Exploring Vorticity Stretching Statistics in Isotropic Turbulence With the Johns Hopkins Turbulence Databases

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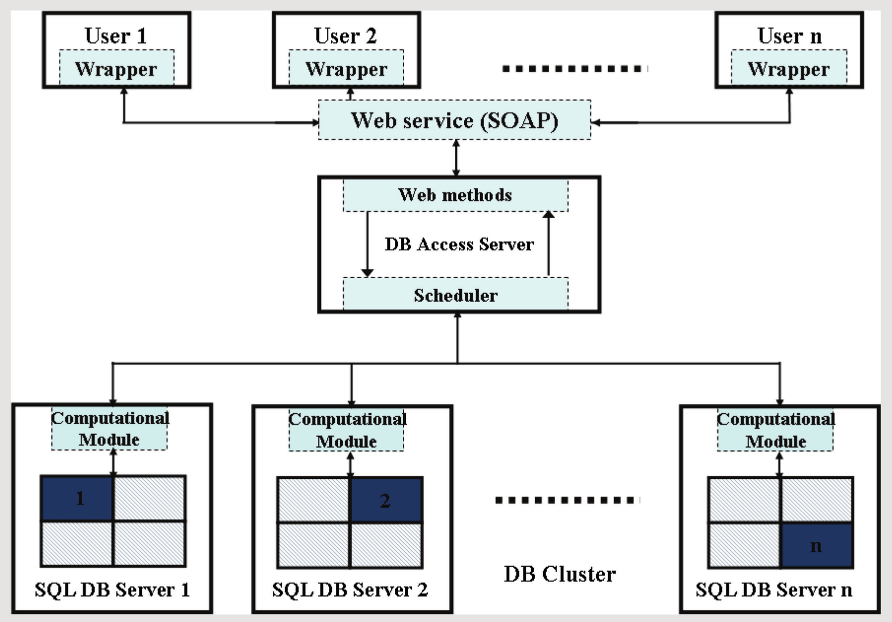
Johns Hopkins Turbulence Databases (JHTDB)

- http://turbulence.pha.jhu.edu/
- access via web services
- Fortran, C, Matlab, HDF5 cutout
- built-in functions
  - e.g. getVelocity, getPressureHessian
  - interpolation & finite-differencing
- Currently hosts four datasets:
  - Isotropic: 1024^4
  - Magnetohydrodynamics: 1024^4
  - Channel: $2048 \times 512 \times 1536 \times 1997$
  - Mixing: $1024^3 \times 1012$

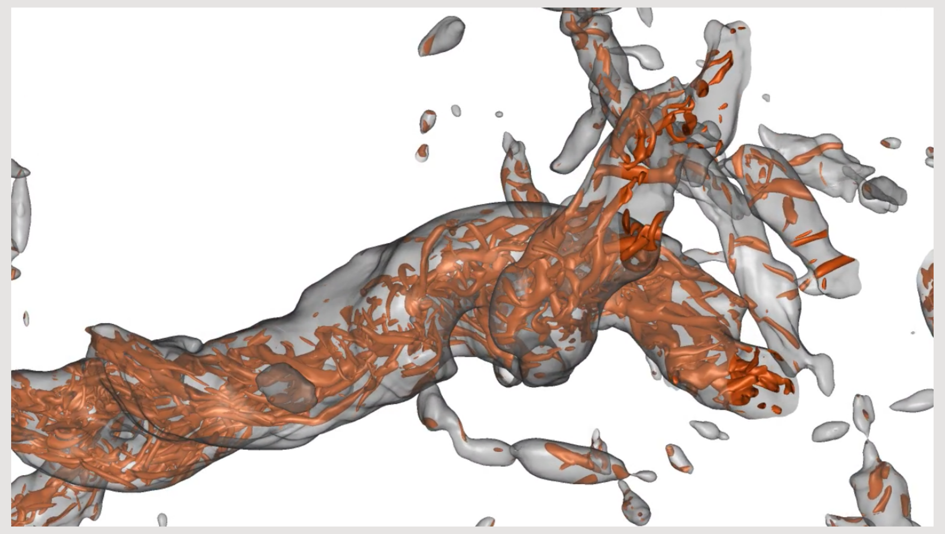


Homogeneous Isotropic Turbulence (HIT)

