BS101 Project Report:

Visual Analysis of BJT with 3D Graph Using Go

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1 INTRODUCTION

1.1 Proposal Background

In BS101 class of this semester, we covered the definition and several theories such as theorems, propositions of representing and analyzing multivariable function (a.k.a. mvf). These are called ‘Multivariable Calculus’ (a.k.a. MC) in total. Learning these, I have constantly sought instances which MC can assist to analyze and understand. By the end of exploring it, I reached this topic, visual analysis of BJT (Bipolar Junction Transistor) with 3D Graph using Go.

This topic is closely related to my future career. To briefly introduce it, I'm planning to research AI-accelerating chips after graduation, especially based on the division of analog circuit. Since I haven't deeply studied AI-accelerating circuit yet, there are no firm evidence whether MC is the fundamental tool in the whole area.

However, it seems certain that MC is powerful tool to evaluate and analyze certain phenomenon (mvf) in several standards (variable). In circuit system where various elements such as ideal variables *e.g., input voltage and current* and non-ideal variables *e.g., temperature, moisture, fabrication methods, etc.* exist, MC acts as truly essential tools to understand the impact of each element to system.

In sum, MC is expected to analyze and grasp some ideas of how to minimize errors, optimize performance and energy consumption of circuit system, regardless of various extreme environment and various fabrication of chips. These are the ultimate goals of research topics for circuits and systems.

To be specific, I also expect MC to solve my problem that I underwent in tutoring session. As a tutor of subject Electronic Circuits, I have several chances to instruct tutees who are junior than me. While teaching them, I observed that they had difficulty in understanding I-V characteristics of BJT (Bipolar Junction Transistor) with Early effect. Since it has more than single independent variable, I-V characteristic graph is not simply delivered in planar graph. Consulting and having conversation with my colleagues and tutees, I concluded that tutees' hardship is attributed to the lack of intuitive understand of that characteristic. Therefore, I especially felt the necessity of graphing I-V characteristic of BJT in 3D figure. I attempt to draw and figure out 3D graph of the characteristic, with confidence that this would aid tutees' understanding.

1.2 Theoretical Background

As I am still undergraduate student who have not deeply studied and researched this area, there are still rooms to be scrutinized how MC helps and aids study circuit and system. However, from my hitherto theories I studied, it can analyze some effects in several variables.

Since this is not a project for electronic circuit class, I will not cover detailed and complicated theory of circuit. Instead, only brief theories and equations of model describing circuit will be covered.

1.2.1 Bipolar Junction Transistor (BJT)

A bipolar junction transistor (a.k.a. BJT) is one type of transistors which is the application of p-n junction. In summary, it has two functions, switching and amplifying. These two functions are determined by V_{BE} (Voltage between Base and Emitter) and V_{CE} (Voltage between Collector and Emitter). Switching, determining whether the current flows or not, is controlled by both V_{BE} and V_{CE}, whereas current amplification is decided only by V_{BE}, ideally.

1.2.2 I-V Characteristics and Early Effect in BJT

this is non-ideal traits of BJT in which I_C (Current of Collector) slightly increase in linear proportion to V_{CE}. Ideally, this should be determined only by V_{BE}. Without and with Early effect, function describing I-V characteristics of BJT is defined as equation (1.1) and (1.2). (Razavi, 2013, p. 145)

- Without Early effect (ideal case)

$$I_C = I_S \exp \frac{V_{BE}}{V_T} \cdots (1.1)$$

- With Early effect (non-ideal case)

$$I_C = I_S \exp \frac{V_{BE}}{V_T} \left(1 + \frac{V_{CE}}{V_A}\right) \cdots (1.2)$$

$*$ I_S : Reverse saturation current
V_T : Thermal voltage
V_A : Early voltage

Adopting the concept of mvf, a list of variables is as below.

- Independent variables: V_{BE}, V_{CE}
- Dependent variable: I_C

The other symbols I_S, V_T, V_A can be set as constant since those values are fabricated as such in practice. In textbook (Razavi, 2013, p. 146), I-V characteristics are visually represented in two graphs (See Figure 1), (a) one for $I_C - V_{BE}$ plane and (b) the other for $I_C - V_{CE}$ plane.

