrm{p}}$	Primary Mass Flow Rate	kg/s
$\dot{m}_{\mathrm{s}}$	Secondary Mass Flow Rate	kg/s
NO_x	Nitrogen Oxides	-
OFPR or $\pi_{\mathrm{fan,outer}}$	Outer Fan Pressure Ratio	-
OPR or π_{overall}	Overall Pressure Ratio	-
P	Power	W
p	Pressure	Pa
RCF	Rotor Cooling Fraction	-
ST or F_{spec}	Specific Thrust	daN/kg/s
T	Temperature	K
TIT or T_4	Turbine Inlet Temperature	K
TSFC	Thrust Specific Fuel Consumption	kg/hr/daN
V	Velocity	m/s
W_{engine}	Weight of Engine	N
η_{core}	Core Efficiency	-
η_{prop}	Propulsive Efficiency	-
η_{p}	Polytropic Efficiency	-
γ	Specific Heat Ratio	-

1 Introduction

The project was centred on determining the optimal engine design for a hybrid passenger aircraft, in service of an aeronautical consultancy company. A research contract had been attained from a venture capital fund, and UsydAir was funded in pursuing this development. Using GasTurb software, an analysis of turbofan engine configurations was conducted and comparisons drawn. Starting from the baseline of the CFM56 engine, as per design specifications, analysis was carried out at off-design conditions at the top-of-climb. Trade-off studies were also completed and off-design performance was assessed on advanced designs to identify the engine solution most efficient for use. These studies were undertaken for both separate and mixed turbofans at design point across a range of design parameters; alongside sensitivity studies focused on the impact of design parameters on engine performance.

2 Engine Selection

2.1 Aircraft Parameters and Performance Requirements

The study was completed for NASA's proposed STARC-ABL aircraft [1], for which the critical parameters were given in Table 2.1 and the performance requirements were given in Table 2.2.

Table 2.1: STARC-ABL parameters (metric units).

Number of Engines	2	Cruise Mach Nr	0.8
Date of First Flight	2035	Nr of Passengers	180
Max Gross TO Weight [N]	60,160	Wing Aspect Ratio [–]	8.3
Operational Empty Weight [N]	34,155	Flight Ceiling [m]	13,716
Max Fuel Capacity [N]	8,777		

Table 2.2: STARC-ABL Performance Requirements (metric units).

	Uninstalled Thrust [N]	Mach Nr	Altitude [m]
Take-Off	126,063	0.20	0
Top-Of-Climb	30,248	0.80	10,668

2.2 Materials Research & Development

Material selection is dependant on the intrinsic properties of individual components. For a turbofan architecture - this is centred on improving efficiency and performance. Polymeric composites are used for the inlet and fan primarily due to the weight-saving benefits afforded by their high strength-to-weight ratios. However, this only applies in the lower reaches of the temperature range restricting the usage to colder components.

Titanium alloy is used at the leading edge of the fan blades to mitigate against bird-strike damage, as polymeric composites lag behind in impact resistance make them brittle by comparison [2].

For the compressor material, high-temperature strength is of emphasis; therefore steel, nickel, and titanium-based alloys are chosen. For the combustion chamber, it is ostensibly a selection based on temperature resistance, which leads to cobalt and nickel-based 'super-alloys'. Moving aft, the turbine blades are subjected to extreme temperature conditions, leaving them vulnerable to: fatigue, creep, and corrosion. To combat these challenges, it is common to use nickel-based super-alloys.

Before nickel-based alloys became widespread, cobalt-based super-alloys were more prevalent; developing a nickel-based solution was challenging, despite the abundance of the Nickel making raw-material costs low. More specifically, the emergence of advanced single-crystal fabrication enabled the composition of the super-alloys to be precisely specified. These improved tolerances were a catalyst for greater creep and thermal fatigue resistance.

The metal temperature is assumed not to be in excess of 1250K, as stipulated within the brief, however, in actuality in the average commercial jet engine, the combustion chamber reaches in the region of 2300K, hence in actuality bespoke cooling methods are required to maintain operation [3].

2.3 Production Engines

Table 2.3: Engine Specifications [4]

Parameter	**RCo.12 Mk.509A**	**IAE V2500-A1**	**CFM56-7B24**	**PW6124**	**CFM LEAP-1A**
BPR [-]	0.3	5.4	5.3	4.8	11
Compressor Stages [-]	1, 7, 9	1, 3, 10	1, 3, 9	1, 4, 5	1, 3, 10
Dry Weight [kg]	2061	2404	2366	2449	3150
FPR [-]	1.5	1.65	-	-	-
Net Thrust [kN]	78	110	108	106	97
OPR [-]	14.1	29.4	26	29.6	50
Year [-]	1956	1989	1997	2006	2013

The following ranges were selected for the associated parameters:

- FPR: 1.3 to 1.8
- BPR: 4 to 11

- HPCPR: 8.5 to 12.5
- TIT: 1300K to 1750K

These values were selected based on the data displayed above in Table 2.3. There was a limited set of data available for FPR, so the range was chosen to be larger to ensure a useful set of results. BPR was chosen on the range present, excluding the low bypass ratio Rolls-Royce engine. The OPR range was chosen in a similar manner. However, OPR was made up of the pressure ratios of the individual components. The fan pressure ratios were fixed by their matching and the Kutta condition, so the high pressure compressor (HPC) pressure ratio was used to vary OPR in `GASTURB`. Literature data for the turbine inlet temperature was not able to be obtained, so the limits were defined based on the design criteria. In general, the trend of BPR, FPR and OPR indicated that as the engines become more modern, these ratios increased. As such, the range selected utilised the upper end of the data present to reflect the technological trend. The 1956 Rolls-Royce engine parameters were neglected as modern engine design has far outstripped the technology used in the Rolls-Royce. The Rolls-Royce engine was included the research to provide a baseline example from which design has evolved from. Note that the ranges described above were used as guidelines - some variation was made during the `GASTURB` iteration to increase plot readability.

2.4 Constraints and Assumptions

2.4.1 Efficiencies

In terms of thermal restrictions, the maximum turbine inlet temperature was stipulated, to ensure operation safely within material limits. At take-off and initial climb, the turbine inlet temperature was limited to 1900K, whilst in top-of-climb condition, the temperature could not be greater than 1750K. Similarly, the compressor discharge temperature was limited to 900K, as this controls the thermal strain on the compressor and components further downstream. Limiting the turbine inlet temperatures also pertained to cooling, especially for the high pressure turbine. Seeing as, cooling air originates from the compressor discharge, meaning a greater need for cooling diminishes the turbine efficiency. The diameter of the fan was restricted to 1.8m, because the STARC-ABL is a commercial aircraft designed to replace the Boeing 737-800 and Airbus A320. Meaning, the aircraft must have a similar fan diameter to its predecessors. Further to this, the fan diameter has a considerable impact on the total weight and drag of the engine, therefore increasing this parameter adversely affects fuel consumption and efficiency. The nozzle was assumed to be adiabatic as well, meaning that there is no consideration for the heat transfer of air within the nozzle.

2.4.2 Kutta Condition

The Kutta condition stipulates that the bypass air rejoining and the core flow of air must be of equal pressure. The mixed exhaust engine has the Kutta condition, wherein $p_{t25} = p_{t5}$, whilst the separate engine did not have this constraint. This is because the bypass flow does not rejoin the core flow prior to exiting the engine. Therefore, an additional iteration was necessary to accommodate this condition $p_{t25} = p_{t5}$, using the OFPR. The Kutta condition yields a single FPR for a given OPR, BPR and TIT, thus this was a constraint that had to be considered.

2.4.3 Environmental Impact

From an environmental perspective, nitrogen oxides (NOx) emissions are a constraint, since this an area in which heavy regulation has come to pass in the aviation industry. This is the primary emission of interest, as the extremely high temperatures and pressures in combustion mean conditions are an ideal breeding ground for NOx emissions. The profound impact on air quality and the contribution to acid rain, serve as examples of why these emissions are tightly monitored in society and government. Hence, there is a trade-off between engine performance and environmental impact, as modern aircraft must be deemed sustainable.

2.4.4 Polytropic Efficiencies

The polytropic efficiency of a compressor is reduced for greater pressure ratios [5]. Therefore, a linear reduction of stage pressure ratio was considered, where the pressure ratio of each compressor was determined by $\pi_{stage} = (\pi_{compressor})^{\frac{1}{n}}$ for n stages. The resulting expressions were given in Equation 2.1.

$$\begin{aligned} \eta_{poly,fan} &= -\frac{1}{30}\pi_{stage} + \frac{24}{25} \\ \eta_{poly,IPC} &= -\frac{1}{10}\pi_{stage} + \frac{21}{20} \\ \eta_{poly,HPC} &= -\frac{2}{15}\pi_{stage} + \frac{11}{10} \end{aligned} \tag{2.1}$$

Cooling air also reduces the polytropic efficiency of a turbine. As only the HPT was cooled, the LPT polytropic efficiency was considered constant. For the HPT, the reduction in $\eta_{poly,HPT}$ was approximated as $\eta_{poly,HPT} - \dfrac{W_{cool}}{W}$, where $\dfrac{W_{cool}}{W}$ was the cooling fraction of air circumventing combustion.

3 Results and Discussion

3.1 Parametric Study - Separate Exhaust at Design Point

To explore the viability of a separate-exhaust turbofan configuration for the NASA STARC-ABL, a parametric study was completed. The study was performed for cruise conditions at the 'top of climb' design point (cruise altitude but with some margin for manoeuvres). The four metrics outlined in Section 2 were varied pairwise in all meaningful permutations, forming plots for FPR vs BPR, OPR vs TIT, FPR vs OPR, and BPR vs TIT. Each parametric study was plotted for TSFC and F_{spec}, with installation effects taken into account. Parameters not explicitly varied in each parametric study were held constant at their baseline parameters: $\pi_{IPC} = 1.81$, $\pi_{fan,inner} = \pi_{fan,outer} = 1.6$. The HPC exit, LPT inlet, and fan diameter constraints justified in Section 2 were also implemented.

Throughout the study, F_N was iterated to meet the total thrust requirement for the design point. As F_N remained unchanged ($F_N = C = 15.1kN$), Equation 3.1 must remain balanced [5].

$$F_N = \dot{m}_p\left[(1 + FAR)V_9 - V_0\right] + \dot{m}_s\left(V_{19} - V_0\right) + A_9(p_9 - p_0) + A_{19}(p_{19} - p_0) \qquad (3.1)$$

3.1.1 FPR vs BPR

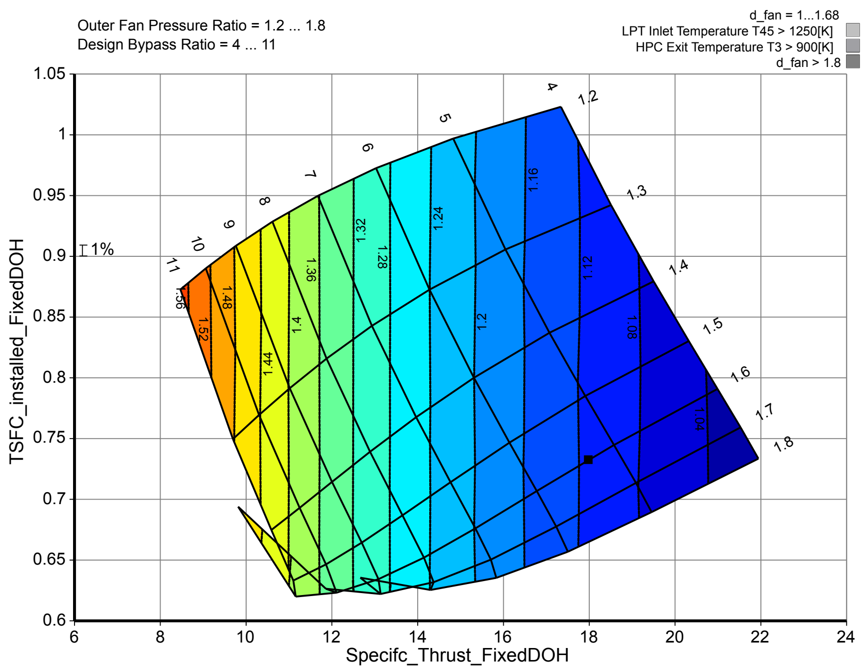

Figure 3.1: A separate exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Bypass Ratio (BPR) and Fan Pressure Ratio (FPR).

For the first investigation, FPR was varied against BPR to produce Figure 3.1. In Figure 3.1, the installed TSFC and F_{spec} ($TSFC_I$ and $F_{spec,I}$) decreased with increasing BPR for a fixed FPR. The decrease in $TSFC_I$ with BPR was greatest along isolines of a lower FPR (towards the top of Figure 3.1), as indicated by their steeper gradient. These isolines flattened for higher values of FPR (towards the bottom), wherein increases in BPR produced smaller decreases of $TSFC_I$, and therefore smaller efficiency gains.

$F_{spec,I}$ decreased with BPR as expected, as $F_N = C$ and $F_N = F_{spec} \cdot \dot{m}_p(1 + BPR)$, F_{spec} must decrease as BPR grows. The size of reduction in F_{spec} also shrank for higher BPRs and hence larger engines, as seen by narrowing contours moving from right to left in Figure 3.1. This was explained by the $(1 + BPR)$ term, which grows rapidly w.r.t to F_{spec} for small BPRs but has a much smaller proportional impact for higher BPRs, trending asymptotically towards zero for an infinitely large engine.

For a fixed BPR, increasing FPR reduced the engine size needed as indicated by BPR-isolines of negative gradient. Increased FPR with fixed BPR also increased efficiency (lowering $TSFC_I$ and raising $F_{spec,I}$) – this was caused by FPR increasing V_{19} in Equation 3.1, such that the bypass flow developed more thrust w.r.t to the core. Therefore for constant BPR, FAR was reduced to satisfy $F_N = C$. This phenomenon led to divergent behaviour for high-FPR performance, producing discontinuities on the lower edge of Figure 3.1, where BPR could not be increased. Along high-FPR isolines (e.g FPR = 1.8), V_{19} dominates and the engine core thrust is low due to low FAR. In the limiting case, the LPT is not able to generate enough power from the core flow to drive the larger fan needed to increase BPR.

Following analysis of Figure 3.1, it was identified that FPR was the critical parameter impacting $TSFC_I$. A conservative FPR of $\approx 1.6 \rightarrow 1.7$ was selected. For BPR, a broad range of BPRs $\approx 5 \rightarrow 8$ were chosen to be refined following the BPR-TIT study outlined in Section 3.1.4.

3.1.2 OPR vs TIT

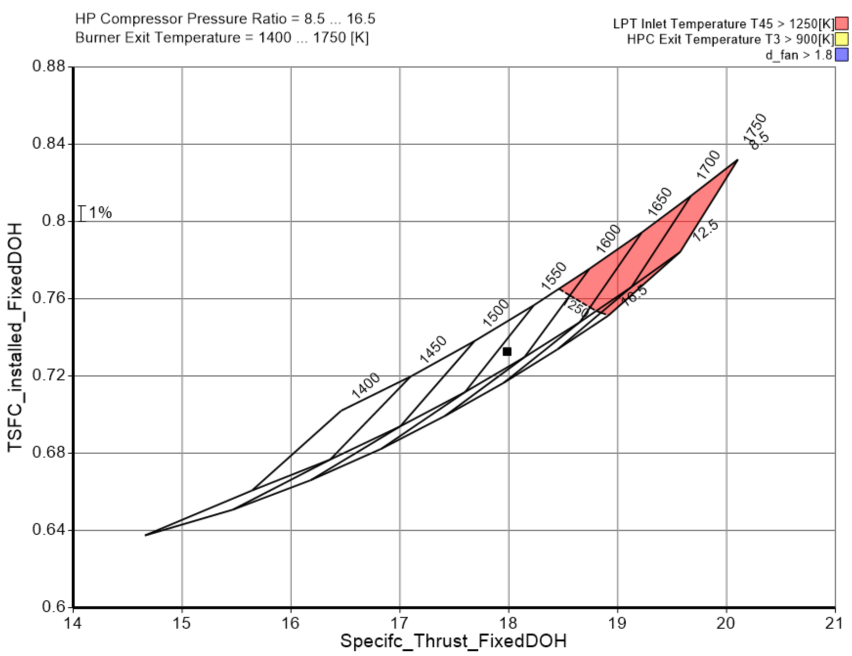

Figure 3.2: A separate exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Overall Pressure Ratio (OPR) and Turbine Inlet Temperature (TIT).