- forced incompressible Navier-Stokes equations in a $(2\pi)^3$ periodic box

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + f_i, \qquad \frac{\partial u_j}{\partial x_j} = 0$$

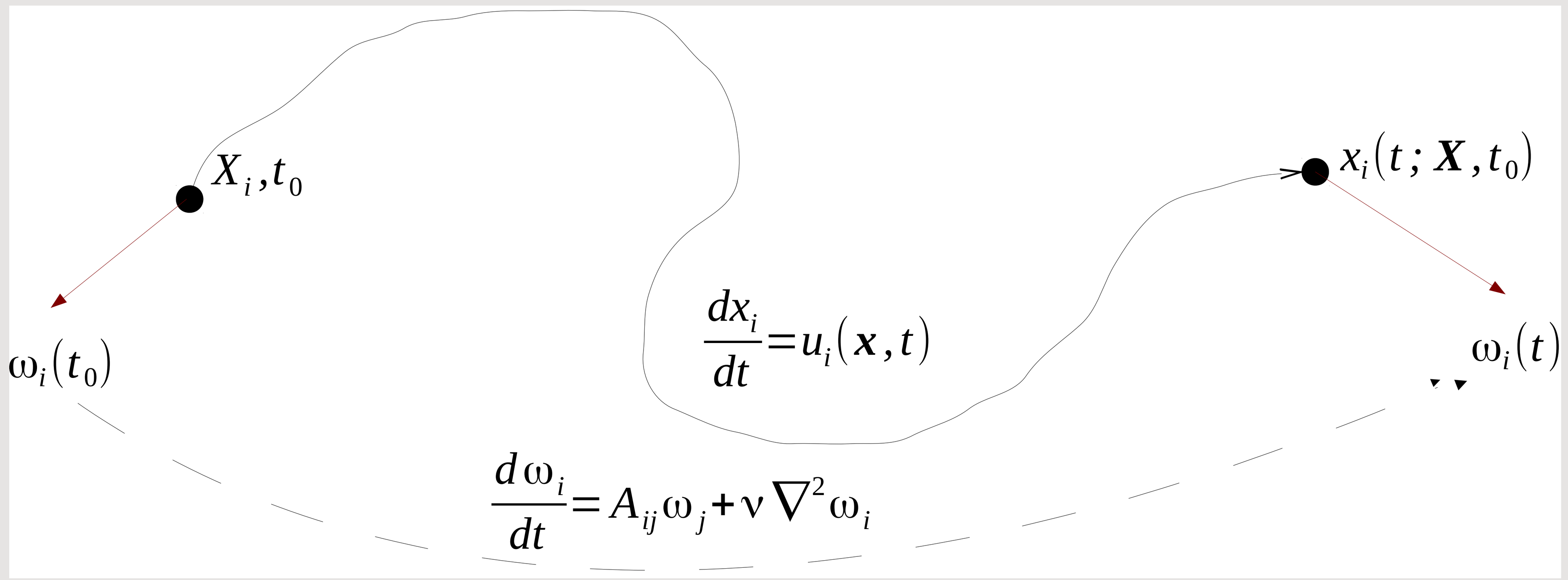
- canonical problem for studying small-scale motions in fluid turbulence
- universal feature of small-scale turbulence: vorticity tubes Bürger et al. 2013



Lagrangian Evolution of Vorticity

- Curl of Navier-Stokes: $\omega = \nabla \times \mathbf{u}$

$$\frac{\partial \omega_i}{\partial t} + u_j \frac{\partial \omega_i}{\partial x_j} = \omega_j \frac{\partial u_i}{\partial x_j} + \nu \frac{\partial^2 \omega_i}{\partial x_j \partial x_j}$$



Hypothesis: Large Deviations for Vorticity Stretching

- Vorticity magnitude increments

$$\ln\left(\frac{|\omega|(t)}{|\omega|(t_0)}\right) = \int_{t_0}^{t} n_i S_{ij} n_j dt + \nu \int_{t_0}^{t} \frac{n_i}{|\omega|}\nabla^2 \omega_i dt$$

