An a-priori test of the integral wall model using the channel flow database

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Motivation

- **Navier-Stokes**, the governing equation of fluid motion:

$$\frac{\partial u}{\partial t}+\nabla\cdot(uu)=-\frac{1}{\rho}\nabla p+\nu\nabla^2 u$$

- **Channel flow** (velocity contour)

- Direct Numerical Simulation (**DNS**): a turbulence simulation methodology; solves Navier-Stokes; resolves all turbulent motions; highly accurate, highly expensive;
- Large Eddy Simulation (**LES**): a turbulence simulation methodology; resolves the large eddies and models the effect of small eddies; accurate, typically not expensive;
- Near wall resolution: small scales of the near wall turbulence structures requires prohibitively high near wall resolution;
- **Wall Model**: models the near wall structures and relieves the resolution requirement.

Objectives

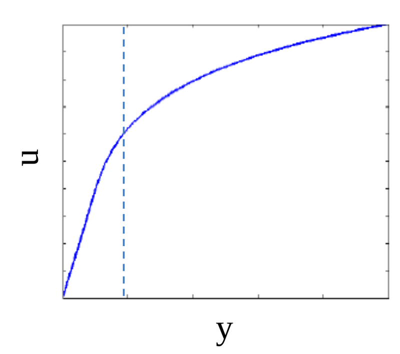
- Conduct an a-priori test of the recently proposed integral **Wall Model**;
- Examine the validity of the proposed velocity shape function;
- Compare the predicted derivatives in wall stress against the equilibrium wall model;

Integral wall model

- A wall model for Large Eddy Simulation (LES);
- Non-equilibrium;
- Based on the von Karman Pohlhausen integral method, no need for numerical integration on a refined mesh near the wall;
- Shape function of the velocity profile:

$$\begin{cases} u=u_\nu\left[\dfrac{y}{\delta_\nu}\right] & 0\le y<\delta_i \\ u=u_\tau\left[C+\dfrac{1}{\kappa}\log\left(\dfrac{y}{\Delta_y}\right)+\boxed{A_2\dfrac{y}{\Delta_y}}\right] & \delta_i\le y<\Delta_y \end{cases}$$

non-equilibrium



- 6 unkowns, 6 constraints

$$u_\nu=u_\tau$$

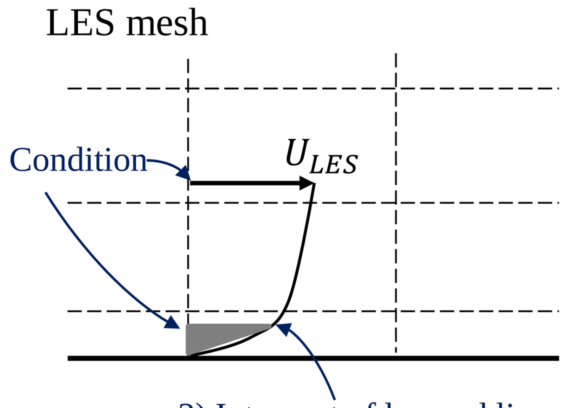
$$\delta_\nu=\nu/u_\tau$$

4,5) Inner layer length/velocity scale

$$\delta_i=11\nu/u_\tau$$

$$A=\frac{1/\kappa\log\left(11\nu/u_\tau\Delta_y\right)+U_{LES}/u_\tau-11}{1-11\nu/u_\tau\,\Delta_y}$$

$$C=U_{LES}/u_\tau-A$$



6) Vertically integrated momentum equation

$$\frac{L^{n+1}-L^n}{dt}+M^n=\tau^n_{\Delta_y}-\tau^n_w$$

- L, vertically integrated velocity; M, vertically integrated convective term; τ_{Δ_y}, momentum flux from the top; τ_w, momentum flux at the wall;

Channel flow database

- Direct numerical simulation (**DNS**) of channel flow;
- Incompressible flow;
- $Re_\tau=1000$;
- Mesh size: $2048\times512\times1536$;
- Grid resolution: $\Delta x^+=12.26$, $\Delta z^+=6.132$, $\Delta y^+=0.01652$ at first grid point, $\Delta y^+=6.155$ at center line;
- Time stored: a whole flow through time;