For the second investigation, OPR was varied against TIT to produce Figure 3.2. As OPR is not available as a parameter to be varied in `GASTURB`, the range of OPR selected in Section 2 were translated to a range of HPCPC using $\pi_{HPC} = \pi_{overall} / (\pi_{fan} \cdot \pi_{booster})$.

For discussion, Figure 3.2 was plotted in a standard view with F_{spec} on the horizontal axis. The same results were also plotted in a side view where $TSFC_I$ trends may be clearer in Figure A.1 (Appendix A.1). Figure 3.2 was limited in high-TIT, low-OPR performance by the material limits of the LPT. As LPT blades were not cooled, a boundary was added to ensure the metal temperature did not exceed the restriction identified in Section 2. The region exceeding this boundary was shaded in red.

Along an isoline of HPCPR (and hence OPR), $TSFC_I$ increased with TIT. This was explained simply – as TIT increases, $\dot{m}_f$ is increased, burning more fuel and reducing TSFC for a fixed thrust. To satisfy $F_N = C$ whilst increasing TIT, the engine size must decrease to counteract the increased V_9 in Equation 3.1, which will in turn increase core thrust. $F_{spec,I}$ also increased with TIT; as engine size was reduced, $\dot{m}_a$ was decreased for $F_N = C$, raising F_{spec}. Conversely, for a fixed TIT with increasing OPR, $TSFC_I$ and $F_{spec,I}$ both decrease. This trend converges towards a single isoline at $\approx HPCPR = 14$. Whilst this trend does reverse (i.e creating a concave parametric plane) for $HPCPR >> 20$, these design points exceeded the HPC exit temperature constraint and so were not considered.

From this study, it was concluded that OPR (HPCPR) should be pushed closed to the convergent region, without being increased unnecessarily (as any $TSFC_I$ gain would be offset by the drag and weight penalties of a larger engine). Therefore, HPCPR should be $\approx 12 \rightarrow 14$. TIT was then selected as a trade-off between improved installation and F_{spec} performance (a smaller engine needed for higher TIT) and an increased TSFC for high TIT, prompting a selected range of $\approx 1550K \rightarrow 1600K$. Whilst this may seem high from Figure 3.2, this was justified in later discussion of a BPR-TIT study, and the impact of hybridisation in Sections 3.1.4 and 3.1.5.

3.1.3 FPR vs OPR

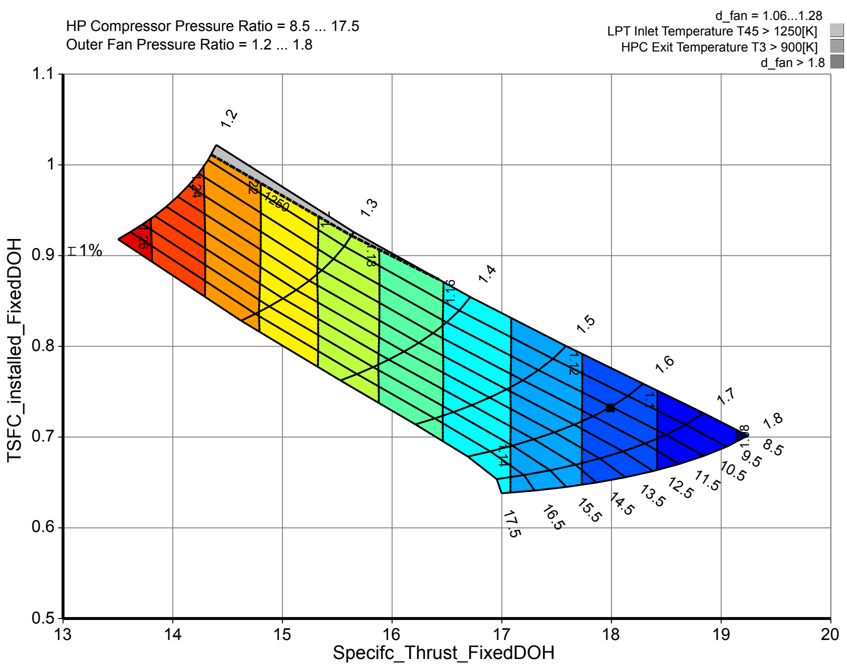

Figure 3.3: A separate exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Overall Pressure Ratio (OPR) and Fan Pressure Ratio (FPR).

For the third investigation, FPR was varied against OPR to produce Figure 3.3. Figure 3.3 showed that for a fixed FPR, $TSFC_I$ and $F_{spec,I}$ both decreased with increasing

HPCPR (and hence OPR). As OPR is increased, T_{t3} grows, therefore the $\dot{m}_f$ required to reach TIT is lowered, improving efficiency and reducing $TSFC_I$. The d_{fan} contour revealed that engine size decreased slightly along these FPR isolines. This was explained by Equation 3.1: as OPR is increased, a leaner FAR is needed whilst the thrust output must be maintained ($F_N = C$). As such, engine size must be increased, increasing $\dot{m}_a$ and explaining the consequent drop in F_{spec}.

For a fixed OPR and increasing FPR, $TSFC_I$ decreased yet F_{spec} increased. This trend was observed in Figure 3.1 and discussed in detail in Section 3.1.1. The divergence seen in Figure 3.1 also produced irregularity in Figure 3.3, presenting a limitation to be carefully considered. For the flatter near-horizontal high-FPR isolines, it was noted that the TSFC penalty for a lower HPCPR (and thus improved F_{spec}) was reduced, encouraging a lower HPCPR for selection. This formed a trade-off to be balanced with the higher HPCPR favoured from Figure 3.2.

Overall, from this study it was identified that FPR was the dominant parameter governing performance in Figure 3.3. As such. it was suggested that FPR should be ≈ 1.65, providing strong performance whilst well-away from the limit explored in Section 3.1.1. This FPR was complimented by the selected HPCPR range of $\approx 12 \rightarrow 14$ from Figure 3.2, for which a wide range of TIT's remained attainable.

3.1.4 BPR vs TIT

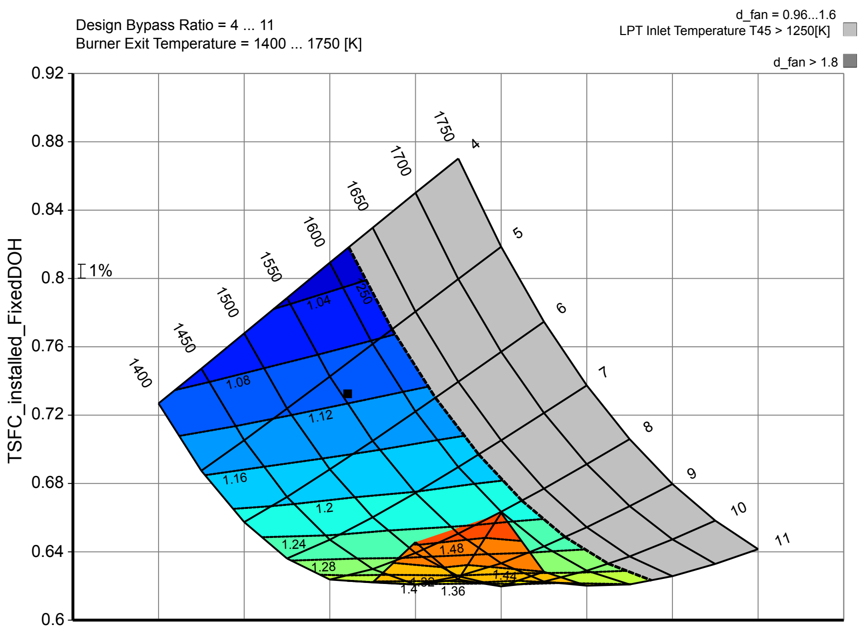

Figure 3.4: A separate exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Bypass Ratio (BPR) and Turbine Inlet Temperature (TIT). This was plotted from the side wherein F_{spec} was aligned into the page.

For the fourth investigation, BPR was varied against TIT to produce Figure 3.4. The same results were also plotted in standard view with F_{spec} on the horizontal as shown in Figure A.2 (Appendix A.1).

In Figure 3.4, an increase in TIT for fixed-BPR resulted in an increased $TSFC_I$ with a reduced engine size as indicated by the d_{fan} contours. This was expected – a richer FAR (and thereby greater $\dot{m}_f$) was needed to maintain $F_N = C$ in the smaller engine (Equation 3.1).

Along a TIT-isoline of increasing BPR, the trend exhibited an inflection point. For low BPRs, $TSFC_I$ initially decreased; the core flow is reduced such that less fuel is required to reach the fixed TIT. The thrust deficit is counteracted by a larger fan, which increases the secondary thrust to maintain $F_N = C$. However at the inflection point (BPR ≈ 9) the reduced core flow is not able to produce enough power in the turbines to drive the fan. Beyond this BPR, the core flow must be increased to feed a larger fan, snowballing the engine-size requirement. This effect was most pronounced for low-TIT isolines

From this study, it was concluded that a high-BPR engine must be carefully bounded in TIT. A TIT $< 1500K$ risked needed an excessively large engine, whilst a TIT > 1620 would exceed the thermal limits (1250K) of the un-cooled LPT blades.

3.1.5 Hybridisation

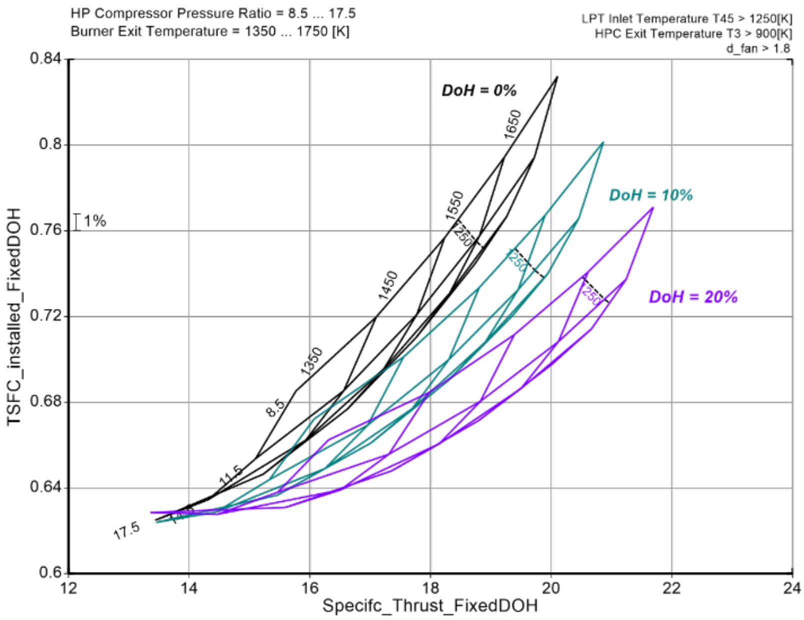

Figure 3.5: A separate exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with HPCPR and TIT. This was repeated for three fixed-degrees of Hybridisation (DoHs).

To investigate the impact of increasing the degree of hybridisation, the study to produce Figure 3.2 was repeated for three hybridisation factors (0%, 10% and 20%) and plotted as Figure 3.5. DoH > 20% failed to converge within `GASTURB`, presenting an upper boundary for testing. For this case, F_N was no longer considered constant. Instead, a total thrust condition was used for iteration, where $F_{TOTAL} = 2F_N + F_{BLI} = 30.2kN$. Using the total thrust condition, F_N was found to reduce with F_{BLI}, and hence DoH.

In Figure 3.5, it was observed that the design point envelope was rotated towards the horizontal, reducing $TSFC_I$ and increasing F_{spec}, significantly improving engine performance. The boundary layer ingestion (BLI) fan was configured to draw power from the main turbofans' turbines such that $2P_{turb} = 2P_{comp} + P_{BLI}$.

To explain the behaviour observed in Figure 3.5, a fixed engine configuration was considered (constant FPR, OPR, BPR, TIT, γ and η), although the engine size itself was not constrained. For an increased DoH, more energy is bled from the engines to be supplied to the BLI fan, reducing V_9 and therefore F_N to maintain the total thrust criterion. Therefore, less power is supplied to the main-engine fan, requiring a small fan diameter (reducing $\dot{m}$) and reducing the fuel flow ($\dot{m}_f$) needed to achieve TIT. Therefore, for a constant $F_{TOTAL\ (per\ engine)} = 15.1kN$, $TSFC$ was reduced whilst F_{spec} was increased.

For a separate exhaust architecture, F_{BLI} is not strongly coupled to F_{bypass} – which produces the majority of the engine thrust. This independence allows hybridisation to be favourable for a separate exhaust design. The upper bound on DoH was then described clearly as the point at which the reduced flow rate through the core is not sufficient to drive air through the engine, preventing `GASTURB` from converging. Overall, it was concluded that the maximum feasible degree of hybridisation should be used for the separate engine. Therefore, DoH was selected = 20%.

3.1.6 Installation Effects

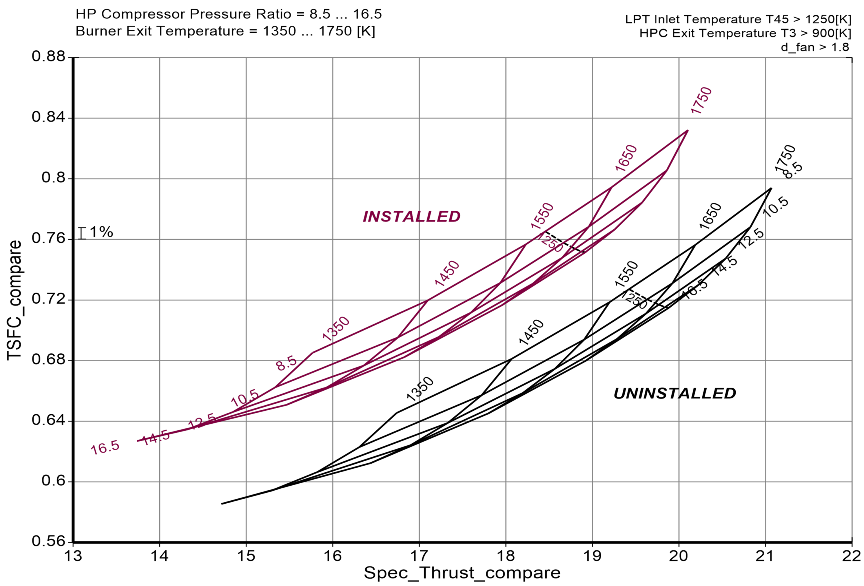

Figure 3.6: A separate exhaust parametric plot comparing the variation in TSFC [kg/hr/-daN] and Specific Thrust [daN/kg/s] with HPCPR and TIT for uninstalled and installed performance.

To explore how installation impacts theoretical performance, the study to produce Figure 3.2 was repeated for installed and uninstalled performance, creating Figure 3.5. Figure 3.5 demonstrated that installation weakens engine performance, as $TSFC$ is increased whilst F_{spec} is decreased. Figure 3.5 showed a pure translation between the two performance maps, as both $TSFC$ and F_{spec} differ from $TSFC_I$ and $F_{spec,I}$ by a subtractive constant – the extra engine thrust required to overcome the nacelle drag and engine weight.