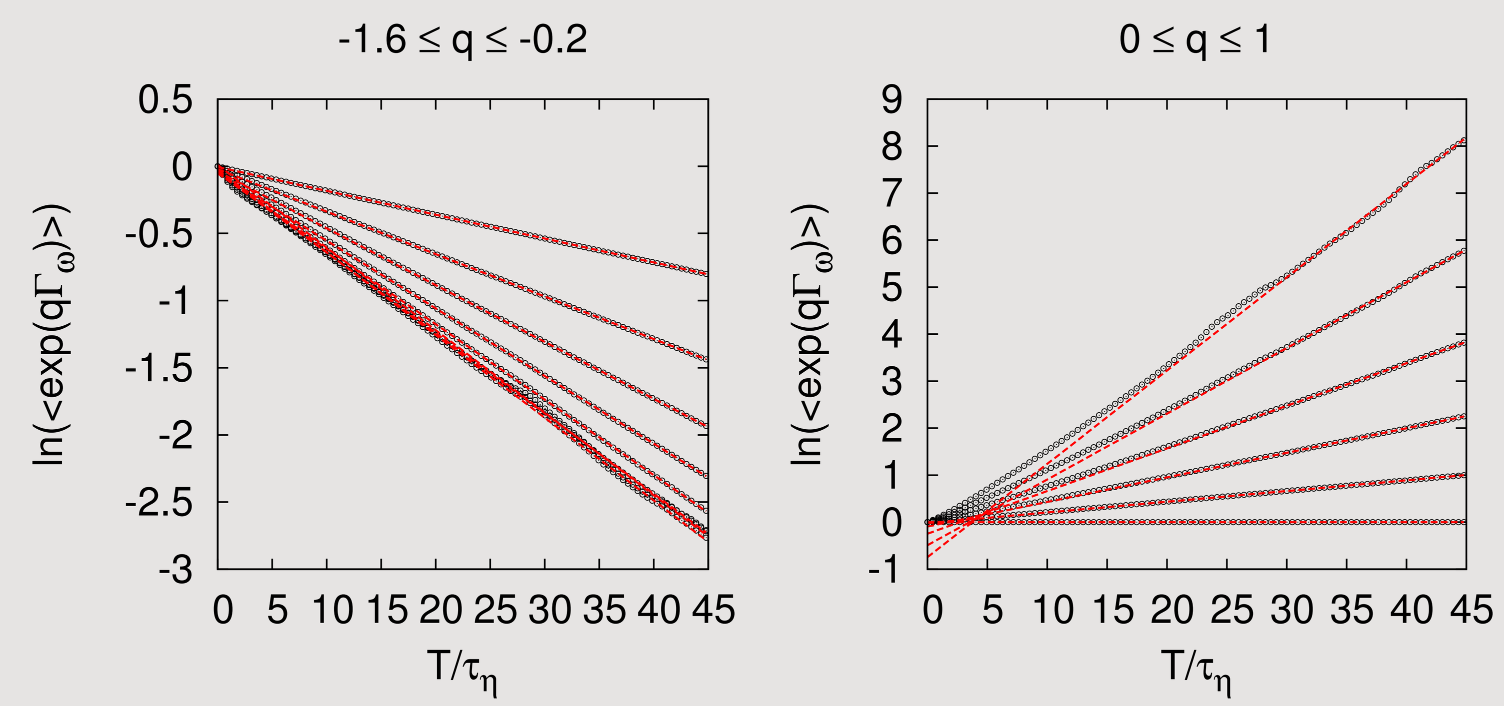
- Stretching as sum of independent random variables

$$\gamma_\omega(T) = \frac{1}{T}\int_0^T n_i S_{ij} n_j dt = \frac{1}{T}\sum_{i=1}^{N}\left[\int_{t_{i-1}}^{t_i} n_i S_{ij} n_j dt\right]$$

- Hypothesis: although ω_i is active variable, treat as passive
- Large-deviation formalism

