Exploring Finite Time Lyapunov Exponents in Isotropic Turbulence With the Johns Hopkins Turbulence Databases

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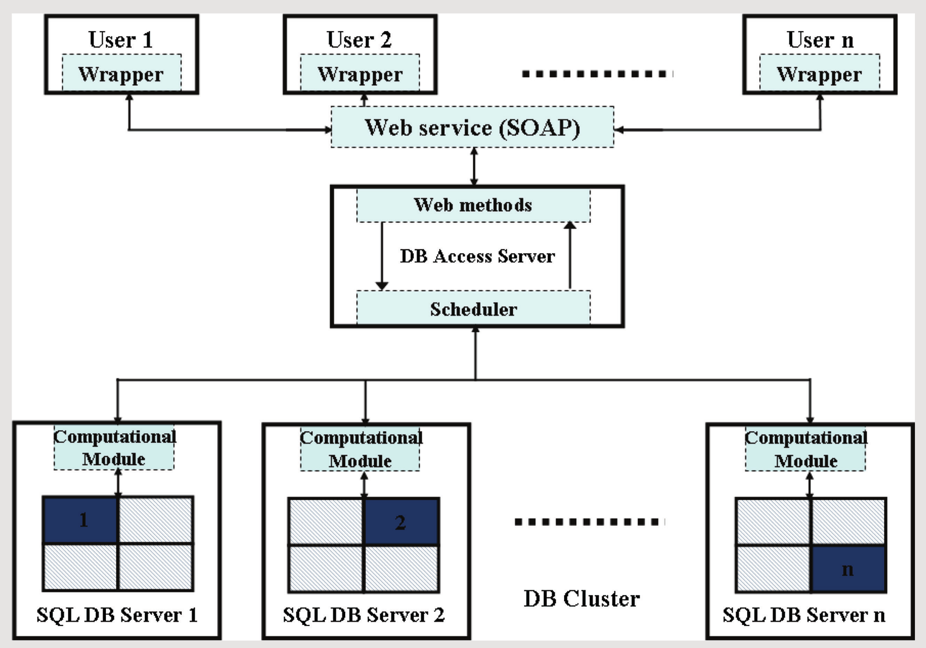
Johns Hopkins Turbulence Databases (JHTDB)

- http://turbulence.pha.jhu.edu/
- access via web services
- Fortran, C, Matlab, HDF5 cutout
- built-in functions
  - e.g. getVelocity, getPressureHessian
  - interpolation (time & space)
  - finite-differencing
- Currently hosts four datasets:
  - Isotropic: 1024^4
  - Magnetohydrodynamics: 1024^4
  - Channel: $2048 \times 512 \times 1536 \times 1997$
  - Mixing: $1024^3 \times 1012$



Homogeneous Isotropic Turbulence (HIT)

- forced incompressible Navier-Stokes equations in a $(2\pi)^3$ periodic box

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial^2 u_i}{\partial x_j \partial x_j} + f_i, \qquad \frac{\partial u_j}{\partial x_j} = 0$$

- canonical problem for studying fluid turbulence

Finite-Time Lyapunov Exponents (FTLE)

- using velocity gradients, $A_{ij} = \partial u_i / \partial x_j$, along fluid particle trajectories

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x}, t), \qquad \mathbf{x}(t_0) = \mathbf{X}, \qquad \frac{d\mathbf{D}}{dt} = \mathbf{AD}, \qquad \gamma_i(T; \mathbf{X}, t_0) = \frac{1}{T}\ln(\sigma_i(T))$$

- exponential rate of separation of neighboring fluid particle trajectories
- deformation of small particles by turbulent flows
- identify coherent motions in fluid turbulence
- converge to the Lyapunov exponent, $\gamma_i(T \to \infty) \to \lambda_i$