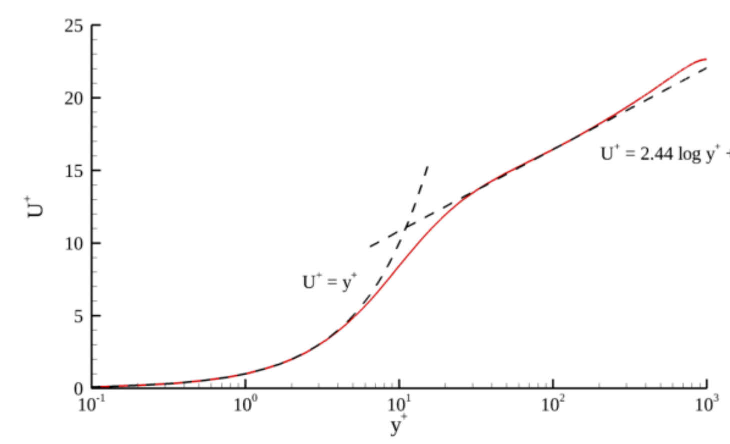


Figure 1. Mean velocity profile in viscous units.

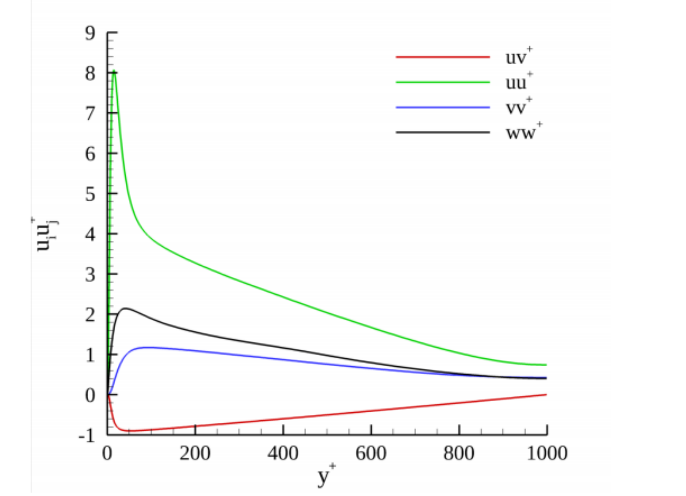
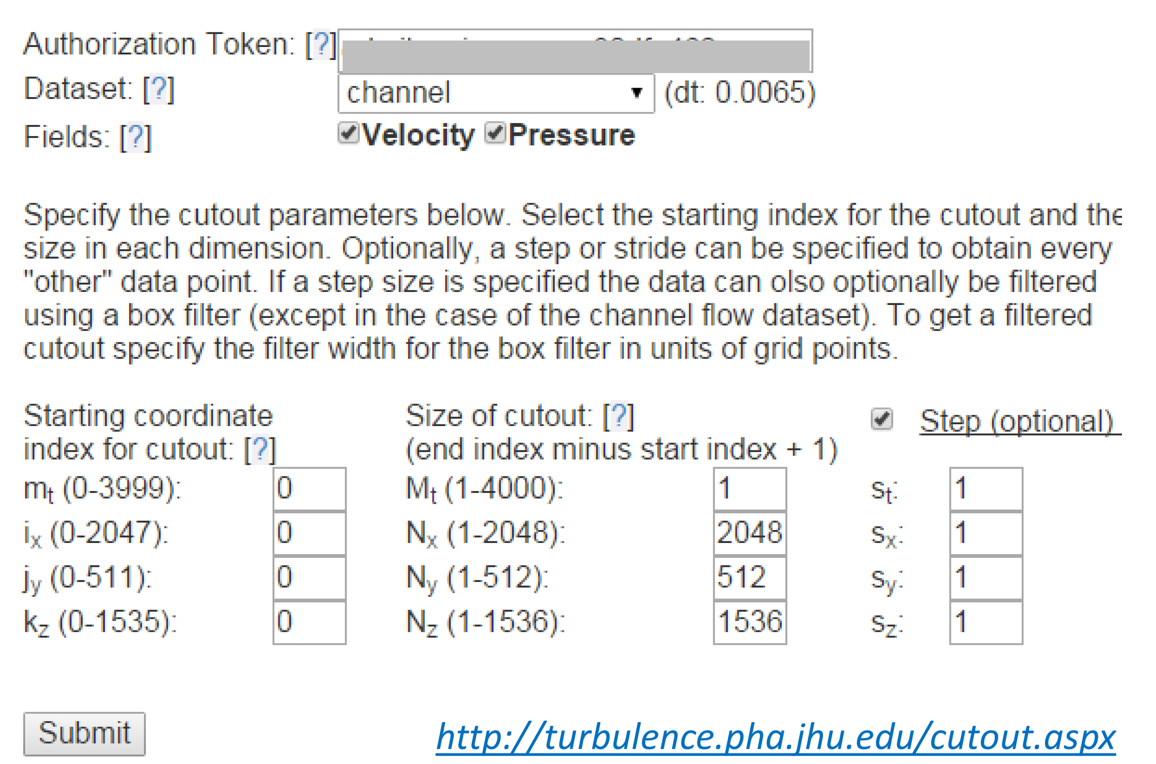


