Convolutional neural networks

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L-layer neural network: reminder

$x \in \mathbb{R}^N, y \in \mathbb{R}^K$, neural network: $\mathbb{R}^N \to \mathbb{R}^K$

$$z^{[1]} = W^{[1]}x + b^{[1]}, \quad W: n^{[1]} \times N, \quad b: n^{[1]} \times 1$$
$$a^{[1]} = g\left(z^{[1]}\right)$$

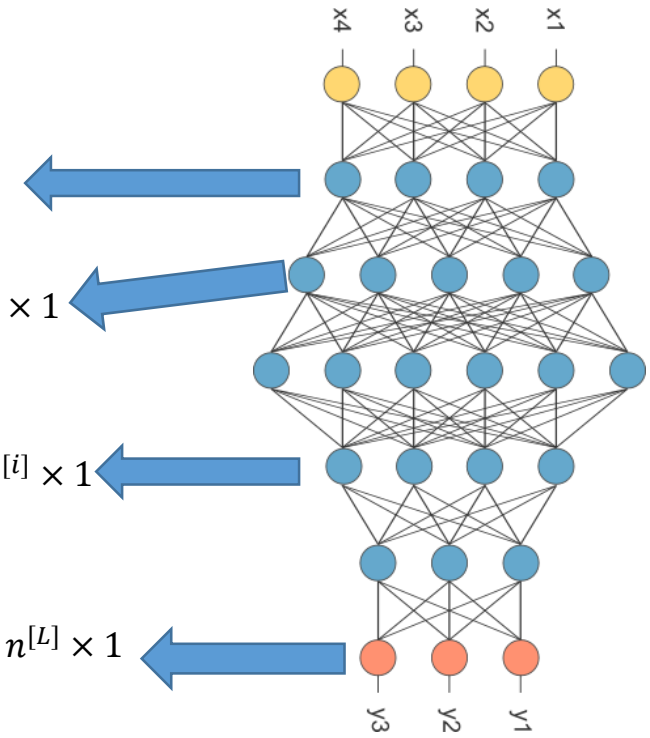
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}, \quad W: n^{[2]} \times n^{[1]}, \quad b: n^{[2]} \times 1$$
$$a^{[2]} = g\left(z^{[2]}\right)$$

$$\vdots$$

$$z^{[i]} = W^{[i]}a^{[i-1]} + b^{[i]}, \quad W: n^{[i]} \times n^{[i-1]}, \quad b: n^{[i]} \times 1$$
$$a^{[i]} = g\left(z^{[i]}\right)$$

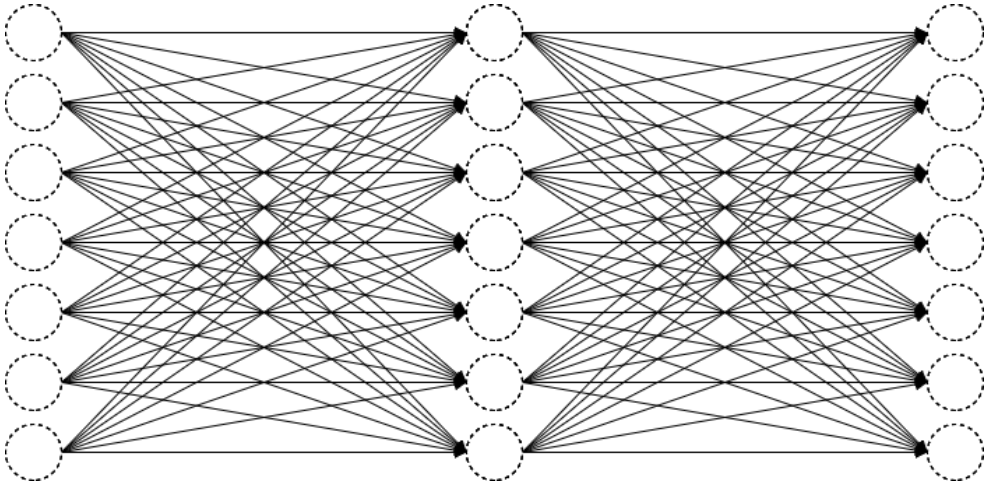
$$\vdots$$

$$z^{[L]} = W^{[L]}a^{[L-1]} + b^{[L]}, \quad W: n^{[L]} \times n^{[L-1]}, \quad b: n^{[L]} \times 1$$
$$y = a^{[L]} = softmax\left(z^{[L]}\right)$$



Credit: OpenNN

Dense neural networks: problems for real world images 1.



- Exploding parameter number:
  - 200x200 pixel input → 40000 input
  - $40000^2 + 40000 \approx 1.6 \cdot 10^9$ parameters per layer
  - float32: 4 byte/number → 6.4 GB/layer
  - color images have 3 color channels (RGB) → 57.6 GB/layer

Dense neural networks: problems for real world images 2.

Translation invariance
- same object appears at different part of the image is still the same object
- shared information -- 'tree detector', 'dog detector'

https://cs.stanford.edu/people/karpathy/ilsvrc/

Convolution in deep learning

filter

w_{00}	w_{01}	w_{02}
w_{10}	w_{11}	w_{12}
w_{20}	w_{21}	w_{22}

Image

a_{00}	a_{01}	a_{02}	a_{03}	a_{04}	a_{05}
a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
a_{40}	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}
a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}

Convolution in deep learning (in math class it is cross-correlation)

filter

w_{00}	w_{01}	w_{02}
w_{10}	w_{11}	w_{12}
w_{20}	w_{21}	w_{22}

Image

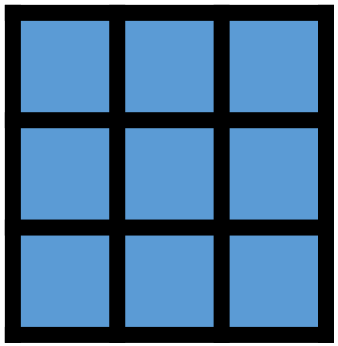
a_{00}	a_{01}	a_{02}	a_{03}	a_{04}	a_{05}
a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{20}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
a_{40}	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}
a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}

Note: Image-processing/math convolution is slightly different.
The kernel/filter is flipped around both axises before the multiplication.

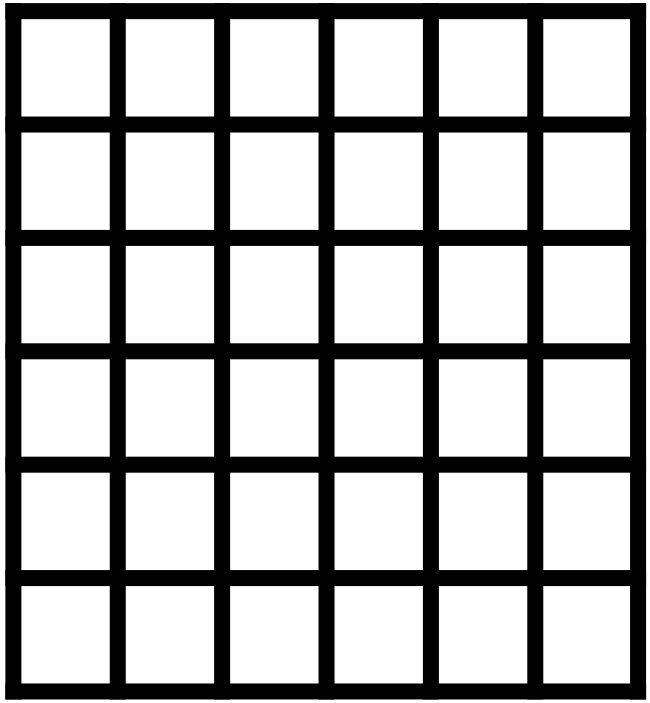
$$a'_{11} = w_{00} \cdot a_{00} + w_{01} \cdot a_{01} + w_{02} \cdot a_{02} + w_{10} \cdot a_{10} + w_{11} \cdot a_{11} + w_{12} \cdot a_{12} + w_{20} \cdot a_{20} + w_{21} \cdot a_{21} + w_{22} \cdot a_{22}$$

Convolution for images

filter

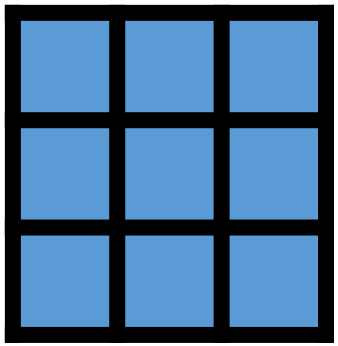


Image



Convolution for images

filter



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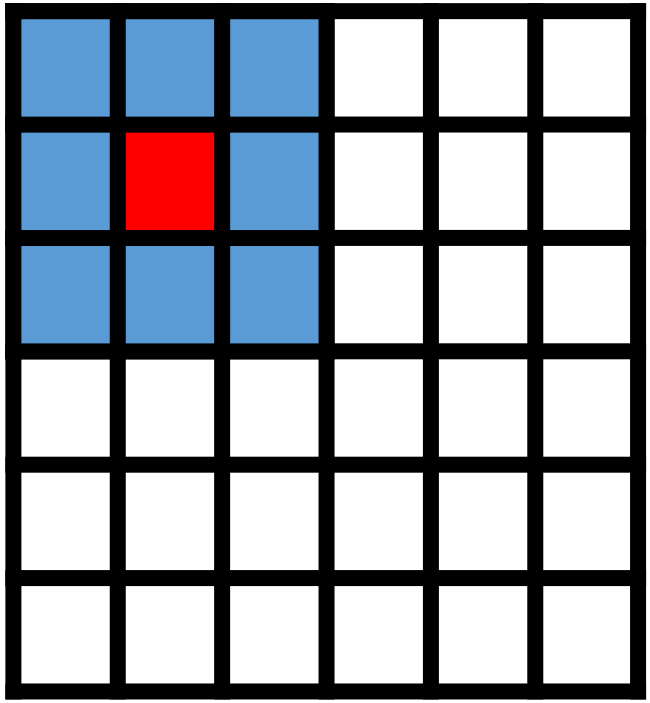
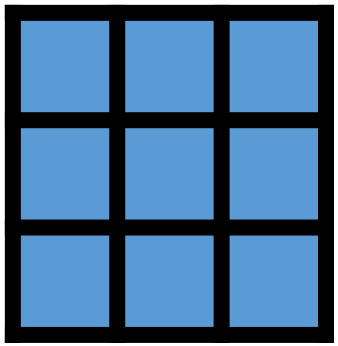


