№1

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$$\int_0^1 dy \int_{\sqrt{y^3}}^{2-\sqrt{2y-y^2}} f(x,y)\,dx \qquad D:\ 0 \le y \le 1,\ \sqrt{y^3} \le x \le 2-\sqrt{2y-y^2}$$

Построим область D:

$$\begin{cases} y=0 \\ y=1 \\ x^2=y^3 \\ x=2-\sqrt{2y-y^2} \end{cases} \quad \begin{cases} y=0 \\ y=1 \\ y=\sqrt[3]{x^2} \\ \sqrt{2y-y^2}=2-x \end{cases} \quad \begin{cases} y=0 \\ y=1 \\ y=\sqrt[3]{x^2} \\ 2y-y^2=4-4x+x^2 \end{cases}$$

$$\begin{cases} y=0 \\ y=1 \\ y=\sqrt[3]{x^2} \\ y^2-2y+4-4x+x^2=0 \end{cases}$$

$$y^2-2y+4-4x+x^2=0$$

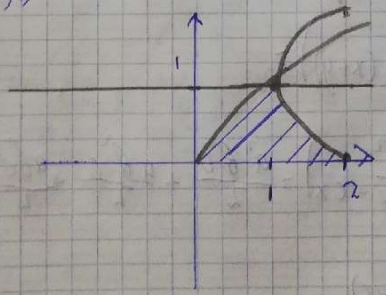
$$y_{1,2}=\frac{2\pm\sqrt{4+4x-x^2\cdot 4+4}}{2}=$$

$$=\frac{2\pm\sqrt{-4x^2+16x-12}}{2}$$

$$y_{1,2}=1\pm\sqrt{-x^2+4x-3}$$

$$y_1=1+\sqrt{-x^2+4x-3}$$

$$y_2=1-\sqrt{-x^2+4x-3}$$



Меняем пределы:

D_1: $0 \le x \le 1$, $0 \le y \le \sqrt[3]{x^2}$

D_2: $1 \le x \le 2$, $0 \le y \le 1-\sqrt{-x^2+4x-3}$

$$\int_0^1 dy \int_{\sqrt{y^3}}^{2-\sqrt{2y-y^2}} f(x,y)\,dx = \int_0^1 dx \int_0^{\sqrt[3]{x^2}} f(x,y)\,dy + \int_1^2 dx \int_0^{\sqrt{-x^2+4x-3}} f(x,y)\,dy$$