Cost-Sensitive Prediction: Applications in Healthcare

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Outline

1. Cost-Sensitive Prediction: Applications in Healthcare
   - The application: prediction of sepsis
   - An exact cost-sensitive modeling formulation of the problem
   - Convex relaxations
   - Nonconvex relaxations and new optimization algorithms

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Definition (sepsis)

Sepsis occurs when chemicals released into the bloodstream to fight an infection trigger inflammatory responses throughout the body.

Definition (septic shock)

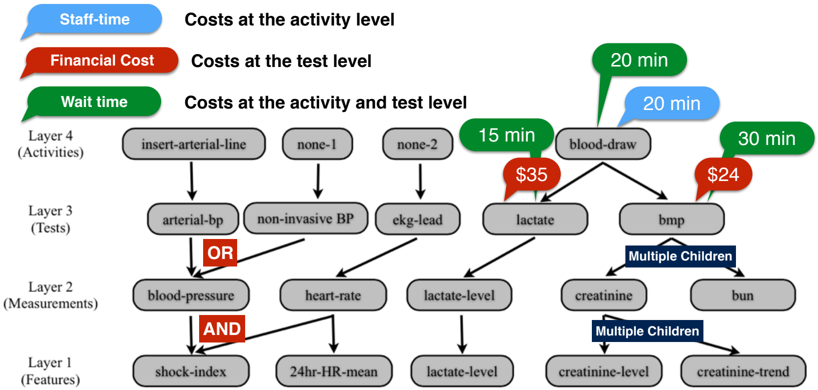
Widespread infection causing organ failure & dangerously low blood pressure

Facts:

- $200{,}000 - 3{,}000{,}000$ cases of sepsis in the USA each year.
- Sepsis is estimated to cost American hospitals $\$20$ billion each year.
- Most common and dangerous in older adults or those with weakened immune systems.
- Early treatment of sepsis with antibiotics and intravenous fluids improves chance of survival.
- 40% of the patients diagnosed with sepsis do not survive.
- Early detection is key to improve survival rates.

The symptoms are not always straightforward or agreed upon.

Cost structure graph



Research goals:

- prediction of septic shock
- model should capture the complex structure of the relevant costs
- evaluate convex/nonconvex relaxations of the exact formulation
- develop new algorithms to solve the resulting optimization problem
- evaluate performance on real healthcare data
- extensions to real-time setting, perhaps to aid in personalized healthcare

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Model prediction: Solve the problem

$$\min_{\beta} \ \mathcal{F}(\beta)$$

- $\beta \in \mathbf{R}^{|f|}$ is the weight vector
- For example, the logistic loss is

$$\mathcal{F}(\beta) = \sum_{n=1}^{N} \log\left(1 + \exp\left(-y^n \beta^T f^n\right)\right)$$

  - $f^n \in \mathbf{R}^{|f|}$ is the n^{th} observed feature vector
  - $y^n \in \{-1, 1\}$ the label of f^n
  - $|f|$ the number of features
  - N the number of observations

To balance predictive accuracy and model complexity:

$$\min_{\beta} \; \mathcal{F}(\beta) + \lambda C(\beta)$$

- λ is a weighting parameter
- Complexity/Cost function $C(\beta)$, e.g.,
  - $C(\beta) = ||\beta||_2^2$
  - $C(\beta) = ||\beta||_1$
  - $C(\beta) = \sum_{i \in G_j} ||\beta_{G_j}||_2$

Key: none of the above cost regularizers accurately model the relevant costs

- first nonzero feature to enter is most predictive
- the most predictive may be the most expensive
- no control over the relevant costs
- a regularizer based on the complicated cost structure is needed!

A cost-sensitive regularizer

- $\mathcal{A}$ nodes at activity layer
- $\mathcal{T}$ nodes at test layer
- $\mathcal{M}$ nodes at measurement layer
- $\mathcal{F}$ nodes at feature layer

Reduce 4 layer boolean circuit (cost graph) to a 3-layer boolean circuit:

- layer-1 contains the nodes $\mathcal{F}$
- layer-2 contains the nodes $\mathcal{Z} := \{f_{i,p} : 1 \leq i \leq n_f$ and $1 \leq p \leq w_i\}$
- layer-3 contain the nodes $\mathcal{A}$
- only **or** gate functions are used in layer-1
- only **and** gate functions are used in layer-2

A cost-sensitive regularizer

We now have that the cost for the lth care-giver activity is given by

$$\mathcal{C}_l^a \, I\left(\bigvee_{(i,p)\in\mathcal{S}_l^a} \beta_{i,p}\right)$$

where the index set $\mathcal{S}_l^a$ is defined as

$$\mathcal{S}_l^a := \{(i,p) : \text{ the output of } g_{f_{i,p}}(\cdot) \text{ depends on } a_l\}$$

Comments:

- The definition of our regularizer follows only from the knowledge of the 3-layer (reduced) boolean circuit
- This procedure generalizes to any cost graph that can be represented as a finite layer boolean circuit

We then have the following optimization problem based on the exact regularizer:

$$\min_{\beta} \ \mathcal{F}(\beta) + \lambda C_{exact}(\beta)$$

- λ is a weighting parameter
- Complexity/Cost function $C(\beta)$:

$$C_{exact}(\beta) := \sum_{l=1}^{|\mathcal{A}|} \mathcal{C}_l^a \, I\left(\bigvee_{(i,p)\in\mathcal{S}_l^a} \beta_{i,p} \right) + \sum_{l=1}^{|\mathcal{T}|} \mathcal{C}_l^t \, I\left(\bigvee_{(i,p)\in\mathcal{S}_l^t} \beta_{i,p} \right)$$

Comments:

- This cost-regularizer is exact, but not tractable for large-scale problems.
- Convex and nonconvex relaxations are possible.