Convolution for images

filter



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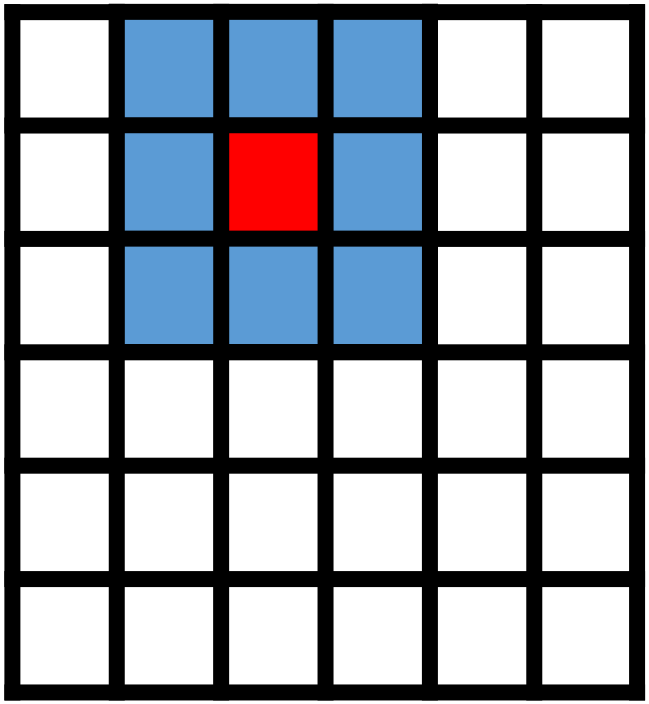
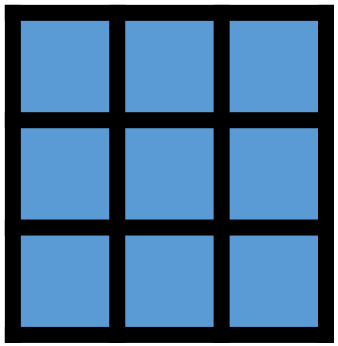
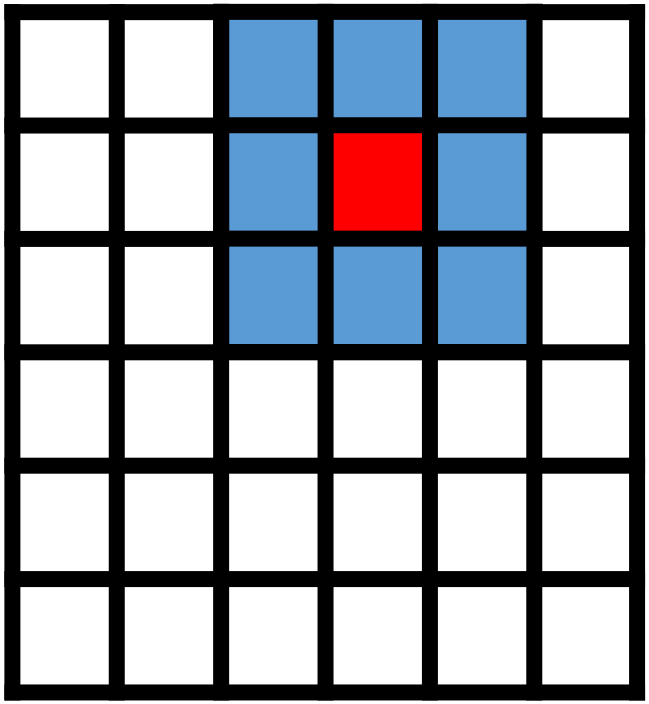


Convolution for images

filter

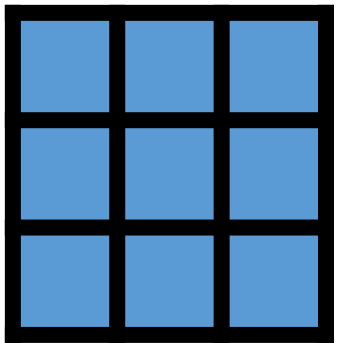


Image



Convolution for images

filter



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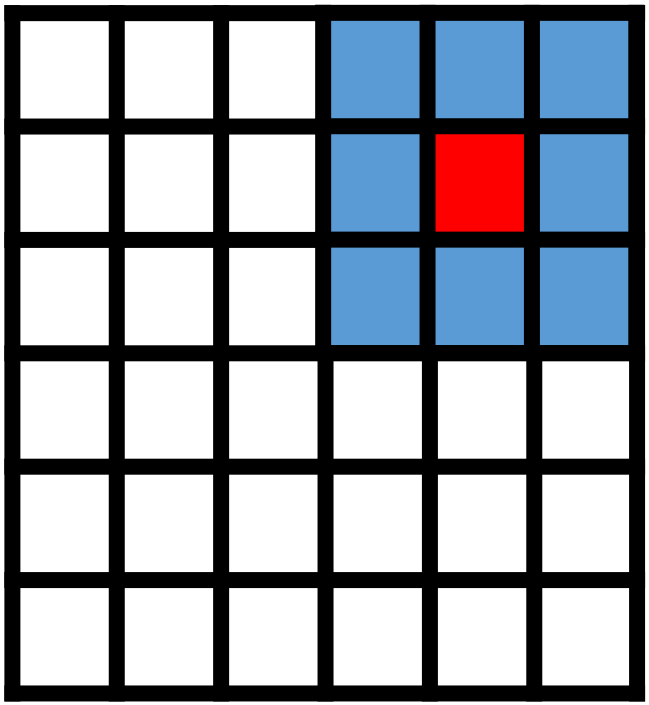
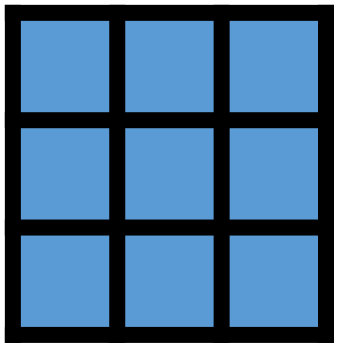


Convolution for images

filter



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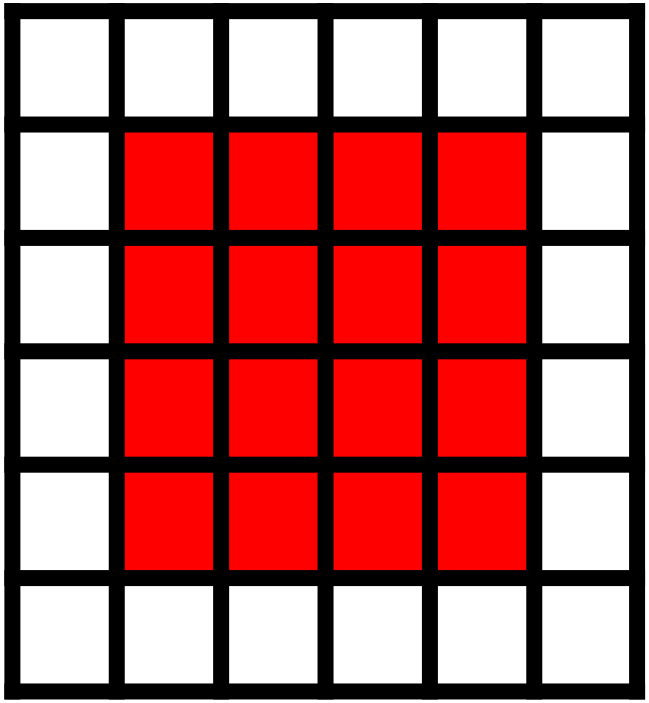
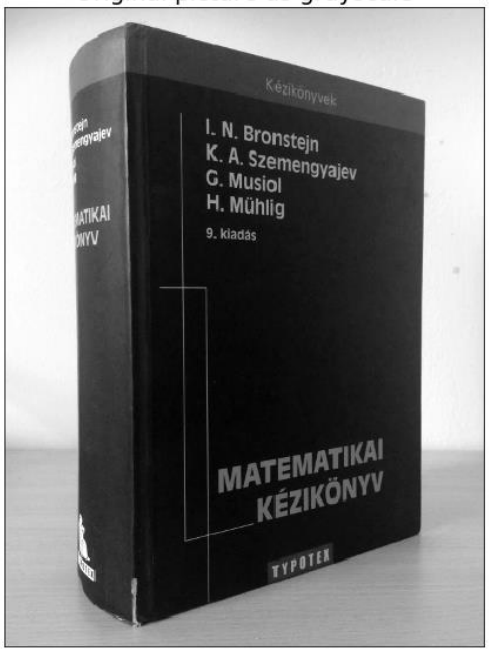


Convolution for images - examples

Original picture as grayscale



*

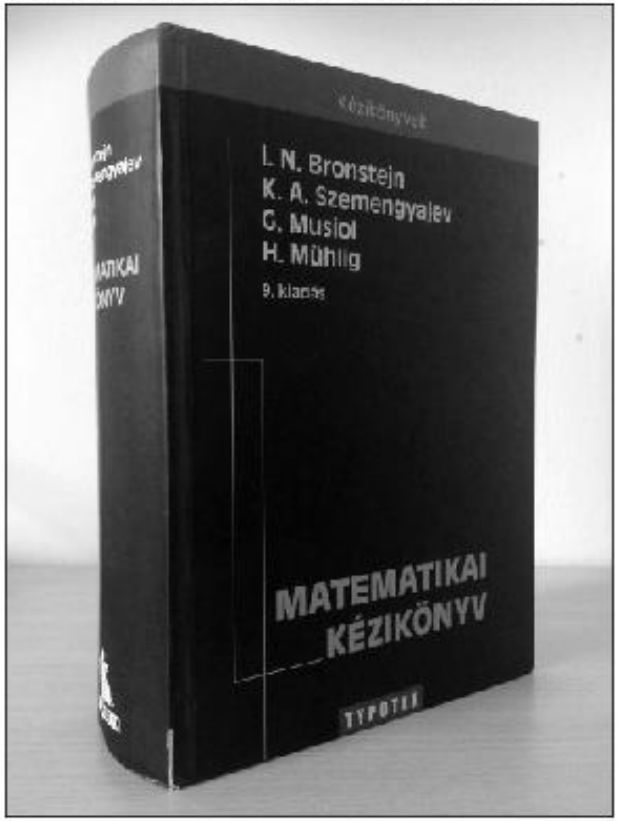
filter

-1	-1	-1
0	0	0
1	1	1

Question: what do you expect?