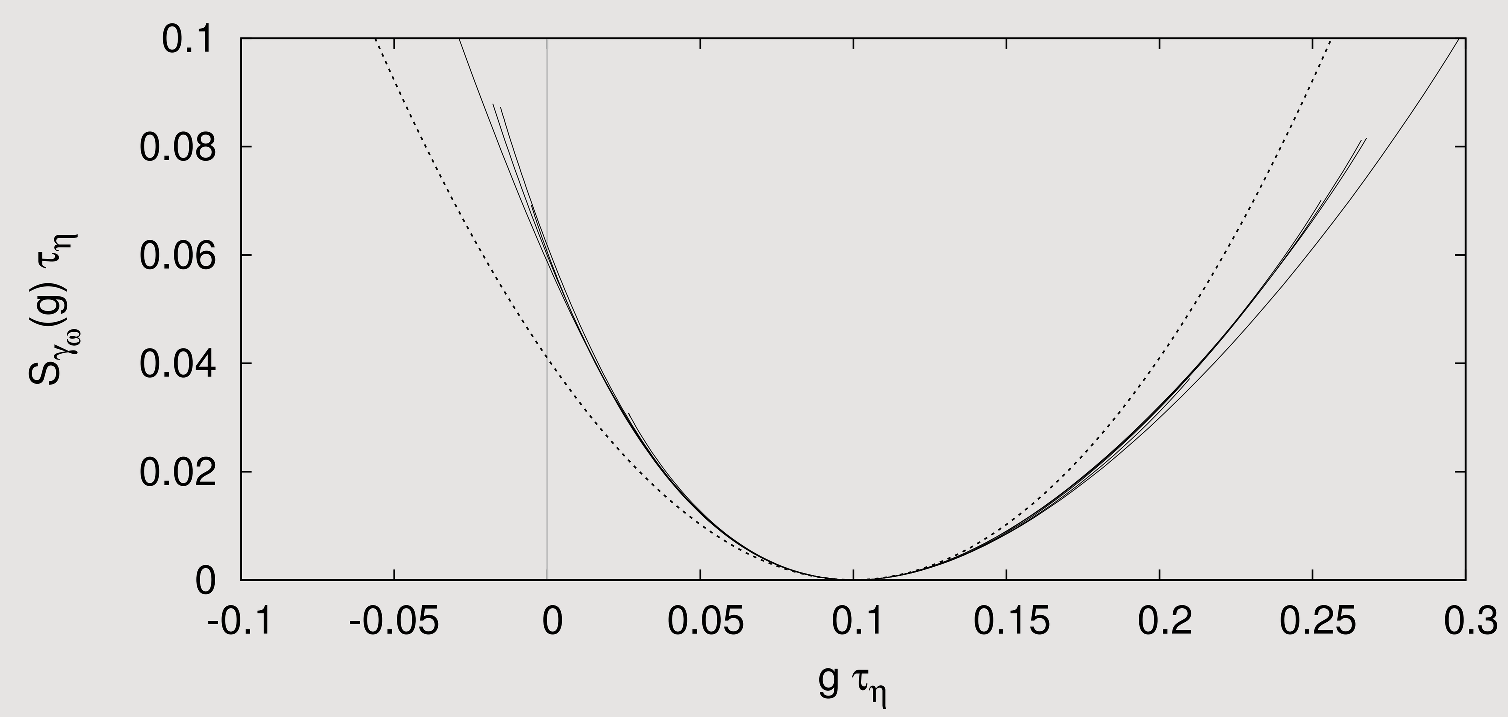
$$p_{\gamma_\omega}(g, T) \sim \exp\left[-T S_{\gamma_\omega}(g)\right]$$

