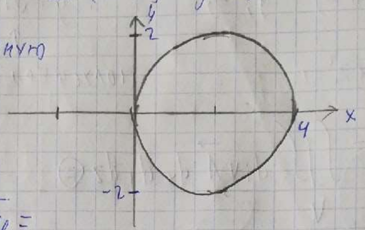
№2

$f(x,y)=\sqrt{x^2+y^3}$ D: $(x-2)^2+y^2\leq 4$

ПЕРЕЙДЁМ В ПОЛЯРНУЮ СИСТЕМУ КООРДИНАТ



$x=r\cdot\cos\varphi$

$y=r\cdot\sin\varphi$

$\sqrt{x^2+y^2}=\sqrt{r^2\cos^2\varphi+r^2\sin^2\varphi}=$

$=r\sqrt{\cos^2\varphi+\sin^2\varphi}=r$ $dx\,dy=r\cdot dr\cdot d\varphi$

$(x-2)^2+y^2\leq 4$

$\cancel{(r\cdot\cos\varphi)^2+r^2\sin^2\varphi\leq 4}$ $(r\cdot\cos\varphi-2)^2+r^2\sin^2\varphi\leq 4$

$r^2\cos^2\varphi-4r\cos\varphi+4+r^2\sin^2\varphi\leq 4$

$r^2+4r\cdot\cos\varphi+4\leq 4$

$r^2\leq -4r\cdot\cos\varphi$ т.к. $r\geq 0$, то

$r\leq\cos\varphi$

$$\iint\limits_D\sqrt{x^2+y^2}\,dx\,dy=\iint\limits_D r\cdot r\,dr\,d\varphi=$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\int_0^{4\cos\varphi}r^2\,dr=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\,\frac{r^3}{3}\Big|_0^{4\cos\varphi}=$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{64\cos^3\varphi}{3}\,d\varphi=\frac{64}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^3\varphi\,d\varphi=\frac{64}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1-\sin^2\varphi)\cdot(\cos\varphi)d\varphi$$

$$=\left|\begin{array}{c}t=\sin\varphi\\ \frac{\pi}{2}\to 1\quad -\frac{\pi}{2}\to -1\end{array}\right|=\frac{64}{3}\int_{-1}^{1}(1-t^2)\,dt=\frac{64}{3}\left(t-\frac{t^3}{3}\right)\Big|_{-1}^{1}=$$

$$=\frac{64}{3}\left(1-\frac{1}{3}-\left((-1)-\frac{-1}{3}\right)\right)=\frac{64}{3}\cdot\frac{4}{3}=\frac{256}{9}$$

ОТВЕТ: $\frac{256}{9}$