Convolution for images - examples

Original picture as grayscale



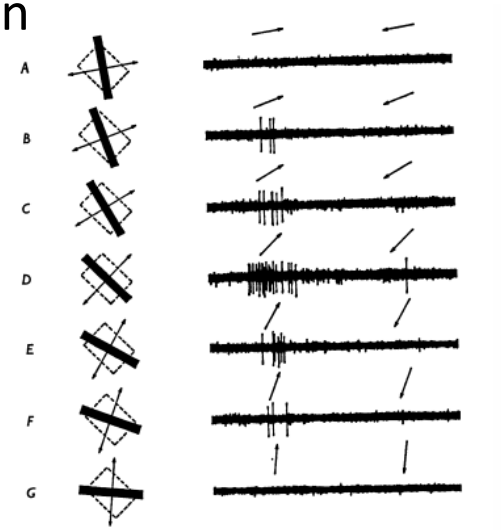
$$* \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} =$$

Horizontal edges

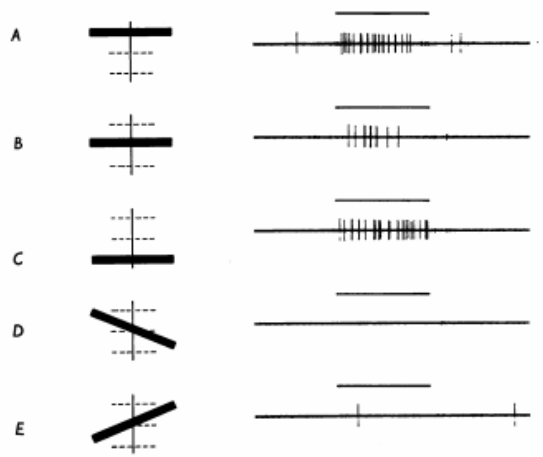
Excellent visualisations:
http://setosa.io/ev/image-kernels/
https://github.com/vdumoulin/conv_arithmetic

Neuroscientific experiments

- Hubel, Wiesel, Sperry
- cat's &monkey's vision
- Electrodes to the brain
- 1981 Nobel prize



Text-fig. 2. Responses of a complex cell in right striate cortex (layer IV A) t various orientations of a moving black bar. Receptive field in the left eye indicate by the interrupted rectangles; it was approximately $\frac{3}{4} \times \frac{3}{4}°$ in size, and was situate 4° below and to the left of the point of fixation. Ocular-dominance group 4. Duratio of each record, 2 sec. Background intensity 1·3 $\log_{10}$ cd/m², dark bars 0·0 log cd/m

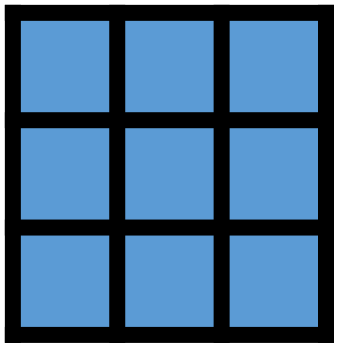


Text-fig. 7. Cell activated only by left (contralateral) eye over a field approximately $5 \times 5°$, situated 10° above and to the left of the area centralis. The cell responded best to a black horizontal rectangle, $\frac{1}{2} \times 6°$, placed anywhere in the receptive field (*A–C*). Tilting the stimulus rendered it ineffective (*D–E*). The black bar was introduced against a light background during periods of 1 sec, indicated by the upper line in each record. Luminance of white background, 1·0 $\log_{10}$ cd/m²; luminance of black part, 0·0 $\log_{10}$ cd/m². A lesion, made while recording from the cell, was found in layer 2 of apical segment of post-lateral gyrus.

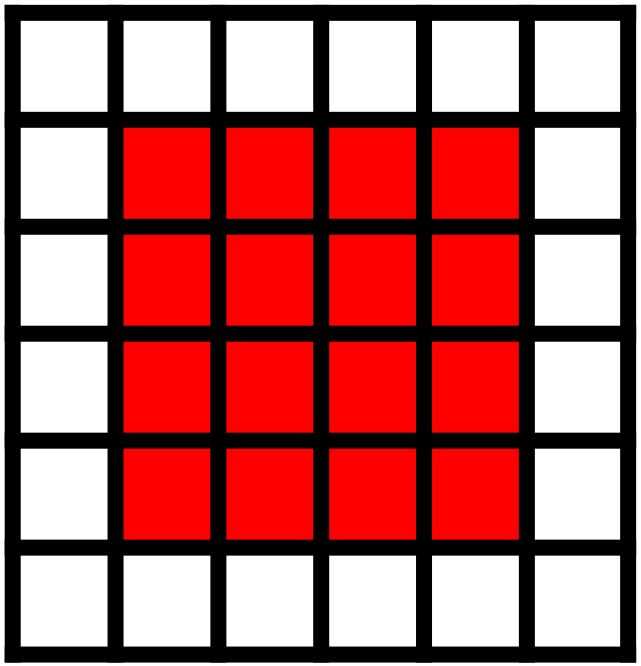
[Hubel, Wiesel: Receptive fields, binocular interaction and functional architecture in the cat's visual cortex, 1961]
[Hubel, Wiesel: Receptive fields and functional architecture of monkey striate cortex, 1968]

Padding

filter

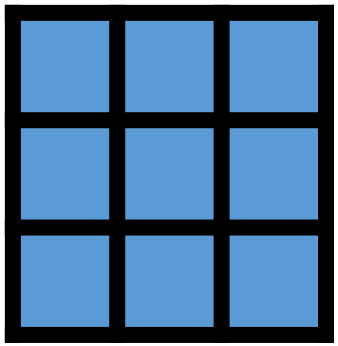


Image



Padding: 1

filter

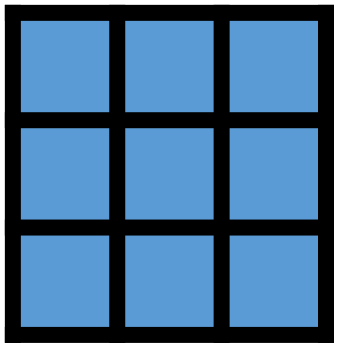


Image

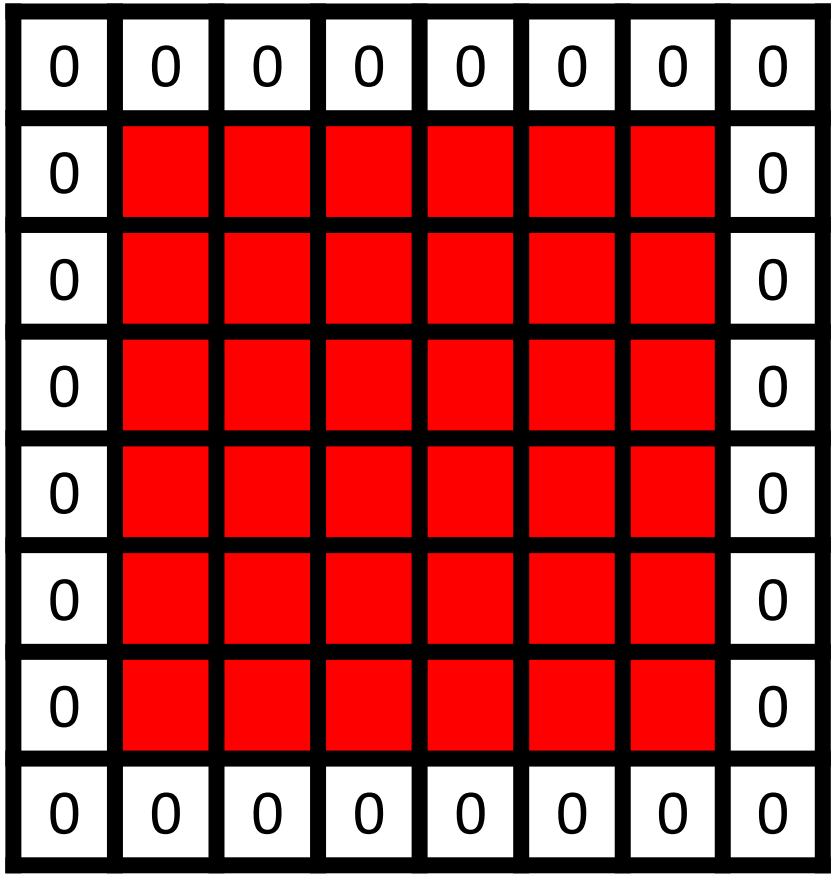
0	0	0	0	0	0	0	0
0							0
0							0
0							0
0							0
0							0
0							0
0	0	0	0	0	0	0	0