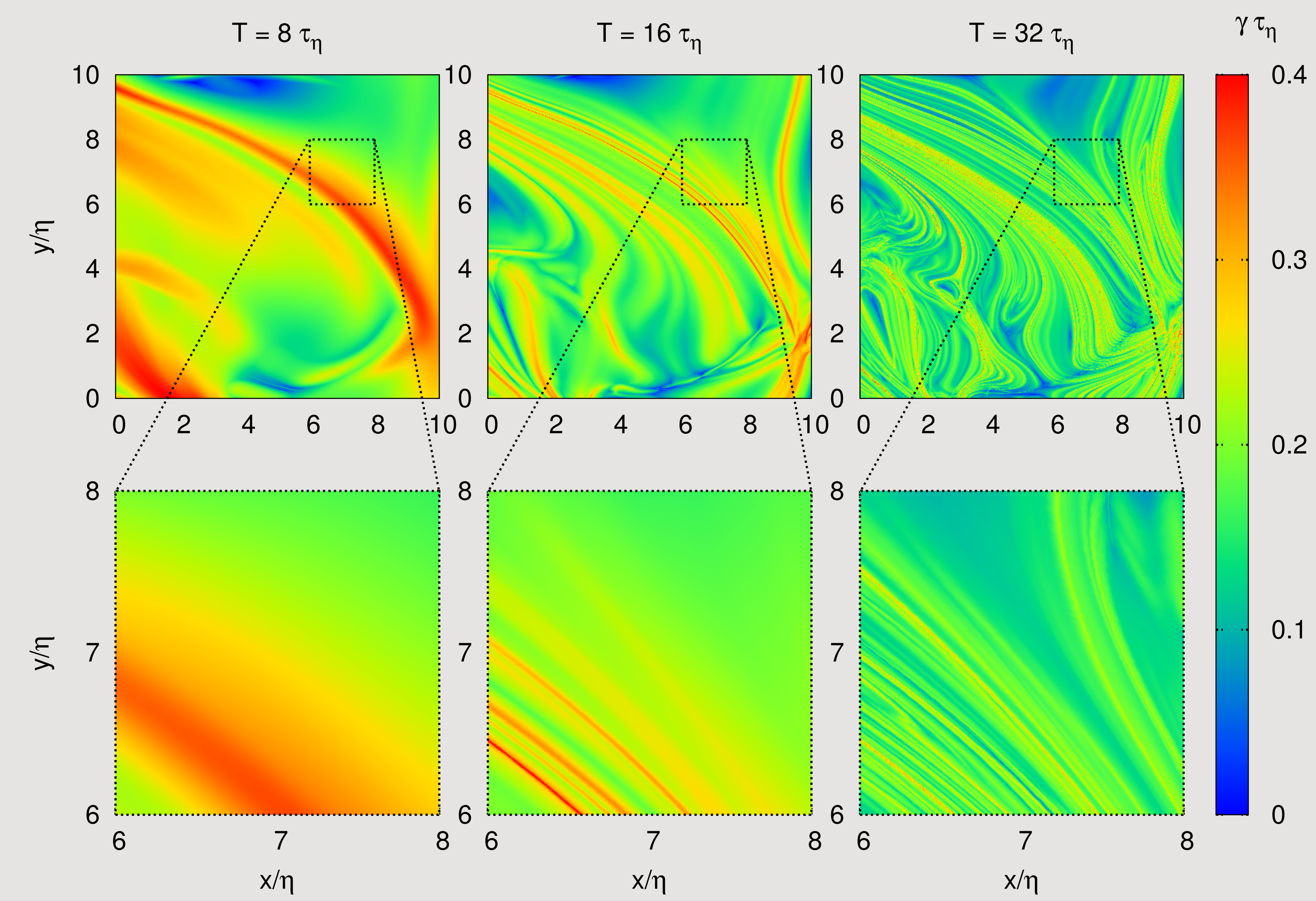
Extracting FTLEs from the JHTDB HIT Simulation

