

Information Theory

Missing Information:

$$S = -k \sum_{i=1}^r p_i \ln p_i$$

Expectation values:

$$\overline{E} = \sum_i E_i p_i$$

Canonical Ensemble:

$$p_i = \frac{1}{Z_c} e^{-\beta E_i} \quad \text{where} \quad Z_c = \sum_i e^{-\beta E_i}$$

Grand-Canonical Ensemble:

$$p_{i,N} = \frac{1}{Z_{gc}} e^{-\beta(E_{iN} - N\mu)}$$

$$\text{where} \quad Z_c = \sum_i e^{-\beta(E_{iN} - N\mu)}$$

Thermodynamics

Potentials:

$$F = \overline{E} - TS$$

$$H = \overline{E} + PV$$

$$G = \overline{E} - TS + PV$$

$$\Phi = F - \overline{N}\mu = \overline{E} - TS - \mu \overline{N}$$

Bridge Equations:

$$F = -kT \ln Z_c$$

$$\Phi = -kT \ln Z_{gc}$$

$$\overline{E} = -\frac{1}{Z_c} \frac{\partial Z_c}{\partial \beta} = -\frac{\partial \ln Z_c}{\partial \beta}$$

Fluctuations

$$\Delta E_i = E_i - \overline{E}$$

$$\overline{(\Delta E)^2} = \overline{E^2} - \overline{E}^2$$

$$\Delta E_{rms} = \overline{E^2}^{\frac{1}{2}}$$

$$\text{relative fluctuation} = \frac{\Delta E_{rms}}{\overline{E}}$$

Weakly Interacting Constituents

Localised particles (distinguishable):

$$Z_c = [Z(1)]^N \quad \text{where} \quad Z(1) = \sum_i e^{-\beta \epsilon_i}$$

Non localised (Fermi-Dirac):

$$Z_{gc} = \prod_j Z_j \quad \text{where} \quad Z_j = \sum_{n_j} e^{\beta n_j (\mu - \epsilon_j)}$$

Fermi-Dirac Statistics:

$$Z_j = (1 \pm e^{\beta(\mu - \epsilon_j)})^{\pm 1} \text{ for } \begin{cases} + \text{ Fermions} \\ - \text{ Bosons} \end{cases}$$

$$\overline{n_j} = \frac{1}{e^{\beta(\epsilon_j - \mu)} \pm 1} \text{ for } \begin{cases} + \text{ Fermions} \\ - \text{ Bosons} \end{cases}$$

Geometric Series:

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{for } |x| < 1$$

B-E Condensation

Density of States:

$$g(\epsilon) = \left(\frac{2m}{\hbar^2} \right)^{\frac{3}{2}} \frac{V}{4\pi^2} \epsilon^{\frac{1}{2}} \equiv AV \epsilon^{\frac{1}{2}}$$

Critical Density:

$$\rho_c(T) = A \frac{\sqrt{\pi}}{2} \zeta(3/2) (kT)^{\frac{3}{2}}$$

$$\rho = \rho_o + \rho_+ = \frac{1}{V} \frac{1}{e^{-\beta\mu} - 1} + A \int_0^{\infty} \frac{\epsilon^{\frac{1}{2}}}{e^{\beta(\epsilon - \mu)} - 1} d\epsilon$$

Thermal Wavelength:

$$\lambda_T = \left(\frac{2\pi\hbar^2}{mkT} \right)^{\frac{1}{2}}$$

Condensation Condition:

$$\left(\frac{V}{\overline{N}} \right)^{\frac{1}{3}} = \rho_c^{-\frac{1}{3}} \sim \lambda_T \ll \text{interparticle spacing}$$

Particle in a Box Energy Levels:

$$\epsilon = \frac{\hbar^2 \pi^2}{2mL^2} (n_x^2 + n_y^2 + n_z^2) + \text{constant}$$

Debye Model

Density of States:

$$g(\omega) = 3 \frac{V}{2\pi^2} \frac{\omega^2}{c_s^3} \equiv AV \omega^2$$

Cutoff Frequency:

$$\omega_{max} = \left(6\pi^2 \frac{N}{V} \right)^{\frac{1}{3}} c_s$$

$$k\Theta_D = \hbar\omega_{max}$$