Padding: 1

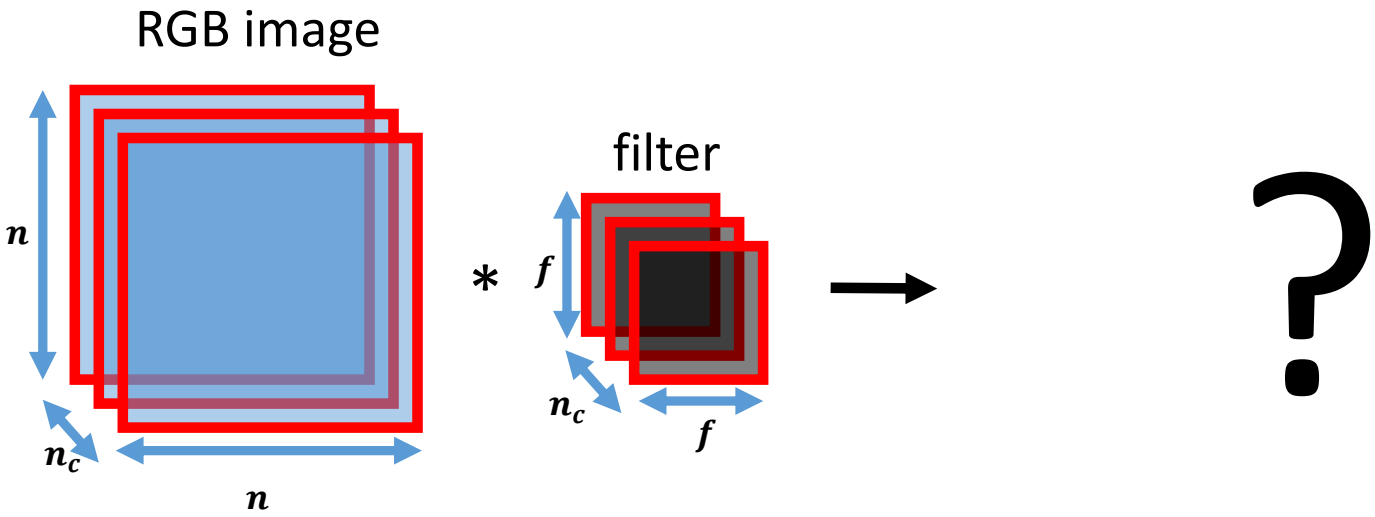
filter



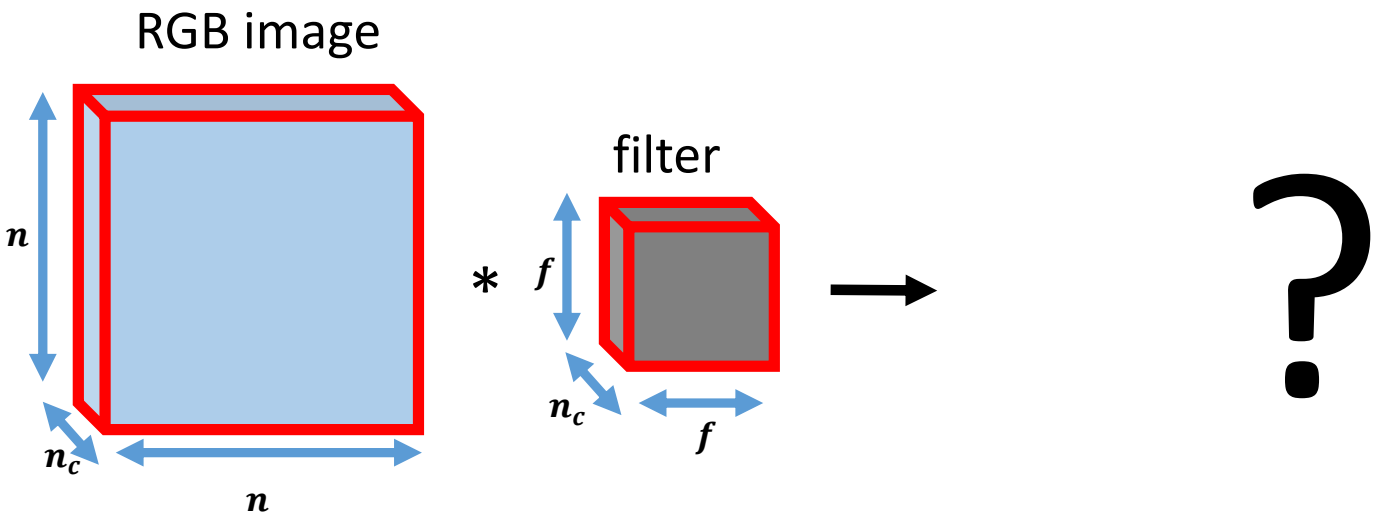
Image



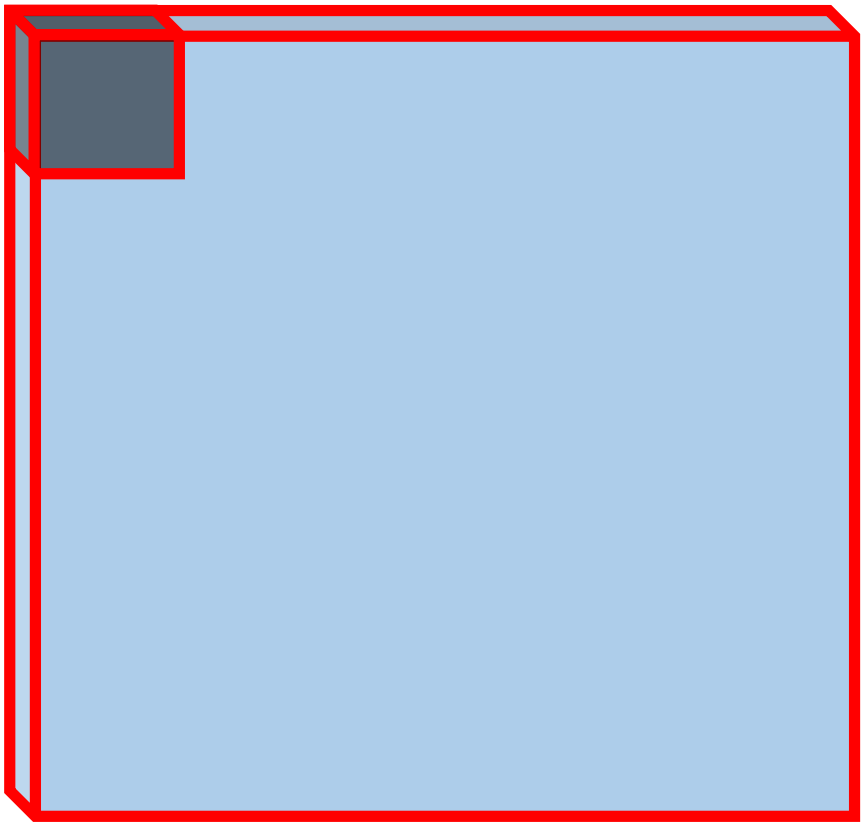
Convolution over volume



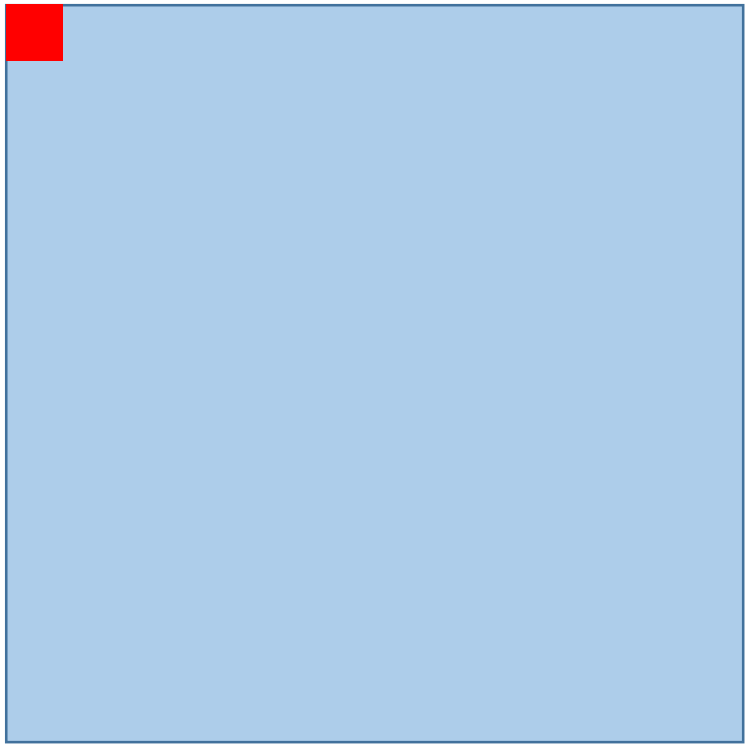
Convolution over volume



Convolution over volume

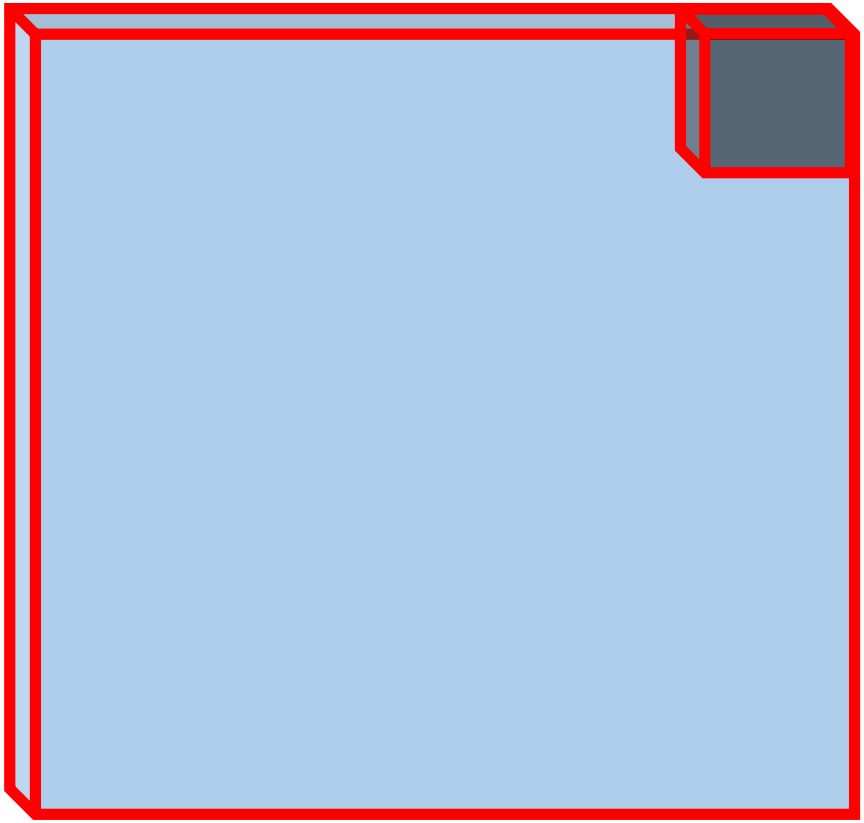


n_c = 3 (RGB)

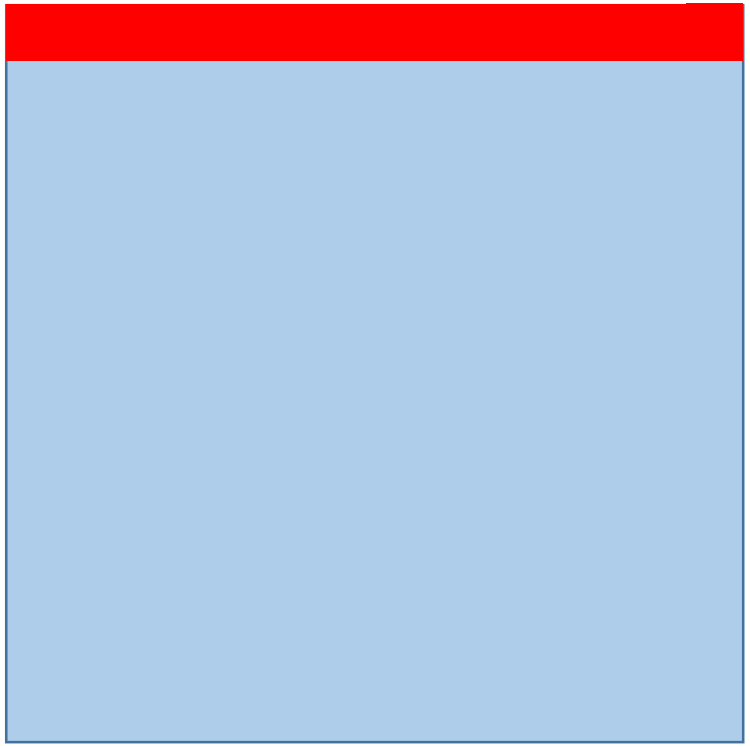


n_c = 1

Convolution over volume

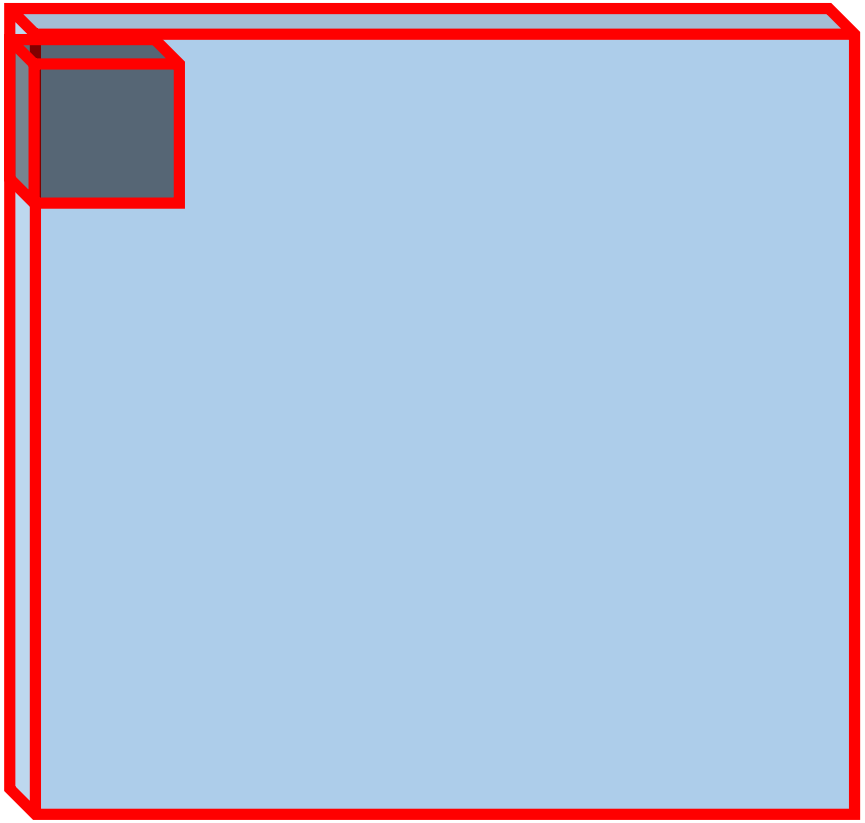


n_c = 3 (RGB)