3.1.7 Uniform FPR discussion

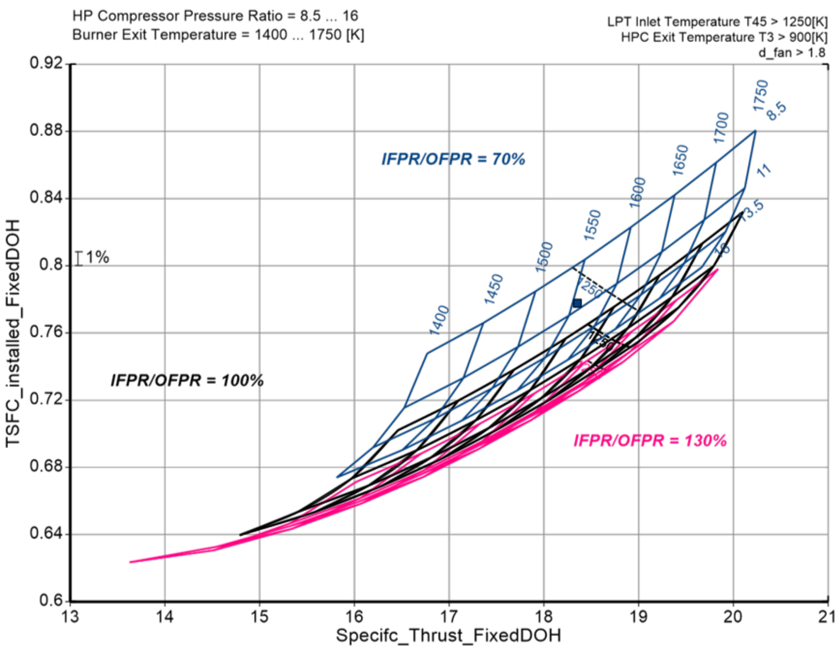

Figure 3.7: A separate exhaust parametric plot comparing the variation in TSFC [kg/hr/-daN] and Specific Thrust [daN/kg/s] with HPCPR and TIT for three different radial pressure distributions.

To explore the validity of matching the inner and outer fan pressure ratio, a short study was completed. Instead of matching IOFP and OFPR in `GASTURB`, the ratio between them was set iteratively to 0.7, 1.0 or 1.3, mimicking a radial-jump of ±30% from hub to tip. The study from Figure 3.2 was repeated for these three testing ratios and plotted as Figure 3.7.

In Figure 3.7 the $TSFC$ increased notably for IFPR $>$ OFPR and decreased slightly for IFPR $<$ OFPR. However, IFPR $>$ OFPR does not correspond to real behaviour; the tip of the fan blades travel faster than the hub relative to the flow, therefore pressure ratio increases radially (excluding the impact of tip-losses). As such, only $IFPR/OFPR > 1$ was considered, for which there was a small decrease in both $TSFC$ and F_{spec}. In Figure 3.7, this offset did not vary significantly throughout the parametric space, implying that any non-uniformity in FPR could be easily accounted for through a simple correction factor. Therefore, the matched FPR condition used in the parametric study was deemed suitable for initial design.

3.1.8 Summary of Selected Design Parameters

Following the parametric study, the engine parameters were selected systematically. Figure 3.1 provided an initial range of BPR and FPR values to be further refined. A value for HPCPR (and hence OPR) was selected using Figure 3.2, in addition to an approximate range for TIT. Figure 3.3 was then used (with reference to the previous studies) to confirm a value of FPR. Finally Figure 3.4 was used to constrain initial estimates for BPR and TIT, whilst Figure 3.5 was used to identified a suitable DOH. Values for OPR, FPR, BPR, TIT and DoH were then used as inputs to the design point cycle in GASTURB, the parameters for which were recorded in Table 3.1.

Table 3.1: Selected design point parameters for the separate-exhaust configuration.

Input Parameter	Value	Units	Output Parameter	Value	Units
BPR	7	-	d_{fan}	1.18	m
FPR	1.65	-	$F_{\text{n,inst}}$	12.3	kN
HPCPR	13	-	F_{spec}	13.6	daN/kg/s
IPCPR	1.81	-	TSFC	0.631	kg/hr/daN
TIT	1575	K	W_{engine}	1291	kg
DoH	0.2	-	η_{IPC}	0.928	-
			$\eta_{\text{p, core}}$	0.461	-
			$\eta_{\text{p, HPC}}$	0.923	-
			$\eta_{\text{p, LPC}}$	0.905	-
			$\eta_{\text{p, propulsive}}$	0.766	-

3.2 Parametric Study - Mixed Exhaust at Design Point

3.2.1 BPR

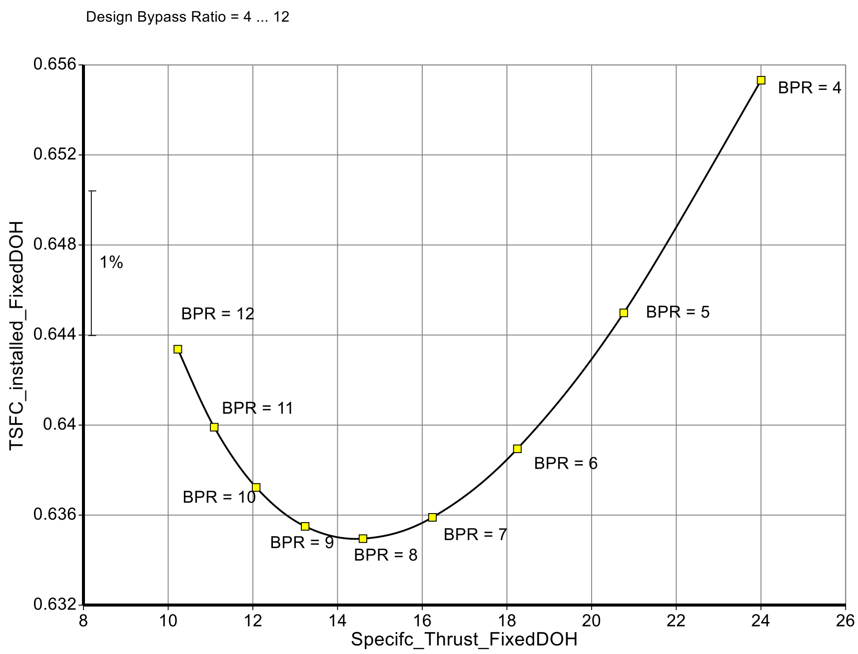

Figure 3.8: A mixed exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with BPR

The relationship between BPR and the installed TSFC and specific thrust was explored in the study and displayed in Figure 3.8. The range of BPR selected was 4-12, with a step size of 1. The figure displayed that increasing BPR causes a decrease in both TSFC and specific thrust until the turning point was reached at a BPR of 8. At a performance level, the bypass ratio represents the ratio of thrust produced by two components - the airflow through the core duct and the airflow through the bypass duct. As BPR increases, more air passes through the bypass, which means more of the thrust is produced by the slower bypass air instead of the faster core stream air. The specific thrust can be expressed as:

$$F_{spec} = (1 + FAR)V_e - V_0 \tag{3.2}$$

As shown in Equation 3.2, the specific thrust depends directly on the exhaust velocity V_e - as such, the increased airflow through the bypass duct will reduce the exhaust velocity and hence the specific thrust. TSFC also decreases with an increase in BPR - the propulsive efficiency increases as more airflow passes through the bypass duct. This is a result of improved efficiency at lower (average) airflow speeds. The energy transfer within the engine is more effective and the reduced exhaust velocity minimises the kinetic energy losses. This in turn requires a lower mass fuel flow rate to achieve the same level of thrust, reducing TSFC. Past the turning point at BPR = 8, the benefits gained by using a lower mass fuel flow are counteracted by design constraints. High BPRs require the fan to compress increasingly large airflows, which, after the turning point, loses efficiency and needs more energy - ultimately this causes an increase in TSFC. From this, the optimal BPR value was chosen to BPR = 8, as this minimises TSFC. However, any values within the range 7-9 still produced satisfactory results, so the value had some opportunity to move based on the other parametric studies.

3.2.2 OPR vs TIT

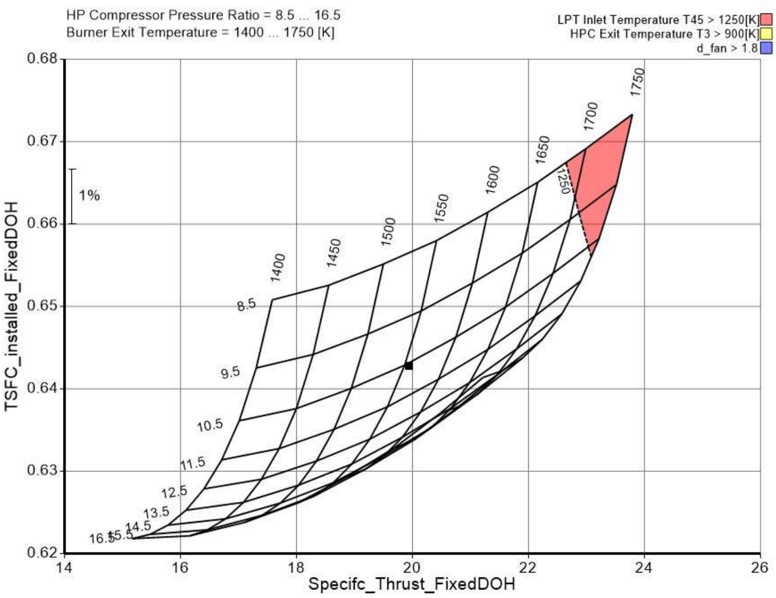

Figure 3.9: A mixed exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Overall Pressure Ratio (OPR) and Turbine Inlet Temperature (TIT).

Figure 3.9 shows a contour plot that depicts the relationship between a gas turbine engine's specific thrust and TSFC under various operating circumstances. The variables of interest are the HPC pressure ratio and the burner exit temperature. Figure 3.9

illustrates how various factors impact engine performance, with a focus on efficiency and thrust production.

The x-axis shows specific thrust, which is expressed in units of thrust per unit airflow. This is an essential measure that indicates how much thrust an engine can generate for a given volume of air moving through it. The y-axis indicates TSFC, which evaluates fuel efficiency. TSFC is the quantity of fuel used to generate one unit of thrust over time, with lower values indicating higher fuel efficiency. The contour lines on the graph reflect constant values for burner exit temperatures ranging from 1400K to 1750K, as well as HPC pressure rations ranging from 8.5 to 16.5.

Analysing the overall patterns in the figure reveals a trade-off between particular thrust and TSFC. As specific thrust grows, so does TSFC, indicating that creating greater thrust consumes more fuel, diminishing efficiency. Higher burner exit temperatures often result in higher specific thrusts, but also in higher TSFC, indicating less fuel efficiency. Furthermore, raising the HPC pressure ratio tilts the contours towards larger specific thrusts for a given TSFC. Higher pressure ratios enhance the engine's capacity to compress air, which increases thrust but at the cost of increased fuel consumption.

Figure 3.9 also shows important zones. The red shaded area represents places where the LPT input temperature (T_{45}) surpasses 1250K. Operating in this zone can cause material and thermal stress on turbine components, limiting engine durability. Although not coloured in Figure 3.9, the labels indicate comparable critical zones for HPC exit temperature ($T3 > 900K$) and fan diameter concerns ($d_{fan} > 1.8$).

3.2.3 FPR vs OPR

A plot of FPR vs OPR was not able to be generated for the mixed exhaust section. In order to accurately model the engine performance in `GASTURB`, the inner and outer fan pressures were matched (for further detail on the `GASTURB` setting view Appendix D). The matching was necessary to ensure that the airflow at fan entry was correct. If the fan pressure ratios are not matched, then airflow separation will occur, which significantly increases drag and reduces efficiency. For further detail on the matching, refer back to Section 3.1.7. For the mixed exhaust, at the point of mixing, the pressures must be equal. This is known as the Kutta condition. In order to achieve this, the pressure in the fan and core ducts were matched - this required iteration on the outer fan pressure ratio. This meant that the outer fan pressure ratio was defined by the Kutta condition, whilst the inner fan pressure ratio was defined by the matching. Hence, for any given design point, only a single FPR value was valid. This meant that FPR was not a design point - it was dependent on the other parameters selected. For further information on FPR for the mixed exhaust, refer to Appendix B. As a plot for FPR vs. OPR was unable to generated, instead a plot for BPR vs. OPR was created. This is displayed in Figure 3.10. The trend for BPR was consistent with the trend displayed in Figure 3.8 - TSFC and specific thrust decreased with BPR until the turning point was reached. Based on Figure 3.10, the ideal range for BPR was selected as 7-10, which is consistent with the conclusion drawn in Section 3.2.1. Similarly, as OPR increased, TSFC decreased. Increasing OPR increased the thermodynamic efficiency of the engine. A higher pressure ratio increased the efficiency of the combustion chamber - combustion was more complete (CO_2 is the primary carbon product of the combustion reaction). Overall, raising OPR increased the

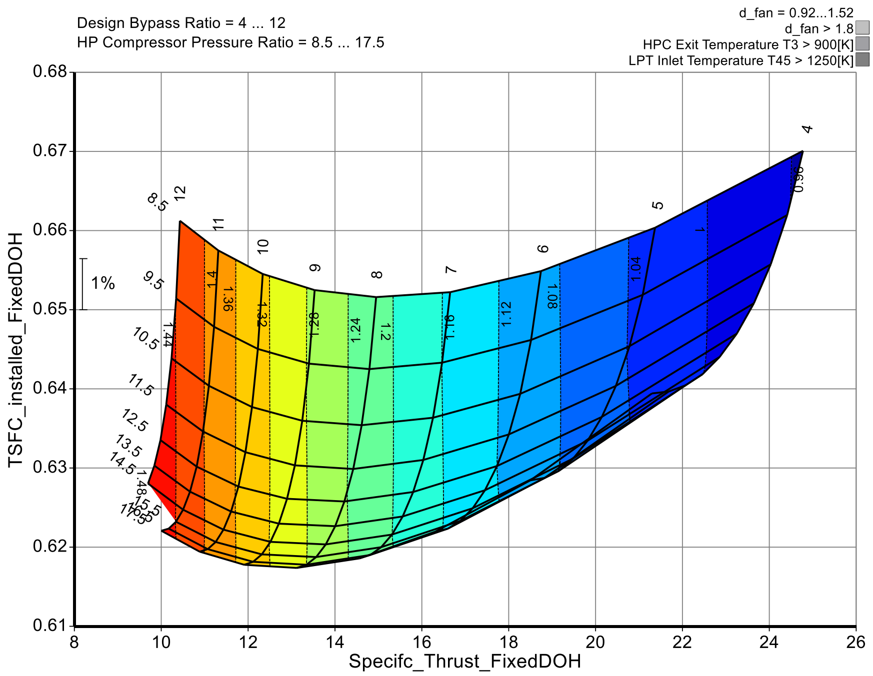

Figure 3.10: A mixed exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Bypass Ratio (BPR) and Overall Pressure Ratio (OPR)

ability of the engine to convert the chemical energy of the fuel into the kinetic energy of the exhaust. Specific thrust was not heavily affected by increasing OPR, although there was a slight trend for it to decrease as OPR increased. As such, the selected range of OPR based on Figure 3.10 was 12.5-17.5, as this minimised TSFC.

3.2.4 BPR vs TIT

In this study, the effects of BPR and TIT were investigated and plotted against TSFC and specific thrust, shown in Figure 3.11. The same result was also plotted with specific thrust on the x-axis, which can be seen in Figure A.3.