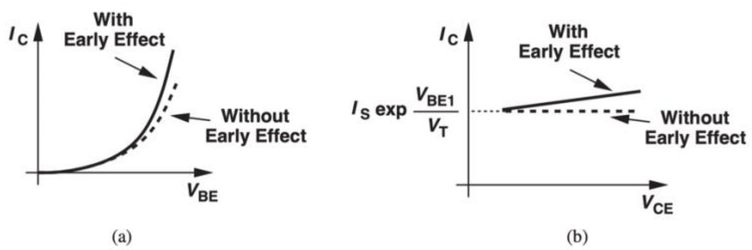

Figure 1. Collector current ($\boldsymbol{I_C}$) as a function of (a) $\boldsymbol{V_{BE}}$ and (b) $\boldsymbol{V_{CE}}$ with and without Early effect

With equation (1.1), (1.2) and two graphs (Figure 1-(a), (b)), we can figure out I-V characteristics as below. (We say it is ideal for the case without Early effect)

In $I_C - V_{BE}$ plane (Figure 1-(a)), I_C increases exponentially according to V_{BE}, both for the case without and with Early effect. With Early effect, the exponential graph slightly scaled up than the one without Early effect, by the amount of $(1 + V_{CE}/V_A)$.

In $I_C - V_{CE}$ plane, I_C is constant regardless of V_{CE} without Early effect. With Early effect, I_C would linearly increase by V_{CE}.

Since two independent variables are separated in each graph, it is easy to miss the overall understanding. Considering the advantage of drawing 3D graph, drawing graph of the characteristic would help students to easily catch the concept of Early effect in BJT. Therefore, this project will use the concept of mvf and the method to draw its graph, to visually understand above characteristics as whole.

2 Methods

2.1 Multivariable Function

2.1.1 Definition

Multivariable function (a.k.a. mvf) is a function that is described with more than one independent or dependent variables. Dependent variable is determined by computing values in function with independent variables. For example, two-variable functions and parametric functions, for example, contain two or more independent or dependent variables, they are thought to be subset of mvf.

2.1.2 Two-variable Function

Two-variable function generally indicates a function that has two independent variables and one dependent variable. A function defined as $f(u, v) = (u^2 - v, v^2 + u)$ can be a good example for mvf. In here, $f(u, v)$ is dependent on two variables, u and v. Since there are more than single independent variables, it can be categorized as mvf.

2.1.3 Definition of Graph and Its Drawing Method

Graph is a collection of points defined by function. Mathematically denoting, it can be defined as below.

$$G(f) = \{(x, f(x)) \mid x \in D\} \text{ where } D \text{ is domain of } x.$$

For two-variable functions, which is our target function to draw graph, the graph of this type of functions can be mathematically denoted as below.

$$G(f) = \{(x, y, f(x, y)) \mid x \in [a, b], y \in [c, d]\}$$

To clarify the surface of graph drawn in 3D figure, we can draw grid on the surface. Below figure (Figure 2) can be a good example.

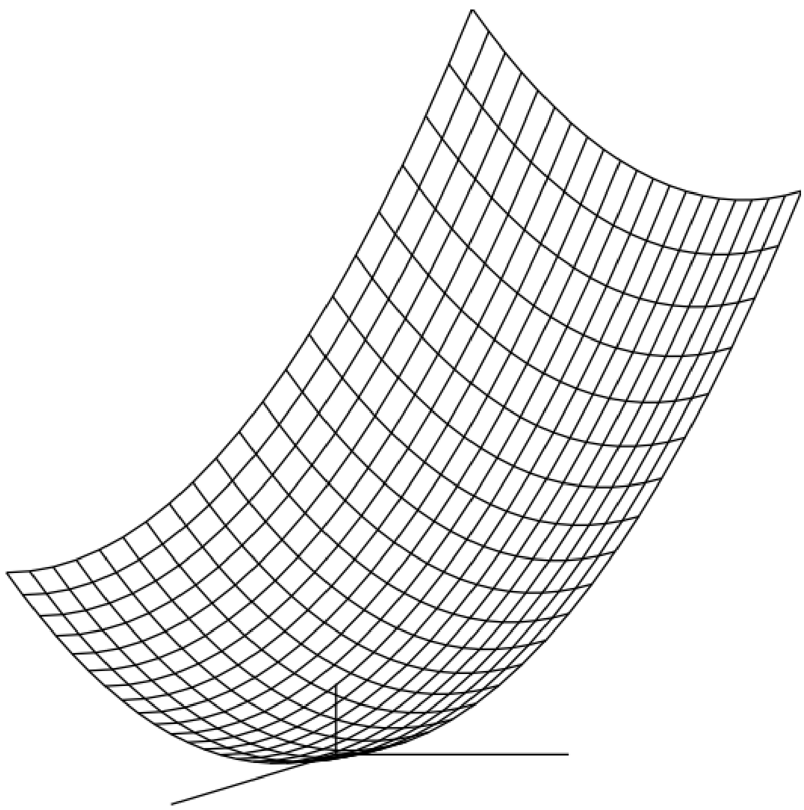

Figure 2. Graph of $\boldsymbol{f(x,y) = x^2 + 2y^2}$ where domain is $\boldsymbol{[0,2] \times [0,2]}$, drawn by Go code