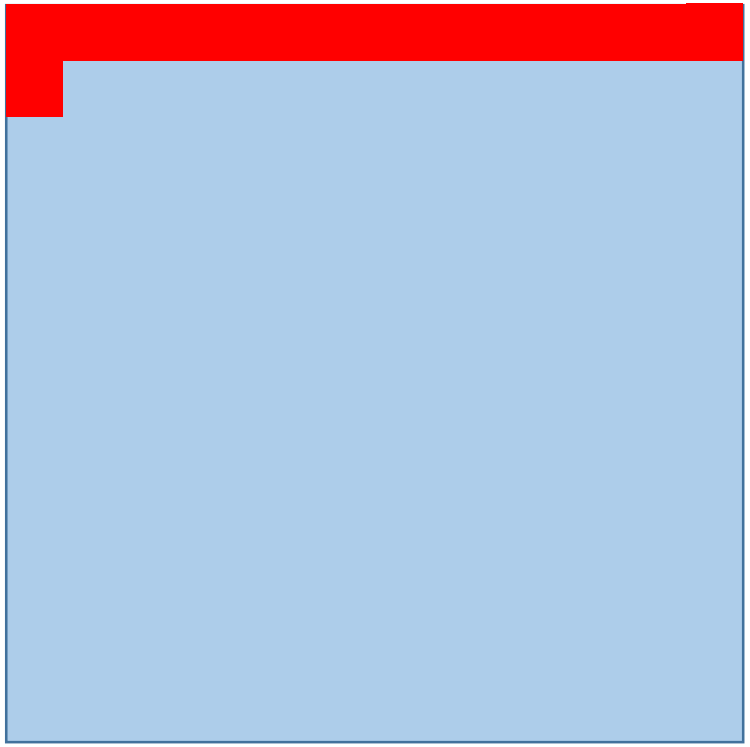


n_c = 1

Convolution over volume

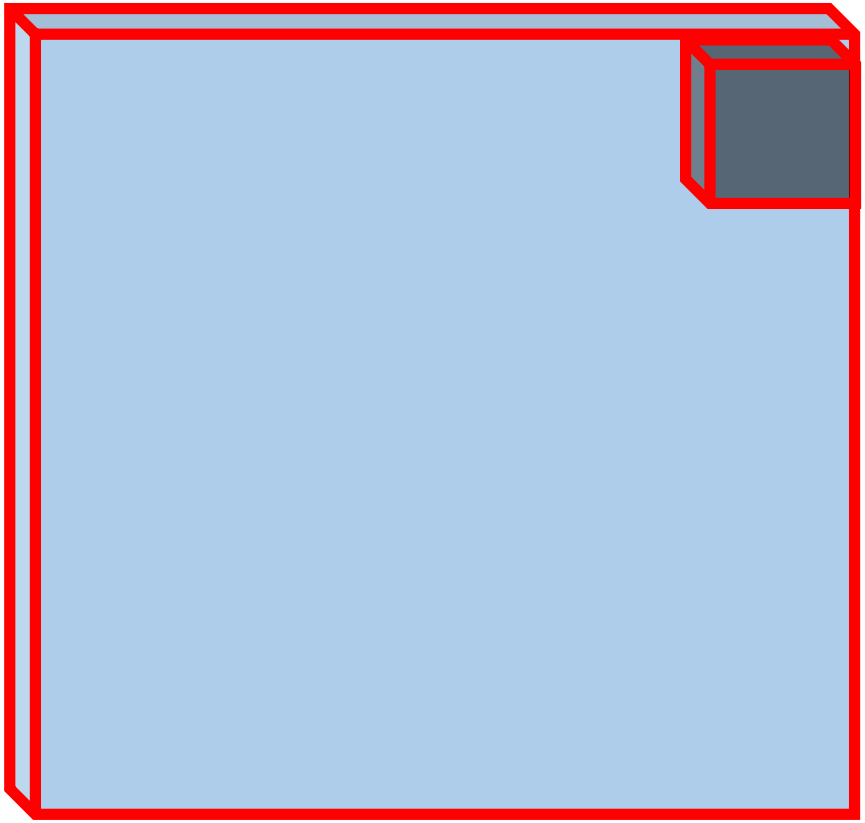


n_c = 3 (RGB)

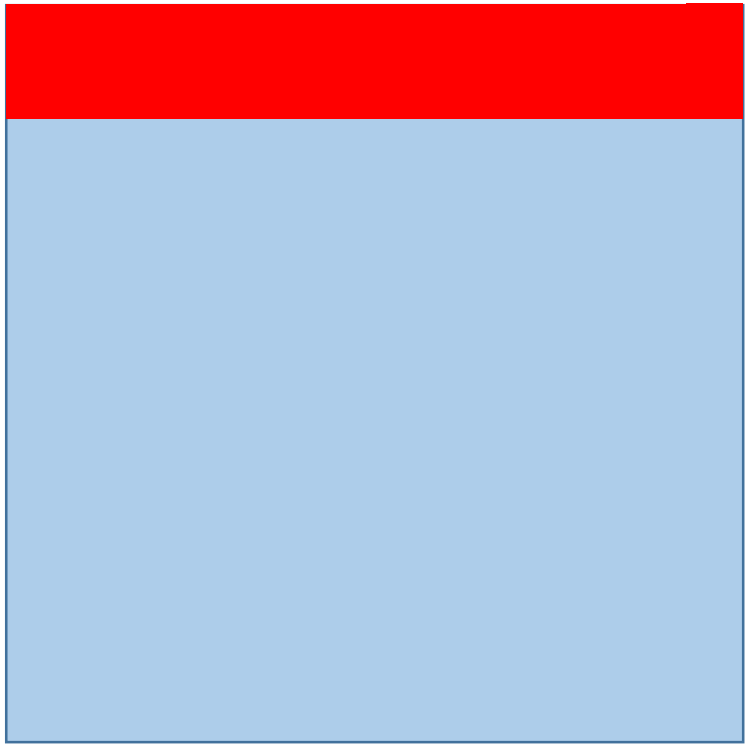


n_c = 1

Convolution over volume

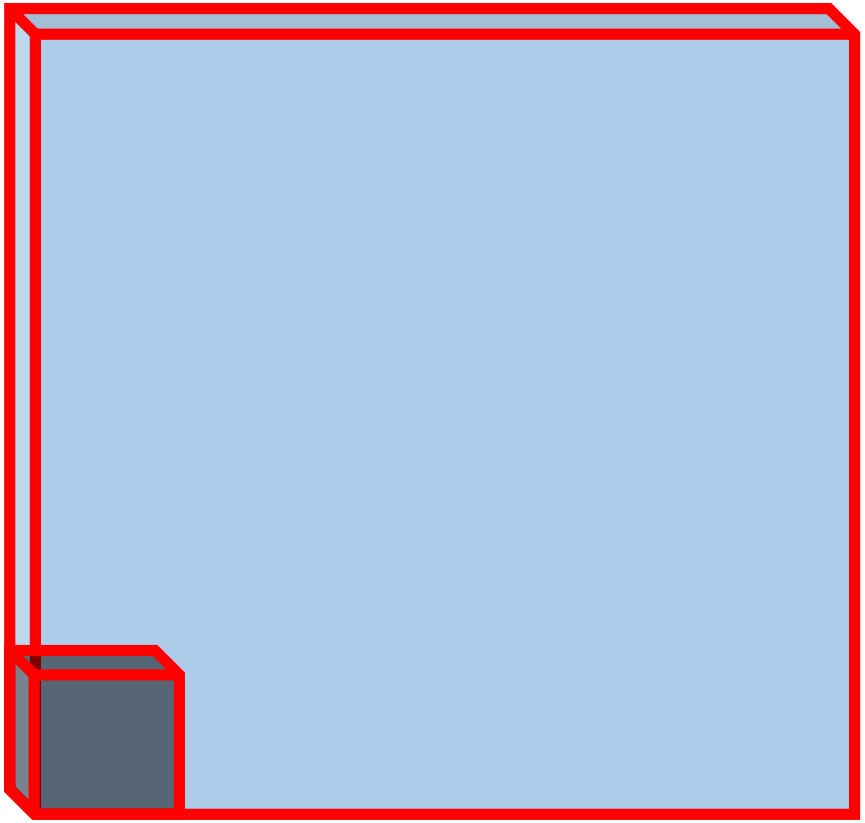


n_c = 3 (RGB)