Figure 3.11 shows a clear trend of the effects of BPR and TIT on TSFC. For a given low to medium BPR value, the lower the TIT the more efficient the engine becomes, this is due to a smaller FAR ratio required to meet the TIT demand. The thrust was compensated for by a larger fan diameter, hence mass flow rate. However, for higher BPRs the effects of reducing TIT were not nearly as strong, in fact at very high BPRs there was a critical point where further reduction in TIT resulted in a decrease in efficiency.

The result in Figure A.3 highlights the high dependence F_{spec} has on BPR. This is an expected result, the larger the BPR leads to an increase in mass flow and as thrust is iterated to be constant, a higher BPR results in a lower F_{spec}. Along a constant TIT, BPR tends to decrease the TSFC a result that stems from the increase in power drawn from the LPT, leading to a lower V9 and hence a higher propulsive efficiency and therefore lower TSFC. At very high BPR and low TIT there remains a critical point where the TSFC increases, this is due to the power required from the fan being too large for the LPT, hence more fuel is burned to increase the power that can be supplied from the LPT.

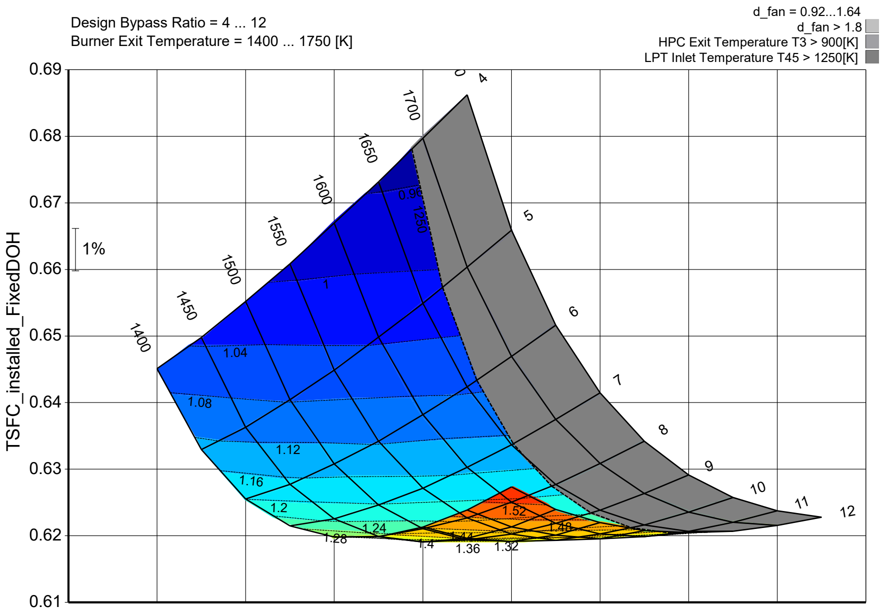

Figure 3.11: A mixed exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Bypass Ratio (BPR) and Turbine Inlet Temperature (TIT). This was plotted from the side wherein F_{spec} was aligned into the page.

It is clear that a larger BPR would have to result in a carefully controlled TIT, as to not exceed material limits while also maintaining a reasonable fan diameter and TSFC. From this study, it is recommended a BPR = 8 and TIT = 1550 for the design point of the mixed exhaust engine.

3.2.5 Hybridisation

Figure 3.12 shows the effects of hybridisation on TSFC and F_{spec} while varying TIT and OPR. Note that as the other compressors are iterated on, varying the HPCPR was treated as varying the OPR. It was found that the `GASTURB` cycle was not able to converge at a hybridisation level above 20% as there was too much power being drawn from the LPT. The hybridisation plots all have a very consistent shape, the discussion of which is in section 3.2.2, but are shown to rotate in a clockwise fashion depending on the degree of hybridisation. At high TIT, there are marginal gains in efficiency when hybridisation is increased. At low TIT, increasing hybridisation has the opposite effect where there are marginal losses. By looking at the power balance in the mixed turbofan, the effects of hybridisation can be explained.

The standard turbofan power balance can be seen in Equation 3.3.

$$P_{HPT} + P_{LPT} = P_{fan} + P_{HPC} + P_{hybrid} \tag{3.3}$$

The standard turbofan power balance can be recast in the form shown in Equation 3.4.

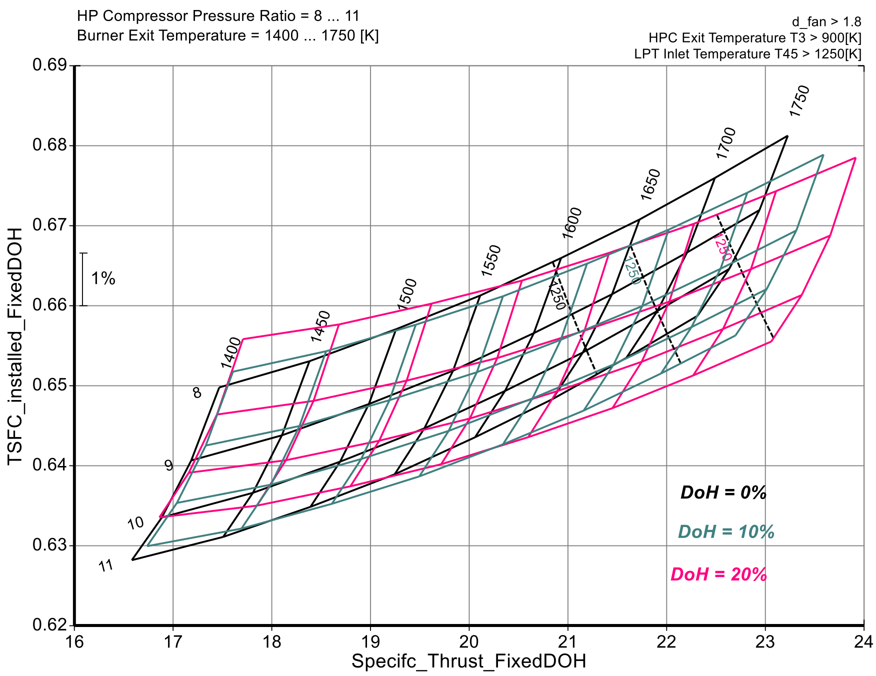

Figure 3.12: A mixed exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with HPCPR and TIT. This was repeated for three fixed-degrees of Hybridisation (DoHs).

$$(1+\mathrm{FAR})\,(\mathrm{TIT})\left[1-\left(\frac{p_{t5}}{p_{t4}}\right)^{\frac{(\gamma-1)\eta_{p,turb}}{\gamma}}\right] = T_{t2}\left[\mathrm{OPR}^{\left(\frac{\gamma-1}{\gamma\eta_{p,comp}}\right)}-1\right] + T_{t2}\left[\mathrm{FPR}^{\left(\frac{\gamma-1}{\gamma\eta_{p,fan}}\right)}-1\right]\mathrm{BPR}+\frac{F_N\ V_0\ \mathrm{DoH}}{2(1-\mathrm{DoH})} \tag{3.4}$$

The hybridisation effect draws power from the LPT to power the electric fan, therefore reduces the power delivered to the bypass fan and hence reduces the OPR. As the other terms on the right-hand side of Equation 3.4 are constant, this means that either the FAR, TIT or LPTPR has to increase so that the equation is satisfied. The most fundamental difference between the separate exhaust and mixed flow is the kutta condition, which fixes $p_{t5} = p_{t25}$. In addition, $p_{t4} \approx p_{t3}$ which is obviously not able to decrease as this would further reduce the OPR. Therefore, for mixed flow, either the TIT or FAR has to increase for a given increase in hybridisation. Figure 3.12 shows the interplay between these two variables to ensure the relationship in Equation 3.4 is satisfied. At low TIT the FAR compensates by increasing; at high TIT the FAR compensates by decreasing. The level of hybridisation dictates how large this compensation is, which explains the more pronounced clockwise rotation at higher levels of hybridisation in Figure 3.12.

Given the TIT design point range of 1500-1600K, it is recommended a DoH of 10% which allows for some gains in efficiency while also much smaller effects if operated at lower TIT values, unlike DoH of 20%.

3.2.6 Installation Effects

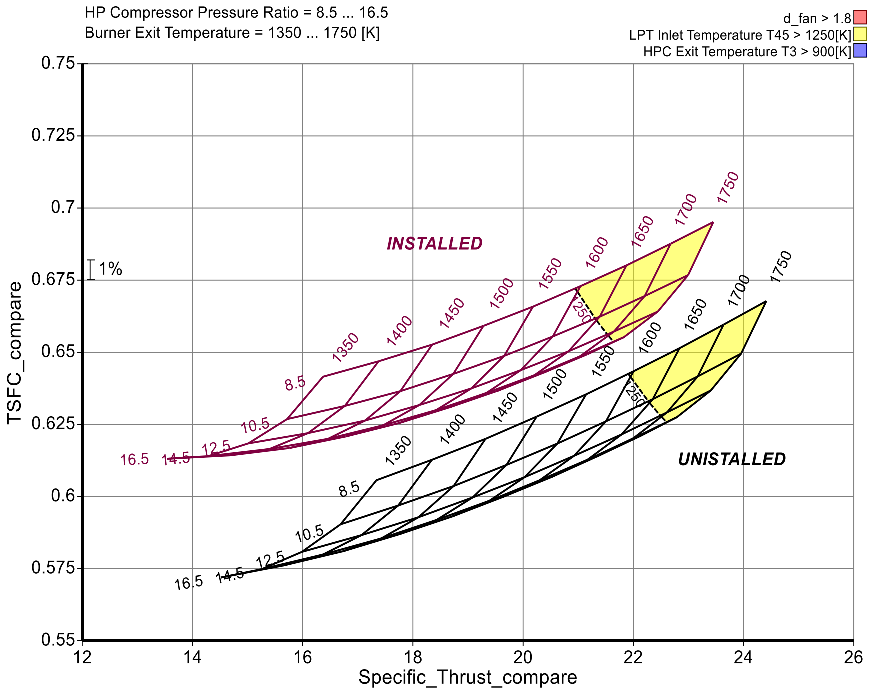

Figure 3.13: A mixed exhaust parametric plot comparing the variation in TSFC [kg/hr/-daN] and Specific Thrust [daN/kg/s] with HPCPR and TIT for uninstalled and installed performance.

Figure 3.13 highlights the installation effects on the TSFC and specific thrust of the engine. The installation effects primarily related to the drag produced by the nacelle of the engine. As BPR was increased, the size of the engine also increased - this increases the size of the nacelle and hence the nacelle drag. Fundamentally, this meant that the engine size was limited by the nacelle drag. Once a sufficiently large engine size was reached, the increase in thrust by raising BPR was counteracted by the increase in nacelle drag. Overall, Figure 3.13 illustrated that the engine performance was reduced when installed, as the increased engine weight and nacelle drag increased the required thrust.

3.2.7 Summary of Selected Design Parameters

Following the mixed exhaust parametric study, the engine parameters were selected. Figure 3.8 outlined the optimal BPR range of 7-9. The BPR range was confirmed in Figure 3.11, which was also used to select a TIT of 1550 as this balanced performance with material limits and tolerance either side. Figure 3.10 was used to determine the HPCPR, which was selected to be 12.5, due to the optimal BPR range and the diminishing returns of further increase. Figure 3.12 showed the effects of hybridisation, in combination with the BPR and TIT, it was discovered that a hybridisation of 10% resulted in a marginal decrease in TSFC, therefore deemed best. Note that the FPR is set for a given OPR due to the kutta condition and is therefore an output for the mixed exhaust study, refer to section B for further discussion on this.

Table 3.2: Selected design point parameters for the mixed-exhaust configuration.

Input Parameter	**Value**	**Units**	**Output Parameter**	**Value**	**Units**
BPR	8	-	d_{fan}	1.22	m
HPCPR	12.5	-	$F_{\text{n,inst}}$	13.3	kN
IPCPR	1	-	F_{spec}	14.9	daN/kg/s
TIT	1550	K	TSFC	0.67	kg/hr/daN
DoH	0.1	-	W_{engine}	1378	kg
			$\eta_{\text{p, IPC}}$	0.950	-
			$\eta_{\text{p, core}}$	0.463	-
			$\eta_{\text{p, HPC}}$	0.924	-
			$\eta_{\text{p, LPC}}$	0.908	-
			$\eta_{\text{p, propulsive}}$	0.763	-
			FPR	1.54	-

3.3 Engine Architecture Comparison

The difference between separate and mixed exhaust engines is: in a separate exhaust the core and bypass streams are expelled in separate nozzles, whilst in a mixed exhaust the two streams are combined before being expelled. Typically, a mixed exhaust is more effective as combining the streams leads to a greater thrust efficiency. Based on the studies performed, the difference between the two is small but not insignificant. Both produced fairly similar output parameters, as shown in Tables 3.1 and 3.2. The major difference was in TSFC and engine weight, both in favour of the separate streams. This meant that despite the increased efficiency of the mixed exhaust, its increased weight requirements caused it to have a worse performance under the selected conditions than the separate engine. The separate exhaust also allowed for FPR to be set, which as aforementioned is not permitted for the mixed exhaust. This meant that the separate exhaust has enhanced design space to deal with any additional requirements that may arise later in the design process.

3.4 Sensitivity Study

Table 3.3: Sensitivity study conducted at the selected design point for separate and mixed exhaust engines at cruise

	Separate Exhaust				**Mixed Exhaust**			
	FPR	**TIT**	**BPR**	**HPCPR**	**FPR**	**TIT**	**BPR**	**HPCPR**
Unit	[-]	[K]	[-]	[-]	[-]	[K]	[-]	[-]
Design Parameters	1.65	1575	7	13	1.54	1550	8	12.5
Delta (Δ)	0.01	15.5	0.05	0.05	0.01	15.5	0.05	0.05
sNOx [%]	0.12	0	0	0.60	0.86	0.01	0	0.55
$\eta_{propulsive}$ [%]	0	-0.31	0.12	0.07	0	-0.31	0.11	0
η_{core} [%]	0	0.08	-0.02	0.08	0.10	-0.01	-0.01	0.06
$\eta_{p,HPT}$ [%]	0	-0.22	0	-0.06	-0.01	-0.17	0	0
$\eta_{p,LPT}$ [%]	0	0	0	0	0	0	0	0
$\eta_{p,HPC}$ [%]	0	0	0	0	0	0	0	0
$\eta_{p,LPC}$ [%]	0	0	0	-0.18	0	0	0.01	0
TSFC [%]	-0.01	0.31	-0.09	-0.57	-0.11	0.18	-0.04	-0.06
F_{spec} [%]	-0.02	1.43	-0.54	-0.3	-0.02	1.42	-0.51	-0.02
d_{fan} [%]	0.01	-0.66	0.25	0.14	0.01	-0.66	0.24	0.01
W_{engine} [%]	0.02	-1.59	0.61	0.34	0.02	-1.58	0.58	0.02
RCF [%]	0.03	4.55	0	1.3	0.18	5.01	0	0.11
BCF [%]	0.03	4.77	0	1.37	0.18	5.17	0	0.12

Table 3.3 shows how sensitive engine parameters are to variations in the chosen design point for the mixed and separate exhaust. The delta of the design parameters was chosen to be $\approx 1\%$ as this would be the upper limits of tolerance in the manufacturing and testing stage of this type of engine. The trends exhibited were largely the same across both engines. For a change in TIT, RCF and BCF were the most sensitive for both engines. RCF and BCF were the most sensitive to TIT, because it directly increases the temperature of flow entering the HPT and therefore a larger cooling fraction is required to maintain blade temperature specified in section 2.2. The propulsive and core efficiencies were most effected by the TIT, since TIT is directly related to throttle setting and because the thrust was iterated to be constant more throttle than required will reduce efficiency. The FPR had minimal effect in both engines as the largest variation was in sNOx where the effect was still fairly negligible, from this it was concluded the FPR was not a design critical parameter. The BPR had the largest effect on F_{spec} and engine weight for both engines, this was simply due to a larger engine which directly increases the weight and the mass flow of air, hence reducing F_{spec}. The TSFC was shown to go down as less fuel has

to be used to meet the thrust requirement. From the BPR sensitivity, it was concluded that it would be better to be slightly above the specified BPR than below. The HPCPR had a large positive effect on sNOx for both engines. The TSFC was seen to go down for both, which is due to the increase in engine weight. From this study it was concluded the TIT is the most critical parameter in terms of its effects of other parts of the engine, the BPR and HPCPR both have a similar degree of ‘knock-on effect’, however it would be preferable to be slightly above the designed BPR and HPCPR than below. The FPR was concluded not to be a sensitive critical parameter. This conclusion can be used to help direct limited resources in the building and testing phase of the engine to ensure it is as close to the designed performance as possible.