Figure 2. Velocity covariance in viscous units.

Methods

1) Download snap shots to local directory using the *data cutout service*;
2) Filter the DNS data to typical LES resolution;
3) Apply both the integral wall model and the equilibrium wall model;
4) Compare the predicted velocity profile from the integral wall model and the filtered velocity profile from DNS data;
5) Compared the predicted time derivative in wall stress from the integral wall model against the equilibrium wall model.

Data cutout service



http://turbulence.pha.jhu.edu/cutout.aspx

Web interface

```
authkey = '[redacted]/';
dataset = 'channel/';
web='http://turbulence.pha.jhu.edu/cutout/download.aspx/';
time=['0,1/']
loc=['0,2048/ 0,1/ 0,1536/'];
url=[web,authkey,dataset,'up/',time,loc];
fname=['data','.h5'];
urlwrite(url,fname);
```

Matlab code

Results

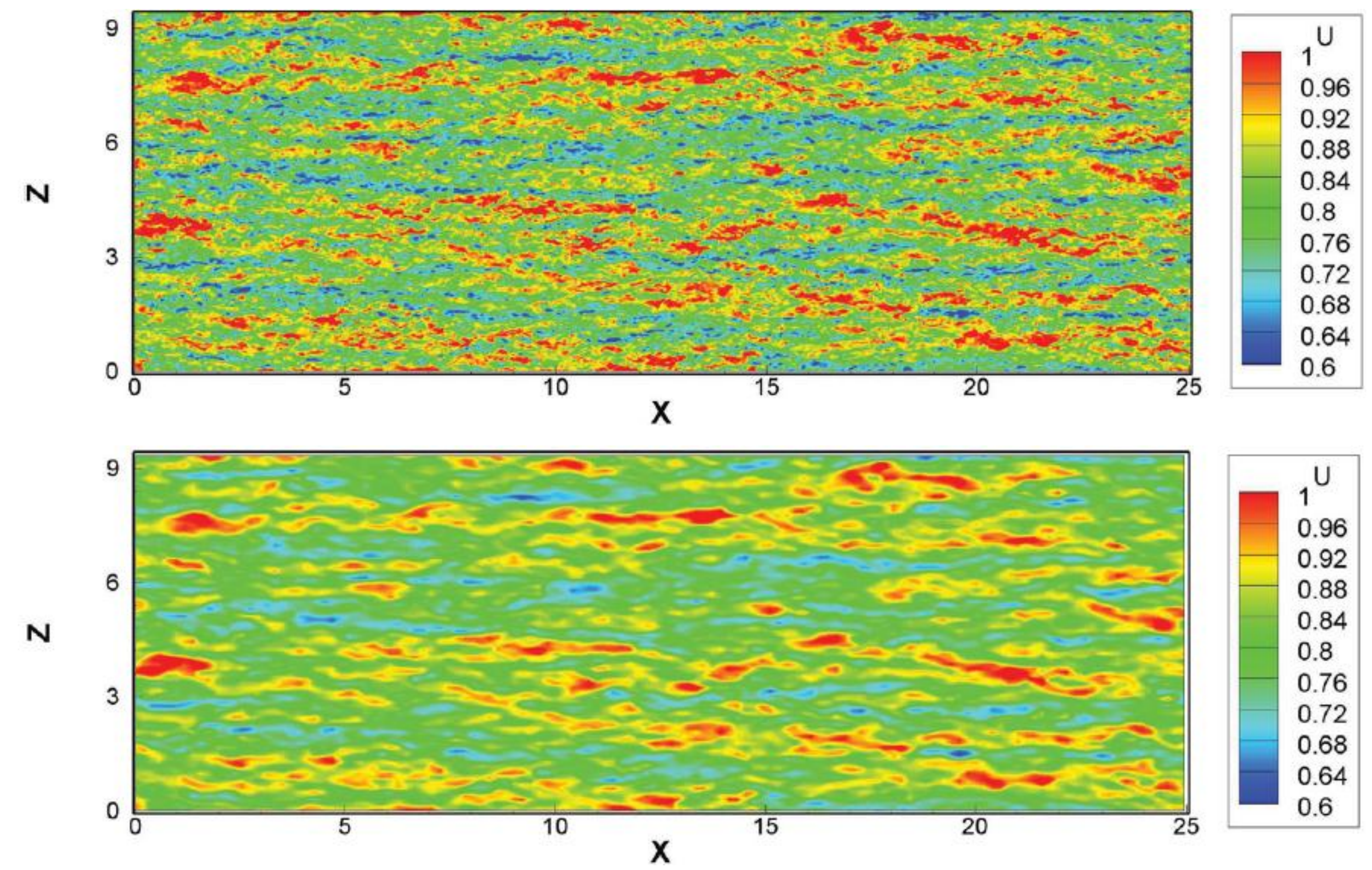


Figure 3. Contour plots of the streamwise velocity at $y^+=100$ (0.1δ) . Top, DNS data; bottom, filtered DNS data. Gaussian filter at sacle $\Delta x^+=192$, $\Delta z^+=97$.

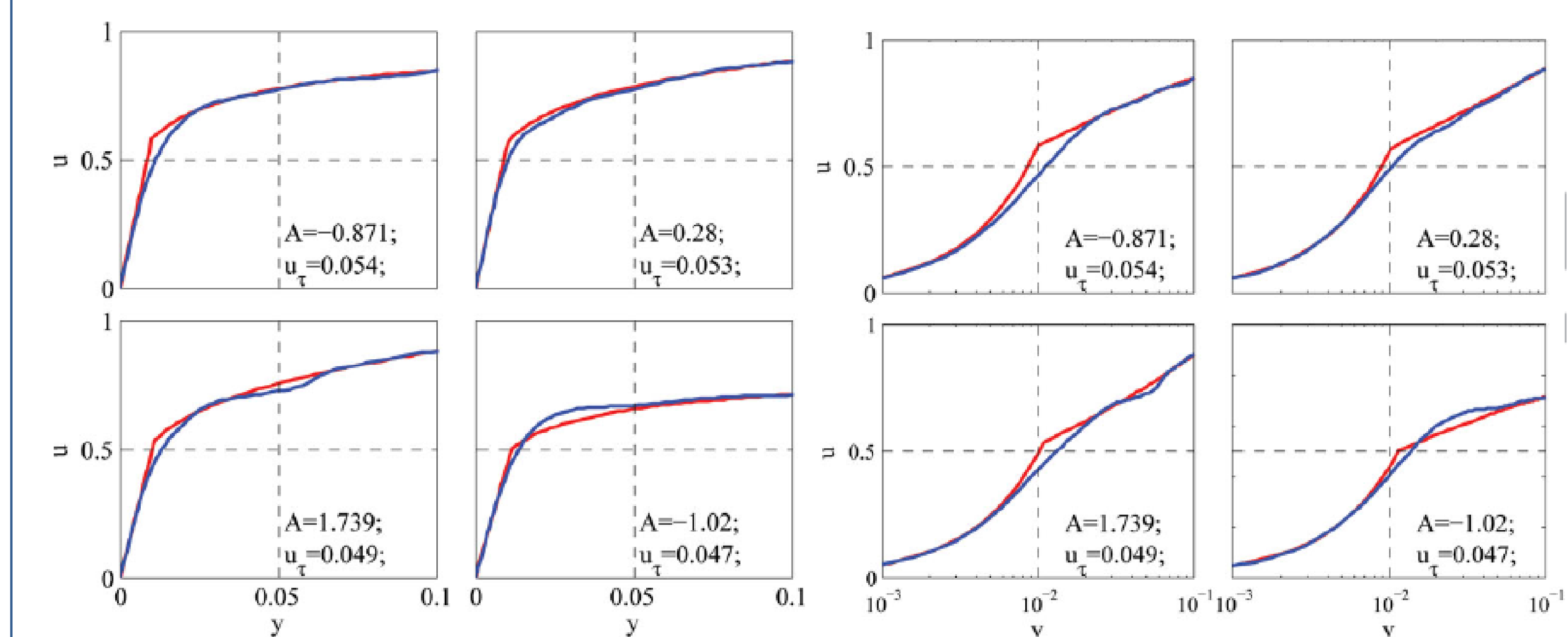


Figure 4. Four sample comparison of the filtered DNS velocity profile near the wall and the predicted profile from the integral wall model. (blue lines: filtered DNS data, red lines: model predicted profiles)