For computer to draw a graph of multivariable function, we need (1) exact formula of mvf, (2) domain of each variable, (3) the number of subintervals. The number of subintervals should be properly large so that the drawn result would approximate ideal graph figure of mvf. We can draw graph by drawing diminutively small straight lines. First, divide each axis into N subintervals. Second, compute the function value in each point. Lastly, draw the line connecting these points.

2.2 Vector

2.2.1 Definition

It is n-tuple of real numbers. It is denoted as below for a vector in $\mathbf{R^n}$. (Colley, 2012, p. 49)

$$\boldsymbol{a} = \underbrace{(a_1, a_2, \dots, a_n)}_{\text{'n' real numbers}}$$

It has two operations as followings:

(1) Vector sum

$$(a_1, a_2, \dots, a_n) + (b_1, b_2, \dots, b_n) = (a_1 + b_1, a_2 + b_2, \dots, a_n + b_n)$$

(2) Scalar multiplication:

$$c \cdot (a_1, a_2, \dots, a_n) = (c \cdot a_1, c \cdot a_2, c \cdot a_n)$$

Moreover, the operation called inner product (or doc product) between two vectors of same dimension are defined as

$$\boldsymbol{a} \cdot \boldsymbol{b} =< (a_1, a_2, \dots, a_n), (b_1, b_2, \dots, b_n) >$$

$$= (a_1, a_2, \dots, a_n) \cdot (b_1, b_2, \dots, b_n) = (a_1 b_1, a_2 b_2, \dots, a_n b_n)$$

$$\text{where } \boldsymbol{a} = (a_1, a_2, \dots, a_n), \boldsymbol{b} = (b_1, b_2, \dots, b_n)$$

The critical property of inner product is

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$$

2.2.2 Three-dimensional Rotations

For vector defined in $\mathbf{R}^3$, rotation matrices are defined as below.

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Recalling the definition of graph, it is a set of points which is independent and dependent variables of a function. Treating each point as graph, we can also rotate graph.

2.2.3 Projection

Projection of vector on a plane can be done as below. (Figure 3)

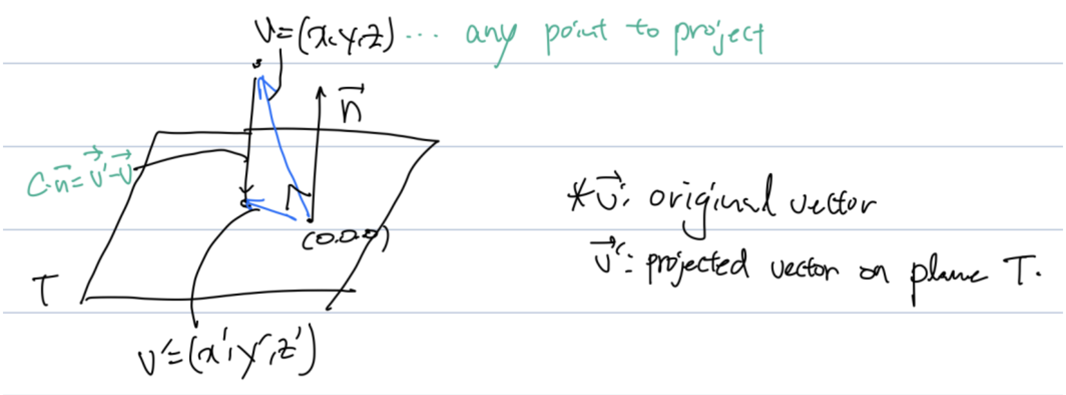

Figure 3. Projection of vector on a plane

Because $\vec{v}' - \vec{v}$ is perpendicular to plane T, it can be written as "$c \cdot \vec{n}$". Below equations are the method to find c.