3.5 Final Design Point Selection

The final design point was chosen to be the separate exhaust design point - the parameters are shown in Table 3.4. The engine was intended to be used primarily at cruise for passenger services - as such, the most critical design point was the cruise TSFC. Optimising TSFC minimises the fuel consumption at cruise, reducing operating costs. The difference in TSFC between the two exhaust types was almost 0.04 kg/hr/daN, meaning that switching between the two was a significant difference. The majority of the other performance parameters were similar, with the exception of specific thrust. The mixed exhaust engine had a much larger specific thrust (14.9 to 13.6 daN/kg/s). However, the seperate engine also had a lower engine weight and fan diameter, which improved the operation of the engine in off-design conditions. Based on all these factors, the seperate engine was considered to be superior and was selected as the ideal engine.

Table 3.4: Selected design point parameters for the final configuration.

Input Parameter	**Value**	**Units**	**Output Parameter**	**Value**	**Units**
BPR	7	-	d_{fan}	1.18	m
FPR	1.65	-	$F_{\text{n,inst}}$	12.3	kN
HPCPR	13	-	F_{spec}	13.6	daN/kg/s
IPCPR	1.81	-	TSFC	0.631	kg/hr/daN
TIT	1575	K	W_{engine}	1291	kg
DoH	0.2	-	η_{IPC}	0.928	-
			$\eta_{\text{p, core}}$	0.461	-
			$\eta_{\text{p, HPC}}$	0.923	-
			$\eta_{\text{p, LPC}}$	0.905	-
			$\eta_{\text{p, propulsive}}$	0.766	-

3.6 Off-Design Performance

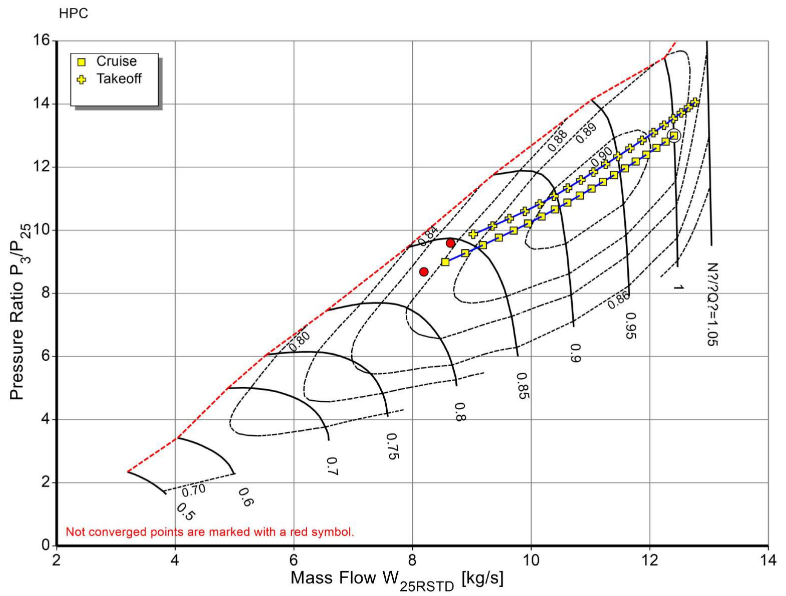

Figure 3.14: The variation in operating line of the HPC at takeoff and cruise

This section examines the selected engine at the off-design point, specifically the take-off situation. The takeoff settings are set at 63 kN thrust per engine at sea level and a Mach number of 0.2. Figure 3.14 shows that the HPC performs similarly at the design and off-design points, with identical pressure ratios. This is due to the HPC's rotor and stator blades, which realign the flow for the next step. As a result of the absence of separation or surge, the pressure ratio between design and off-design locations for a particular RPM will be comparable. As a result, the HPC efficiency remains stable.

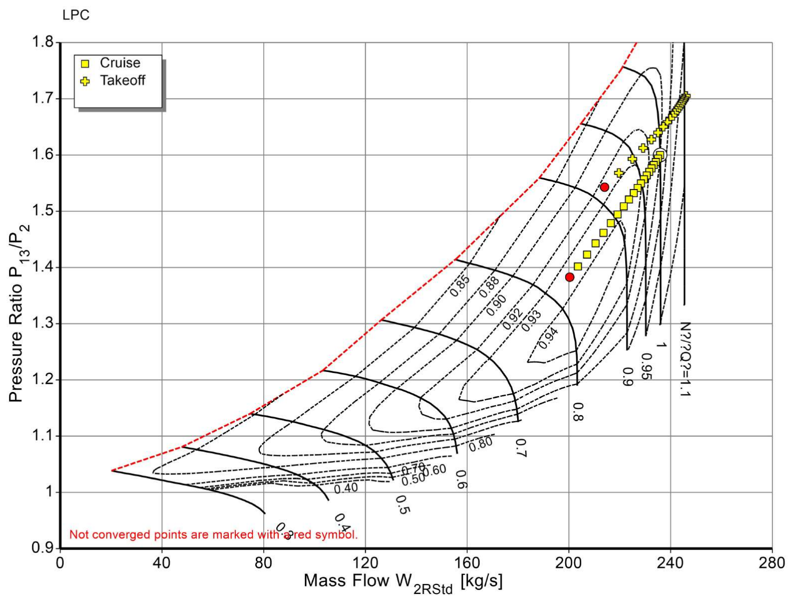

Figure 3.15: The variation in operating line of the LPC at takeoff and cruise

Figure 3.15 shows a contrasting pattern, with the LPC behaving considerably differently at the design and off-design points. Due to the lack of stator blades, the LPC will be more efficient for the design points alone. To correctly interpret figure 3.15, keep in mind that during takeoff, the throttle setting will be as high as possible in order to reduce the distance to takeoff. At cruise, the throttle setting will be somewhat lower, mid to high throttle, around $80 - 90\%$. Full throttle results in a normalised rotational speed line of 1.1. It can be observed that the LPC has an efficiency of 0.94 for a lower throttle in the cruising setting, as it was built for this flying action, however the LPC has an estimated efficiency of 0.90 during takeoff.

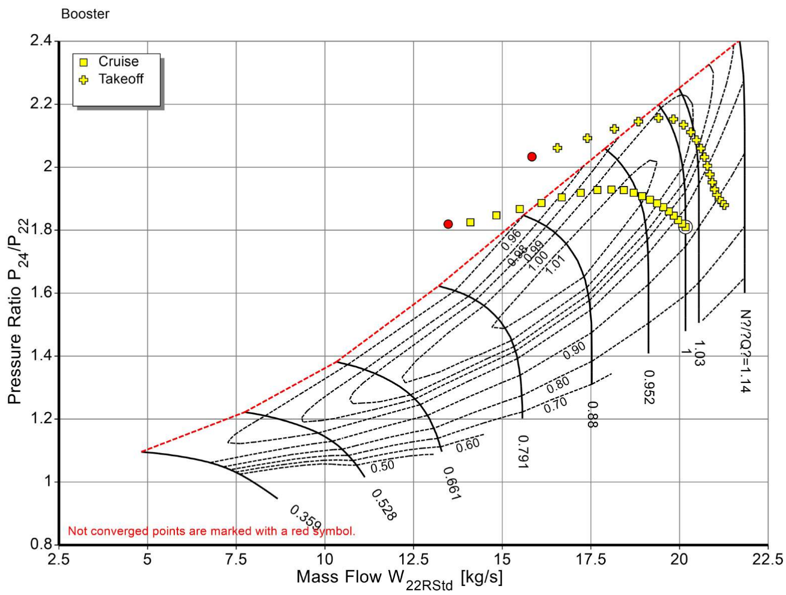

Figure 3.16: The variation in operating line of the booster at takeoff and cruise

The cruise line has a higher positive gradient than the takeoff line due to the limited number of throttle settings. A low throttle setting results in an undesirable pressure ratio, but increasing the throttle leads to a steep increase in the design pressure ratio. A similar approach may be used with the intermediate compressor map. The cruise operating lines lies above the surge line at low RPMs, however this is irrelevant because these RPMs will never be used. There is no possibility of surge at full power during takeoff since a low pressure ratio is achieved. At low mass flow rates, the relative flow vector seen by the compressor blades is too high, posing a surge danger for the design point.

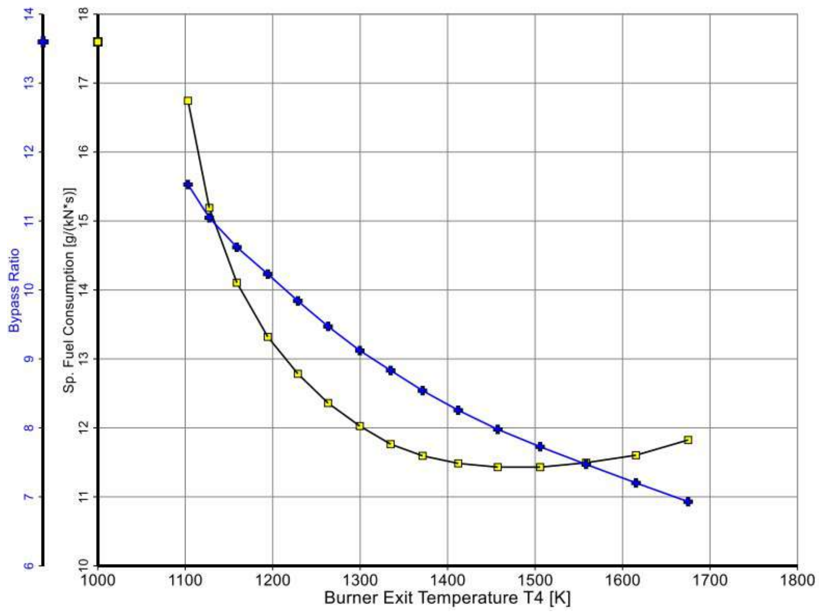

Figure 3.17: The variance in BPR and TSFC with throttle setting.

The thrust required per engine during takeoff is 63 kN. It's worth nothing that as altitude climbs, less thrust is produced. This is due to a reduction in air density with height. As air density declines, fewer molecules are present for compression, resulting in less oxygen available for burning, lowering the amount of thrust that may be created. Figure 3.17 shows a clear association between BPR and TSFC in respect to throttle settings. The growing TIT on the x-axis indicates the throttle setting. At low throttle levels, core thrust is restricted, hence the fan is the major thrust supplier, resulting in high BPR. This occurs when the engine generates additional core thrust to increase overall power, resulting in the ideal BPR at the flying state. As smaller increases in thrust need more $\dot{m}_f$,TSFC is lowered. Because takeoff accounts for a minor amount of operational time, a small TSFC increase is permitted to boost F_{net}.

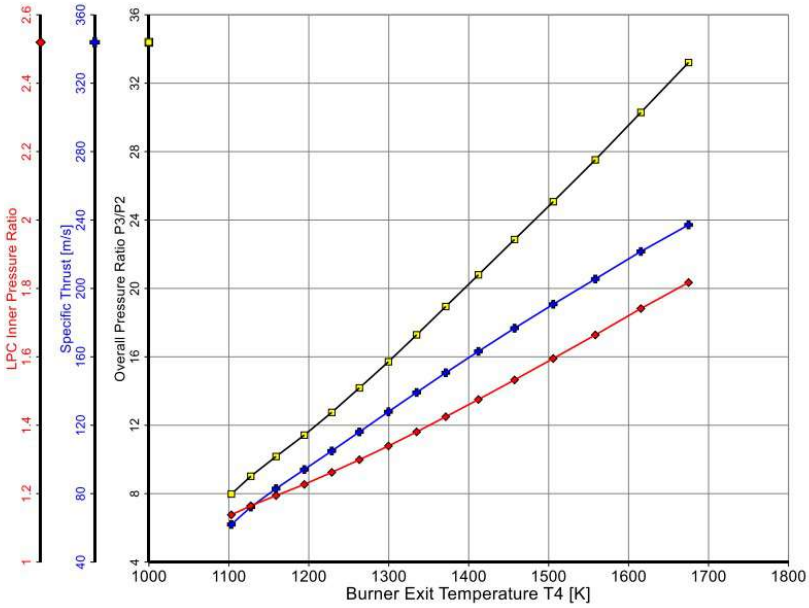

Figure 3.18: The variance in LPR, OPR and specific thrust with throttle settings.

Figure 3.18 shows how LPR, OPR and specific thrust vary with throttle setting. LPR and OPR both rise with throttle setting, when the engine spools up, the compressor RPM increases, resulting in increased fan and overall pressure. Increased pressure allows for greater energy extraction from the air-fuel combination, resulting in higher specific thrust with less air mass flow.

4 Conclusion

In summary, the ideal engine for the design brief is a separate exhaust turbofan with 20% DoH, as conveyed in Table 3.1. The deciding benefit of the separate configuration was the notable difference in TSFC and engine weight portrayed by the studies. Whilst, the selection of an engine with hybridisation was due to the greater efficiency observed with an increased DoH. Hence, the report successfully evaluated separate and mixed turbofan engines across various configurations to ascertain the optimum layout for the design brief.

References

[1] AIAA Foundation, *A hybrid-electric propulsion system using fuselage boundary layer ingestion for a single-aisle commercial aircraft*, AIAA Foundation Student Design Competition 2022/23, Undergraduate Team - Engine, 2022. [Online]. Available: https://www.aiaa.org.

[2] T. Okura, "Materials for aircraft engines," University of Colorado Boulder, Tech. Rep., 2015. [Online]. Available: https://www.colorado.edu/faculty/kantha/sites/default/files/attached-files/73549-116619_-_takehiro_okura_-_dec_17_2015_1027_am_-_asen_5063_2015_final_report_okura.pdf.

[3] I. Halliwell *et al.*, "Materials for aircraft engines," AIAA, Tech. Rep., 2022. [Online]. Available: https://www.aiaa.org/docs/default-source/uploadedfiles/membership-and-communities/university-students/design-competitions/rfp---a-hybrid-electric-propulsion-system-for-a-single-aisle-commercial-aircraft_final-85.pdf.

[4] N. Meier. "Civil turbofan specifications." Accessed: 2024-05-12. (2024), [Online]. Available: https://jet-engine.net/civtfspec.htm.

[5] D. Verstraete, *Aerospace Propulsion: AERO3261*. USYD Aero, 2013.