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One could consider convex relaxations:

$$\min_{\beta} \; \mathcal{F}(\beta) + \lambda C(\beta)$$

where

$$C(\beta) := \sum_{l=1}^{|\mathcal{A}|} \mathcal{C}_l^a \, \| \bigvee_{(i,p)\in\mathcal{S}_l^a} \beta_{i,p} \|_\infty + \sum_{l=1}^{|\mathcal{T}|} \mathcal{C}_l^t \, \| \bigvee_{(i,p)\in\mathcal{S}_l^t} \beta_{i,p} \|_\infty$$

Comments:

- Sum of (overlapping) group regularizers.
- Can use software such as SPAMS to solve this problem.
- Tried this approach on the following data ...

Setup:

- Used MIMIC-II, a large publicly available dataset containing electronic health records from patients admitted to the ICU at the Beth Israel Deaconess Medical Center over a period of seven years.
- Processed the data from all adults (age > 15) with at least one measurement of blood urea nitrogen, hematocrit, and heart rate.
- This yielded data from $16{,}232$ patients.
- Septic shock onset was identified using the 2012 Surviving Sepsis Campaign definitions, which resulted in $2{,}291$ patients.
- Patients with severe sepsis who never developed septic shock but received treatment, were removed since their outcome was confounded.
- Split individuals into training (75%) and test (25%) sets.
- Since the dataset is highly unbalanced, during training, we subsample the negative pairs to generate a balanced training set.

Table : Various costs for different models obtained by using our structured regularizer.

Models	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$
Sensitivity at $\mathbf{0.85}$ specificity	0.61	0.66	0.65	0.72
AUC	$\mathbf{82.79 \pm 0.55}$	$\mathbf{84.45 \pm 0.64}$	$\mathbf{84.75 \pm 0.55}$	$\mathbf{87.21 \pm 0.46}$
Financial Cost	**$0**	**$0**	**$72**	**$170**
Care-giver Time	0 minutes	**10** minutes	0 minutes	**30** minutes
Result Time	0 minutes	**10** minutes	50 minutes	**50** minutes
Tests Required	routine	routine, urine	abg, routine	abg, cbc, cmp, hct, hemoglobin, routine, urine
Activities Required	none	urine	arterial stick	arterial stick, blood draw, urine

Comments:

- Diverse models are easily obtained using this cost-sensitive regularizer.
- Easily can adjust model preferences.

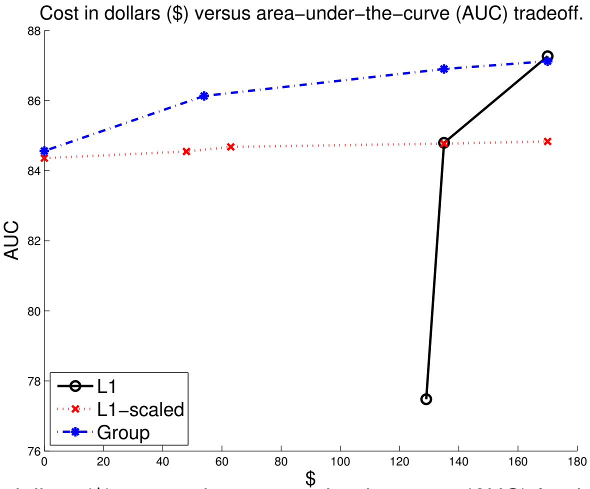

Figure : Cost in dollars ($) versus the area under the curve (AUC) for the ℓ_1 (L1), the scaled-ℓ_1 (L1-scaled), and group regularizer (Group).

Incorporating End-User Preferences in Predictive Models Via Structured Regularizers, submitted to AAAI, 2015.

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Solve the problem

$$\min_{\beta} \ \mathcal{F}(\beta) + \lambda C(\beta)$$

for choices such as

$$C(\beta) := \sum_{l=1}^{|\mathcal{A}|} \mathcal{C}_l^a \sqrt{\sum_{(i,p)\in\mathcal{S}_l^a} |\beta_{i,p}|} + \sum_{l=1}^{|\mathcal{T}|} \mathcal{C}_l^t \sqrt{\sum_{(i,p)\in\mathcal{S}_l^t} |\beta_{i,p}|}$$

Other choices are possible:

- Sigmoid functions.
- Min functions such as $\min\{0, \sum_{(i,p)\in\mathcal{S}_l} |\vec{\beta}_{i,p}|\}$.

The general problem formulation

$$\min_{\beta \in \mathbb{R}^{|f|}} \quad \mathcal{F}(\beta) + \sum_{j=1}^{|G|} g_j\Big(\sum_{i \in U_j} |\beta_i|\Big)$$

- $f : \mathbb{R}^{|f|} \to \mathbb{R}$ is convex
- $g_i : \mathbb{R} \to \mathbb{R}$ is concave and increasing on $[0, \infty)$

Comments:

- Difference of convex functions.
- Known algorithms when the gradient of $\mathcal{F}$ is cheap to compute.
- Methods become inefficient in the big data setting.
- We are developing methods that ...
  - only use stochastic or batch gradients of $\mathcal{F}$ (scalable)
  - utilize the difference of convex functions structure
  - utilize projections onto cones
  - inexact subproblem solvers (efficiency)
  - have convergence guarantees
  - details...hopefully next year!

Thank You