$$\begin{cases} \overrightarrow{v'} = \vec{v} + c \cdot \vec{n} \\ \vec{n} \cdot \overrightarrow{v'} = 0 \end{cases} \Longrightarrow c = -\frac{\vec{n} \cdot \vec{v}}{\vec{n} \cdot \vec{n}} = -\frac{\vec{n} \cdot \vec{v}}{|\vec{n}|^2}$$

2.2.4 Rotating Graph

To graph $z = f(x, y)$ with custom perspective, we can do steps as followings. First, rotate default canvas, where we want to project 3D graph, by y-axis, and then by z-axis. Normal vector of it is $(1,0,0)$. Second, project the 3D graph onto the 2D canvas. Lastly, draw the object in canvas to image file.

2.3 Go Code

2.3.1 Package math

Package named math provides basic constants and mathematical functions, such as function of sine and cosine, or square root, and the others. [3] To eliminate excuses to define every mathematical function in hand, it is crucial when writing Go code handling mathematical problem. Of course, I will import math package to benefit convenience of using pre-defined mathematical functions. Modules such as 'sin()', 'cos()', 'sqrt()' are to be frequently used in my code.

2.3.2 Package plotter

Package plotter defines a variety of standard Plotters for the plot package. Plotters use the primitives provided by the plot package to draw to the data area of a plot. This package provides some standard data styles such as lines, scatter plots, box plots, labels, and more. [4] In this project, I will import and make use of this package to use modules such as 'XY', 'XYs', and the others.

2.4 Code Implementation

Doing methods described above step by step, I implemented code for drawing 3D figure of I-V characteristic of BJT. Briefly explaining my code, this contains several '.go' extension files.

Files in 'canvas' folder are codes for package 'canvas'. This package contains the code for customized struct-type 'Canvas', which indicates the plot instance, functions for drawing each axis according to rotating angles, drawing rotated graph including its meshgrid on 'Canvas'. When drawing meshgrid, the color of each line is distinguished according to what axis the line is parallel with. If single line in meshgrid is parallel with x-axis, the color will be red. Otherwise, parallel with y-axis, the color of line will be blue. The function for rotation is designed as explained above in **Methods** section. Note that the very default rotation angles are all set to be zeros. In other words, 3D graph of 'mvf' is initially seen from canvas perpendicular to x-axis, the normal vector of which is $(1,0,0)$.

Files in 'mvf' folder are codes for package 'mvf'. This package contains the code for customized struct-type 'mvf', which indicates the multivariable function, the graph of which is the one we desire to draw.

Files in 'vector' folder are codes for package 'vector'. This package contains the code for customized slice-type 'vector' and 'matrix', and rotation matrix to rotate vectors or whole graph. This eases the burden of writing iterative statements for vector and matrix operation in every usage.

'main.go' in root directory is the code for defining mvf describing I-V characteristic and some constants that describes actual characteristics of BJT, calling new instance of the type 'canvas' and the function for drawing graph, and save graph, which is statements for execution which is defined in packages. For detailed code implementation, see 'Code Repository' on Result section of my Notion page.

3 Results & Analysis

With written code, several figures of graph are obtained. I modified and saved figures by changing rotating angle of canvas. With these obtained figures, I-V characteristics of BJT without and with Early effect can be analyzed as below.

Initially, $I_S = 8 \times 10^{-16}A,\ V_T = 0.026V, V_A = 5V$, choosing some conditions in problems included in textbook (Razavi, 2013, p. 161). The axis of V_{BE}, V_{CE}, and I_C are set to be general axis of x, y, and z in 3D graph axis. The range of each domain of variables is initially set to be $[0, 1.5] \times [0, 1.5]$. The number of subdivisions is set to be 50. The constants contained in formula of mvf are set to be realistic. To figure out and evaluate the characteristic, I will try several cases by changing angles and domains.