A Supporting Figures

A.1 Separate Exhaust Study

A.1.1 OPR vs TIT

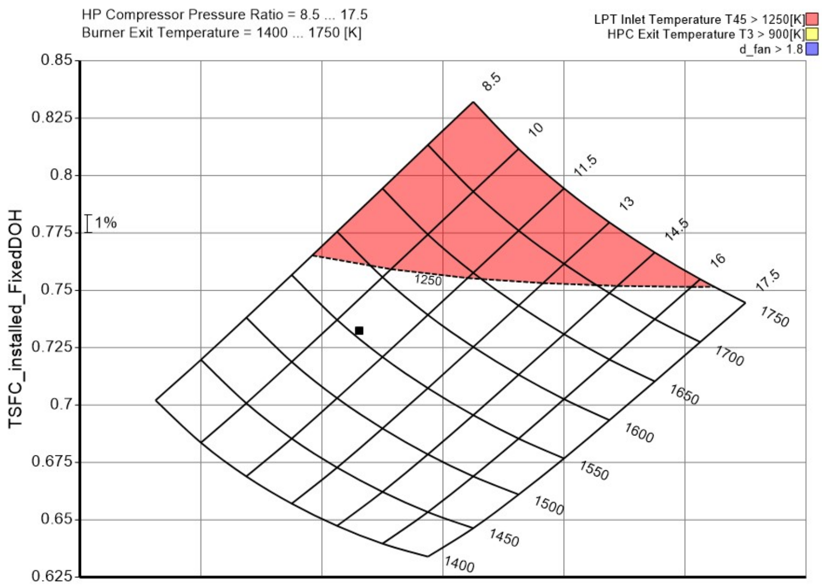

Figure A.1: A separate exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Overall Pressure Ratio (OPR) and Turbine Inlet Temperature (TIT). This was plotted from the side wherein F_{spec} was aligned into the page.

A.1.2 BPR vs TIT

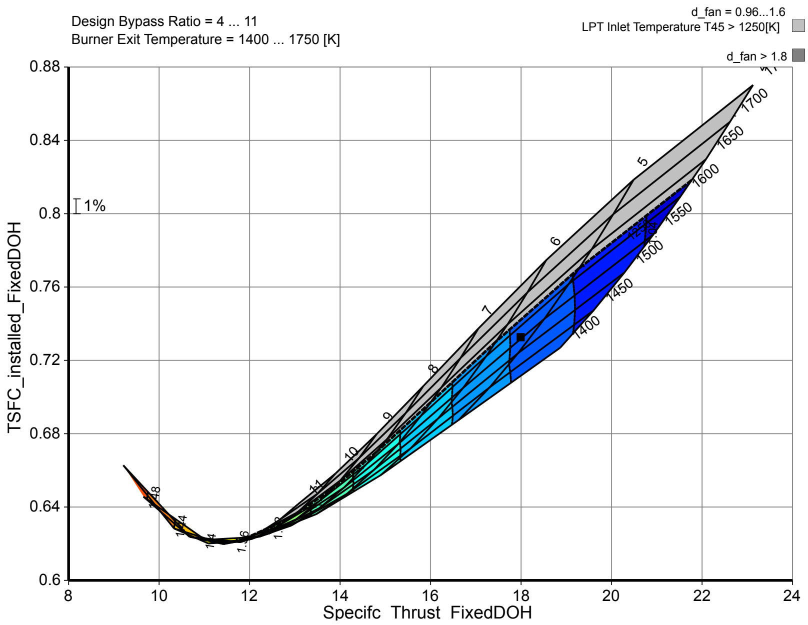

Figure A.2: A separate exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Bypass Ratio (BPR) and Turbine Inlet Temperature (TIT) in a standard view.

A.2 Mixed Exhaust Study

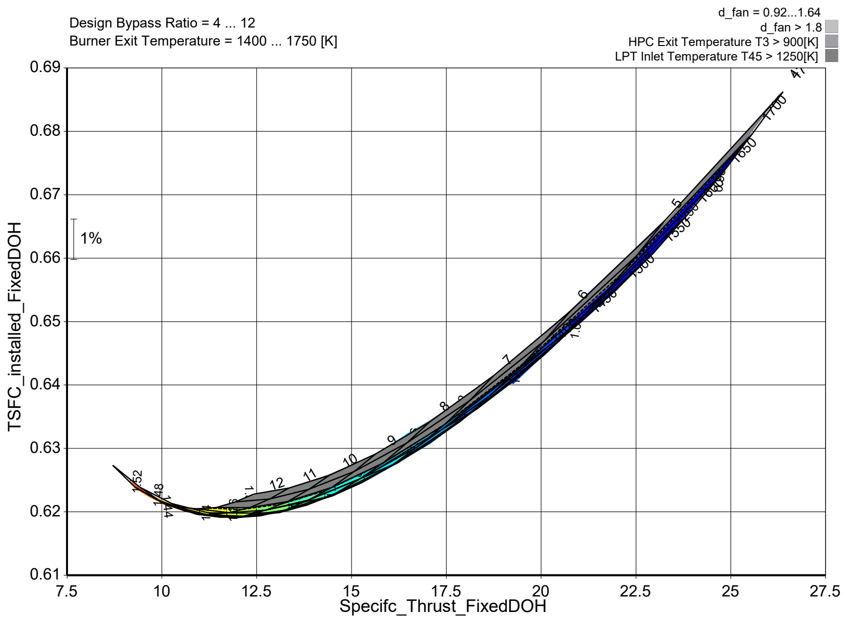

Figure A.3: A mixed exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and installed Specific Thrust [daN/kg/s] with Bypass Ratio (BPR) and Turbine Inlet Temperature (TIT) in standard view.

B Mixed Fan Pressure Ratio Notes

B.1 Figures

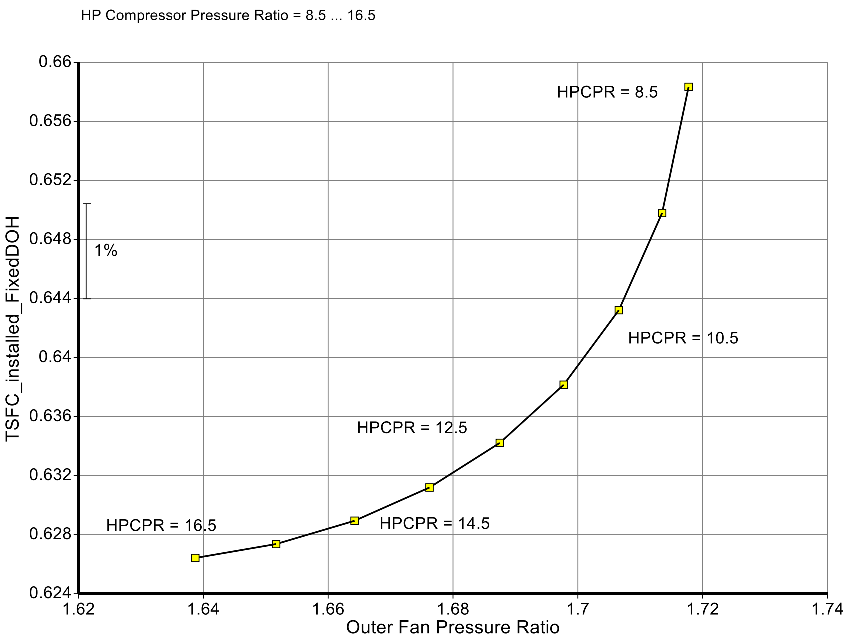

Figure B.1: A mixed exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and Fan Pressure Ratio with Overall Pressure Ratio (OPR)

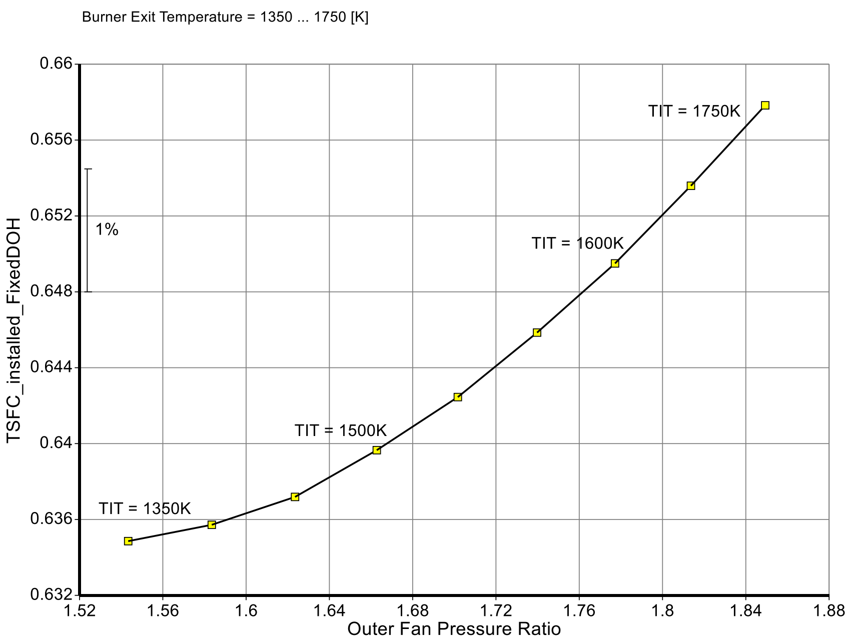

Figure B.2: A mixed exhaust parametric plot describing the variation in installed TSFC [kg/hr/daN] and Fan Pressure Ratio with Turbine Inlet Temperature (TIT) [K]

B.2 Discussion

Figures B.1 and B.2 display the effect of changing OPR and TIT on FPR. The figures show that there was a limited change in FPR with these parameters and that there was a corresponding change in TSFC, which represented the change in TSFC with FPR. However, it is disingenuous to consider the change in FPR with TSFC as a design parameter - for the mixed exhaust, as discussed in Section 3.2.3, FPR is fixed by the other parameters selected. Hence FPR functions as an output of the design parameters inputs, and so should not be considered as a design parameter for the mixed exhaust.

C GASTURB for Separate

C.1 Iteration Parameters

Variable	min	max	Target	Value
Polytr.Inner LPC Efficiency	0.7	1	IterLPCin_PolyEff=E221pol/PolyE	1
Polytr.Outer LPC Efficiency	0.7	1	IterLPCout_PolyEff=E213pol/Poly	1
Inner Fan Pressure Ratio	1	10	IterFanRatio=ZP21q2/ZP13q2	1
Polytr.IPC Efficiency	0.7	1	IterIPC_PolyEff=E2224pol/PolyEf	1
Polytr.HPC Efficiency	0.7	1	IterHPC_PolyEff=E253pol/PolyEff	1
HPT NGV 1 Cooling Air / W25	0	0.3	IterNGV=CoolFrNGV-WCHN1q25	0
HPT Rotor 1 Cooling Air / W25	0	0.3	IterRot=CoolFrRotor-WCHR1q25	0
HPT NGV 2 Cooling Air / W25	0	0.3	IterNGV2=CoolFrNGV-WCHN2q25	0
HPT Rotor 2 Cooling Air / W25	0	0.3	IterRot2=CoolFrRotor-WCHR2q25	0
Inlet Corr. Flow W2Rstd	0	1000	Total_Thrust_fixed=F_fan_fixed+FN*2	30.2
Power Offtake	0	1000	PWX_fixed_Iter=PWX-Power_Outake_DOH_fixed	0
Polytr.HPT Efficiency	0.7	1	IterHPT_PolyEff=E444pol/PolyEff_HPT_corrected	1

Figure C.1: Iteration variables in for separate exhaust GASTURB window.

C.2 Composed Variables

```
cp_val1: COOLING_PARAMETERS_APPENDIX_B=1  // ************************
cp_val2: T_metal=1250  // Metal Temperature
cp_val3: epsNGV=(T4_D-T_metal)/(T4_D-T3)  // Epsilon NGV
cp_val4: epsRotor=(T41-T_metal)/(T41-T3)  // Epsilon Rotor
cp_val5: CoolFrNGV=0.06*epsNGV/(1-epsNGV)  // Cooling Fraction of NGV
cp_val6: CoolFrRotor=0.06*epsRotor/(1-epsRotor)  // Cooling Fraction of
    Rotor
cp_val7: IterNGV=CoolFrNGV-WCHN1q25  // Iteration Variable for NGV 1
   Cooling (Target = 0)
cp_val8: IterRot=CoolFrRotor-WCHR1q25  // Iteration Variable for Rotor
   1 Cooling (Target = 0)
cp_val9: IterNGV2=CoolFrNGV-WCHN2q25  // Iteration Variable for NGV 2
   Cooling (Target = 0)
cp_val10: IterRot2=CoolFrRotor-WCHR2q25  // Iteration Variable for
   Rotor 2 Cooling (Target = 0)
cp_val11: POLYTROPIC_EFFICIENCY_APPENDIX_B=1  //
   ************************
cp_val12: PolyEff_HPT_corrected=0.8679-CoolFrNGV  // HPT Efficiency
   reduced with cooling fraction correction
cp_val13: IterHPT_PolyEff=E444pol/PolyEff_HPT_corrected  // Iteration
   Variable for HPT Polytropic Efficiency (Target = 1)
cp_val14: n_stag_LPC=1  // Number of Fan Stages
cp_val15: n_stag_IPC=3  // Number of Booster Stages
cp_val16: n_stag_HPC=9  // Number of HPC Stages
```

```
cp_val17: PiCstagLPC=ZP13q2^(1/n_stag_LPC)  // Stage Pressure Ratio of
   Fan
cp_val18: PiCstagIPC=ZP24q22^(1/n_stag_IPC)  // Stage Pressure Ratio of
    Booster
cp_val19: PiCstagHPC=ZP3q25^(1/n_stag_HPC)  // Stage Pressure Ratio of
   HPC
cp_val20: IterFanRatio=ZP21q2/ZP13q2  // Iteration Variable for Inner
   and Outer Fan Matching (Target = 1)
cp_val21: PolyEffLPC=(-1/30)*PiCstagLPC+24/25  // Polytropic Efficiency
    of Fan
cp_val22: PolyEffIPC=(-1/10)*PiCstagIPC+21/20  // Polytropic Efficiency
    of Booster
cp_val23: PolyEffHPC=(-2/15)*PiCstagHPC+11/10  // Polytropic Efficiency
    of HPC
cp_val24: IterLPCin_PolyEff=E221pol/PolyEffLPC  // Iteration Variable
   for LPC (inner) Polytropic Efficiency (Target = 1)
cp_val25: IterLPCout_PolyEff=E213pol/PolyEffLPC  // Iteration Variable
   for LPC (outer) Polytropic Efficiency (Target = 1)
cp_val26: IterIPC_PolyEff=E2224pol/PolyEffIPC  // Iteration Variable
   for IPC Polytropic Efficiency (Target = 1)
cp_val27: IterHPC_PolyEff=E253pol/PolyEffHPC  // Iteration Variable for
    HPC Polytropic Efficiency (Target = 1)
cp_val28: INSTALLATION_EFFECTS=1  // ************************
cp_val29: kutta=St16_P/St6_P  // Kutta Condition (Target = 1)
cp_val30: NACELLE_DRAG=1  // ************************
cp_val31: k_var=0.04  // Constant
cp_val32: NThrust=FN  // FN is given in kN
cp_val33: Dnac=k_var*V0*(FN/FNqW2)  // Nacelle drag
cp_val34: Fnet_eff=NThrust*(1-k_var*V0/FNqW2)  // FN effective
cp_val35: MTOW=60160  // MTOW
cp_val36: Area=W2/(rho0*V0)  // Area that air passes through
cp_val37: d_fan=(4*Area/3.1415/(1-0.3^2))^0.5  // Diameter of fan
cp_val38: W_engine=860*d_fan^2.4  // Engine weight
cp_val39: Fnet_installed=Fnet_eff-(W_engine/(MTOW*0.95/Dnac))  // FN
   CORRECTED
cp_val40: TSFC_installed=(WF/Fnet_installed)*36  // TSFC corrected
cp_val41: FIXED_DOH=1  // ************************
cp_val42: P_propulsive=FN*V0  // Propulsive power
cp_val43: DoH_fixed=0.2  // Fixed degree of hybridisation
cp_val44: P_ex_fixed=DoH_fixed*P_propulsive/(1-DoH_fixed)  // Power
   extracted from fan
cp_val45: FanEff=0.585  // Fan efficiency
cp_val46: F_fan_fixed=FanEff*P_ex_fixed/V0*2  // Thrust produced by fan
cp_val47: Total_Thrust_fixed=F_fan_fixed+FN*2  // Total thrust produced
cp_val48: TSFC_fixed=(WF/Total_Thrust_fixed)*36/2  // TSFC in daN/kg/hr
cp_val49: Power_Outake_DOH_fixed=111.8+P_ex_fixed  // Power offtake
cp_val50: PWX_fixed_Iter=PWX-Power_Outake_DOH_fixed  // Total power
   offtake (Target = 0)
cp_val51: INSTALL_AND_FAN_EFFECTS=1  // ************************
cp_val52: F_specific_installed=(Fnet_installed*100)/W2  // Specific
   Thrust in daNs/kg
cp_val53: Fixed_TSFC_installed=WF/(Fnet_installed+F_fan_fixed/2)*36  //
    TSFC in daN/kg/hr
cp_val54: Fixed_Specifc_Thrust=((Fnet_installed+F_fan_fixed/2)*100)/W2
    // Specific Thrust in daNs/kg
cp_val55: ZP3q25  // HP comp ratio
cp_val56: ZP13q2  // Outer fan ratio
```