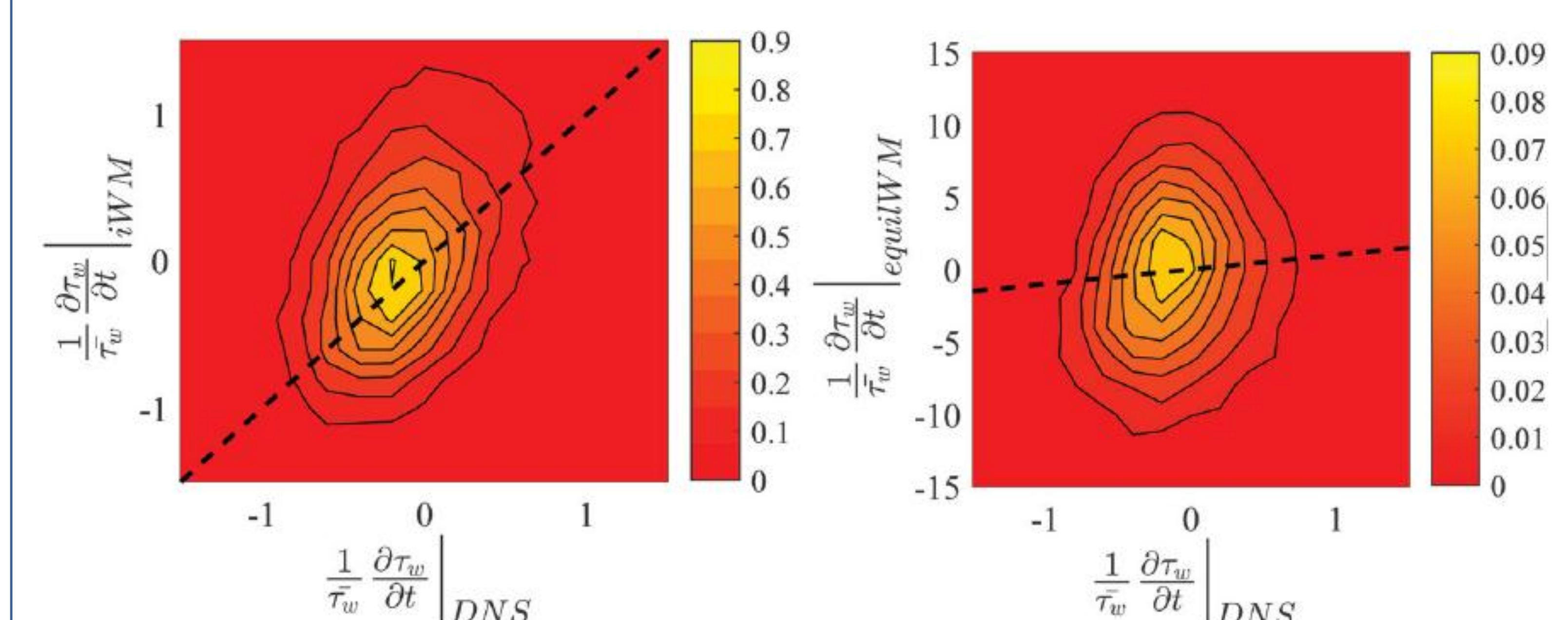


Figure 5. Correlation of the time derivative of the wall stress between the integral wall model and filtered DNS wall stress (on left). The comparison of the equilibrium wall model is on the right. The commonly used equilibrium wall model displays much weaker correlation.

References J. Graham, K. Kanov, X. I. A. Yang, M. K. Lee, N. Malaya, C. C. Lalescu, R. Burns, G. Eyink, A. Szalay, R. D. Moser and C. Meneveau, A Web services accessible database of turbulent channel flow and its use for testing a new integral wall model for LES , *Journal of turbulence*, 2015, in press

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