For convenience, I will arbitrarily denote below 'rotate plane of Angle = $\{\theta_y, \theta_z\}$' the plane whose normal vector is rotated from $(1,0,0)$ as the amount of

Angle = $\{\theta_y, \theta_z\}$, 'plane of (x, y, z)' or 'seen from (x, y, z)' the plane whose normal vector is (x, y, z).

i) Angle = $\{0, -\pi/2\}$ (i.e. projected to plane of (0, -1, 0))

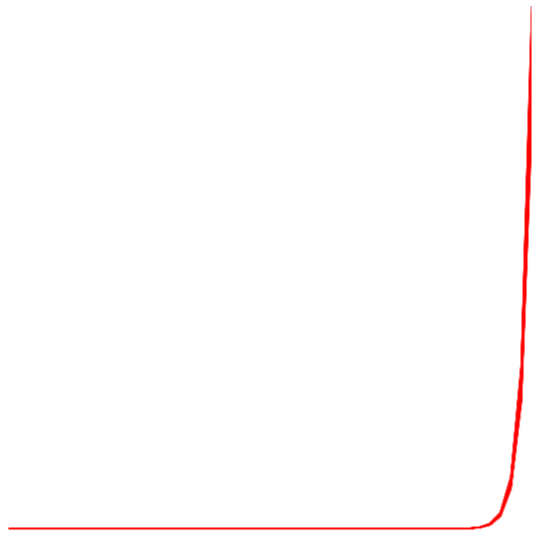

Figure 4. 3D graph seen from (0, -1, 0)

For comparison between the graph from textbook, I will first observe this case. Since x-axis ((1,0,0)) and y-axis ((0,1,0)) are defined as V_{BE} and V_{CE}, horizontal and vertical axis of Figure 4 indicate V_{BE} and I_C respectively. That is, the axis of V_{CE} is ignored in this case.

Observing $I_C - V_{BE}$ lines (red lines) in Figure 4, it exponentially increases as equation (1.2) implies. However, the line seems to get thicker as horizontal value (V_{BE}) gets larger to its maximal limit of domain. Considering meshgrid drawn on the surface of graph, this implies slight increase of I_C by V_{CE}. This increase was small than I expected than it was in Figure 1-(a). Exponential line slightly increased by V_{CE} was almost approximated to original line ($V_{CE} = 0$). However, in application, small increase in current value might perturb the operation point by a great deal. Therefore, it can be evaluated that this code shows good figure of $I_C - V_{BE}$

characteristics of BJT with Early effect quite well. However, the error due to Early effect does not seem obvious than expected.

ii) Angle = {0, 0} (i.e. projected to plane of (0, 1, 0))

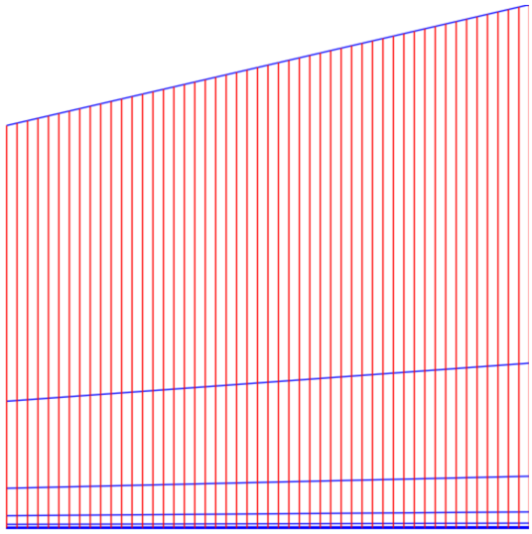

Figure 5. I-V characteristic graph of BJT seen from (1, 0, 0)

Since x-axis ((1,0,0)) and y-axis ((0,1,0)) are defined as V_{BE} and V_{CE}, horizontal and vertical axis of Figure 5 indicate V_{CE} and I_C respectively. That is, the axis of V_{BE} is ignored in this case.

According to the equation (1.2) (See Introduction - Theoretical Background), I_C linearly increases as V_{CE} increases. Observing $I_C - V_{CE}$ lines (blue lines) in Figure 5, however, as the line get close to horizontal axis, it seems almost parallel to the axis, that is, it does not seem to increase by V_{CE}. To understand this, remind equation (1.2) to understand I_C increases exponentially as V_{BE} increases. This means slope of the $I_C - V_{CE}$ line is exponentially related to V_{BE}. Therefore, it seems almost parallel if the value of V_{BE} is relatively small, whereas not appearing parallel if the value is not small.