Verification of Hypothesis: Linear Growth of Cumulants

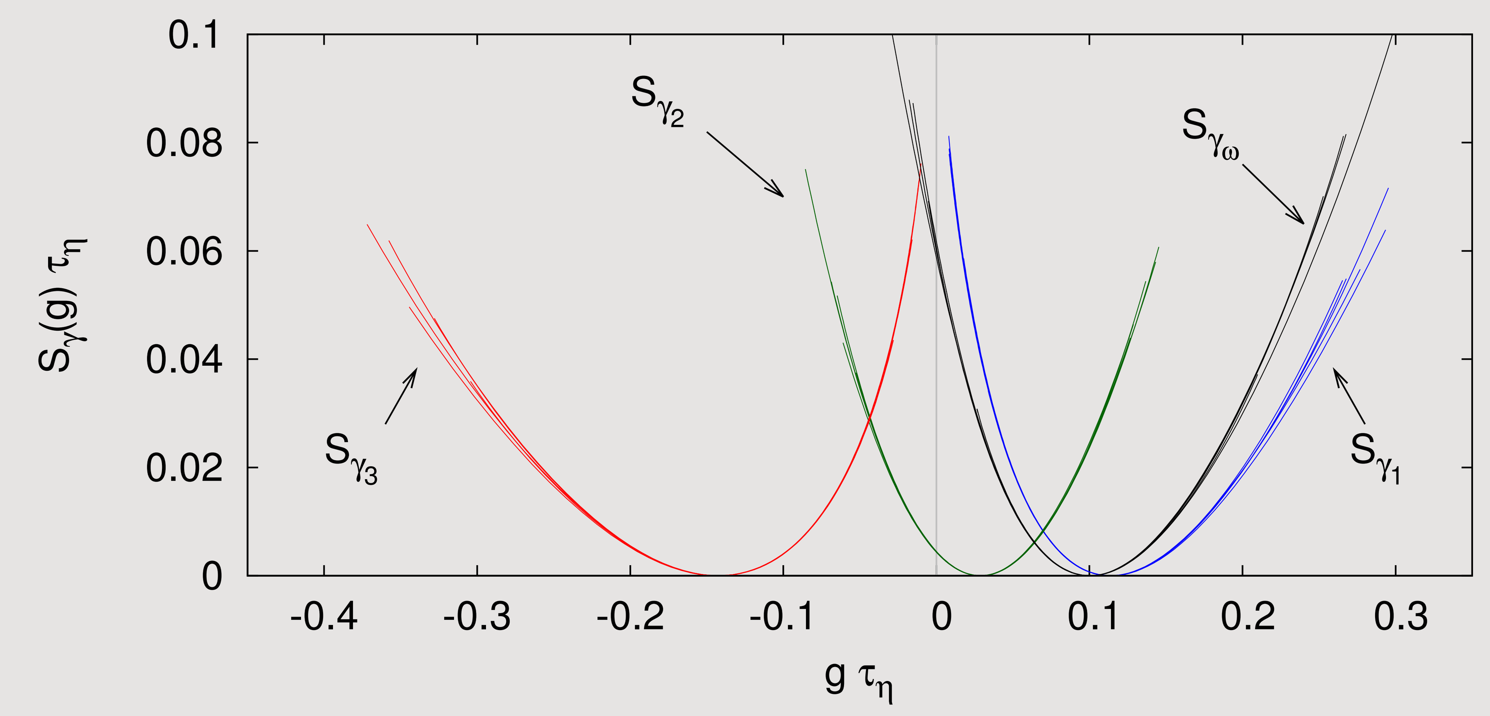


Cramér Function by Legendre Transform

$$L_\omega(q) = \lim_{T\to\infty}\frac{1}{T}\ln\left\langle \exp\left(q\gamma_\omega T\right)\right\rangle = \sup_g\left[qg - S_{\gamma_\omega}(g)\right]$$