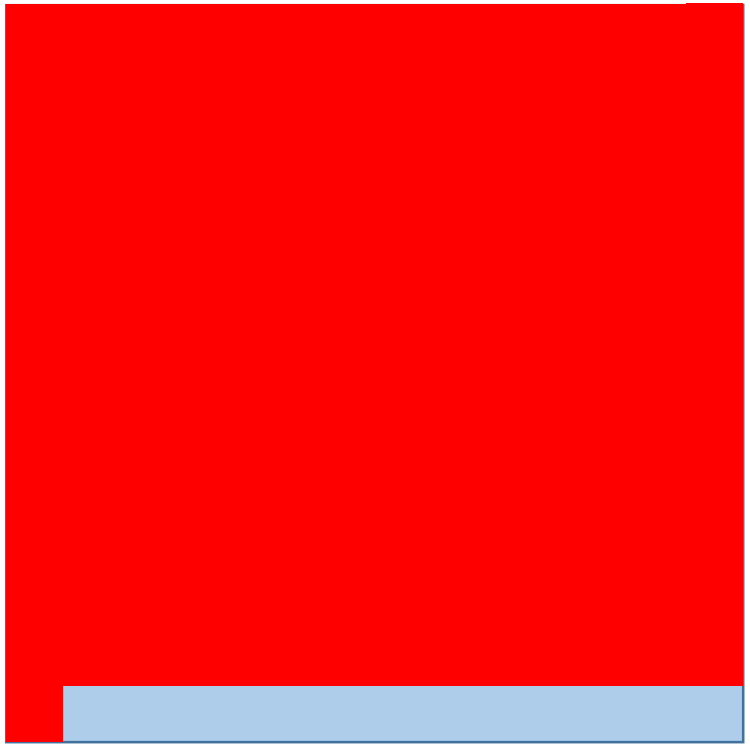


n_c = 1

Convolution over volume

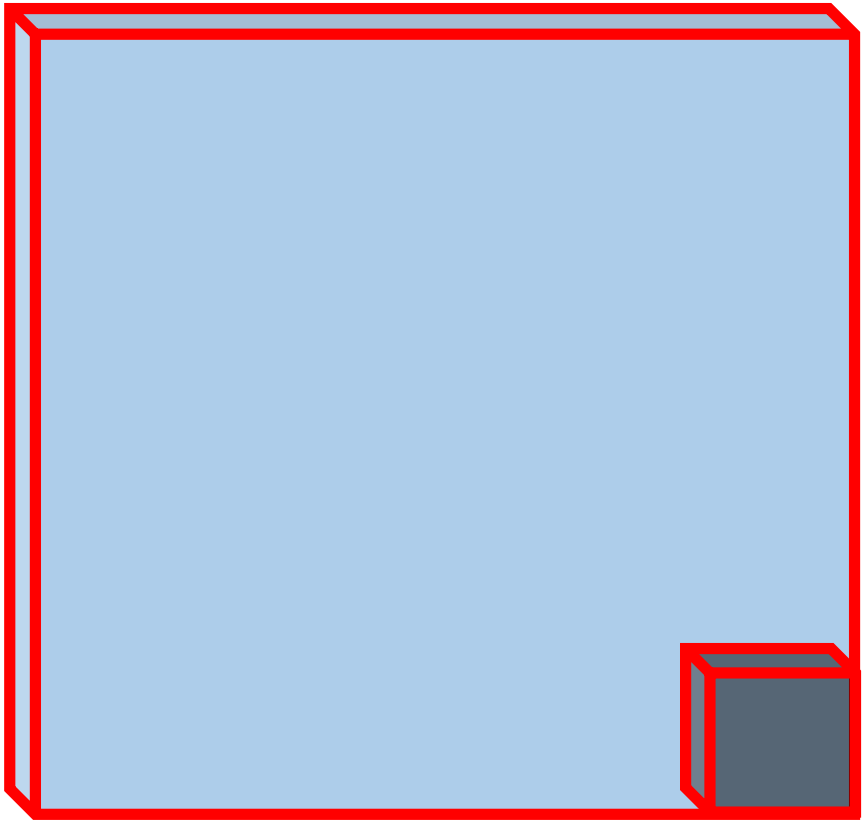


n_c = 3 (RGB)

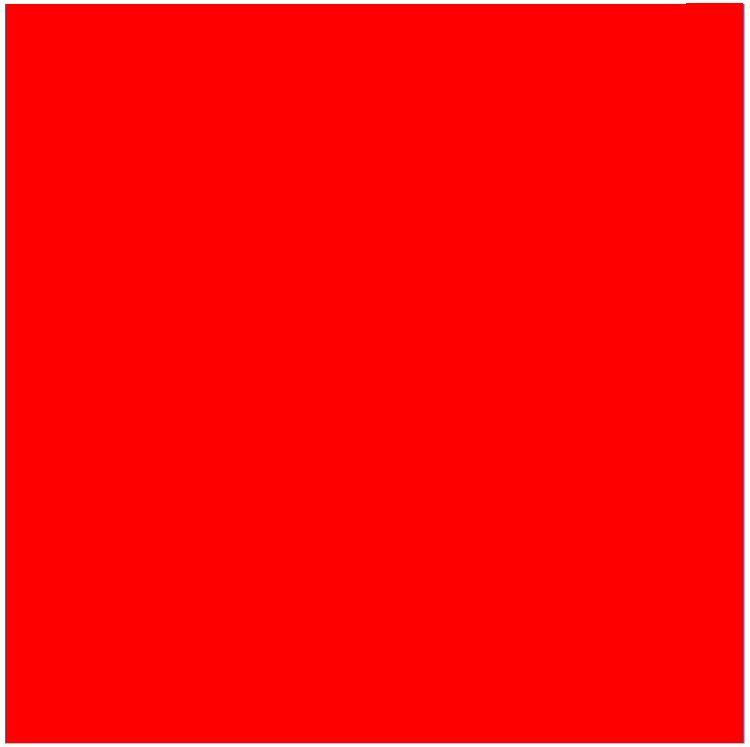


n_c = 1

Convolution over volume

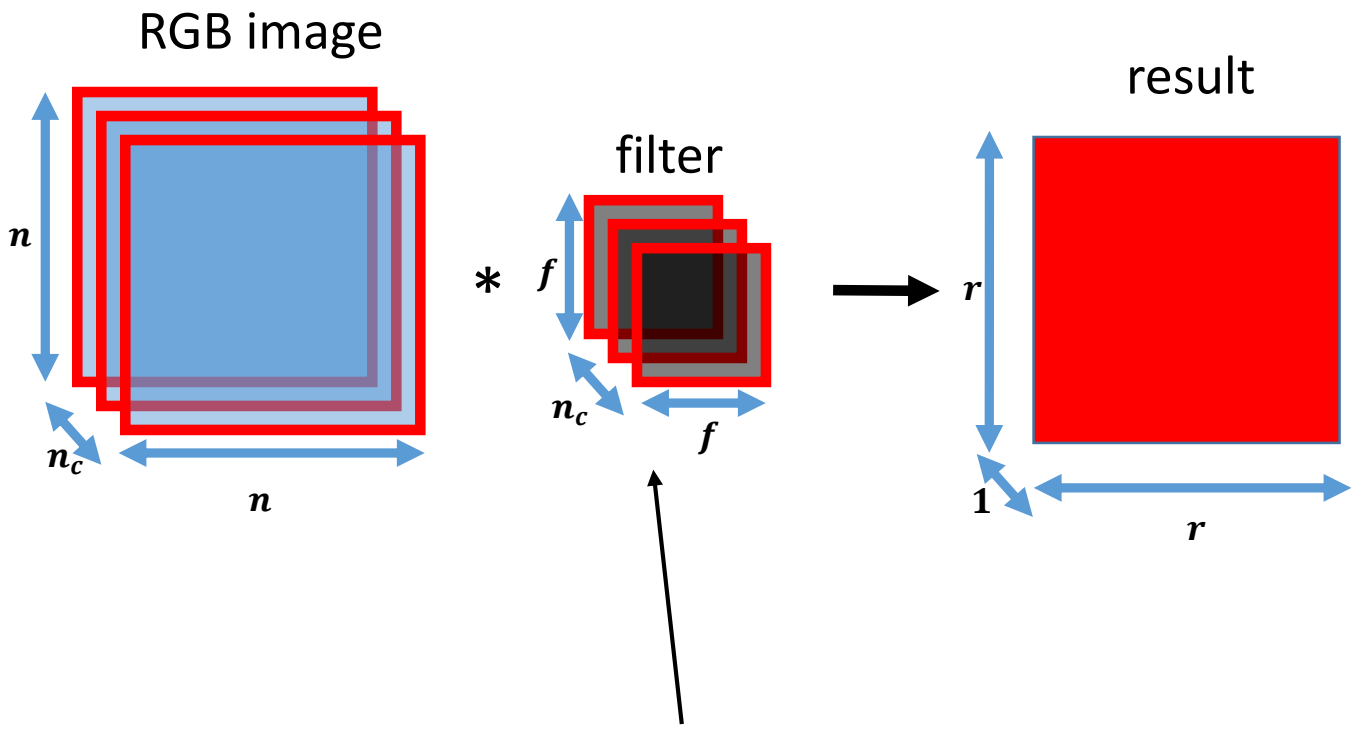


n_c = 3 (RGB)