Comparing the graph with Figure 1-(b), each blue line shows linear line of constant slope, which is determined by V_{BE} value. Therefore, it can be evaluated that this code shows good figure of $I_C - V_{CE}$ characteristics of BJT with Early effect quite well.

iii) Angle = $\{\pi/6,\ -\pi/4\}$

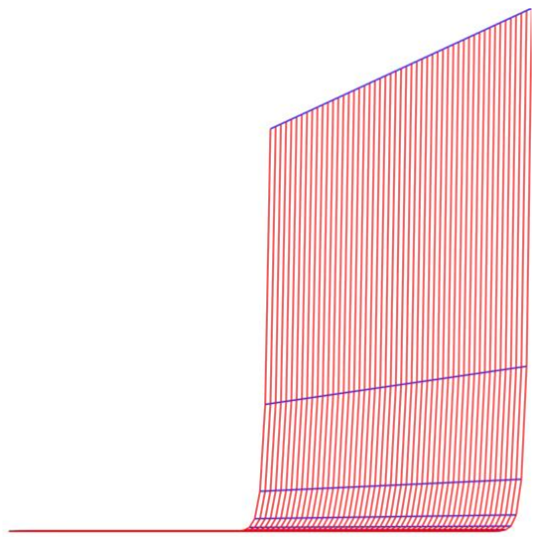

Figure 6. I-V characteristic graph of BJT seen from rotated plane of Angle = $\{\boldsymbol{\pi/6},\ \boldsymbol{-\pi/4}\}$

I expected 3D graph seen from overall perspective of I-V characteristic of BJT. However, the xy-plane is not seen in Figure 6. I assume the reason of this is that the plane is almost approximated to single line since the maximum I_C is quite a lot bigger than zero. Therefore, I tried to narrow domain from $[0, 1.5] \times [0, 1.5]$ to $[0, 0.9] \times [0, 0.9]$ so that the maximum I_C decreases.

iv) Angle = $\{\pi/6,\ -\pi/4\}$, Domain = $[0, 0.9] \times [0, 0.9]$

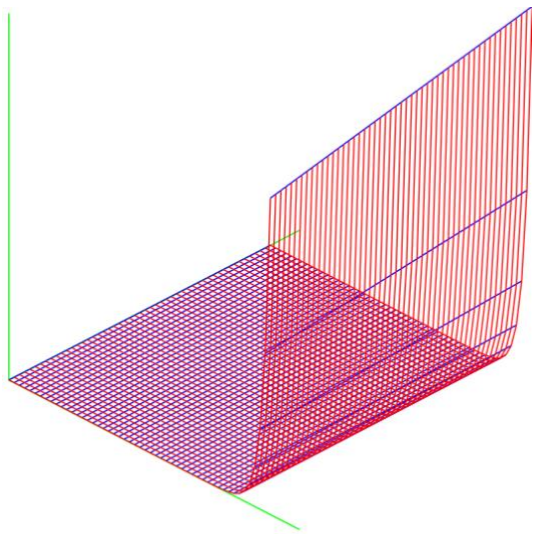

Figure 7. I-V characteristic graph of BJT seen from rotated plane of

Angle = $\{\boldsymbol{\pi/6},\ \boldsymbol{-\pi/4}\}$, with the domain = $[\mathbf{0}, \mathbf{0.9}] \times [\mathbf{0}, \mathbf{0.9}]$

Now we can see the graph in total perspective. However, the graph still seems flat before it exponentially increases by V_{BE}. This is good for showing the figure of graph in total view, but I desire to emphasize the impact of linear increase by V_{CE}. Therefore, I will try to (1) change V_A to smaller value so that the mvf sensitively recognize the change of V_{CE}, and (2) narrow down the domain to $[0.2, 0.63] \times [0, 0.9]$. The range of domain is properly chosen so that the I_C of which is neither so small nor so large.

v) Angle = $\{\pi/6,\ -\pi/4\}$, Domain = $[0.2, 0.63] \times [0, 0.9]$, $V_A = 5 \times 10^{-5} V$