C.3 Design Point Cycle

```
W          T          P          WRstd
Station  kg/s       K          kPa        kg/s        FN       =    13.17
   kN
 amb              218.81    23.842                    TSFC     =  18.9499
    g/(kN*s)
   2     90.350    246.88     35.990   235.444         WF       =    0.2496
       kg/s
  13     79.056    289.22     59.384   135.142         s NOX    =    0.5534
  21     11.294    289.22     59.384    19.306
  22     11.294    289.22     58.790    19.501         Core Eff =    0.4606
  24     11.294    347.03    106.411    11.802         Prop Eff =    0.7663
  25     11.294    347.03    104.283    12.043         BPR      =    7.0000
   3     11.068    749.09   1355.673     1.334         P2/P1    =    0.9900
  31     10.230    749.09   1355.673                   P3/P2    =     37.67
   4     10.479   1575.00   1301.446     1.907         P5/P2    =    1.3327
  41     10.919   1544.56   1301.446     1.968
  43     10.919   1127.59    252.918                   P16/P6   =   1.26387
  44     11.318   1115.20    252.918                   P16/P2   =   1.61700
  45     11.487   1108.14    247.859     9.209         P6/P5    =   0.96000
  49     11.487    769.45     47.965                   A8       =   0.17773
   5     11.543    768.81     47.965    39.831         A18      =   0.58559
   8     11.543    768.81     46.046    41.491         XM8      =   1.00000
  18     79.056    289.22     58.197   137.900         XM18     =   1.00000
 Bleed    0.000    749.09   1355.673                   WBld/W2  =   0.00000
 ---------------------------------------------         CD8      =   0.97976
 Efficiency       isentr  polytr    RNI     P/P        CD18     =   0.97600
  Outer LPC       0.8980  0.9050  0.426   1.650        PWX      =     893.4
  Inner LPC       0.8980  0.9050  0.426   1.650        V18/V8,id=   0.69938
  IP Compressor   0.9219  0.9281  0.578   1.810        WBLD/W22 =   0.00000
  HP Compressor   0.8928  0.9227  0.825  13.000        Wreci/W25=   0.00000
  Burner          0.9995                  0.960        Loading  =    100.00
  HP Turbine      0.8550  0.8290  1.806   5.146        WCHN/W25 =   0.03893
  LP Turbine      0.9174  0.9000  0.504   5.167        WCHR/W25 =   0.03528
 ---------------------------------------------         WCLN/W25 =   0.01500
 HP Spool mech Eff 0.9900  Nom Spd  18931 rpm          WCLR/W25 =   0.00500
 LP Spool mech Eff 0.9900  Nom Spd   4000 rpm          WBLD/W25 =   0.00000
 ---------------------------------------------         P6/P5    =   0.96000
 P22/P21=0.9900 P25/P24=0.9800 P45/P44=0.9800          P16/P13  =    0.9800
 ---------------------------------------------
 hum [%]     war0       FHV     Fuel
    0.0   0.00000    43.153    Generic

  Composed Values:
  1: COOLING_PARAMETERS_APPENDIX_B
     = 1          *************************
  2: T_metal
     = 1250       Metal Temperature
  3: epsNGV
     = 0.393507   Epsilon NGV
  4: epsRotor
     = 0.370296   Epsilon Rotor
  5: CoolFrNGV
     = 0.0389294   Cooling Fraction of NGV
  6: CoolFrRotor
     = 0.0352828   Cooling Fraction of Rotor
```

```
 7: IterNGV
    = 0         Iteration Variable for NGV 1 Cooling (Target = 0)
 8: IterRot
    = 0         Iteration Variable for Rotor 1 Cooling (Target = 0)
 9: IterNGV2
    = 0         Iteration Variable for NGV 2 Cooling (Target = 0)
10: IterRot2
    = 0         Iteration Variable for Rotor 2 Cooling (Target = 0)
11: POLYTROPIC_EFFICIENCY_APPENDIX_B
    = 1         ************************
12: PolyEff_HPT_corrected
    = 0.828971  HPT Efficiency reduced with cooling fraction
       correction
13: IterHPT_PolyEff
    = 1         Iteration Variable for HPT Polytropic Efficiency (
       Target = 1)
14: n_stag_LPC
    = 1         Number of Fan Stages
15: n_stag_IPC
    = 3         Number of Booster Stages
16: n_stag_HPC
    = 9         Number of HPC Stages
17: PiCstagLPC
    = 1.65      Stage Pressure Ratio of Fan
18: PiCstagIPC
    = 1.21869   Stage Pressure Ratio of Booster
19: PiCstagHPC
    = 1.32975   Stage Pressure Ratio of HPC
20: IterFanRatio
    = 1         Iteration Variable for Inner and Outer Fan Matching (
       Target = 1...
21: PolyEffLPC
    = 0.905     Polytropic Efficiency of Fan
22: PolyEffIPC
    = 0.928131  Polytropic Efficiency of Booster
23: PolyEffHPC
    = 0.922699  Polytropic Efficiency of HPC
24: IterLPCin_PolyEff
    = 1         Iteration Variable for LPC (inner) Polytropic
       Efficiency (Targe...
25: IterLPCout_PolyEff
    = 1         Iteration Variable for LPC (outer) Polytropic
       Efficiency (Targe...
26: IterIPC_PolyEff
    = 1         Iteration Variable for IPC Polytropic Efficiency (
       Target = 1)
27: IterHPC_PolyEff
    = 1         Iteration Variable for HPC Polytropic Efficiency (
       Target = 1)
28: INSTALLATION_EFFECTS
    = 1         ************************
29: kutta
    = 1.26387   Kutta Condition (Target = 1)
30: NACELLE_DRAG
    = 1         ************************
31: k_var
    = 0.04      Constant
32: NThrust
```

```
     = 13.1734   FN is given in kN
 33: Dnac
     = 0.857657  Nacelle drag
 34: Fnet_eff
     = 12.3157   FN effective
 35: MTOW
     = 60160     MTOW
 36: Area
     = 1.00294   Area that air passes through
 37: d_fan
     = 1.18462   Diameter of fan
 38: W_engine
     = 1291.48   Engine weight
 39: Fnet_installed
     = 12.2964   FN CORRECTED
 40: TSFC_installed
     = 0.730853  TSFC corrected
 41: FIXED_DOH
     = 1         ************************
 42: P_propulsive
     = 3126.24   Propulsive power
 43: DoH_fixed
     = 0.2       Fixed degree of hybridisation
 44: P_ex_fixed
     = 781.561   Power extracted from fan
 45: FanEff
     = 0.585     Fan efficiency
 46: F_fan_fixed
     = 3.85322   Thrust produced by fan
 47: Total_Thrust_fixed
     = 30.2      Total thrust produced
 48: TSFC_fixed
     = 0.148789  TSFC in daN/kg/hr
 49: Power_Outake_DOH_fixed
     = 893.361   Power offtake
 50: PWX_fixed_Iter
     = 7.28116E-07  Total power offtake (Target = 0)
 51: INSTALL_AND_FAN_EFFECTS
     = 1         ************************
 52: F_specific_installed
     = 13.6097   Specific Thrust in daNs/kg
 53: Fixed_TSFC_installed
     = 0.631853  TSFC in daN/kg/hr
 54: Fixed_Specifc_Thrust
     = 15.7421   Specific Thrust in daNs/kg
 55: ZP3q25
     = 13        HP comp ratio
 56: ZP13q2
     = 1.65      Outer fan ratio

Iteration converged after 18 loops.

Iteration Variables:
1:  Polytr.Inner LPC Efficiency  (0.7...1)          = 0.905
2:  Polytr.Outer LPC Efficiency  (0.7...1)          = 0.905
3:  Inner Fan Pressure Ratio  (1...10)              = 1.65
4:  Polytr.IPC Efficiency  (0.7...1)                = 0.928131
5:  Polytr.HPC Efficiency  (0.7...1)                = 0.922699
```

```
6:  HPT NGV 1 Cooling Air / W25  (0...0.3)          = 0.0389294
7:  HPT Rotor 1 Cooling Air / W25  (0...0.3)        = 0.0352828
8:  HPT NGV 2 Cooling Air / W25  (0...0.3)          = 0.0389294
9:  HPT Rotor 2 Cooling Air / W25  (0...0.3)        = 0.0352828
10:  Inlet Corr. Flow W2Rstd kg/s (0...1000)         = 235.444
11:  Power Offtake kW (0...1000)                     = 893.361
12:  Polytr.HPT Efficiency  (0.7...1)                = 0.828971

Iteration Targets:
1:  cp_val24                                        = 1
2:  cp_val25                                        = 1
3:  cp_val20                                        = 1
4:  cp_val26                                        = 1
5:  cp_val27                                        = 1
6:  cp_val7                                         = 0
7:  cp_val8                                         = 0
8:  cp_val9                                         = 0
9:  cp_val10                                        = 0
10:  cp_val47                                        = 30.2
11:  cp_val50                                        = 0
12:  cp_val13                                        = 1
```

D GASTURB for Mixed

D.1 Iteration Parameters

Variable	min	max	Target	Value	act
Polytr.Inner LPC Efficiency	0.7	1	IterLPCin_PolyEff=E221pol/PolyE	1	✓
Polytr.Outer LPC Efficiency	0.7	1	IterLPCout_PolyEff=E213pol/Poly	1	✓
Inner Fan Pressure Ratio	1	10	IterFanRatio=ZP21q2/ZP13q2	1	✓
Polytr.IPC Efficiency	0.7	1	IterIPC_PolyEff=E2224pol/PolyEf	1	✓
Polytr.HPC Efficiency	0.7	1	IterHPC_PolyEff=E253pol/PolyEff	1	✓
HPT NGV 1 Cooling Air / W25	0	0.3	IterNGV=CoolFrNGV-WCHN1q25	0	✓
HPT Rotor 1 Cooling Air / W25	0	0.3	IterRot=CoolFrRotor-WCHR1q25	0	✓
HPT NGV 2 Cooling Air / W25	0	0.3	IterNGV2=CoolFrNGV-WCHN2q25	0	✓
HPT Rotor 2 Cooling Air / W25	0	0.3	IterRot2=CoolFrRotor-WCHR2q25	0	✓
Inlet Corr. Flow W2Rstd	0	1000	Total_Thrust_fixed=F_fan_fixed+	30.2	✓
Power Offtake	0	1000	PWX_fixed_Iter=PWX-Power_Outake	0	✓
Polytr.HPT Efficiency	0.7	1	IterHPT_PolyEff=E444pol/PolyEff	1	✓
Outer Fan Pressure Ratio	1	10	kutta=St16_P/St6_P	1	✓

Figure D.1: Iteration variables in for mixed exhaust GASTURB window.

D.2 Composed Variables

```
Composed Values for Cycle Design

cp_val1: COOLING_PARAMETERS_APPENDIX_B=1  // ************************
cp_val2: T_metal=1250  // Metal Temperature
cp_val3: epsNGV=(T4_D-T_metal)/(T4_D-T3)  // Epsilon NGV
cp_val4: epsRotor=(T41-T_metal)/(T41-T3)  // Epsilon Rotor
cp_val5: CoolFrNGV=0.06*epsNGV/(1-epsNGV)  // Cooling Fraction of NGV
cp_val6: CoolFrRotor=0.06*epsRotor/(1-epsRotor)  // Cooling Fraction of
    Rotor
```

```
cp_val7: IterNGV=CoolFrNGV-WCHN1q25  // Iteration Variable for NGV 1
   Cooling (Target = 0)
cp_val8: IterRot=CoolFrRotor-WCHR1q25  // Iteration Variable for Rotor
   1 Cooling (Target = 0)
cp_val9: IterNGV2=CoolFrNGV-WCHN2q25  // Iteration Variable for NGV 2
   Cooling (Target = 0)
cp_val10: IterRot2=CoolFrRotor-WCHR2q25  // Iteration Variable for
   Rotor 2 Cooling (Target = 0)
cp_val11: POLYTROPIC_EFFICIENCY_APPENDIX_B=1  //
   ************************
cp_val12: PolyEff_HPT_corrected=0.8679-CoolFrNGV  // HPT Efficiency
   reduced with cooling fraction correction
cp_val13: IterHPT_PolyEff=E444pol/PolyEff_HPT_corrected  // Iteration
   Variable for HPT Polytropic Efficiency (Target = 1)
cp_val14: n_stag_LPC=1  // Number of Fan Stages
cp_val15: n_stag_IPC=3  // Number of Booster Stages
cp_val16: n_stag_HPC=9  // Number of HPC Stages
cp_val17: PiCstagLPC=ZP13q2^(1/n_stag_LPC)  // Stage Pressure Ratio of
   Fan
cp_val18: PiCstagIPC=ZP24q22^(1/n_stag_IPC)  // Stage Pressure Ratio of
    Booster
cp_val19: PiCstagHPC=ZP3q25^(1/n_stag_HPC)  // Stage Pressure Ratio of
   HPC
cp_val20: IterFanRatio=ZP21q2/ZP13q2  // Iteration Variable for Inner
   and Outer Fan Matching (Target = 1)
cp_val21: PolyEffLPC=(-1/30)*PiCstagLPC+24/25  // Polytropic Efficiency
    of Fan
cp_val22: PolyEffIPC=(-1/10)*PiCstagIPC+21/20  // Polytropic Efficiency
    of Booster
cp_val23: PolyEffHPC=(-2/15)*PiCstagHPC+11/10  // Polytropic Efficiency
    of HPC
cp_val24: IterLPCin_PolyEff=E221pol/PolyEffLPC  // Iteration Variable
   for LPC (inner) Polytropic Efficiency (Target = 1)
cp_val25: IterLPCout_PolyEff=E213pol/PolyEffLPC  // Iteration Variable
   for LPC (outer) Polytropic Efficiency (Target = 1)
cp_val26: IterIPC_PolyEff=E2224pol/PolyEffIPC  // Iteration Variable
   for IPC Polytropic Efficiency (Target = 1)
cp_val27: IterHPC_PolyEff=E253pol/PolyEffHPC  // Iteration Variable for
    HPC Polytropic Efficiency (Target = 1)
cp_val28: INSTALLATION_EFFECTS=1  // ************************
cp_val29: kutta=St16_P/St6_P  // Kutta Condition (Target = 1)
cp_val30: NACELLE_DRAG=1  // ************************
cp_val31: k_var=0.04  // Constant
cp_val32: NThrust=FN  // FN is given in kN
cp_val33: Dnac=k_var*V0*(FN/FNqW2)  // Nacelle drag
cp_val34: Fnet_eff=NThrust*(1-k_var*V0/FNqW2)  // FN effective
cp_val35: MTOW=60160  // MTOW
cp_val36: Area=W2/(rho0*V0)  // Area that air passes through
cp_val37: d_fan=(4*Area/3.1415/(1-0.3^2))^0.5  // Diameter of fan
cp_val38: W_engine=860*d_fan^2.4  // Engine weight
cp_val39: Fnet_installed=Fnet_eff-(W_engine/(MTOW*0.95/Dnac))  // FN
   CORRECTED
cp_val40: TSFCinstalled=(WF/Fnet_installed)*36  // TSFC corrected
cp_val41: FIXED_DOH=1  // ************************
cp_val42: P_propulsive=FN*V0  // Propulsive power
cp_val43: DoH_fixed=0.1  // Fixed degree of hybridisation
cp_val44: P_ex_fixed=DoH_fixed*P_propulsive/(1-DoH_fixed)  // Power
   extracted from fan
```

```
cp_val45: FanEff=0.585  // Fan efficiency
cp_val46: F_fan_fixed=FanEff*P_ex_fixed/V0*2  // Thrust produced by fan
cp_val47: Total_Thrust_fixed=F_fan_fixed+FN*2  // Total thrust produced
cp_val48: TSFC_fixed=(WF/Total_Thrust_fixed)*36/2  // TSFC in daN/kg/hr
cp_val49: Power_Outake_DOH_fixed=111.8+P_ex_fixed  // Power offtake
cp_val50: PWX_fixed_Iter=PWX-Power_Outake_DOH_fixed  // Total power
   offtake (Target = 0)
cp_val51: INSTALL_AND_FAN_EFFECTS=1  // ************************
cp_val52: F_specific_installed=(Fnet_installed*100)/W2  // Specific
   Thrust in daNs/kg
cp_val53: TSFC_installed_FixedDOH=WF/(Fnet_installed+F_fan_fixed/2)*36
    // TSFC in daN/kg/hr
cp_val54: Specifc_Thrust_FixedDOH=((Fnet_installed+F_fan_fixed/2)*100)/
   W2  // Specific Thrust in daNs/kg
cp_val55: ZP3q25  // HP comp ratio
cp_val56: ZP13q2  // Outer fan ratio
```