- Initialize particle locations $\mathbf{x}(t_0) = \mathbf{X}$
- Loop through time:
  - Use getVelocityGradient database function to retrieve $A_{ij}(\mathbf{x}, t) = \partial u_i / \partial x_j$
  - Use getPosition database function to advance trajectory to next time step, $\dot{\mathbf{x}} = \mathbf{u}(x, t)$
- Use second simulation to advance $\mathbf{D}$ along each trajectory, $\dot{\mathbf{D}} = \mathbf{AD}$
- Periodically use Gram-Schmidt (**QR decomposition**) to compute orthogonal stretching rates

$$\gamma_i = \frac{1}{T}\ln(R_{ii})$$

Lagrangian Coherent Structures (LCS)

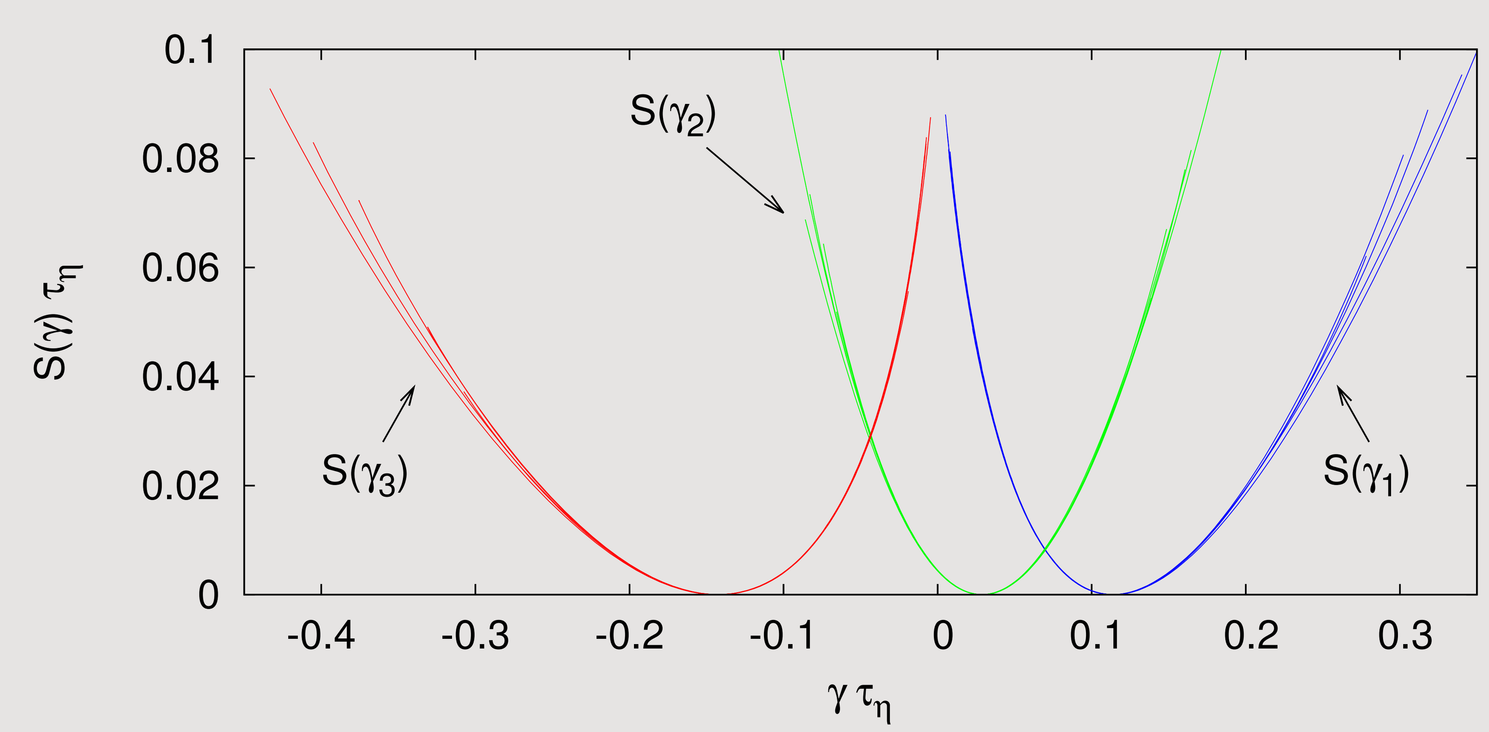
- technique for identification of coherent fluid motions
- attracting/repelling material surfaces
- ridges in the FTLE field $\gamma(\mathbf{X}, t_0)$ for a fixed integration time T
- JHTDB database example:



Large Deviation Formalism

- describes the behavior of the PDFs for sums of i.i.d. variables
- applies to FTLEs
- extends the central-limit theorem for $T \to \infty$ by introducing the Cramér function, $S(\gamma)$

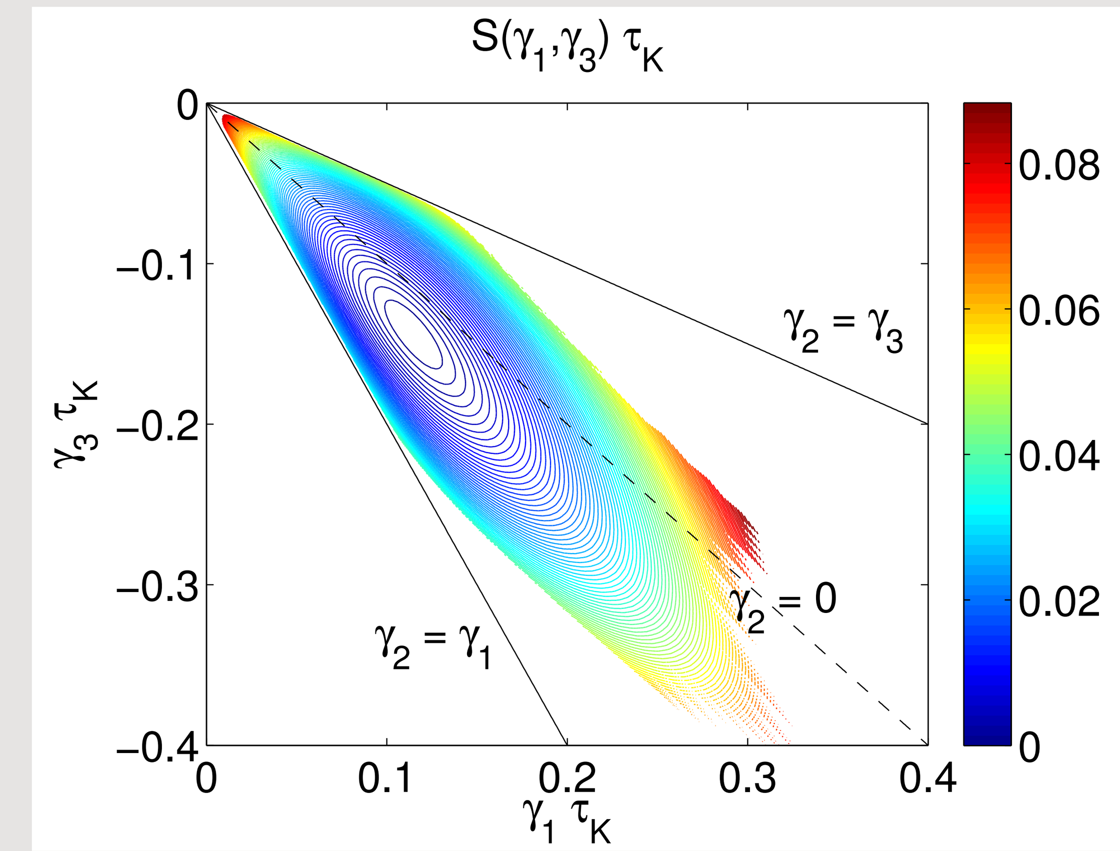
$$p(\gamma, T) \sim \exp(-TS(\gamma))$$



Large Deviations for Joint-Statistics

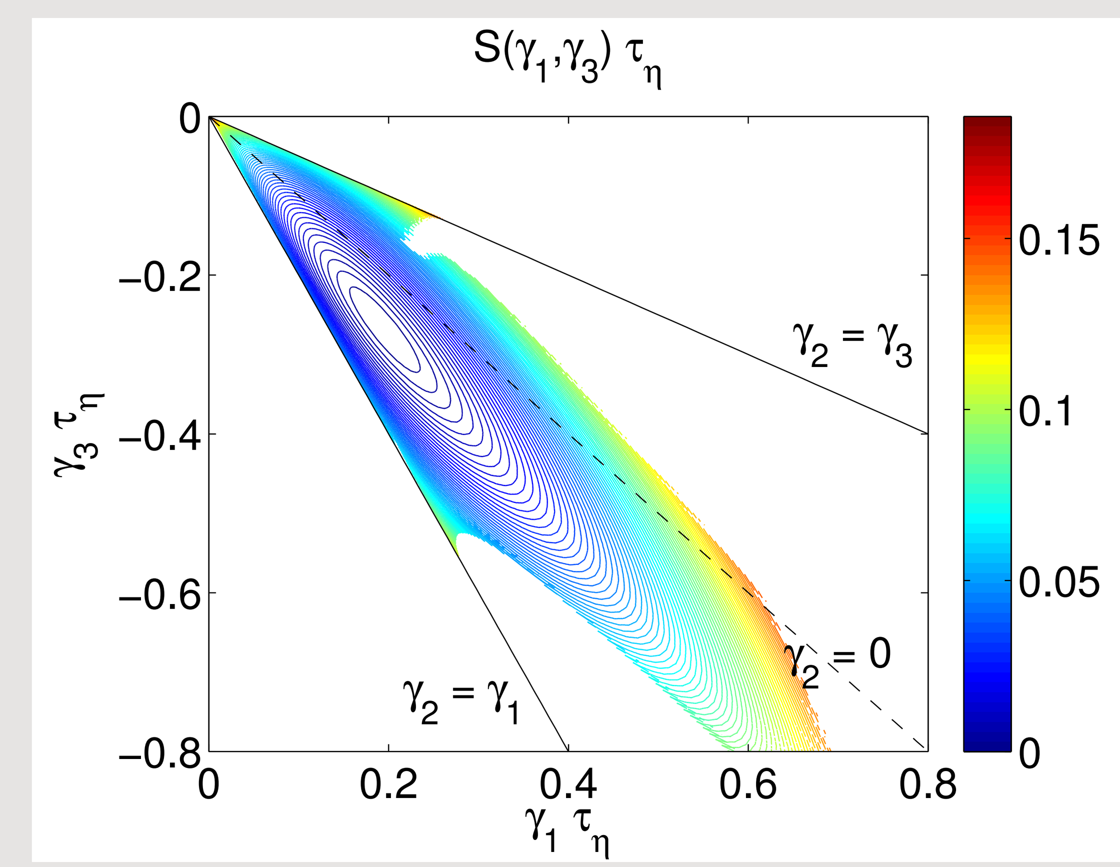
- the large deviation formalism is easily extended to joint statistics

$$p(\gamma_i, , \gamma_j, T) \sim \exp(-TS(\gamma_i, \gamma_j))$$



Effect of Rotation

- Advance deformation tensor with strain-rate tensor, $\dot{\mathbf{D}} = \mathbf{SD}$



Conclusions

- $\{\lambda_1, \lambda_2, \lambda_3\}\tau_\eta = \{0.114, 0.029, -0.143\}$, $\lambda_1 : \lambda_2 : \lambda_3 \approx 4 : 1 : -5$
- Bias for both weakly and strongly deformed particles: $\gamma_2 > 0$.
- Without particle rotation, deformation approximately doubled.