Задания на неопределенный интеграл.

№1660.

$$\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx = -\int \frac{\sqrt[5]{x^2-2x+1}}{x-1} = -\int \frac{\sqrt[5]{(x-1)^2}}{x-1}\,dx =$$

$$= -\int (x-1)^{-\frac{3}{5}}\,dx = -\frac{5}{2}(x-1)^{\frac{2}{5}} + C$$

№1875.

$$\int \frac{dx}{x^5+x^4-2x^3-2x^2+x+1} = \int \frac{dx}{x(x^4-2x^2+1)+x^4-2x^2+1} =$$

$$= \int \frac{dx}{(x^4-2x^2+1)(x+1)} = \int \frac{dx}{(x^2-1)^2(x+1)} = \int \frac{dx}{(x-1)^2(x+1)^2(x+1)} =$$

$$= \int \frac{dx}{(x-1)^2\cdot(x+1)^3} =$$

$$= \int \left(\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3}\right)dx$$

$$A(x-1)(x+1)^3 + B(x+1)^3 + C(x-1)^2(x+1)^2 + D(x-1)^2(x+1) + E(x-1)^2 = 1$$

$$A(x^2-1)(x^2+2x+1) + B(x^3+3x^2+3x+1) + C(x^4-2x^2+1) + D(x^2-1)(x-1) +$$
$$+ E(x^2+1-2x) = 1$$

$$A(x^4+2x^3-x^2-1+\cancel{x}) + B(x^3+3x^2+3x+1) + C(x^4-2x^2+1) +$$
$$+ D(x^3-x^2-x+1) + E(x^2-2x+1) = 1$$

$$x^4(A+C) + x^3(2A+C+D) + x^2(3B-2C-D+E) + x(3B-2A-D-$$
$$-2E) + D+E-A+B+C = 1$$

$$\begin{cases} A+C=0 \\ 2A+C+D=0 \\ 3B-2C-D+E=0 \end{cases};\quad -2A+3B-D-2E=0;\quad -A+B+C+D+E=1$$

$$\begin{cases} A = -\frac{3}{16} \\ B = \frac{1}{8} \\ C = \frac{3}{16} \\ D = \frac{1}{4} \end{cases} \quad E = \frac{1}{4}$$

$$\int \frac{dx}{x^5+x^4-2x^3-2x^2+x+1} = \int\left(-\frac{3}{16(x-1)} + \frac{1}{8(x-1)^2} + \frac{3}{16(x+1)} + \frac{1}{4(x+1)^2} + \right.$$

$$\left. + \frac{1}{4(x+1)^3}\right)dx = -\frac{3}{16}\ln(x-1) + \frac{1}{8(x-1)} - \frac{1}{4(x-1)} - \frac{1}{8(x+1)^2} + \frac{3}{16}\ln(x+1)$$

N 1969

$$\int \frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}}\,dx =$$

$u = x+\sqrt{x^2+3x+2}$

$x = \frac{u^2-2}{2u+3}$

$dx = \left(\frac{2u}{2u+3} - \frac{2(u^2-2)}{(2u+3)^2}\right)du$

$$= \int \frac{-2(3u+4)(u^2+3u+2)}{u(2u+3)^3}\,du =$$

$$\frac{(3u+4)(u^2+3u+2)}{u(2u+3)^3} = \frac{A}{u} + \frac{B}{2u+3} + \frac{C}{(2u+3)^2} + \frac{D}{(2u+3)^3}$$

$$(3u+4)(u^2+3u+2) = A(2u+3)^3 + Bu(2u+3)^2 + Cu(2u+3) + Du = 27A + (8A+4B)u^3 + (36A+12B+2C)u^2 + (54A+9B+3C+D)u$$

$$\begin{cases} 27A = 8 & (1) \\ 54A + 9B + 3C + D = 18 & (2) \\ 36A + 12B + 2C = 13 & (3) \\ 8A + 4B = 3 & (4) \end{cases}$$

$54\cdot\frac{8}{27} + 9\cdot\frac{17}{108} + 3\cdot\frac{2}{9} + D = 18$

$36\cdot\frac{8}{27} + 12\cdot\frac{17}{18} + 2C = 13$

$8\cdot\frac{8}{27} + 4B = 3$

$A = \frac{8}{27}$

$B = \frac{17}{108}$

$C = \frac{2}{9}$

$D = -\frac{1}{12}$

$$= -2\int\left(\frac{17}{108(2u+3)} + \frac{8}{27u} + \frac{2}{9(2u+3)^2} - \frac{1}{12(2u+3)^3}\right)du =$$

$$= -\frac{17}{54}\int\frac{1}{2u+3}\,du - \frac{4}{9}\int\frac{1}{(2u+3)^2}\,du + \frac{1}{6}\int\frac{1}{(2u+3)^3}\,du - \frac{16}{27}\int\frac{1}{u}\,du =$$

$$= \frac{-17\ln(2u+3)}{108} + \frac{2}{9(2u+3)} - \frac{1}{24(2u+3)^2} - \frac{16\ln u}{27} =$$

$$= \frac{-17 \ln(2 \cdot (\sqrt{x^2+3x+2} + x) + 3)}{108} +$$

$$+ \frac{2}{9 \cdot (2\sqrt{x^2+3x+2} + x) + 3} - \frac{1}{(24(2\sqrt{x^2+3x+2} + x) + 3)^2} -$$

$$- \frac{16 \ln(\sqrt[2]{x^2+3x+2} + x)}{27} + C$$

№2007.

$$\int \frac{dx}{\sqrt{\sin^3 x \cdot \cos^5 x}} = \left| \begin{array}{l} \sin x = tg\, x \cdot \cos x \\ \frac{1}{\cos^2 x} = tg^2 x + 1 \end{array} \right| =$$

$$= \int \frac{dx}{tg^{\frac{3}{2}} x \cdot \cos^{\frac{3}{2}} x \cdot \cos^{\frac{5}{2}} x} = \left| \begin{array}{l} t = tg\, x \\ dt = \frac{1}{\cos^2 x}\, dx \end{array} \right| =$$

$$= \int \frac{t^2+1}{t^{\frac{3}{2}}}\, dt = \int \frac{t^2}{t^{\frac{3}{2}}}\, dt + \int \frac{1}{t^{\frac{3}{2}}}\, dt = \int \sqrt{t}\, dt + \int t^{\frac{3}{2}}\, dt$$

$$= \frac{2}{3} \cdot t^{\frac{3}{2}} - 2t^{-\frac{1}{2}} + C$$

№2038.

$$\int \frac{\sin x \cdot \cos x}{1 + \sin^4 x}\, dx = \left| \begin{array}{l} t = \sin^2 x \\ dt = 2\sin x \cdot \cos x\, dx \end{array} \right| =$$

$$= \frac{1}{2} \int \frac{dt}{1+t^2} = \frac{1}{2}\, arctg\, t = \frac{1}{2}\, arctg(\sin^2 x) + C$$