D.3 Design Point Cycle

```
          W         T         P        WRstd
Station  kg/s       K        kPa       kg/s       FN        =     14.18
    kN
 amb             218.81    23.842                 TSFC      =   18.6696
     g/(kN*s)
  2    95.394    246.88    35.990   248.588       WF        =    0.2647
      kg/s
 13    84.794    283.41    55.781   152.757       s NOX     =    0.2068
 21    10.599    283.41    55.781    19.095
 22    10.599    283.41    55.223    19.288       Core Eff  =    0.4631
 24    10.599    283.42    55.228    19.286       Prop Eff  =    0.7628
 25    10.599    283.42    54.124    19.679       BPR       =    8.0000
  3    10.281    611.84   676.546     2.244       P2/P1     =    0.9900
 31     9.708    611.84   676.546                 P3/P2     =     18.80
  4     9.973   1550.00   649.485     3.608       P5/P2     =    1.5822
 41    10.272   1525.46   649.485     3.687       P16/P13   =    0.9800
 43    10.272   1210.83   195.804                 P16/P6    =   1.00000
 44    10.546   1196.64   195.804                 P16/P2    =   1.51887
 45    10.846   1178.70   191.887    11.584       P6/P5     =   0.96000
 49    10.846    906.04    56.943                 P63/P6    =   1.00000
  5    10.864    905.46    56.943    34.268       P163/P16  =   1.00000
  6    10.864    905.46    54.665                 XM63      =   0.48182
 16    84.794    283.41    54.665                 XM163     =   0.47050
 64    95.659    359.46    54.070                 XM64      =   0.50000
  8    95.659    359.46    54.070   200.215       A64       =   1.11307
Bleed   0.000    611.84   676.545                 WBld/W2   =   0.00000
--------------------------------------------      A8        =   0.84738
Efficiencies:    isentr   polytr    RNI    P/P    CD8       =   0.98000
 Outer LPC       0.9025   0.9083   0.426  1.550   XM8       =   1.00000
 Inner LPC       0.9025   0.9083   0.426  1.550   PWX       =     485.7
     kW
 IP Compressor   0.9500   0.9500   0.556  1.000   WBLD/W22 =   0.00000
 HP Compressor   0.8937   0.9235   0.545 12.500   Wreci/W25=   0.00000
 Burner          0.9995                   0.960   Loading  =    100.00
     %
 HP Turbine      0.8578   0.8397   0.914  3.317   e444 th  =   0.84975
```

```
 LP Turbine      0.9128  0.9000  0.363  3.370      WBLD/W25 =   0.00000
 Mixer           0.6000                            WCHN/W25 =   0.02821
-------------------------------------------        WCHR/W25 =   0.02590
HP Spool mech Eff 0.9900  Nom Spd  16082 rpm       WCLN/W25 =   0.02833
LP Spool mech Eff 0.9900  Nom Spd   4000 rpm       WCLR/W25 =   0.00167
P22/P21=0.9900 P25/P24=0.9800 P45/P44=0.9800       P6/P5    =    0.9600
-------------------------------------------
hum [%]     war0       FHV     Fuel
   0.0   0.00000    43.153    Generic

 Composed Values:
 1: COOLING_PARAMETERS_APPENDIX_B
    = 1          ************************
 2: T_metal
    = 1250       Metal Temperature
 3: epsNGV
    = 0.319774  Epsilon NGV
 4: epsRotor
    = 0.301501  Epsilon Rotor
 5: CoolFrNGV
    = 0.0282059  Cooling Fraction of NGV
 6: CoolFrRotor
    = 0.0258984  Cooling Fraction of Rotor
 7: IterNGV
    = 1.53974E-12  Iteration Variable for NGV 1 Cooling (Target = 0)
 8: IterRot
    = 4.04923E-13  Iteration Variable for Rotor 1 Cooling (Target = 0)
 9: IterNGV2
    = 1.53974E-12  Iteration Variable for NGV 2 Cooling (Target = 0)
10: IterRot2
    = 4.04923E-13  Iteration Variable for Rotor 2 Cooling (Target = 0)
11: POLYTROPIC_EFFICIENCY_APPENDIX_B
    = 1          ************************
12: PolyEff_HPT_corrected
    = 0.839694  HPT Efficiency reduced with cooling fraction
       correction
13: IterHPT_PolyEff
    = 1          Iteration Variable for HPT Polytropic Efficiency (
       Target = 1)
14: n_stag_LPC
    = 1          Number of Fan Stages
15: n_stag_IPC
    = 3          Number of Booster Stages
16: n_stag_HPC
    = 9          Number of HPC Stages
17: PiCstagLPC
    = 1.54987    Stage Pressure Ratio of Fan
18: PiCstagIPC
    = 1          Stage Pressure Ratio of Booster
19: PiCstagHPC
    = 1.32397    Stage Pressure Ratio of HPC
20: IterFanRatio
    = 1          Iteration Variable for Inner and Outer Fan Matching (
       Target = 1...
21: PolyEffLPC
    = 0.908338  Polytropic Efficiency of Fan
22: PolyEffIPC
    = 0.95       Polytropic Efficiency of Booster
```

```
23: PolyEffHPC
    = 0.92347   Polytropic Efficiency of HPC
24: IterLPCin_PolyEff
    = 1         Iteration Variable for LPC (inner) Polytropic
       Efficiency (Targe...
25: IterLPCout_PolyEff
    = 1         Iteration Variable for LPC (outer) Polytropic
       Efficiency (Targe...
26: IterIPC_PolyEff
    = 1         Iteration Variable for IPC Polytropic Efficiency (
       Target = 1)
27: IterHPC_PolyEff
    = 1         Iteration Variable for HPC Polytropic Efficiency (
       Target = 1)
28: INSTALLATION_EFFECTS
    = 1         *************************
29: kutta
    = 1         Kutta Condition (Target = 1)
30: NACELLE_DRAG
    = 1         *************************
31: k_var
    = 0.04      Constant
32: NThrust
    = 14.1784   FN is given in kN
33: Dnac
    = 0.905535  Nacelle drag
34: Fnet_eff
    = 13.2729   FN effective
35: MTOW
    = 60160     MTOW
36: Area
    = 1.05893   Area that air passes through
37: d_fan
    = 1.21724   Diameter of fan
38: W_engine
    = 1378.47   Engine weight
39: Fnet_installed
    = 13.251    FN CORRECTED
40: TSFCinstalled
    = 0.719142  TSFC corrected
41: FIXED_DOH
    = 1         *************************
42: P_propulsive
    = 3364.75   Propulsive power
43: DoH_fixed
    = 0.1       Fixed degree of hybridisation
44: P_ex_fixed
    = 373.861   Power extracted from fan
45: FanEff
    = 0.585     Fan efficiency
46: F_fan_fixed
    = 1.84319   Thrust produced by fan
47: Total_Thrust_fixed
    = 30.2      Total thrust produced
48: TSFC_fixed
    = 0.157771  TSFC in daN/kg/hr
49: Power_Outake_DOH_fixed
    = 485.661   Power offtake
```

```
 50: PWX_fixed_Iter
     = 2.94196E-05  Total power offtake (Target = 0)
 51: INSTALL_AND_FAN_EFFECTS
     = 1          *************************
 52: F_specific_installed
     = 13.8909    Specific Thrust in daNs/kg
 53: TSFC_installed_FixedDOH
     = 0.672379   TSFC in daN/kg/hr
 54: Specifc_Thrust_FixedDOH
     = 14.857     Specific Thrust in daNs/kg
 55: ZP3q25
     = 12.5       HP comp ratio
 56: ZP13q2
     = 1.54987    Outer fan ratio

Iteration converged after 1 loops.

  Iteration Variables:
  1:  Polytr.Inner LPC Efficiency  (0.7...1)          = 0.908338
  2:  Polytr.Outer LPC Efficiency  (0.7...1)          = 0.908338
  3:  Inner Fan Pressure Ratio  (1...10)              = 1.54987
  4:  Polytr.IPC Efficiency  (0.7...1)                = 0.95
  5:  Polytr.HPC Efficiency  (0.7...1)                = 0.92347
  6:  HPT NGV 1 Cooling Air / W25  (0...0.3)          = 0.0282059
  7:  HPT Rotor 1 Cooling Air / W25  (0...0.3)        = 0.0258984
  8:  HPT NGV 2 Cooling Air / W25  (0...0.3)          = 0.0282059
  9:  HPT Rotor 2 Cooling Air / W25  (0...0.3)        = 0.0258984
 10:  Inlet Corr. Flow W2Rstd kg/s (0...1000)         = 248.588
 11:  Power Offtake kW (0...1000)                     = 485.661
 12:  Polytr.HPT Efficiency  (0.7...1)                = 0.839694
 13:  Outer Fan Pressure Ratio  (1...10)              = 1.54987

  Iteration Targets:
  1:  cp_val24                                        = 1
  2:  cp_val25                                        = 1
  3:  cp_val20                                        = 1
  4:  cp_val26                                        = 1
  5:  cp_val27                                        = 1
  6:  cp_val7                                         = 0
  7:  cp_val8                                         = 0
  8:  cp_val9                                         = 0
  9:  cp_val10                                        = 0
 10:  cp_val47                                        = 30.2
 11:  cp_val50                                        = 0
 12:  cp_val13                                        = 1
 13:  cp_val29                                        = 1
```

E GASTURB for Off-design

E.1 Off-Design Point Cycle

```
         W          T          P         WRstd
```

```
Station  kg/s       K        kPa       kg/s       FN        =     13.17
    kN
 amb             218.81    23.842                 TSFC      =   19.1505
     g/(kN*s)
  2     90.508   246.88    35.990   235.857       WF        =    0.2523
      kg/s
 13     79.195   286.35    57.585   138.913       s NOX     =    0.5276
 21     11.314   286.35    57.585    19.845       P5/P2     =    1.4900
     EPR
 22     11.314   286.35    57.009    20.045       Core Eff  =    0.4617
 24     11.314   343.59   103.186    12.131       Prop Eff  =    0.7666
 25     11.314   343.59   101.122    12.379       BPR       =    7.0000
  3     11.087   742.21  1314.591     1.372       P2/P1     =    0.9900
 31     10.259   742.21  1314.591                 P3/P2     =     36.53
  4     10.511  1575.00  1262.007     1.973       P5/P2     =    1.4900
 41     10.946  1544.77  1262.007     2.035
 43     10.946  1131.79   249.885                 P16/P6    =   1.09621
 44     11.340  1119.21   249.885                 P16/P2    =   1.56800
 45     11.509  1112.02   244.888     9.355       P6/P5     =   0.96000
 49     11.509   794.78    53.625                 A8        =   0.16194
  5     11.566   793.98    53.625    36.276       A18       =   0.60193
  8     11.566   793.98    51.480    37.787       XM8       =   1.00000
 18     79.195   286.35    56.433   141.748       XM18      =   1.00000
Bleed    0.000   742.21  1314.591                 WBld/W2   =   0.00000
--------------------------------------------      CD8       =   0.98000
Efficiency       isentr  polytr    RNI    P/P     CD18      =   0.97600
 Outer LPC       0.9003  0.9067  0.426  1.600     PWX       =     893.4
     kW
 Inner LPC       0.9003  0.9067  0.426  1.600     V18/V8,id=   0.62780
 IP Compressor   0.9219  0.9281  0.567  1.810     WBLD/W22 =   0.00000
 HP Compressor   0.8928  0.9227  0.810 13.000     Wreci/W25=   0.00000
 Burner          0.9995                 0.960     Loading   =    100.00
     %
 HP Turbine      0.8552  0.8295  1.751  5.050     WCHN/W25 =   0.03840
 LP Turbine      0.9161  0.9000  0.496  4.567     WCHR/W25 =   0.03483
--------------------------------------------      WCLN/W25 =   0.01500
HP Spool mech Eff 0.9900  Speed    18673 rpm      WCLRW25  =   0.00500
LP Spool mech Eff 0.9900  Speed     4000 rpm      WBLD/W25 =   0.00000
--------------------------------------------      P6/P5     =   0.96000
P22/P21=0.9900 P25/P24=0.9800 P45/P44=0.9800      P16/P13   =    0.9800
--------------------------------------------
hum [%]     war0       FHV     Fuel
   0.0   0.00000    43.153    Generic


  All iteration variables estimated correctly - no iteration needed.

Spool Speeds:                 LP Spool  IP Spool  HP Spool
Absolute [RPM]                4000.0    4000.0   18672.6
Relative                      1.0000    1.0000    1.0000
                               LPC       IPC       HPC
Surge Margin [%]              45.477    56.467    24.104
Handling Bleed WB,hd/W22                0.0000

Map Coordinates:              LPC       IPC       HPC       HPT
   LPT
Map Speed                     1.0000    1.0000    1.0000    1.0000
   1.0000
```

```
Map Coordinate Beta           0.5000     0.4000     0.5000     0.5000
   0.5000
Reynolds Corrections:         LPC        IPC        HPC        HPT
   LPT
Efficiency                    1.0000     0.9877     0.9954     1.0000
   0.9848
Flow                          1.0000     0.9938     0.9977     1.0000
   0.9924
  Modifiers:                    Bypass-LPC-Core    IPC      HPC      HPT
         LPT
Delta Efficiency [%]           0.000     0.000    0.000    0.000    0.000
   0.000
Delta Flow Capacity [%]             0.000         0.000    0.000    0.000
   0.000
Delta Core Nozz Area [%]       0.000
Delta Byp Nozz Area [%]        0.000
Delta Core Inl Duct P/P [%]    0.000
Delta Comp Interd P/P [%]      0.000
Delta Burner P/P [%]           0.000
Delta Turb Interd P/P [%]      0.000
Delta Turbine Exit P/P [%]     0.000
Delta Bypass P/P [%]           0.000
```