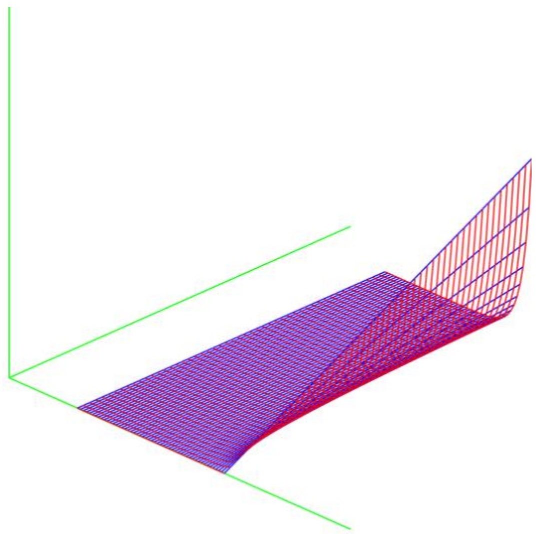

Figure 8. I-V characteristic graph of BJT seen from rotated plane of

Angle = $\{\boldsymbol{\pi/6},\ \boldsymbol{-\pi/4}\}$, with the domain = $[\mathbf{0.2}, \mathbf{0.63}] \times [\mathbf{0}, \mathbf{0.9}]$ $\boldsymbol{and}\ \boldsymbol{V_A} = \mathbf{5} \times \mathbf{10^{-5}}\boldsymbol{V}$

The mvf is still insensitive to change V_{CE} while V_{BE} is fixed to be small. However, as V_{BE} goes large, the linear relation between I_C and V_{CE} gets more obvious. Though this mvf does not reflect the real value of constants, Figure 8 is nice figure in that it properly shows the characteristic of both $I_C - V_{CE}$ (linear) and $I_C - V_{BE}$ (exponential).

4 Discussions

In project, I reminded the definition of mvf and its graph, and methods to draw graph. In addition, I also reminded the vector projection and rotation in three-dimensional space by doing this project. Moreover, I applied those concepts to analyze I-V characteristic of BJT with Early effect by varying some domains and rotating angles of graph, with adding some customized features such as distinguished line color by direction.

Programming language 'Go' is quite burdensome language to define and describe needed functions to draw and rotate graphs all in hand. In fact, MATLAB can draw this graph by less than ten lines of script[1] (Figure 9). Moreover, this supports 'surf()' [5], which enables users to zoom in and out, show coordinate value, add marker on specific point, rotate graph using mouse, and provide graph with varying color on surface by z-value (I_C). However, thanks to these traits of Go, it gave me a good chance to scrutinize whole step-by-step methods to draw 3D graph and rotate it.

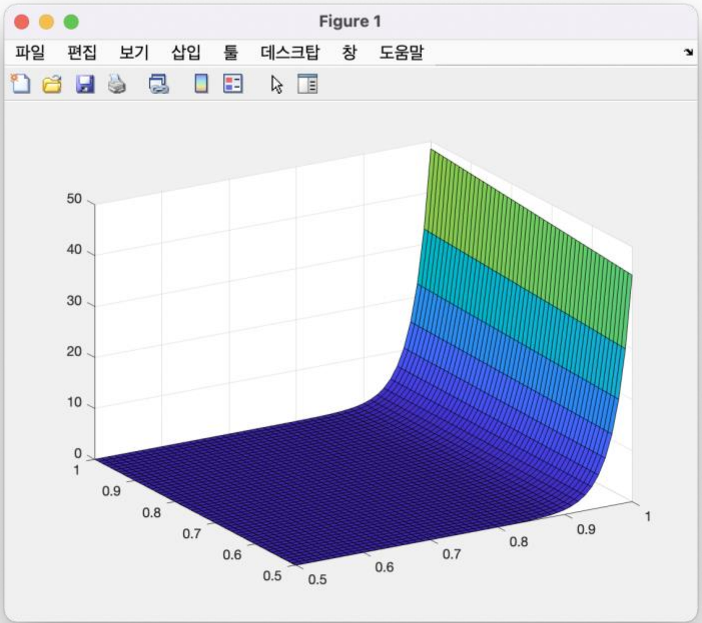

Figure 9. I-V characteristic graph of BJT with the domain = $[\mathbf{0}, \mathbf{1}] \times [\mathbf{0}, \mathbf{1}]$, drawn by MATLAB

However, there are more rooms to be detailed up. As compared with obtained graph from MATLAB, more to be supplemented are (1) zooming in and

[1] This MATLAB code is also included in my Notion page.

out, (2) showing coordinate values and adding marker on point that cursor indicates, (3) rotating graph using mouse, (4) providing graph with varying color on surface by z-value, (5) receiving constants via input terminal or vertical bar. If there is a chance to learn Go codes to implement above features in further days, I would try to improve my code in such ways.

5 References

[1] Razavi, B. (2013). *Fundamentals of Microelectronics.* Wiley

[2] Colley, S. J. (2012). Vector Calculus. Pearson.

[3] https://pkg.go.dev/math

[4] https://pkg.go.dev/gonum.org/v1/plot/plotter

[5] https://www.mathworks.com/help/matlab/ref/surf.html