Newtonian Mechanics

N1 - *If no forces act on a body, it remains at rest or moves with constant velocity:* $\dot{\underline{v}} = 0$

N2 - $\dot{\underline{p}} = \underline{F}$

N3 - $\underline{F_{ab}} = -\underline{F_{ba}}$

$$\underline{L} \equiv \underline{r} \times \underline{p}$$

$$W_{BA} \equiv \int_A^B \underline{F} \cdot d\underline{r} = T_B - T_A$$

Orbits (cylindrical polars):

$$\underline{e_r} = \cos\phi \underline{i} + \sin\phi \underline{j}$$

$$\underline{e_\phi} = -\sin\phi \underline{i} + \cos\phi \underline{j}$$

$$\dot{\underline{r}} = \dot{r}\underline{e_r} + r\dot{\phi}\underline{e_\phi}$$

$$E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + V(r)$$

Newton to Lagrange

Holonomic constraint is an algebraic relation between coordinates:

$$f(\underline{r_a}, \underline{r_b}, ..., \underline{r_N}; t) = 0$$

For system with N cartesian cordinates x_i, M constraints, and $3N - M$ generalised coordinates q_i, and $x_i = x_i(\{q\}, t)$

Virtual displacement:

$$\delta x_i = \sum_i \frac{\partial x_i}{\partial q_j} \delta q_j + 0$$

Generalised forces:

$$Q_j = \sum_i F_i \frac{\partial x_i}{\partial q_j}$$

for a function $f = f(\{q\}, \{\dot{q}\}, t)$

$$df = \sum_j \frac{\partial f}{\partial q_j} + \sum_j \frac{\partial f}{\partial \dot{q}_j} d\dot{q}_j + \frac{\partial f}{\partial t} dt$$

Lagrange's equations (general form):

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$$

or

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial q_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = 0$$

where $\mathcal{L}(\{q\}, \{\dot{q}\}.t) = T(\{q\}.\{\dot{q}\}, t) - V(\{q\}, t)$

Calculus of Variations