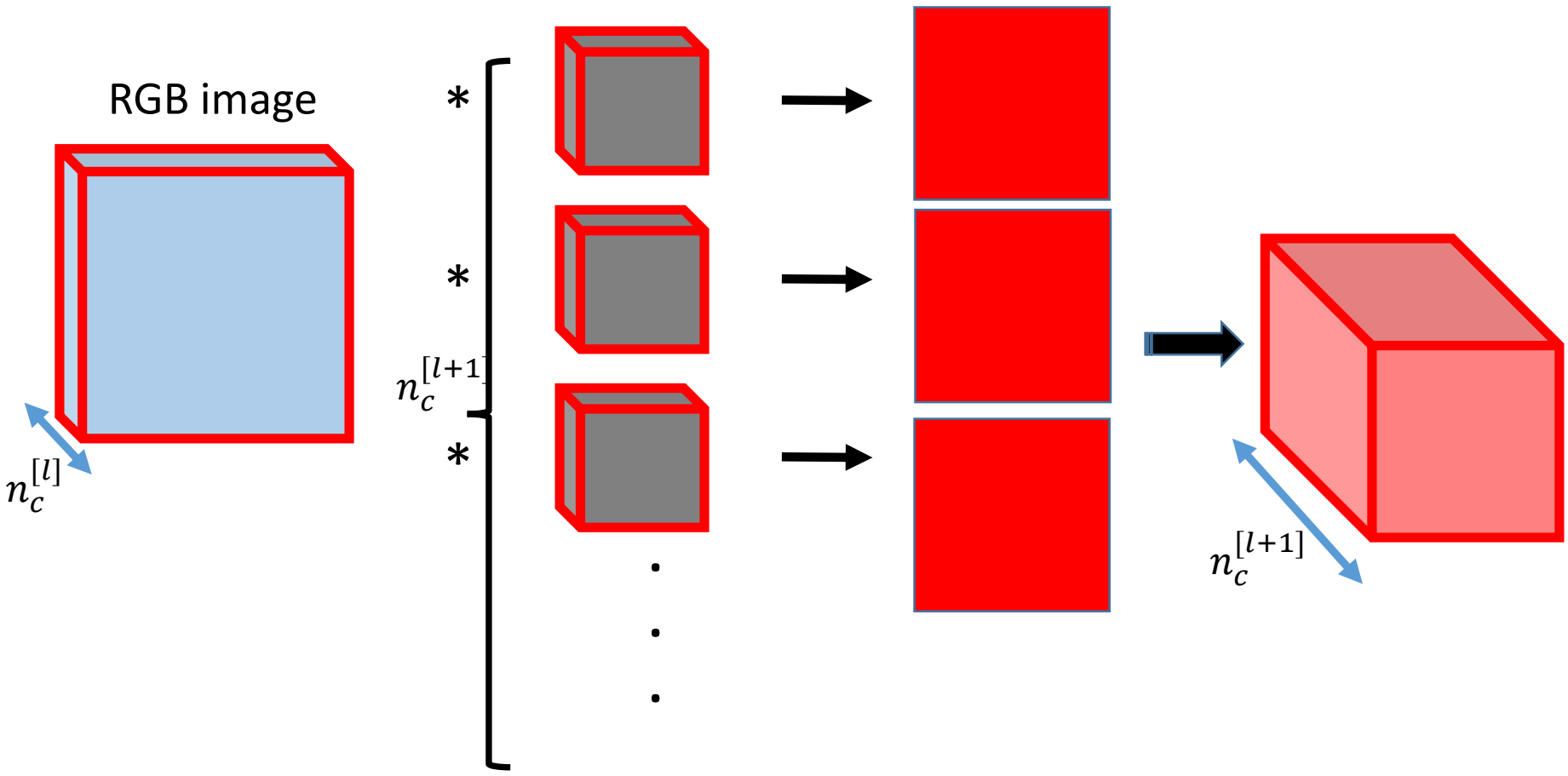
n_c = 1

Convolution over volume



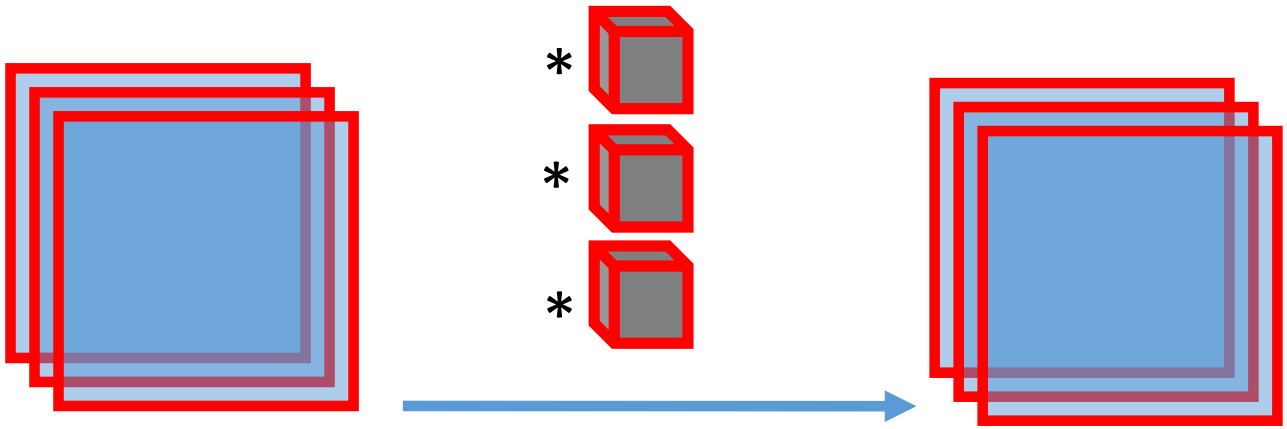
these are n_c independent filters each with different parameters

Convolution over volume



1 layer of convolution in neural networks

$\#channels\ in\ layer\ l: n_c^{[l]}$



$n_c^{[0]} = 3$ $n_c^{[1]} = 3$

Difference from the previous: 1 bias per filter

Each filter has a parameter number of: $f \cdot f \cdot n_c + 1$

1 layer of convolution in neural networks

Input: 200 x 200 pixel RGB image, in the first layer we want 200 x 200 x 3 neurons

How many parameters does it have?

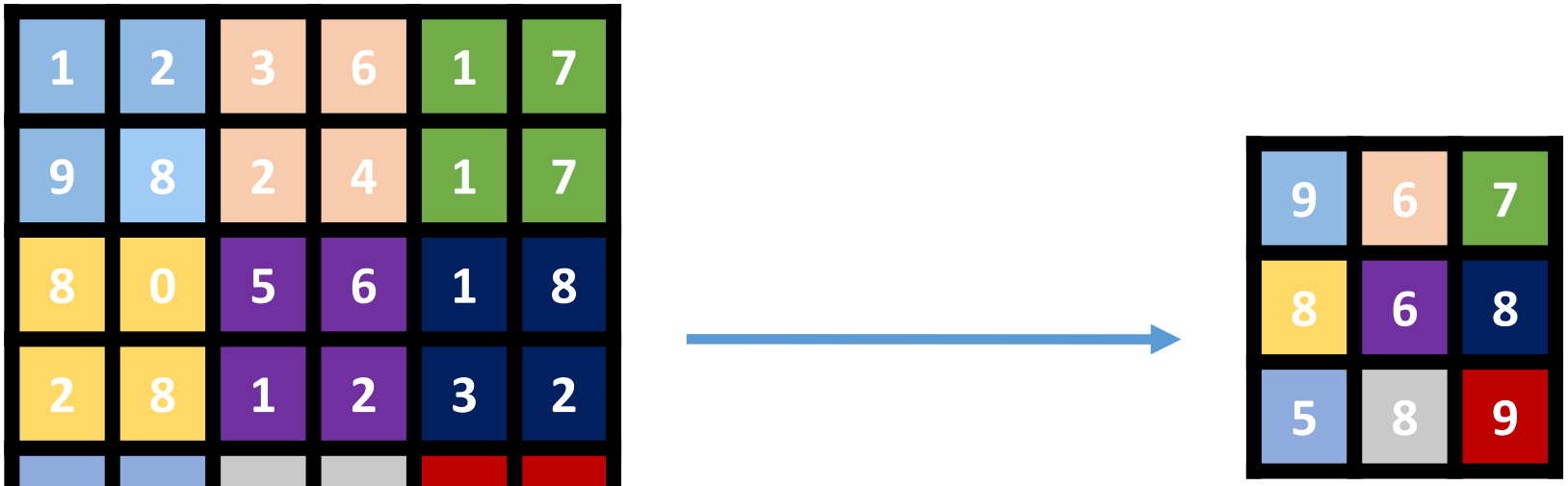
Fully connected layer

- weights: $(200 \cdot 200 \cdot 3)^2$
- bias: $200 \cdot 200 \cdot 3$
- $Total \approx 1.4 \cdot 10^{10}$

Convolutional layer (f = 3)

- Weights per filter: $f \cdot f \cdot n_c^{[0]} + 1$
- Number of filters: $n_c^{[1]}$
- $Total = 84$

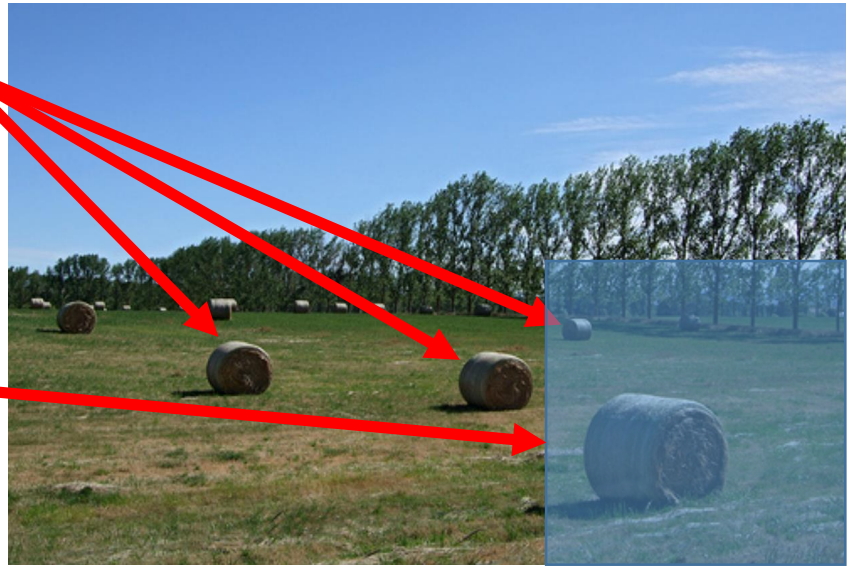
Maxpooling: a special filter to reduce parameters



Translation invariance

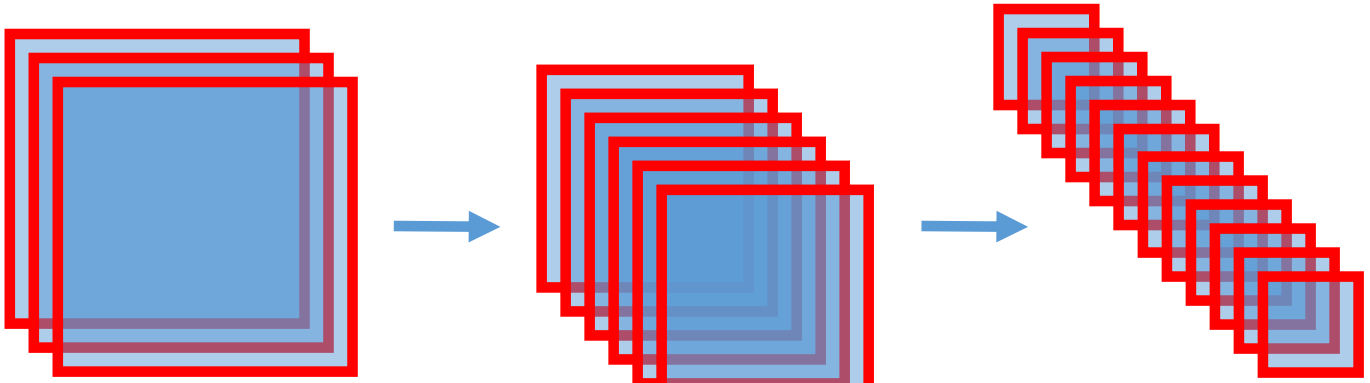
- Objects are not position dependents

✓ A convolution filter
✓ Maxpooling: best value from a region (exact position doesn't matter for image classification)



Convolution in neural networks - representation

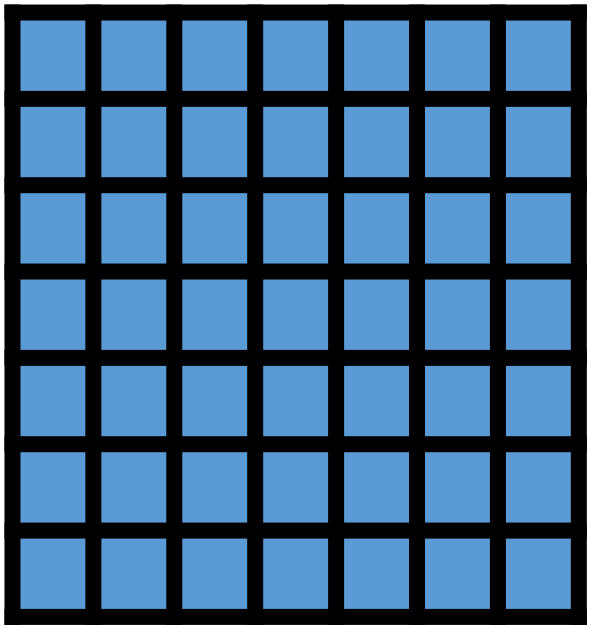
$\#channels\ in\ layer\ l: n_c^{[l]}$



$n_c^{[0]} = 3$ $n_c^{[1]} = 6$ $n_c^{[2]} = 12$

Small f vs large f – receptive field

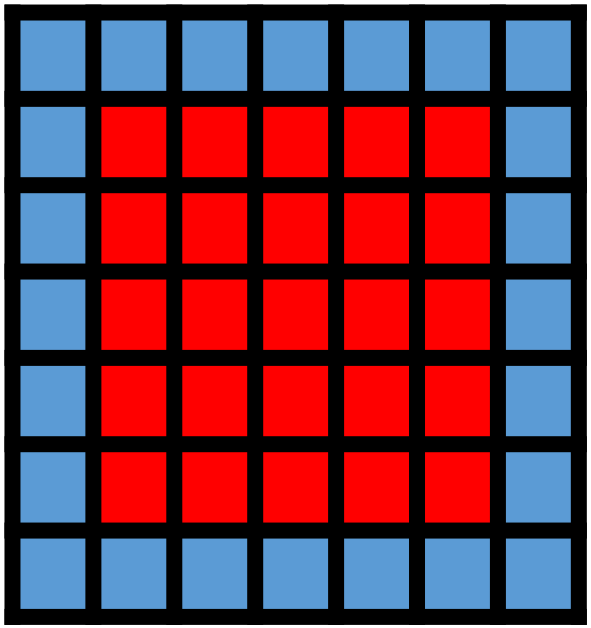
Three 3 x 3 convolution after each other
No padding, stride=1, f=3