Comparison with Material Deformation



Stochastic Model for the Enstrophy PDF

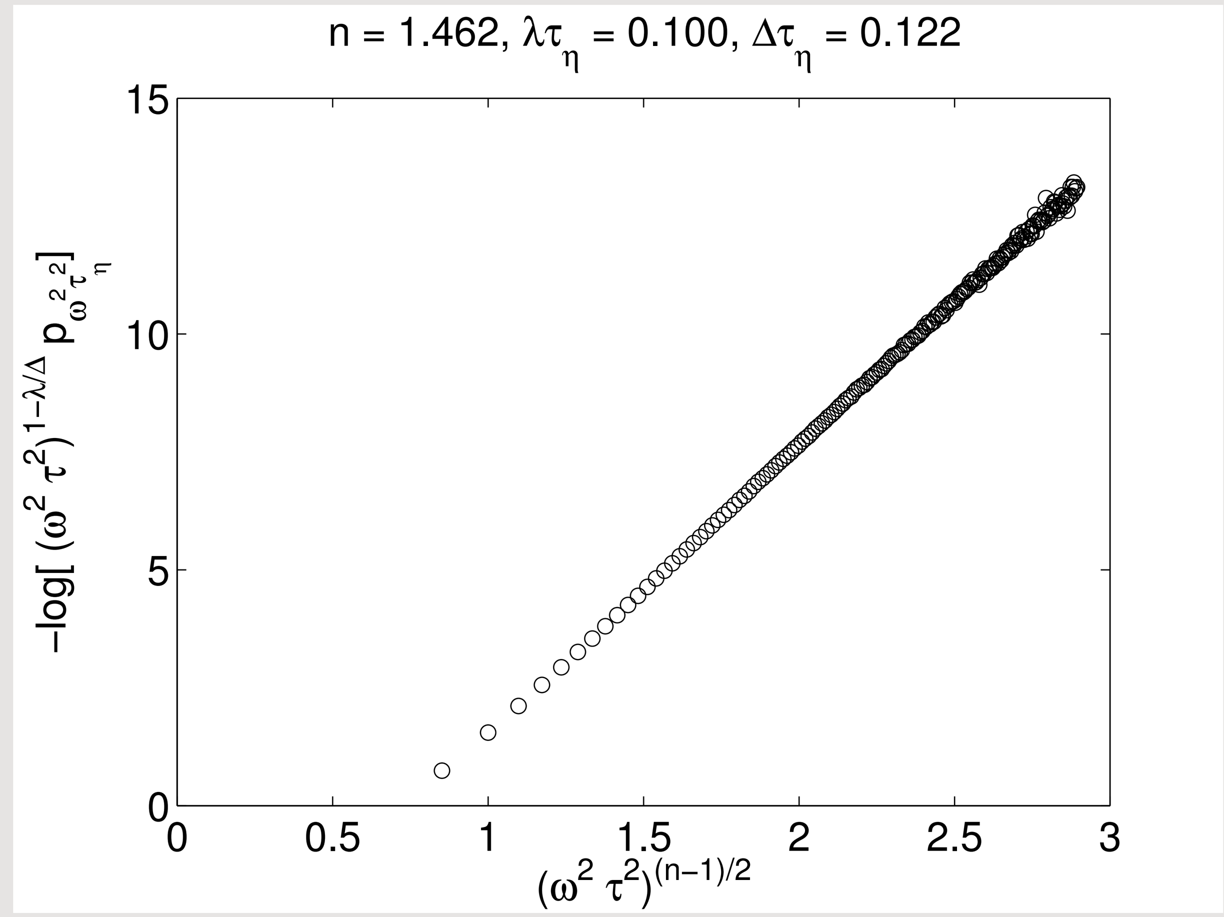
- Viscous diffusion: deterministic relaxation function $f(|\omega|)$
- Vorticity stretching by strain-rate: Cramér function statistics $d\mathcal{W}$

$$d\ln(|\omega|) = \left[\lambda - \tilde{f}(\ln|\omega|)\right]dt + d\mathcal{W}$$

- Fokker-Planck equation with stationary solution:

$$p_{\omega^2\tau_\eta^2}(\xi) = C'\xi^{-1+\frac{\lambda_\omega}{\Delta_\omega}}\exp\left(-\frac{2A}{(n-1)\Delta_\omega\tau_\eta}\xi^{(n-1)/2}\right)$$

- Test model against numerical results:



Conclusions

- Hypothesis confirmed: large-deviation form applied to vorticity stretching.
- Vorticity stretching, γ_ω qualitatively similar to material line stretching, γ_1.
- Stochastic model constructed using Cramér function
- Prediction of stretched exponential enstrophy PDF.
- Accurate exponent, but pre-factor off by 35%.