Small f vs large f – receptive field

Three 3 x 3 convolution after each other
No padding, stride=1, f=3

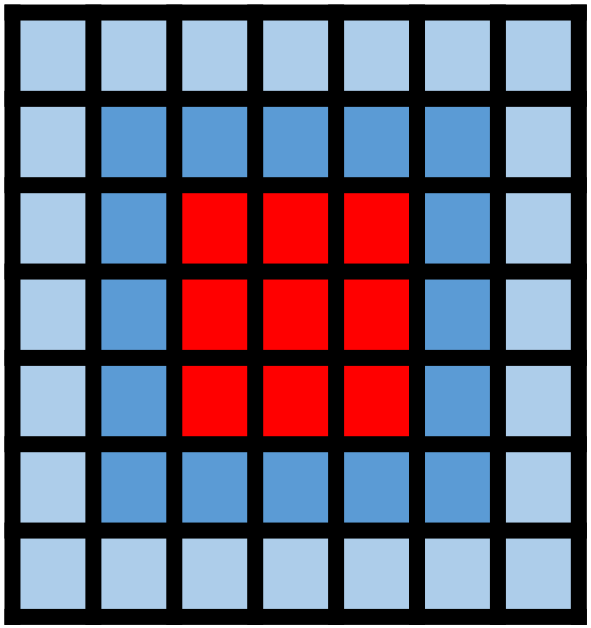
After 1 layer:



Small f vs large f – receptive field

Three 3 x 3 convolution after each other
No padding, stride=1, f=3

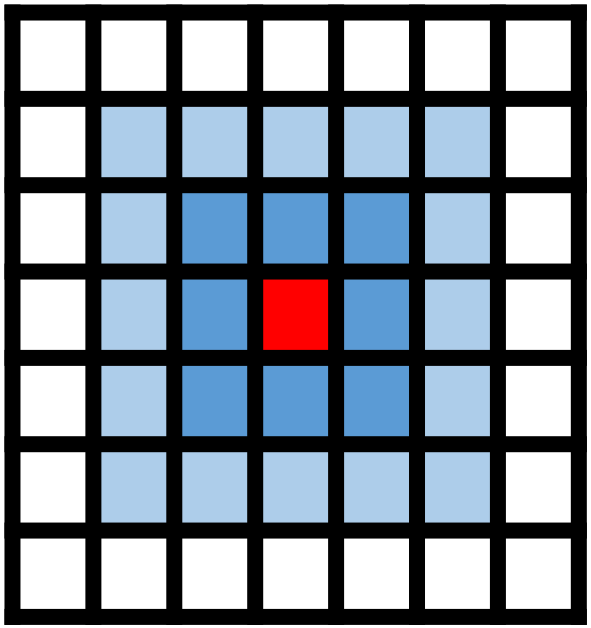
After 2 layer:



Small f vs large f – receptive field

Three 3 x 3 convolution after each other
No padding, stride=1, f=3

After 3 layer:



Small f vs large f – receptive field

After three 3 x 3 convolution each neuron can see 7 x 7 field from the input

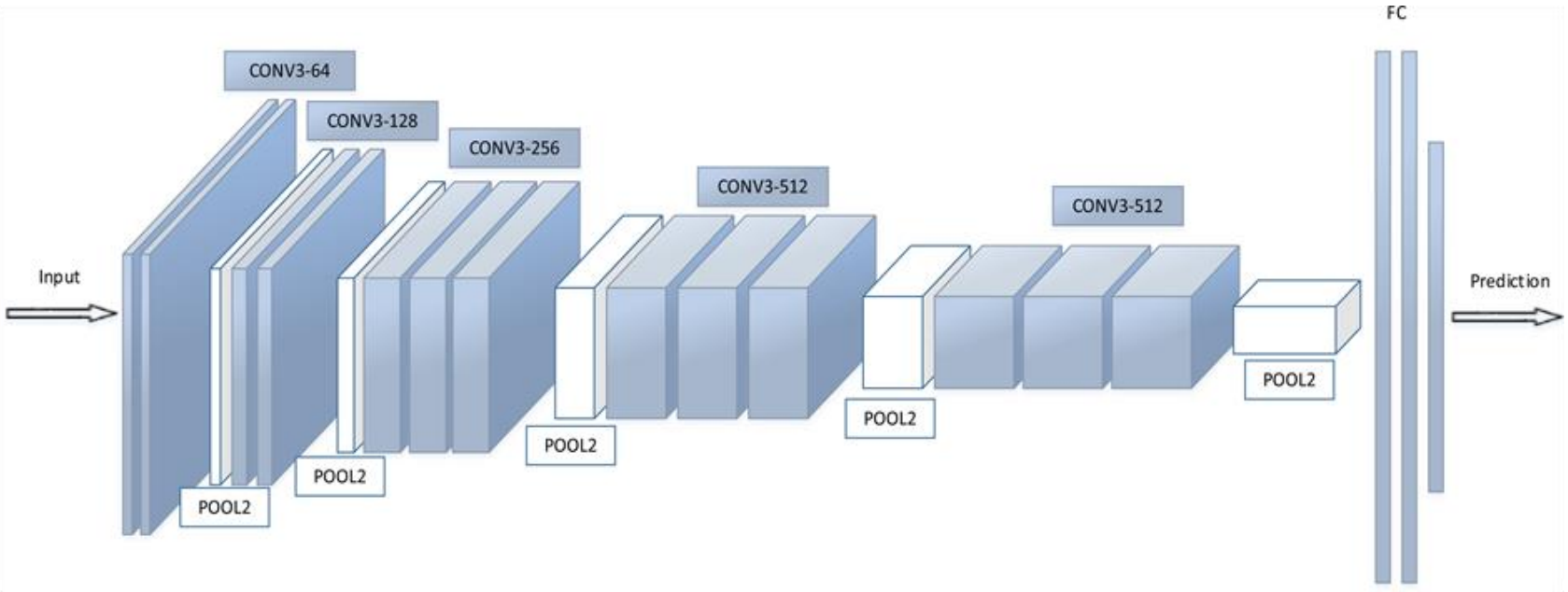
#parameters in three 3 x 3 conv: $3 \cdot (3 \cdot 3 + 1) = 30$

#parameters in one 7 x 7 conv: $1 \cdot (7 \cdot 7 + 1) = 50$

3 convolutions → more 'non-linearity'

VGG16

This is actually a smart way to restrict our universal function.



[http://file.scirp.org/Html/4-7800353_65406.htm]

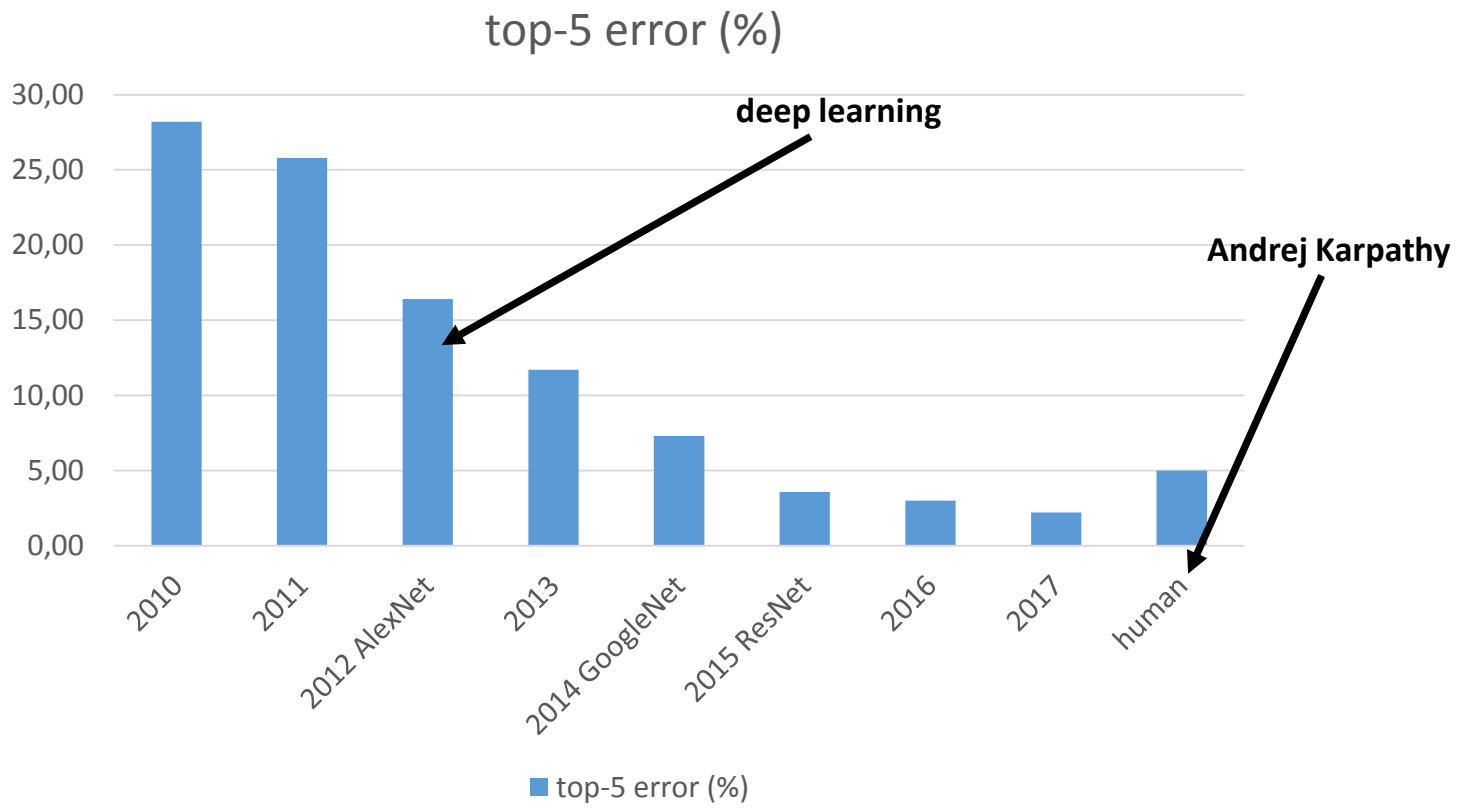
Nice visualisation of CNNs:
http://scs.ryerson.ca/~aharley/vis/conv/flat.html

https://ujjwalkarn.me/2016/08/11/intuitive-explanation-convnets/

ImageNet Large Scale Visual Recognition Challange

- 2010 –
- 1.2M images (100K test set)
- 1000 categories
- 'Image classification world cup',
- top-5 error (still not that easy…)

ImageNet Large Scale Visual Recognition Challange



What about you?

https://cs.stanford.edu/people/karpathy/ilsvrc/

Keras CNN notebook

https://github.com/patbaa/physdl/blob/master/notebooks/05/cnn.ipynb